State variable analysis of four tank system

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Abstract- Many industrial processes of manufacturing, refining, etc., typically requires control over at least two variables. This could use at least two control loops and makes it as multi-input multi-output (MIMO) or multivariable systems. Quadruple tank process (QTP) is one of the multivariable laboratory processes with four interconnected water tanks. This novel deals with mathematical modeling of the QTP by linearization principles and Jacobian matrix formation to represent the system in state space. The linearized dynamics of the system have a multivariable zero that can be placed in both the left and right half of s-plane. The interaction between the control loops are studied using RGA and analysed using LabVIEW. This process is ideal for illustrating multivariable control concept and also the performance limitations due to multivariable right half-plane zeros.

Keywords: QTP, Quadruple tank process, mathematical modeling, MIMO, State space analysis.

I. INTRODUCTION

A System may be classified into two types, SISO (Single input Single Output) and MIMO (Multi Input Multi Output). A SISO is a simple system with a single input and single output while MIMO system is a system with more than one input and outputs. The QTP design is a well-known MIMO system suitable for analysis of various control schemes used in real-time which have nonlinear dynamics. Some systems cannot be represented by a linear model and require the use of nonlinear models. The nonlinearity in QTP is due to the square root term in mass flow relationship, between flow and level of the tank. The nonlinear models create more difficulty in optimizing the system and also its performance becomes poor. The linearization of this type of system requires a stationary point around which the system operates. Taylor series expansion is one of the methods used for linearization which approximates the system at a given stationary point. Generally a system can be represented by state-space or Input-output model. This paper uses the former model where A, B, C and D of the state-space representation are obtained using Jacobian matrices.

The benefits of the state-space model are

- It can be applied to Linear and Non-linear systems.
- Applicable toTime-variant and Time-Invariant Systems.
- Can be applied to MIMO systems.
- It provides information about the internal variables of a system.

The stability of the nonlinear system can be analyzed by various methods like Lyapunov direct method, Popov criterion, method of linearization. It is possible to define the stability of the nonlinear system by also using the zeros by mapping them in real and imaginary axis known as Pole-Zero Map. **Pole-Zero Map** is a plot of the poles and zeros of a system model on the complex plane, where the real values are on the x-axis, and imaginary values are on the y-axis.

Many real-time MIMO systems have interactions that occur due to inputs affecting more than one controlled variable. In QTP, a strong interaction exists between the tank 1 and tank 3 and also between tank 2 and tank 4. This happens due to input from pump 1 filling tank 1 and 3 along with output from tank 3 filling tank 1. Similarly interaction is present between tank 2 and tank 4. This interaction has adverse effect on effective control. The Relative Gain Array (RGA) is a measure of interaction between the control loops in multivariable control system.

When the MIMO system is such that each input only affects one particular output, different from the outputs affected by other inputs, the system is decoupled or non-interacting.

II. FOUR TANK PROCESS

The Quadruple tank is a laboratory process with four interconnected tanks and two pumps as shown in Figure 1. The process inputs are u_1 and u_2 (input voltages to pumps, (0-10 V)) and the outputs are y_1 and y_2 (voltages from level measurement devices (0-10V)). The target is to control the level of the lower two tanks with inlet flow rates.

The output of each pump is split into two using a three-way valve. Pump 1 is shared by tank 1 and tank 3, while pump 2 is shared by tank 2 and tank 4. Thus each pump output goes to two tanks, one lower and another upper diagonal tank and the flow to these tanks are controlled by the position of the valve represented as γ. The position of the two valves determines whether the system is in the minimum phase or in the non-minimum phase. Let the parameter γ be determined by how the valves are set.

Each tank has a discharge valve at the bottom. The discharge from tank 4 goes to tank 1 while discharge of tank 3 goes to tank 2. This interaction creates a strong coupling between the tanks which makes it a multivariable control system. Due to its strong nonlinear behavior, the problem of identification and control of QTP is always a challenging task for control systems engineer.Discharge from tank1 and tank 2 goes to the reservoir tank at the bottom.

