Ambiguities in waveforms

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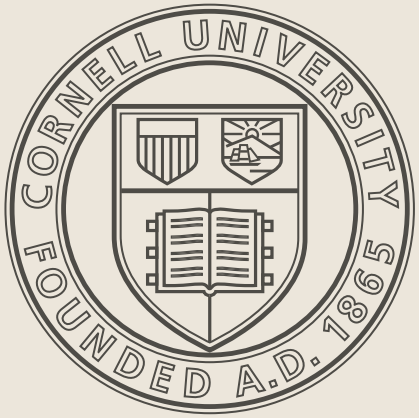

Comparing waveforms

- Testing accuracy of numerical results
- Comparing analytical models
- Constructing hybrids
- Matched filtering

Standard ambiguities

- Time translation
- Phase rotation

Symmetry transformations

- Time translation
- Space translation
- Supertranslation
- Rotation
- Boost

Symmetry transformations

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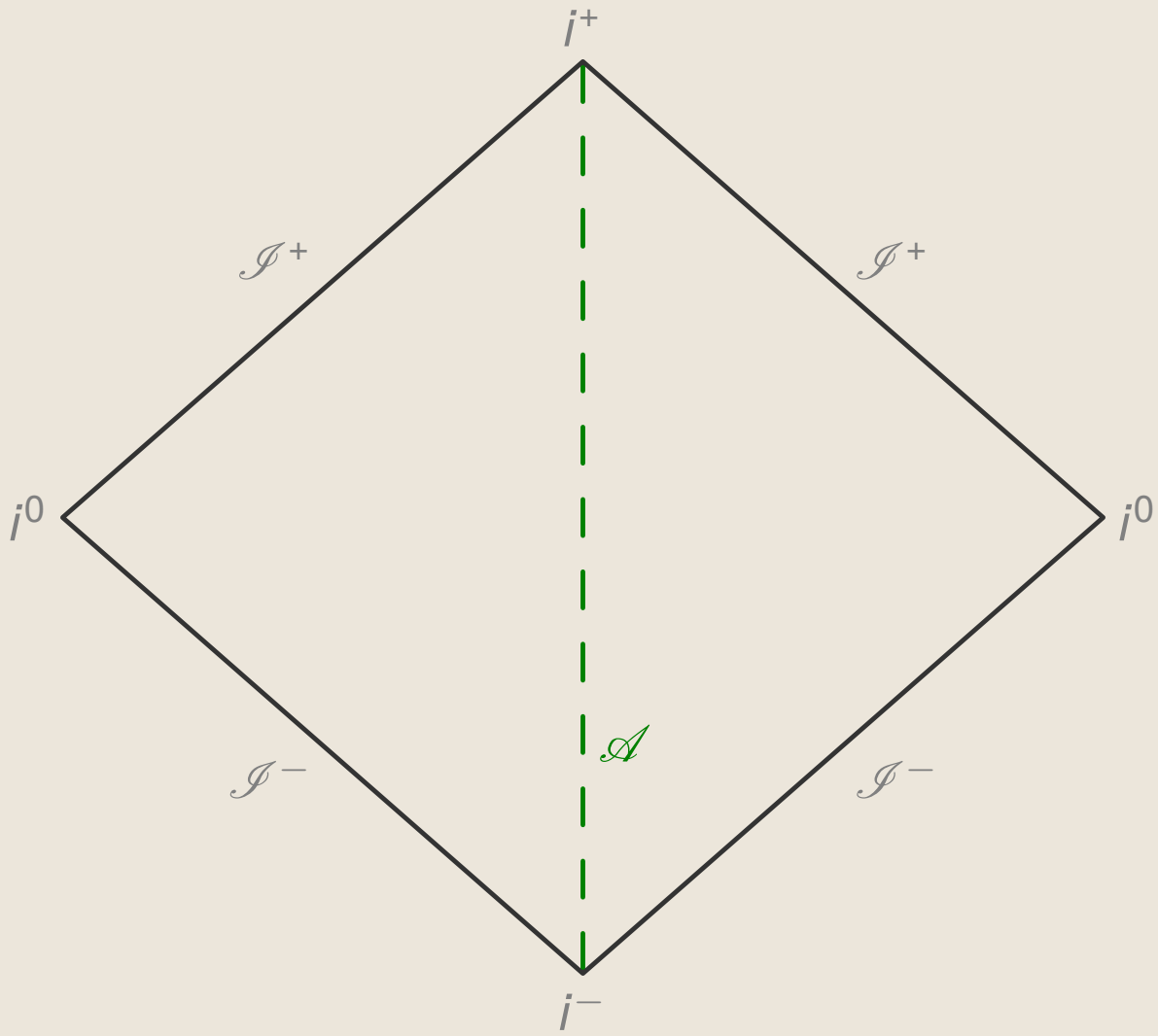
Bondi-Metzner-Sachs (BMS) group

