

CALCULUS BC

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

**Directions:** Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

**In this test:** Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

1. Which of the following is a  $y$ -coordinate for the equation  $y = \frac{1}{2}x^4 + \frac{2}{3}x^3 - 2x^2 + 6$  when the tangents to the curve equal zero?

(A)  $-\frac{35}{6}$

(B)  $-6$

(C)  $0$

(D)  $\frac{34}{3}$

(E)  $36$

$$y' = 2x^3 + 2x^2 - 4x = 0$$

$$2x(x^2 + x - 2) = 0$$

$$2x(x-1)(x+2)$$

$$x = 0, 1, -2$$

$$y(0) = 6$$

$$y(1) = \frac{1}{2} + \frac{2}{3} - 2 + 6 = \frac{31}{6}$$

$$y(-2) = 8 - \frac{16}{3} + 8 + 6 = \frac{50}{3}$$

2. What is the sum of the series  $\sqrt{5} - \frac{5}{2} + \frac{5\sqrt{5}}{3} - \frac{25}{4} + \dots + (-1)^n \frac{\sqrt{5}^{n+1}}{n+1} + \dots$ ?

(A)  $\ln(1 + \sqrt{5})$

(B)  $e^{\sqrt{5}}$

(C)  $\ln(\sqrt{5})$

(D)  $\sqrt{5}$

(E) The series diverges.

$$\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{5}^{n+1}}{n+1}$$

3.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} + 2x-4}{x^3}$

(A)  $0$

(B)  $\frac{3\sqrt{2}}{64}$

(C)  $\frac{\sqrt{2}}{24}$

(D)  $\frac{\sqrt{2}}{18}$

(E) Undefined

$$\lim_{x \rightarrow 0} \frac{1}{x^3} (3x-2)^{-\frac{1}{2}}$$

$$\lim_{x \rightarrow 0} \frac{-\frac{1}{2}(3x-2)^{-\frac{3}{2}} \cdot 3}{2x^3} = \frac{-\frac{3}{4}}{0} = \infty$$

$$\lim_{x \rightarrow 0} \frac{-\frac{3}{8}(3x-2)^{-\frac{5}{2}} \cdot 3}{2x^3} = \frac{-\frac{9}{8}}{0} = \infty$$

4. Find  $\frac{d^2y}{dx^2}$  at  $x = 1$  for  $y^2 - y = 2x^3 - 3x^2 - 4x + 7$ .

$$\frac{dy}{dx} = \frac{6x^2 - 6x - 4}{2y - 1}$$

(A)  $-\frac{26}{9}$

(B)  $-\frac{22}{27}$

(C)  $-\frac{22}{25}$

(D)  $-\frac{10}{9}$

(E)  $\frac{26}{9}$

$y^2 y = 2$

$(2y-1)(10y-1) = (2y-1)(\frac{-4}{2y-1})$

$\frac{20y}{7} = \frac{10}{7} = \frac{2}{(2y-1)}$

$20y - 10 = \frac{8}{2y-1}$

$20y - 10 = \frac{8}{2y-1}$

$4y^2 - 4y - 1$

5.  $\lim_{h \rightarrow 0} \frac{(2x^2 + 4xh + 2h^2) - 2x^2}{h}$

(A)  $2x^2$

(B)  $-2x^2$

(C)  $4x$

(D)  $4$

(E) Undefined

6.  $\int \frac{dx}{4x^2 - 20x + 26} = \frac{1}{2} \int \frac{dx}{2x^2 - 10x + 13}$

(A)  $\tan^{-1}(2x - 5) + C$

(B)  $\sin^{-1}(x - 5) + C$

(C)  $\tan^{-1}(x - 5) + C$

(D)  $\frac{1}{2} \tan^{-1}(2x - 5) + C$

(E)  $\frac{1}{2} \sin^{-1}(2x - 5) + C$

7.  $\int \frac{14x - 12}{(x^2 + 9)(x + 3)} dx = 2 \int \frac{7x - 6}{(x^2 + 9)(x + 3)}$

(A)  $\left(\frac{3}{2}\right) \ln|x^2 + 9| + \left(\frac{5}{3}\right) \tan^{-1} \frac{x}{3} + 3 \ln|x + 3| + C$

(B)  $\left(\frac{3}{2}\right) \ln|x^2 + 9| + \left(\frac{5}{3}\right) \tan^{-1} \frac{x}{3} - 3 \ln|x + 3| + C$

(C)  $\left(\frac{3}{2}\right) \ln|x^2 + 9| - \left(\frac{5}{3}\right) \tan^{-1} \frac{x}{3} - 3 \ln|x + 3| + C$

(D)  $\left(\frac{3}{2}\right) \ln|x^2 + 9| - \left(\frac{5}{3}\right) \tan^{-1} \frac{x}{3} + 3 \ln|x + 3| + C$

(E)  $\left(\frac{3}{2}\right) \ln|x^2 + 9| + \left(\frac{5}{3}\right) \tan^{-1} \frac{x}{3} - 3 \ln|x + 3| + C$