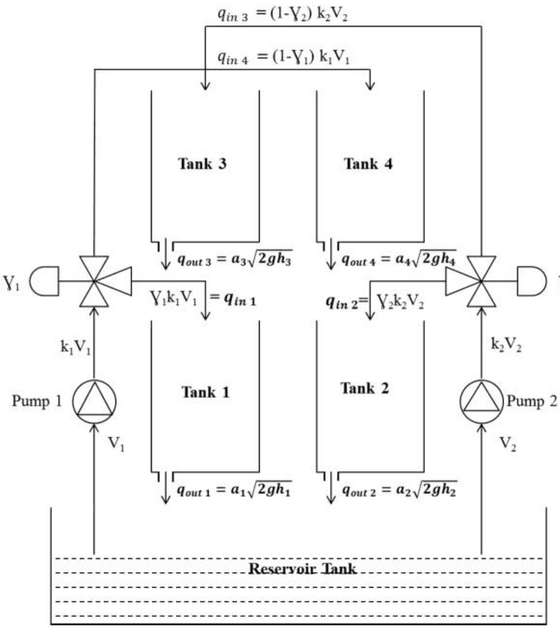

Fig 1: Schematic diagram of a four tank MIMO Process

III. Model Development

Modelling of a process is necessary to investigate how the behaviour of a process changes with time under influence of changes in the external disturbances and manipulated variables and to consequently design an appropriate controller. This uses two different approaches, one is experimental and the other is theoretical. In such case a representation of the process is required in order to study its dynamic behaviour. This representation is usually given in terms of a set of mathematical equations whose solution gives the dynamic behaviour of the process.

For each tank i=1…4, the mathematical modelling is done by consideration of mass balance equation and Bernoulli's law yields:

$$\left.\begin{matrix}\text{Rate of Accumulation}\\ \text{of Mass in system}\end{matrix}\right\} = \left.\begin{matrix}\textit{Mass flow rate into}\\ \textit{the system}\end{matrix}\right\} - \left.\begin{matrix}\textit{Mass flow out}\\ \textit{of the system}\end{matrix}\right\}$$

Before deriving the mathematical equations of the system lets consider,

- The input to the pump 1 be V_1& for pump 2 be V_2.
- The valve priority set for the flow is $\gamma_1\ \gamma_2\ \varepsilon$ [0, 1].
- The flow through the pump 1 when V_1voltage is applied is k_1V_1 and for pump 2 when V_2 voltage is applied is k_2V_2.
- The flow through the pump is directly proportional to the input voltage applied for the pump.
- The flow in the tank 1 after crossing the valve 1 is $\gamma_1k_1V_1$ and for tank 2 after crossing the valve 2 is $\gamma_2k_2V_2$.
- The flow in the tank 4 after crossing the valve 1 is (1-γ_1) k_1V_1 and for tank 3 after crossing the valve 2 is (1-γ_2) k_2V_2.

The non-linear model of the Quadruple tank process is given below. Mass balance equation states that
[Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

Using the law of conservation of mass,

$$\frac{dm_T}{dt} = m_{in} - m_{out} \qquad [1]$$

Where, m_T= mass accumulated in the tank
m_{in}= input mass flow rate
m_{out}= output mass flow rate
Mass accumulated, m_T= volume of tank (v) * density of liquid in the tank(ρ)
Input mass flow rate (m_{in}) = volumetric flow rate(q_{in})* density of liquid in the inlet stream(ρ_1)
Output mass flow rate (m_{out}) = volumetric flow rate (q_{out})* density of liquid in the outlet stream(ρ_2)

$$\frac{d\rho v}{dt} = \rho q_{in} - \rho q_{out} \qquad [2]$$

Since liquid using is same throughout the system, then $\rho = \rho_1 = \rho_2$.

Modelling of non-linear Quadruple tank process is,

$$A_i\frac{dh_i}{dt} = q_{in_i} - q_{out_i} \qquad [3]$$

Where A_i denotes the cross sectional area of the tank, h_i is the water level, $q_{in_i}$is the in-flow of the tank and $q_{out_i}$is out-flow of the tank i=1...4.

The q_{in_i} onlydepends on the input voltage supplied to the pump and the q_{out_i}, depends on the acceleration due to gravity and the head of the water in the tank. The q_{out_i}can be determined by the using Bernoulli's equation and flow rate of the liquid.

Therefore,

$$q_{in_1} = k_1V_{11}$$
$$q_{in_2} = k_2V_{22}$$
$$q_{in_3} = k_2V_2(1-{}_2)$$
$$q_{in_4} = k_1V_1(1-{}_1) \qquad [4]$$

Where k_1 , $k_2$are the pumps constant, V_1, V_2 are the velocity of the flow of water through pump 1 and 2, $\gamma_1\ \gamma_2$are the valve ratio.

$$q_{out_i} = a_i\sqrt{2gh_i} \qquad [5]$$

Where,
a_i = cross sectional area of the outlet pipes, g = acceleration due to gravity, h_i = represents level of the water in each tanks i=1...4.