Coordinates on $\mathscr{I}^+$

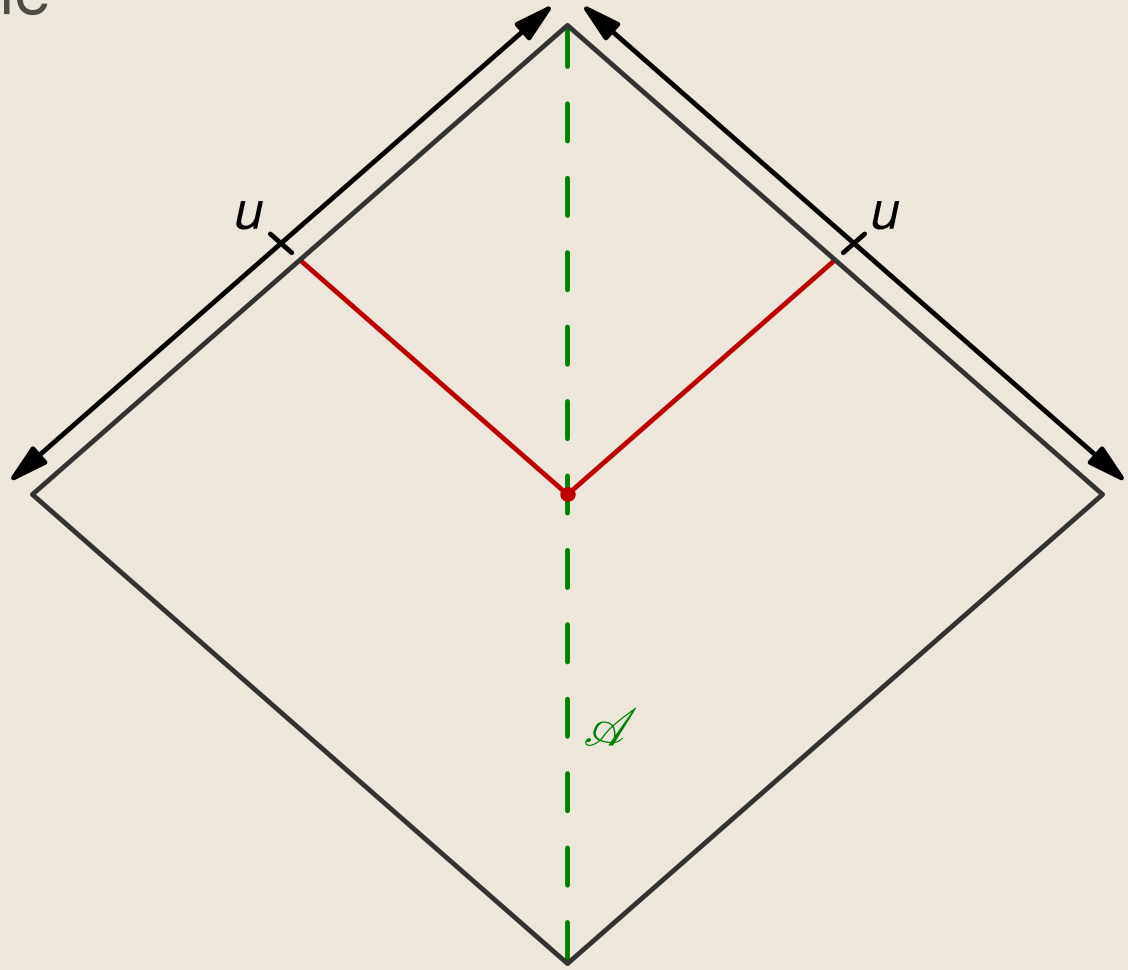
Penrose diagram



Coordinates on $\mathscr{I}^+$

Local time $\rightarrow$ retarded time

$u = t - r$

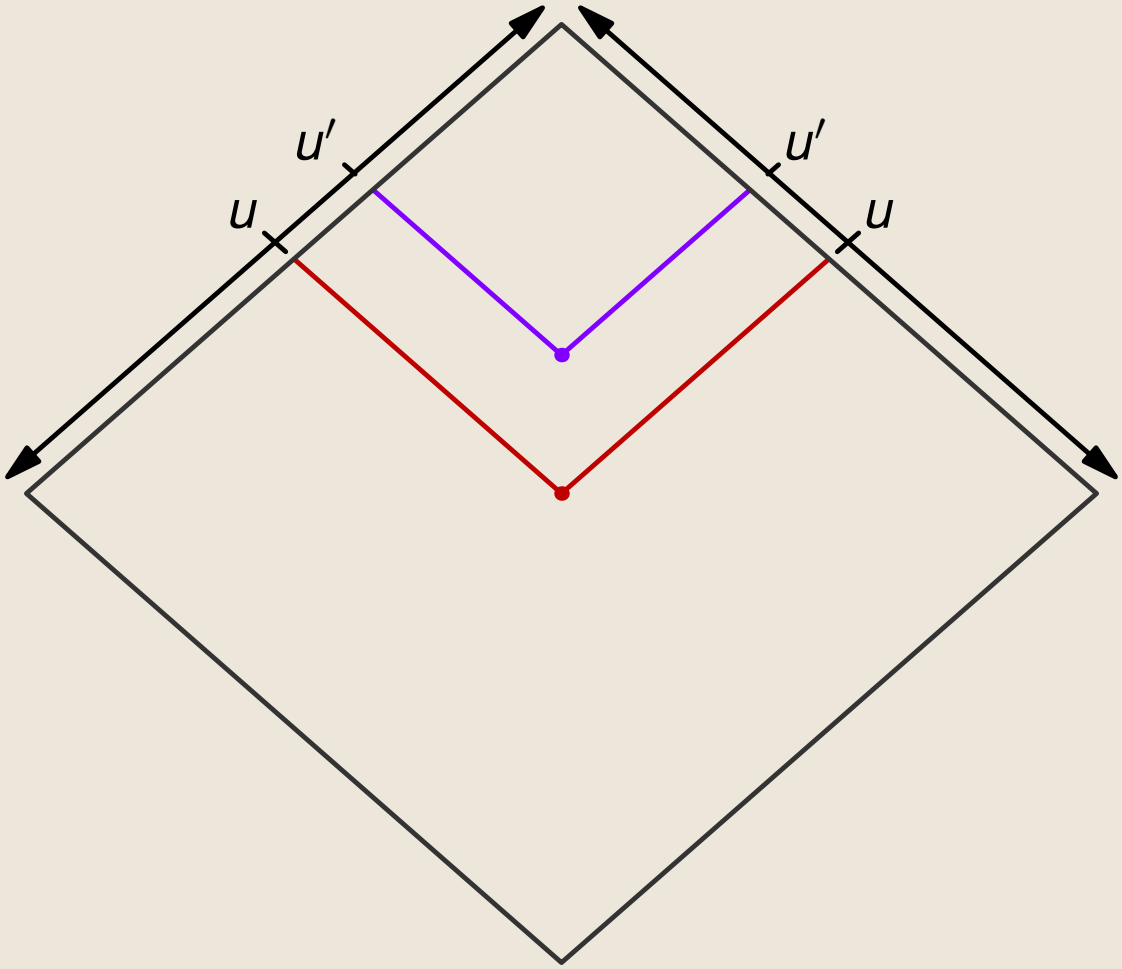


Coordinates on $\mathscr{I}^+$

Time translation

$t \mapsto t + \delta t$