$\frac{Ax+B}{x^2+9} + \frac{C}{x+3}$

$Ax + B(x+3) + C(x^2+9) = 7x-6$

$2x^2 + 3Ax + Bx + 3B + Cx^2 + 9C = 7x - 6$

$A = -C$

$3A + B = 7$

$3A = 7 - \frac{B}{2}$

$2B + 9C = -6$

$-9C + 3B = 21$

$A = \frac{27}{2}$

$9C + 3B = 10$

$C = -\frac{27}{2}$

$6B = 17$

$B = \frac{17}{6}$

$U = x^2 + 9$

8. If  $\frac{dy}{dx} = 2x^3y$  and  $y(0) = 4$ , find an equation for  $y$  in terms of  $x$ .

(A)  $y = e^{2x^4}$

(B)  $y = 4e^{2x^4}$

(C)  $y = 4e^{x^4}$

(D)  $y = e^x$

(E)  $y = 4e^x$

$y - 4 = 2x^3 y (x - 0)$

$y = 4e^{2x^4}$

$y = 4e^{2x^4}$

$y = 4e^{2x^4}$

$y = 4e^{2x^4}$

$\int \frac{27x}{x^2+9} = \int \frac{8}{x^2+9} = \int \frac{27}{x^2+9}$

(C)  $\csc^2 x - \sec^2 x$

(D)  $\sec^2 x + \csc^2 x$

(E)  $\csc x + \sec x$

14.  $\frac{d}{dx} \left( \tan \left( \frac{x^3}{x+1} \right) \right) = \sec^2 \left( \frac{x^3}{x+1} \right) \cdot \left( \frac{(x+1)3x^2 - x^3}{(x+1)^2} \right)$

(A)  $\frac{3x^3 + 2x^2}{(x+1)^2} \sec^2 \left( \frac{x^3}{x+1} \right)$

(B)  $\frac{2x^3 + 3x^2}{(x+1)^2} \sec^2 \left( \frac{x^3}{x+1} \right)$

(C)  $\frac{2x^3 - 3x^2}{x+1} \sec^2 \left( \frac{x^3}{x+1} \right)$

(D)  $\frac{2x^3 - 3x}{(x+1)^2} \sec^2 \left( \frac{x^3}{x+1} \right)$

(E)  $\frac{2x^3 + 3x^2}{x+1} \sec^2 \left( \frac{x^3}{x+1} \right)$

15.  $\lim_{x \rightarrow 2} \frac{2x^3 + 4x^2 - 6x + 7}{2x^3 + 2x^2 + 4x - 9} = \frac{1}{2}$

(A) 0

(B)  $\frac{1}{6}$

(C)  $\frac{1}{3}$

(D)  $\frac{1}{2}$

(E) The limit is undefined.

16. Where is the tangent line perpendicular to the y-axis for the curve  $y = 2x^4 - 4x^2 + 7$  located?

(A)  $y = 5$

(B)  $y = -7$

(C)  $x = 5$

(D)  $y = 1$

(E)  $x = 7$

17. If  $f$  is continuous on the interval  $[-3, 3]$  and differentiable everywhere on  $(-3, 3)$ , find  $x = c$ , where  $f(c)$  is the mean value of  $f(x) = x^3 - 3x^2 + x - 4$ .

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

18. A toy manufacturer has determined the total profit for a month can be determined by the equation  $P = -3x^2 + 30x + 150$ , where  $x$  is the number of thousands of toys sold. How many thousands of toys

9. Find the derivative of  $y^3 = (x + 2)^2(2x - 3)^3$

(A)  $\frac{y}{3} \left( \frac{2}{x+2} + \frac{3}{2x-3} \right)$

(B)  $\frac{y}{3} \left( \frac{2}{x+2} + \frac{6}{2x-3} \right)$

(C)  $\frac{3}{y} \left( \frac{2}{x+2} + \frac{3}{2x-3} \right)$

(D)  $\frac{3}{y} \left( \frac{2}{x+2} + \frac{6}{2x-3} \right)$

(E)  $\frac{y}{3} \left( \frac{2}{x+2} - \frac{6}{2x-3} \right)$

10.  $\frac{dy}{dx} = (x^3 - 3)y^2$  and  $f(2) = \frac{1}{2}$ . Find an equation for  $y$  in terms of  $x$ .

(A)  $y = \frac{4}{12x - x^4}$

(B)  $y = \frac{4}{x^4 - 12x}$

(C)  $y = \frac{1}{3x - x^4} - \frac{1}{2}$

(D)  $y = \frac{1}{x^4 - 3x}$

(E)  $y = \frac{4}{12x - x^4} + 2$

11. Find the derivative of  $y = \cos^{-1}(x^2 + 2x)$ .

(A)  $\frac{-2x-2}{\sqrt{1-(x^2+2x)^2}}$

(B)  $\frac{2x+2}{\sqrt{1-(x^2+2x)^2}}$

(C)  $\frac{-1}{\sqrt{1-(2x+2)^2}}$

(D)  $\frac{1}{\sqrt{1-(2x+2)^2}}$

(E)  $\frac{-1}{\sqrt{1-(x^2+2x)^2}}$

12.  $\lim_{x \rightarrow 2} (x^3 - 5x + 3) =$

(A) 1

(B) 3

(C) 8

(D) 10

(E) 18

13.  $\frac{d}{dx}(\csc x \sec x) =$

(A)  $\sec^2 x - \csc^2 x$

(B)  $\sec x - \csc x$