Tank 1

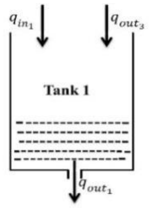

Fig 2: Mass-balance equation for Tank 1

Using the law of conservation of mass,
[Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

$$A_1\frac{dh_1}{dt} = q_{in_1} + q_{out_3} - q_{out_1}$$

$$= \gamma_1k_1V_1 + a_3\sqrt{2gh_3} - a_1\sqrt{2gh_1} \qquad [6]$$

Tank 2

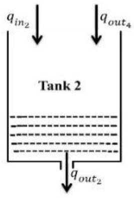

Fig 3: Mass-balance equation for Tank 2

Using the law of conservation of mass,
[Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

$$A_2 \frac{dh_2}{dt} = q_{in_2} + q_{out_4} - q_{out_2}$$

$$= \gamma_2 k_2 V_2 + a_4\sqrt{2gh_4} - a_2\sqrt{2gh_2} \qquad [7]$$

Tank 3

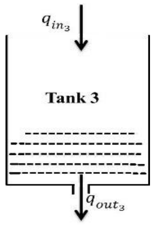

Fig 4: Mass-balance equation for Tank 3

Using the law of conservation of mass,
[Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

$$A_3 \frac{dh_3}{dt} = q_{in_3} - q_{out_3}$$

$$= (1-\gamma_2) k_2 V_2 - a_3\sqrt{2gh_3} \qquad [8]$$

Tank 4

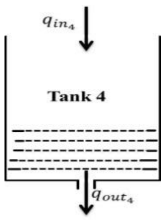

Fig 5: Mass-balance equation for Tank 4

Using the law of conservation of mass,
[Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

$$A_4 \frac{dh_4}{dt} = q_{in_4} - q_{out_4}$$

$$= (1-\gamma_1) k_1 V_1 - a_4\sqrt{2gh_4} \qquad [9]$$

The Final equations,

$$\frac{dh_1}{dt} = \frac{{}_1\mathrm{k}_1\mathrm{V}_1}{A_1} + \frac{a_3\sqrt{2gh_3}}{A_1} - \frac{a_1\sqrt{2gh_1}}{A_1}$$

$$\frac{dh_2}{dt} = \frac{{}_2\mathrm{k}_2\mathrm{V}_2}{A_2} + \frac{a_4\sqrt{2gh_4}}{A_2} - \frac{a_2\sqrt{2gh_2}}{A_2}$$

$$\frac{dh_3}{dt} = \frac{(1-{}_2)\mathrm{k}_2\mathrm{V}_2}{A_3} - \frac{a_3\sqrt{2gh_3}}{A_3}$$

$$\frac{dh_4}{dt} = \frac{(1-{}_1)\mathrm{k}_1\mathrm{V}_1}{A_4} - \frac{a_4\sqrt{2gh_4}}{A_4} \qquad [10]$$

The above non-linear differential equation represents mathematical model of the four-tank system. The tank is being mathematically modelled by using law of conservation of mass. It is always enough to develop a controller for a particular process using its mathematical model. But here in QTP there is a challenge, that due to its non-linearity and uncertainty it is difficult to develop a controller that will take a proper control action.

IV. Taylor Series and Jacobian Conversion

The non-linear relationship in the equation [10] is due to the square root term present in those equations which makes the controller design difficult. To overcome the difficulty the linearization is required.The equation [10]is solved using Taylor series followed by Jacobian matrix transformation to obtain a state space form of the QTP. After obtaining the State space model of QTP the state space to transfer function conversion is done by using a simple conversion technique. The initial step is to obtain a linear approximation of the differential equations which is done by Taylor series.

If the mathematical model of QTP is being integrated to obtain h_1, h_2, h_3 and $h_4$it produces an infinite series of values. It is common practice to approximate a function by using a finite number of terms of its Taylor series.The general form of differential equation can be represented by,

$$\frac{dx_1}{dt} = f_1(h_1, h_2, \ldots, h_n, u_1, u_2, \ldots, u_n)$$

$$\vdots$$

$$\frac{dx_n}{dt} = f_1(h_1, h_2, \ldots, h_n, u_1, u_2, \ldots, u_n) \qquad [11]$$

The general vector form representation is,

$$\dot{x} = f(x, u)$$

Let $H_e = h_e + \Delta \mathrm{h}$

$U_e = u_e + \Delta \mathrm{u}$

We use the Taylor series to yield the linear approximation.

$$\dot{x} = \frac{dx}{dt} = f(H_e, U_e) \qquad [12]$$

$$= f(h_e + \Delta \mathrm{h}, u_e + \Delta \mathrm{u}) \qquad [13]$$

$$= f(h_e, u_e) + \frac{df}{dh}(h_e, u_e)\Delta \mathrm{h} + \frac{df}{du}(h_e, u_e)\Delta \mathrm{u} + \text{higher order terms.}$$

For simplification, the higher order terms are neglected.