$u \mapsto u + \delta t$

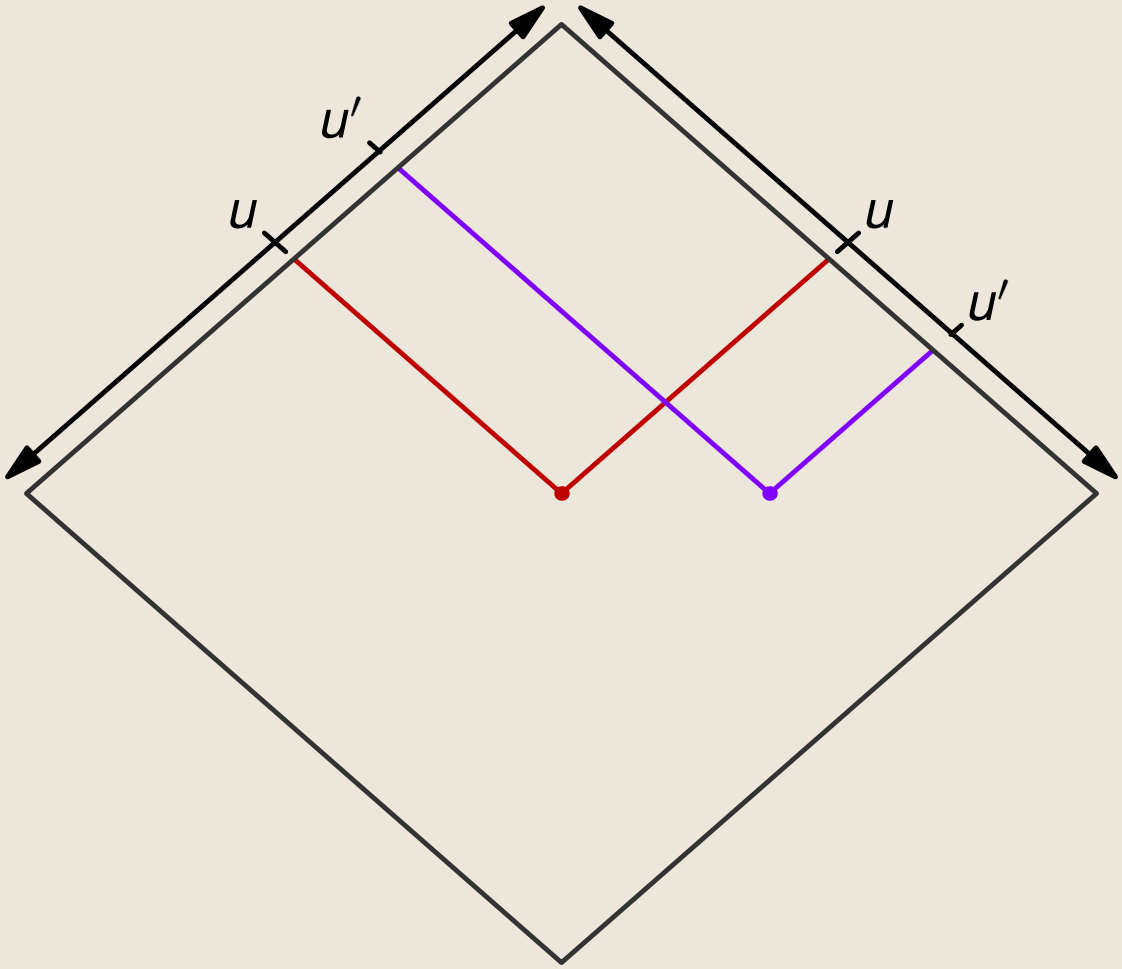


Coordinates on $\mathscr{I}^+$

Space translation

$\boldsymbol{x} \mapsto \boldsymbol{x} + \delta\boldsymbol{x}$

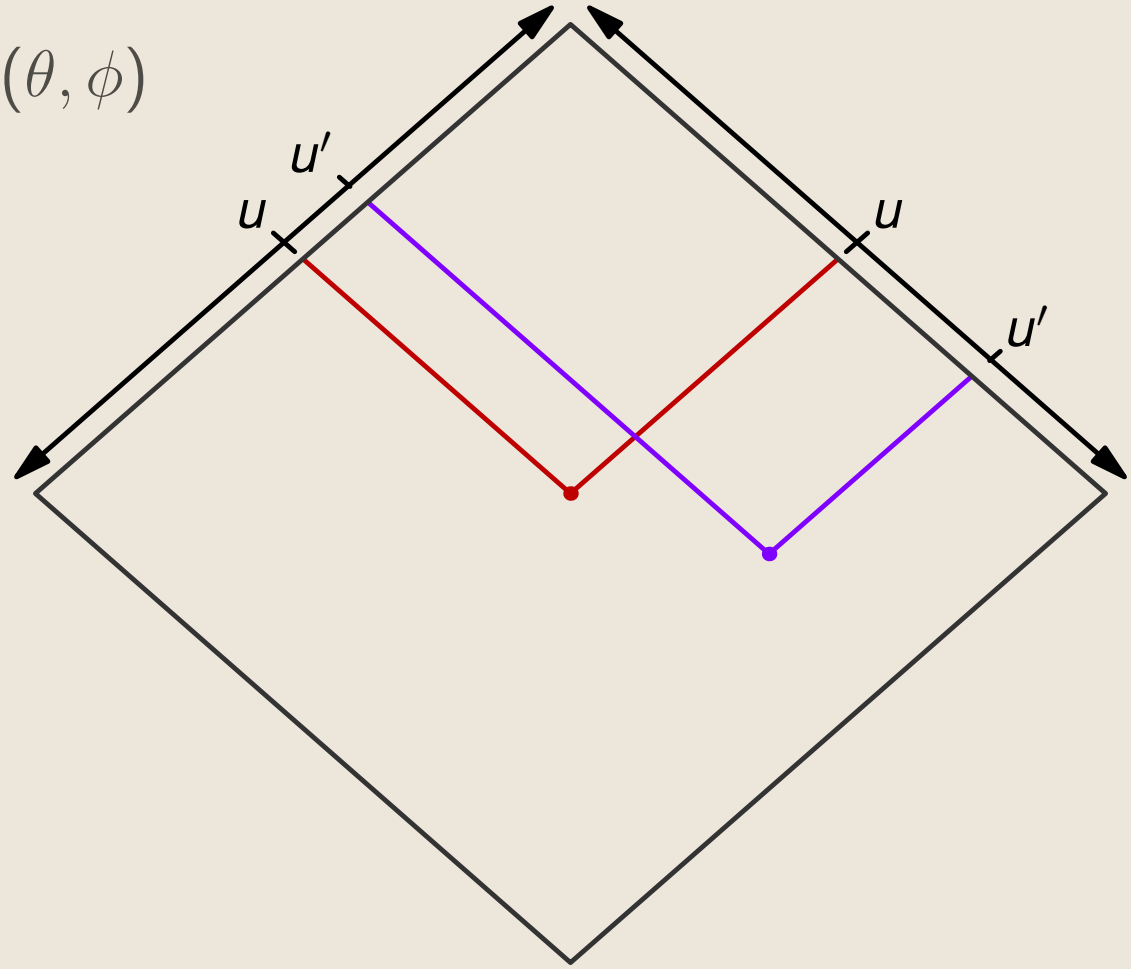
$u \mapsto u - \delta\boldsymbol{x} \cdot \hat{n}$



Coordinates on $\mathscr{I}^+$

Spacetime translation

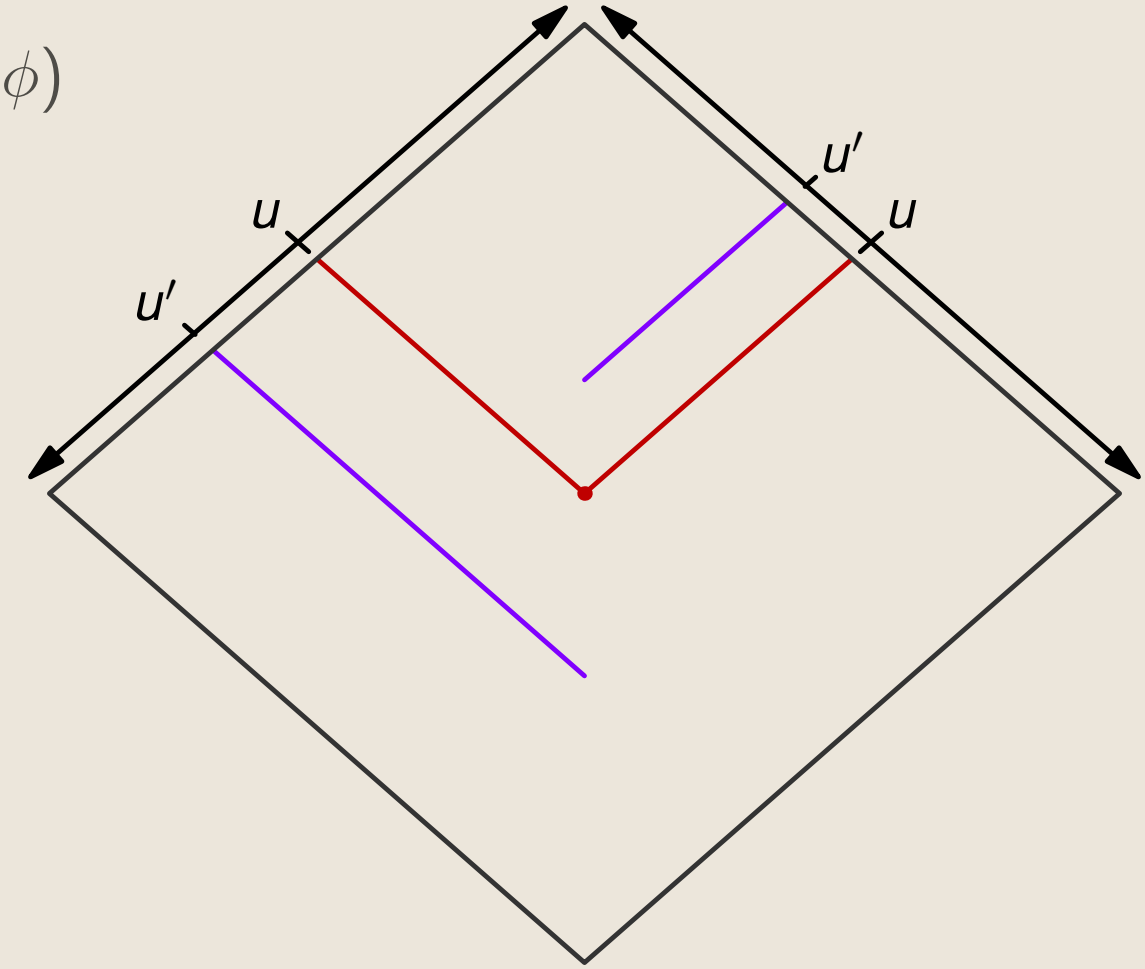
$u \mapsto u + \sum_{\ell=0,1;m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



Coordinates on $\mathscr{I}^+$

Supertranslation

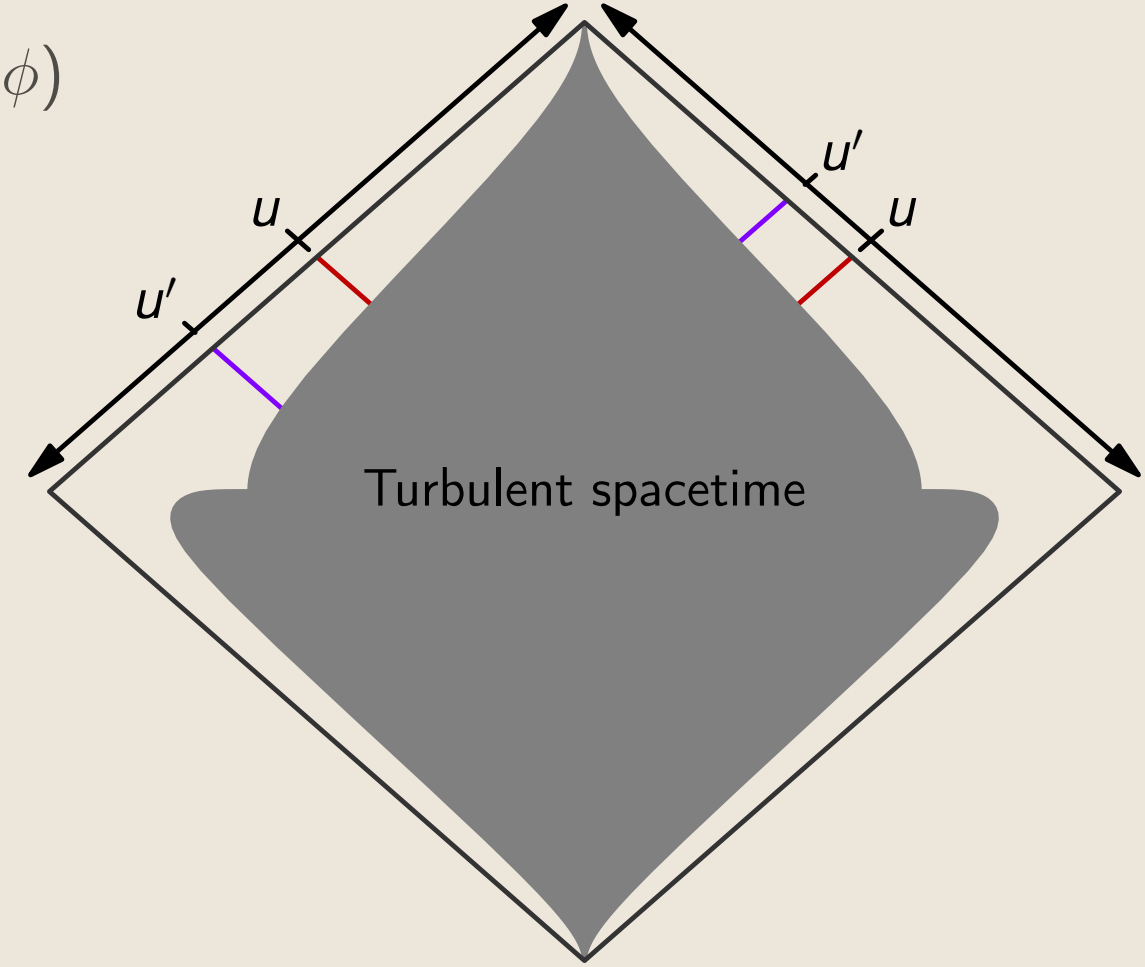
$u \mapsto u + \sum_{\ell,m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