Let,

$$A = \frac{\delta f}{\delta h}(h_e, u_e) = \begin{bmatrix} \frac{\delta f_1}{\delta h_1} & \cdots & \frac{\delta f_1}{\delta h_n} \\ \vdots & \cdots & \vdots \\ \frac{\delta f_n}{\delta h_1} & \cdots & \frac{\delta f_n}{\delta h_n} \end{bmatrix}$$

and

$$B = \frac{\delta f}{\delta u}(h_e, u_e) = \begin{bmatrix} \frac{\delta f_1}{\delta u_1} & \cdots & \frac{\delta f_1}{\delta u_n} \\ \vdots & \cdots & \vdots \\ \frac{\delta f_n}{\delta u_1} & \cdots & \frac{\delta f_n}{\delta u_m} \end{bmatrix} \qquad [14]$$

are the Jacobian matrices of f with respect to h and u, evaluated at the equilibrium point, $[H_e^T, u_e^T]^T$

The Jacobian is used to solve systems of differential equations at an equilibrium point or approximate solutions near an equilibrium point.

Note that $\frac{dH_e}{dt} = \frac{d}{dt}h_e + \frac{d}{dt}\Delta h$ [15]

$= \frac{d}{dt}\Delta h$ as h_e is constant.

Furthermore,

$f(h_e.u_e) = 0.$

$$\frac{dx}{dt} = \frac{df}{dh}(h_e,u_e)\,\Delta h + \frac{df}{du}(h_e,u_e)\,\Delta u \quad [16]$$

Let

$$A = \frac{\delta f}{\delta h}(h_e,u_e) \quad \text{and} \quad B = \frac{\delta f}{\delta u}(h_e,u_e) \quad [17]$$

Neglecting the higher-order terms, we arrive at the linear approximation.

$$\frac{d}{dt}\Delta x = A\Delta x + B\Delta u \quad [18]$$

Similarly, if the outputs of the non-linear system is of the form,

$$y_1 = g_1(h_1,h_2,\ldots\ldots,h_n,u_1,u_2,\ldots\ldots,u_m)$$

$$y_2 = g_2(h_1,h_2,\ldots\ldots,h_n,u_1,u_2,\ldots\ldots,u_m)$$

.

.

$$y_p = g_p(h_1,h_2,\ldots\ldots,h_n,u_1,u_2,\ldots\ldots,u_m) \quad [19]$$

Taylor series yields the following linear approximation.

$$\frac{dy}{dt} = g(H_e,U_e) \quad [20]$$

$$= g(h_e,u_e) + \frac{dg}{dh}(h_e,u_e)\Delta h + \frac{dg}{du}(h_e,u_e)\Delta u + \text{higher order terms.}$$

Let,

$$C = \frac{\delta g}{\delta h}(h_e,u_e) = \begin{bmatrix} \frac{\delta g_1}{\delta h_1} & \cdots & \frac{\delta g_1}{\delta h_n} \\ \vdots & \cdots & \vdots \\ \frac{\delta g_p}{\delta h_1} & \cdots & \frac{\delta g_p}{\delta h_n} \end{bmatrix}$$

and

$$D = \frac{\delta g}{\delta u}(h_e,u_e) = \begin{bmatrix} \frac{\delta g_1}{\delta u_1} & \cdots & \frac{\delta g_1}{\delta u_n} \\ \vdots & \cdots & \vdots \\ \frac{\delta g_p}{\delta u_1} & \cdots & \frac{\delta g_p}{\delta u_m} \end{bmatrix} \quad [21]$$

Neglecting the higher-order terms, we arrive at the linear approximation.