Coordinates on $\mathscr{I}^+$

Supertranslation

$u \mapsto u + \sum_{\ell,m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



BMS transformations

$$\theta \mapsto \theta'$$

$$\phi \mapsto \phi'$$

$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

BMS transformations

$$\theta \mapsto \theta'$$

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$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

$$h \mapsto \frac{e^{-2i\lambda}}{K^b}\left[h - \bar{\eth}^2\alpha\right]$$

BMS transformations

$$\theta \mapsto \theta'$$

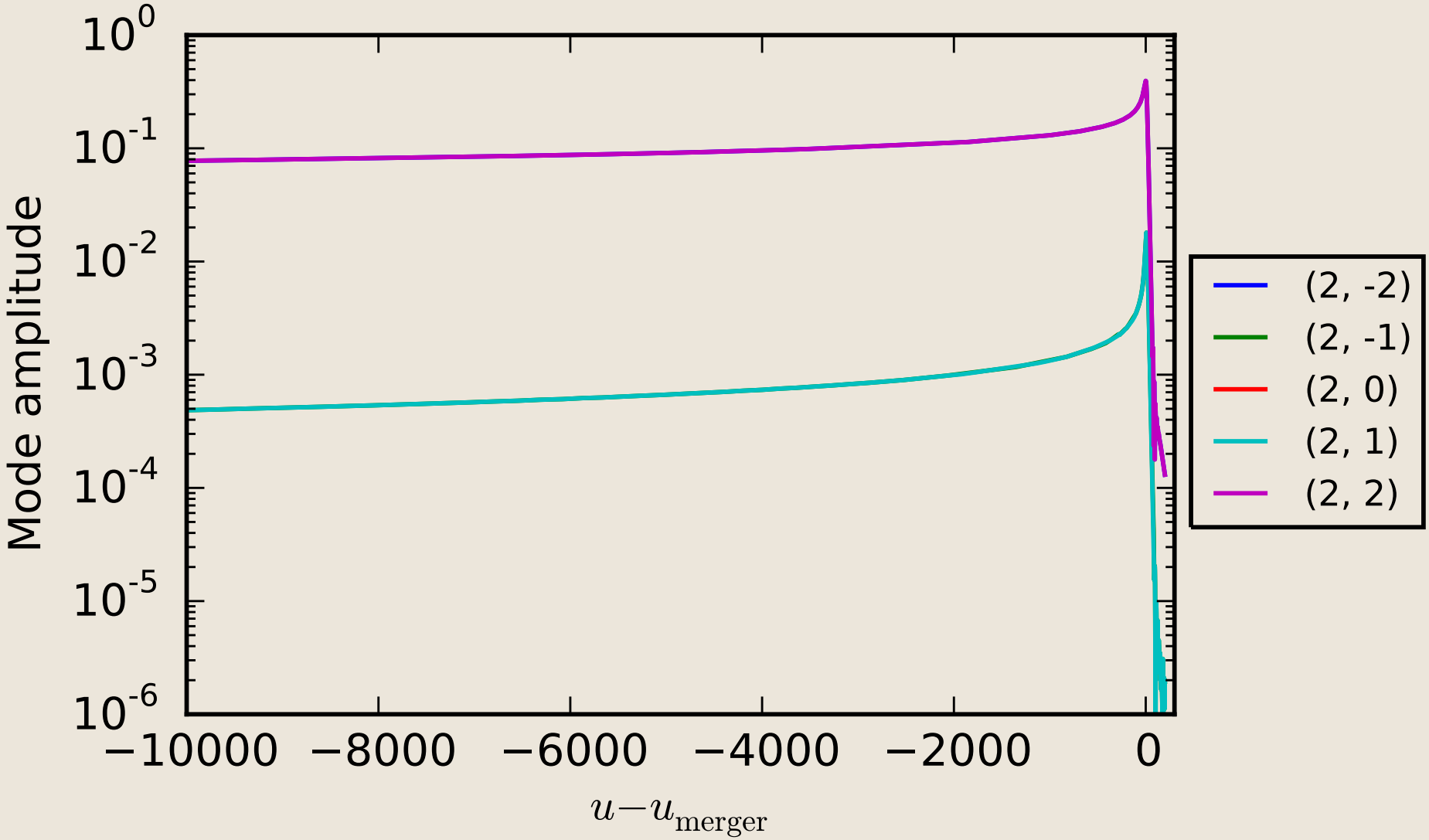
$$\phi \mapsto \phi'$$

$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

$$h(u, \theta, \phi) \mapsto \frac{e^{-2i\lambda(\theta', \phi')}}{K^b(\theta', \phi')}\left[h(u', \theta', \phi') - \bar{\eth}^2\alpha(\theta', \phi')\right]$$

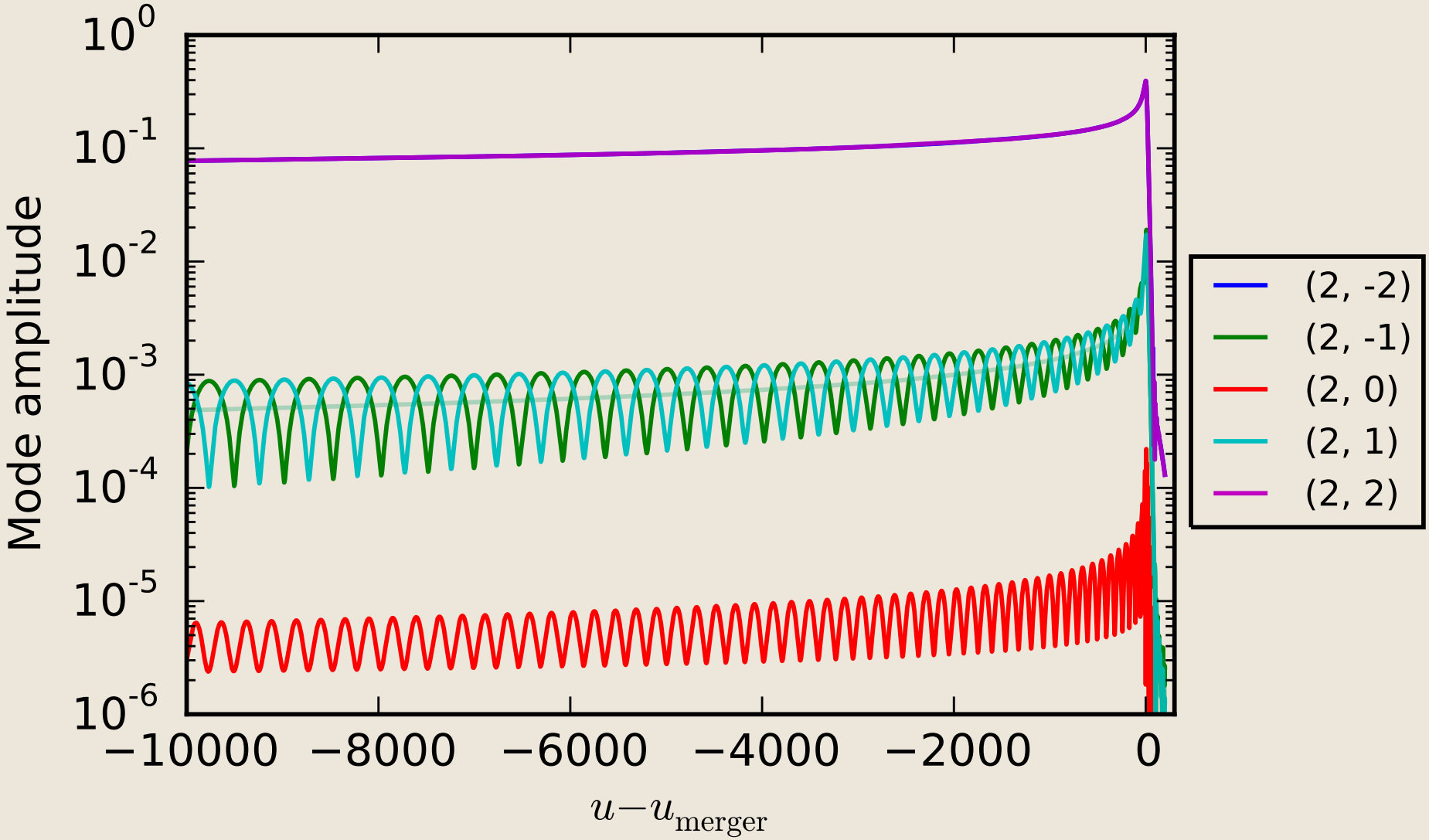
Effects on waveforms

Simplified waveform (zero $\ell > 2$)