$$y = C\Delta x + D\Delta u$$

Linearization,

$$A = \frac{df}{dh} = \begin{bmatrix} \frac{\delta f_1}{\delta h_1} & \frac{\delta f_1}{\delta h_2} & \frac{\delta f_1}{\delta h_3} & \frac{\delta f_1}{\delta h_4} \\ \frac{\delta f_2}{\delta h_1} & \frac{\delta f_2}{\delta h_2} & \frac{\delta f_2}{\delta h_3} & \frac{\delta f_2}{\delta h_4} \\ \frac{\delta f_3}{\delta h_1} & \frac{\delta f_3}{\delta h_2} & \frac{\delta f_3}{\delta h_3} & \frac{\delta f_3}{\delta h_4} \\ \frac{\delta f_4}{\delta h_1} & \frac{\delta f_4}{\delta h_2} & \frac{\delta f_4}{\delta h_3} & \frac{\delta f_4}{\delta h_4} \end{bmatrix}$$

The results of partial differentiation of the A matrix is given below:

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} \quad [22]$$

$$B = \frac{df}{du} = \begin{bmatrix} \frac{\delta f_1}{\delta u_1} & \frac{\delta f_1}{\delta u_2} \\ \frac{\delta f_2}{\delta u_1} & \frac{\delta f_2}{\delta u_2} \\ \frac{\delta f_3}{\delta u_1} & \frac{\delta f_3}{\delta u_2} \\ \frac{\delta f_4}{\delta u_1} & \frac{\delta f_4}{\delta u_2} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_1 k_1}{A_1} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \quad [23]$$

The output equations are,

$$y = C\Delta x + D\Delta u \quad [24]$$

There are two outputs from the process. They are the level of the lower two tanks.

$$Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{pmatrix} x$$

$$y_1 = k_c h_1$$

$$y_2 = k_c h_2$$

$$C = \begin{pmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{pmatrix} \quad [25]$$

Actually the total output which is obtained from the system will be the levels of all the four tanks. And it is necessary that only the levels of the lower two tank is enough to be considered.

State space method generates a matrix from the system differential equations by making each order of the derivatives into a variable.The state space representation serves as an alternative to transfer function representation of a system so that a SISO or a MIMO process can be treated equally. The state-space representation is best suited both for the theoretical treatment of control systems and for numerical calculations. The determination of the system response in the homogeneous case with the initial condition $x(t_o)$is very simple. For these many advantages the state space representation is carried out for the quadruple tank process. The linearized state space equation of a quadruple tank process is given as,

$$\dot{X} = \frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u \quad [26]$$

$$y = \begin{pmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{pmatrix} x \quad [27]$$

The equation [26] and [27] gives the state space analysis ofthe QTP which is obtained from the developed mathematical model.

V. TRANSFER FUNCTION MODEL

The transfer function method applies a Laplace transformation to the differential equations, which allows handling them as a single algebraic equation.The key advantage of transfer functions is in their compactness, which makes them suitable for frequency-domain analysis and stability studies. However, the transfer function approach suffers from neglecting the initial conditions.To determine the transfer function for the QTP the following formula is used,

$$G\ (s) = C\ (sI - A)^{-1}B + D \quad [28]$$

Here,

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_1 k_1}{A_1} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix}$$

$$C = \begin{pmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{pmatrix}$$

So,

$$(sI - A)^{-1} = \begin{bmatrix} \frac{T_1}{1+sT_1} & 0 & \frac{A_3T_1}{A_1(1+sT_1)(1+sT_3)} & 0 \\ 0 & \frac{T_2}{1+sT_2} & 0 & \frac{A_4T_2}{A_2(1+sT_2)(1+sT_4)} \\ 0 & 0 & \frac{T_3}{1+sT_3} & 0 \\ 0 & 0 & 0 & \frac{T_4}{1+sT_4} \end{bmatrix} \quad [29]$$

$$C\ (sI - A)^{-1}B = \begin{bmatrix} \frac{T_1k_1k_{c1}}{A_1\ (1+sT_1)} & \frac{T_1k_1k_ck_2(1-{}_2)}{k_1A_1(1+sT_1)(1+sT_3)} \\ \frac{T_2k_1k_ck_2(1-{}_1)}{k_2A_2(1+sT_2)(1+sT_4)} & \frac{T_2k_2k_{c2}}{A_2\ (1+sT_2)} \end{bmatrix} \quad [30]$$

$$G(s) = \begin{bmatrix} \frac{C_{11}}{(1+sT_1)} & \frac{C_1k_2(1-{}_2)}{k_1(1+sT_1)(1+sT_3)} \\ \frac{C_2k_1(1-{}_1)}{k_2(1+sT_2)(1+sT_4)} & \frac{C_{22}}{(1+sT_2)} \end{bmatrix} \quad [31]$$

Here the ratio ${}^{k_1}/_{k_2}$ and ${}^{k_2}/_{k_1}$ are approximately equal to 1.