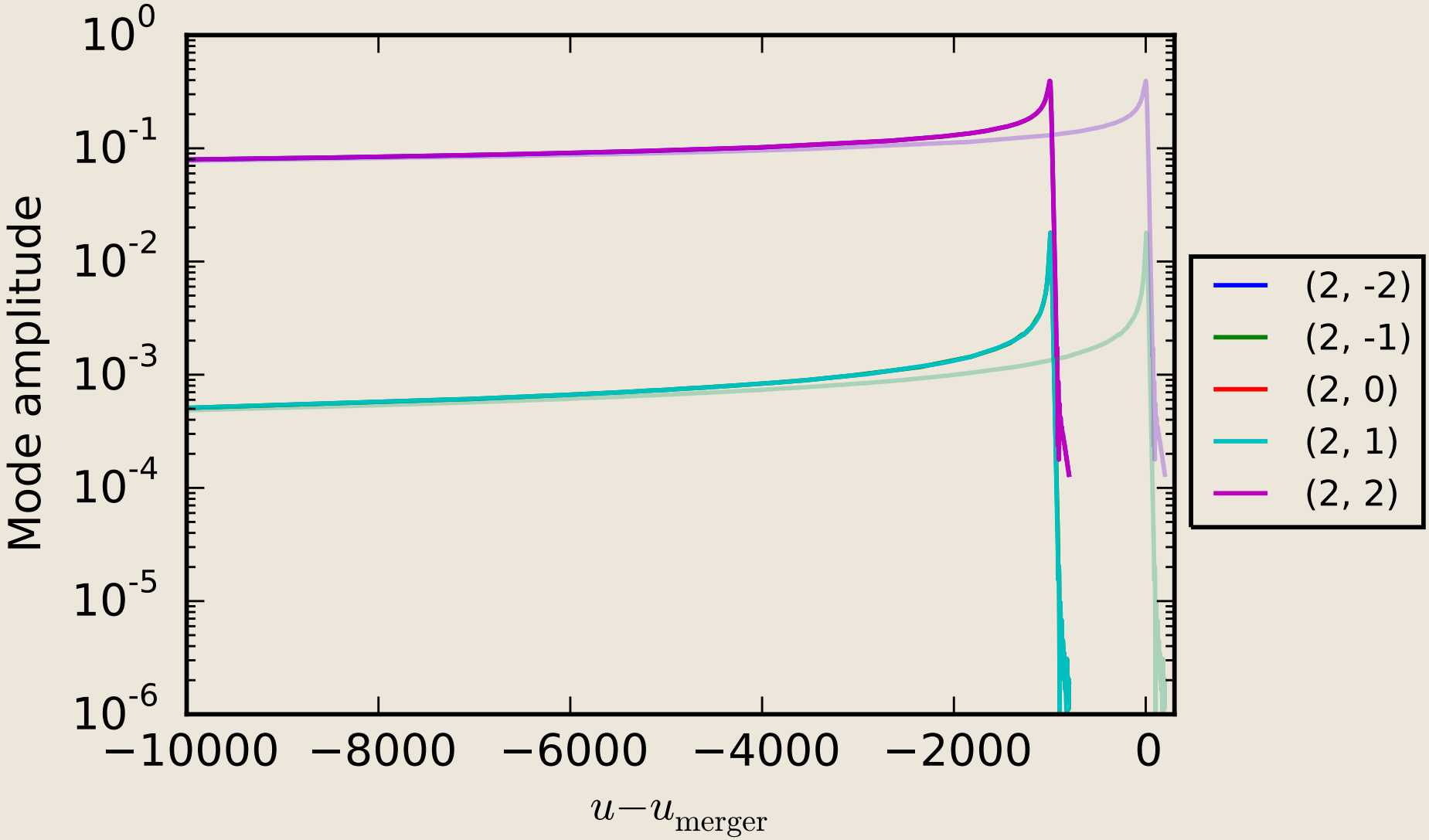
Effects on waveforms

Rotation 10^{-3} about $\hat{x}$



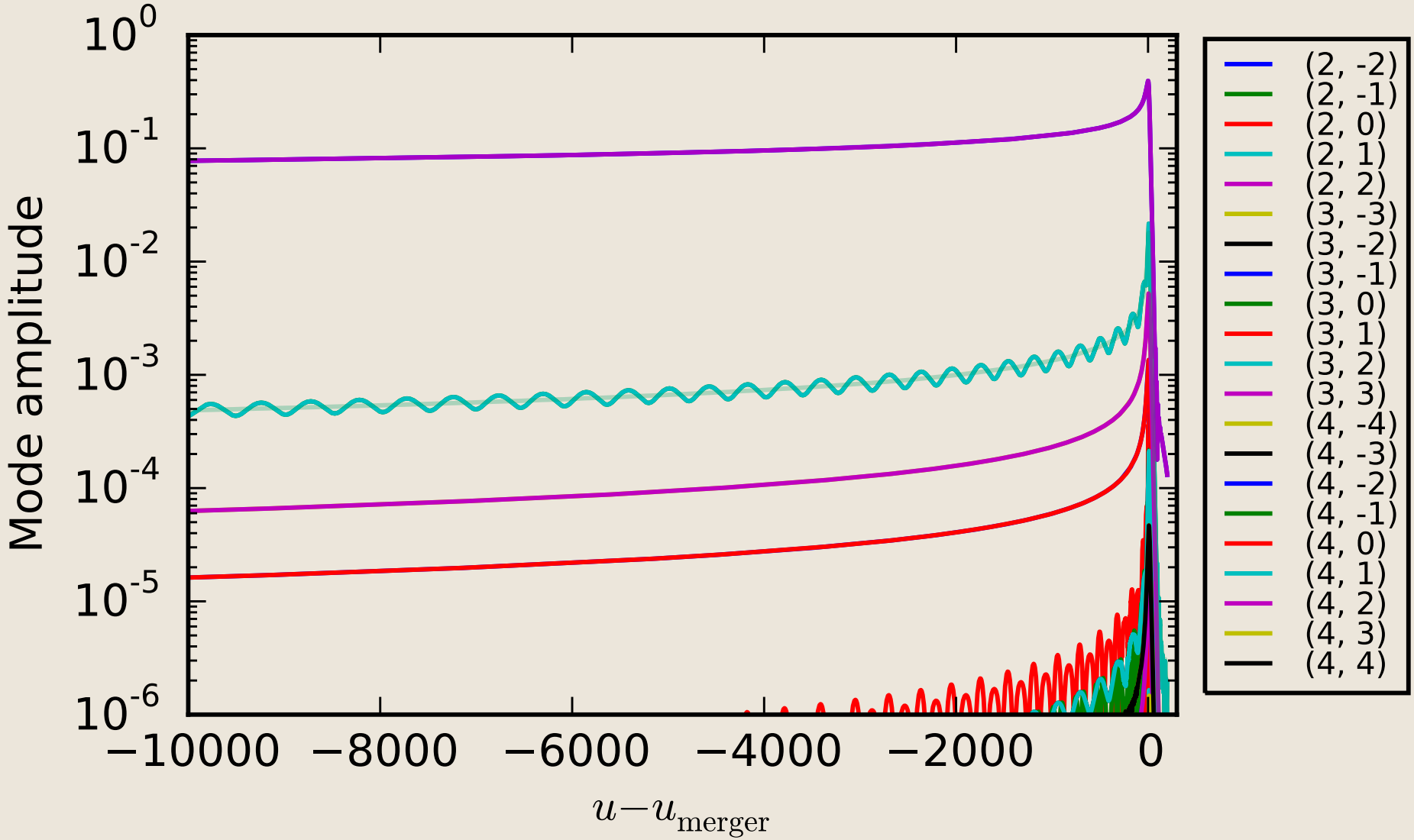
Effects on waveforms

Time translation $\delta t = -1000$