The corresponding transfer matrix is

$$\mathrm{G}(s) = \begin{bmatrix} \frac{\mathrm{C}_{11}}{(1+\mathrm{sT}_1)} & \frac{\mathrm{C}_1(1-{}_2)}{(1+\mathrm{sT}_1)(1+\mathrm{sT}_3)} \\ \frac{\mathrm{C}_2(1-{}_1)}{(1+\mathrm{sT}_2)(1+\mathrm{sT}_4)} & \frac{\mathrm{C}_{22}}{(1+\mathrm{sT}_2)} \end{bmatrix} \quad [32]$$

Where

$$\mathrm{C}_1 = \frac{\mathrm{T}_1\mathrm{k}_c\mathrm{k}_1}{\mathrm{A}_1} \text{and} \mathrm{C}_2 = \frac{\mathrm{T}_2\mathrm{k}_c\mathrm{k}_2}{\mathrm{A}_2}$$

Thus the equation [32] shows the transfer function of QTP which has been derived from state space equation.

VI. MINIMUM PHASE AND NON-MINIMUM PHASE

The system is said to be in minimum phase or non-minimum phase based on the location of the multivariable zeros of G(s). These zeros are the zeros of the numerator polynomial given as follows:

$$\det G(s) = \frac{c_1c_2}{\gamma_1\gamma_2\prod_{i=4}^{4}(1+sT_i)} \times \left[(1 + sT_3)(1 + sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_1}\right] \quad [33]$$

The system is found to be in non-minimum phase if

$$0 < \gamma_1 + \gamma_2 < 1$$

And found to be in minimum phase if

$$1 < \gamma_1 + \gamma_2 < 2$$

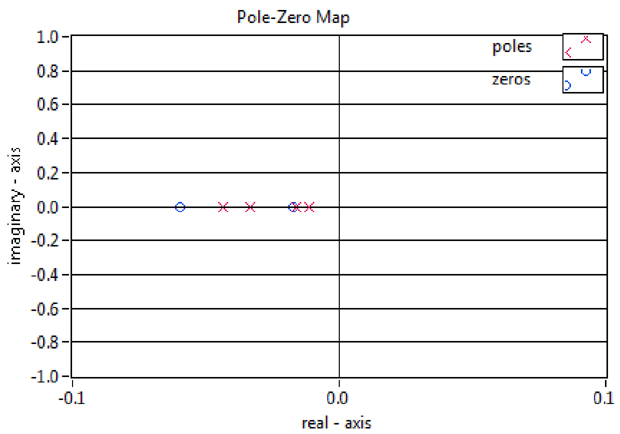

Fig 6: Pole-Zero map for minimum phase

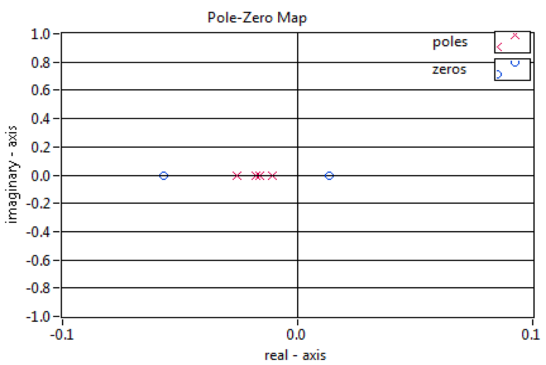

Fig 7: Pole-Zero map for non-minimum phase

The figure 6 and 7 clearly shows the pole and zero map of the system when it is operated in minimum phase (i.e. $1<\gamma_1+\gamma_2<2$) and non-minimum phase (i.e. $0<\gamma_1+\gamma_2<1$). There is a pole shift to the right half of the s-plane when the system is operated in non-minimum phase. This causes the uncertainty of the process under study.

VII. SIMULATION RESULT

The result is obtained for the minimum phase and non-minimum phase operation of the quadruple tank process in both open loop and closed loop. The transfer function analysis and state space analysis are compared and the performance of the system is studied for the PI controller which is implemented. The x-axis of the graph shown will be the height of the water level in the tank and y-axis will be the simulation time.

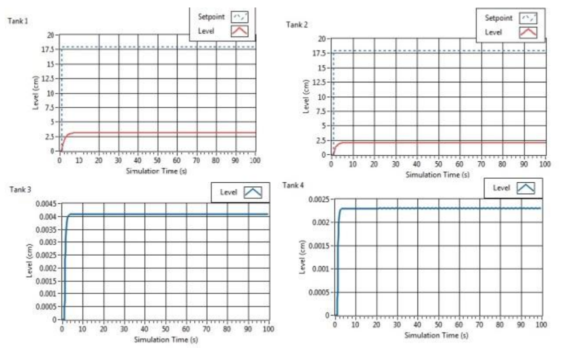

Fig 8: Open loop response of the system in transfer function analysis in minimum phase

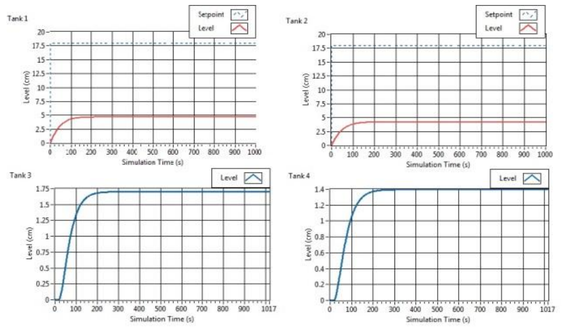

Fig 9: Open loop response of the system in State Variable analysis in minimum phase

The figure 8 and 9 distinguishes the simulated output of the quadruple tank process in transfer function and state space analysis for the open loop operation. It is clear that the response obtained for the system is the tanks 1 and 2 takes short time to settle in transfer function model rather than the state space analysis of the system under study.

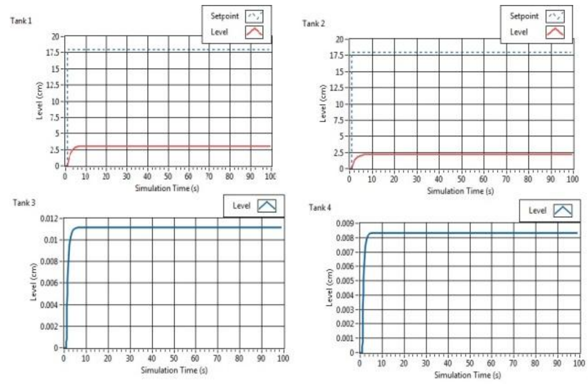

Fig 10: Open loop response of the system in transfer function analysis in non-minimum phase

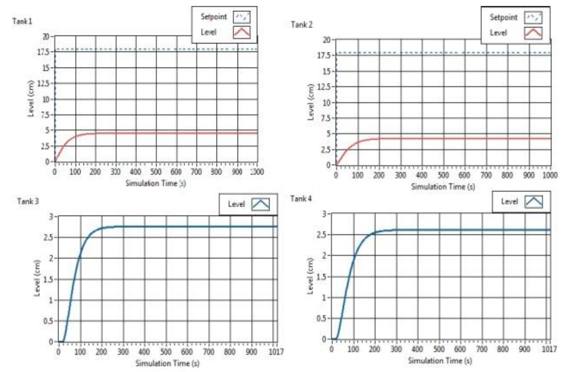

Fig 11: Open loop response of the system in State Variable analysis in non-minimum phase

The figure 10 and 11 shows the simulated output of the quadruple tank process in transfer function and state space analysis for the open loop operation in non-minimum phase. In figure 10 the tank 1 and 2 settles quickly and sharply at 10 and 12. And in the state space analysis the system takes much time of 200 sec to start to settle down. But the fact is, in state space analysis all the four tanks has given proper response in open loop operation.

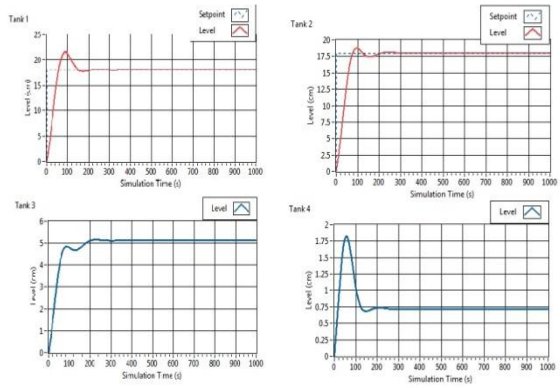

Fig 12: Closed loop response of the system in transfer function analysis in minimum phase

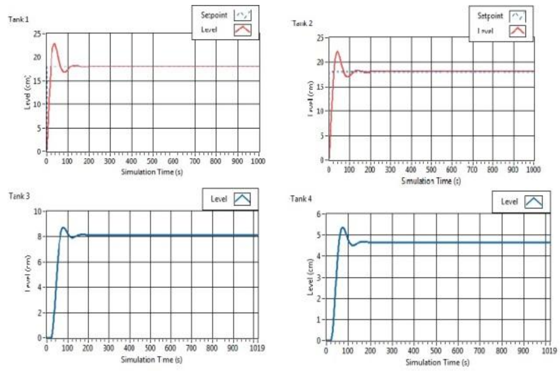

Fig 13: Closed loop response of the system in State Variable analysis in minimum phase

The figure 12 and 13 shows the simulated output of the quadruple tank process in transfer function and state space analysis for the closed loop PI controller operation in minimum phase. In figure 12 the tank 1 and 2 settles after it produce a small peak over shoot at 125 sec. In transfer function analysis the settling time takes 350 sec to settle down and has little oscillation. In the state space analysis the system produces a sharp peak over shoot at 75 sec and takes much lesser time of 135 sec to start to settle down. Here in figure 13 all the four tanks settles and response is good for the PI controller. The value of PI controller is chosen using the root locus technique.The minimum phase response for both the transfer function and state space analysis provides a better and satisfactory result.

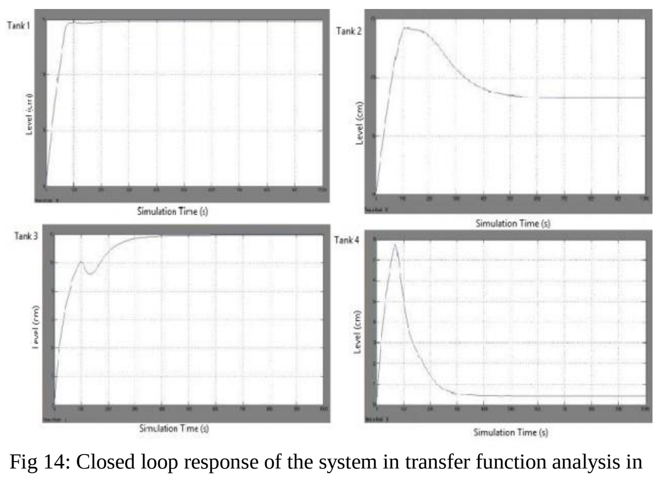

Fig 14: Closed loop response of the system in transfer function analysis in non-minimum phase

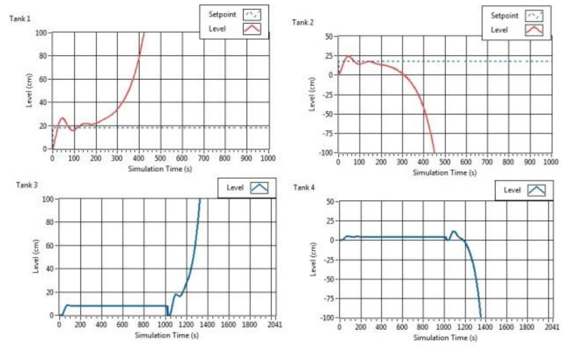

Fig 15: Closed loop response of the system in State Variable analysis in non-minimum phase

The figure 14 and 15 shows the simulated output of the quadruple tank process in transfer function and state space analysis for the closed loop PI controller operation in non-minimum phase. The tank 1 and 2 response is very poor as it never settles in non-minimum phase operation for PI controller. The Controller fails to take action and the performance of the controller is not satisfactory. Figure 14 and 15 clear shows there is a strong interaction between the tanks. The impact of the interaction and uncertainty is being clearly stated from the obtained response of the tanks both in transfer function and state space analysis. When comparing both the response the state space analysis provides a better and clear representation that the system can be studied in detail and much more clearly in state space analysis.

VIII. CONCLUSION

The state space method works better with complex time domain responses, while the transfer function method is a frequency domain model. The QTP is very well suited for demonstrating minimum phase and non-minimum phase system. The study of transfer function and state space representation of the QTP helps in attaining a clear idea of the how the zero location in multivariable control systems affects the performance and act as the limitation for the controller performance. This is ultimately due to the effects of coupling and strong interaction effect between the tanks when it if operated in non-minimum phase. The response obtained in state space representation is much better than the transfer function.

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