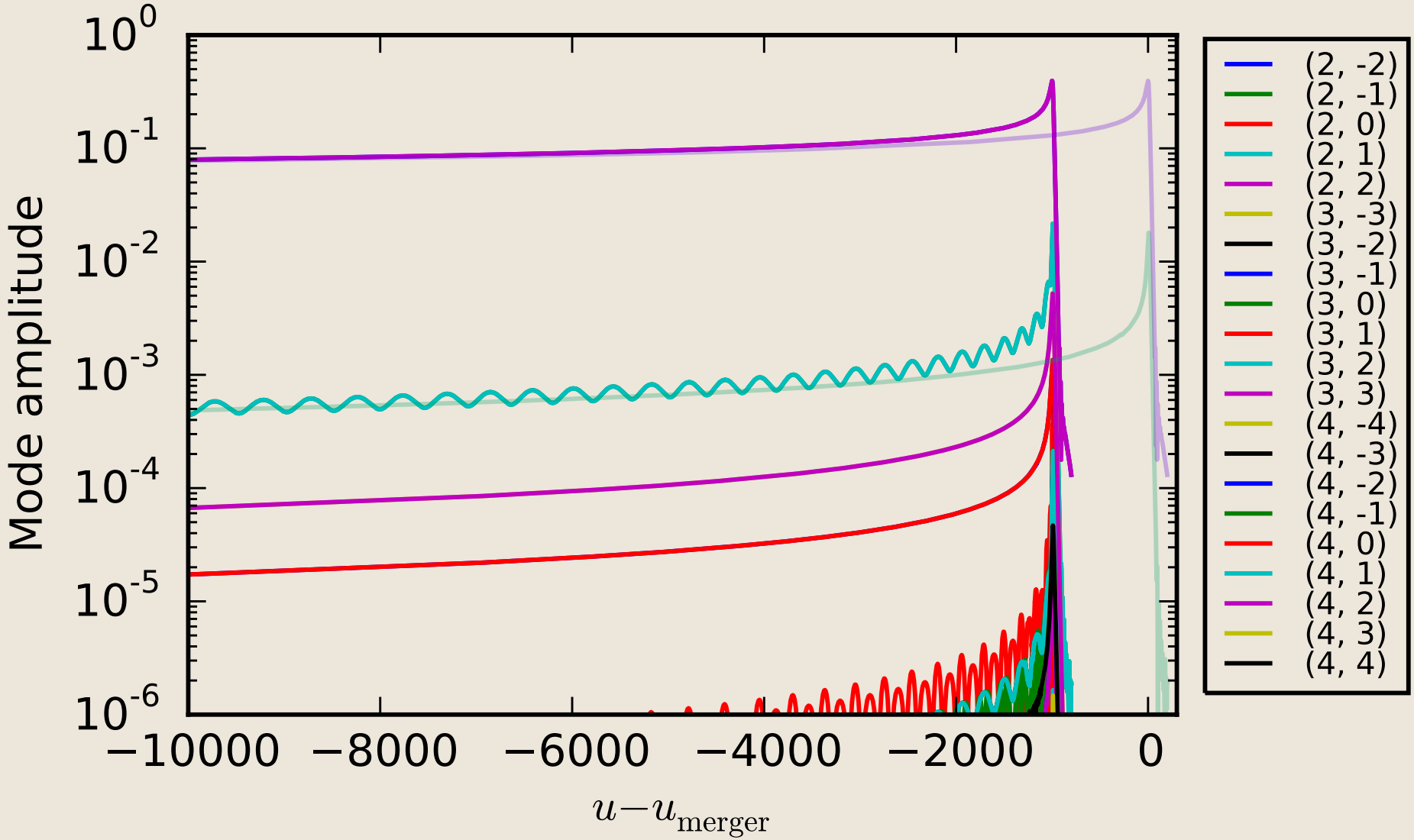
Effects on waveforms

Space translation $\delta \boldsymbol{x} = (0.1, 0, 0)$



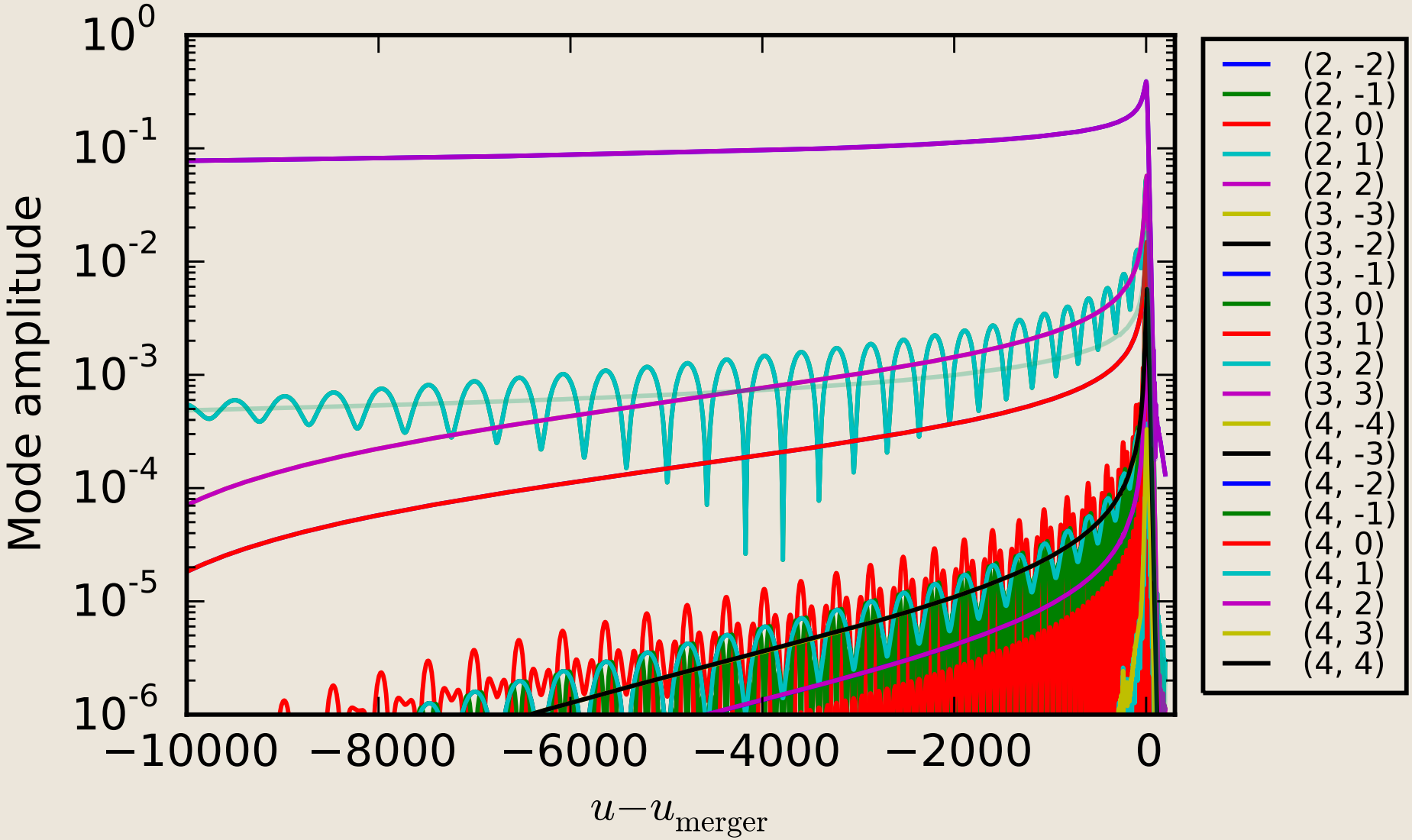
Effects on waveforms

Spacetime translation $\delta = (-1000, 0.1, 0, 0)$



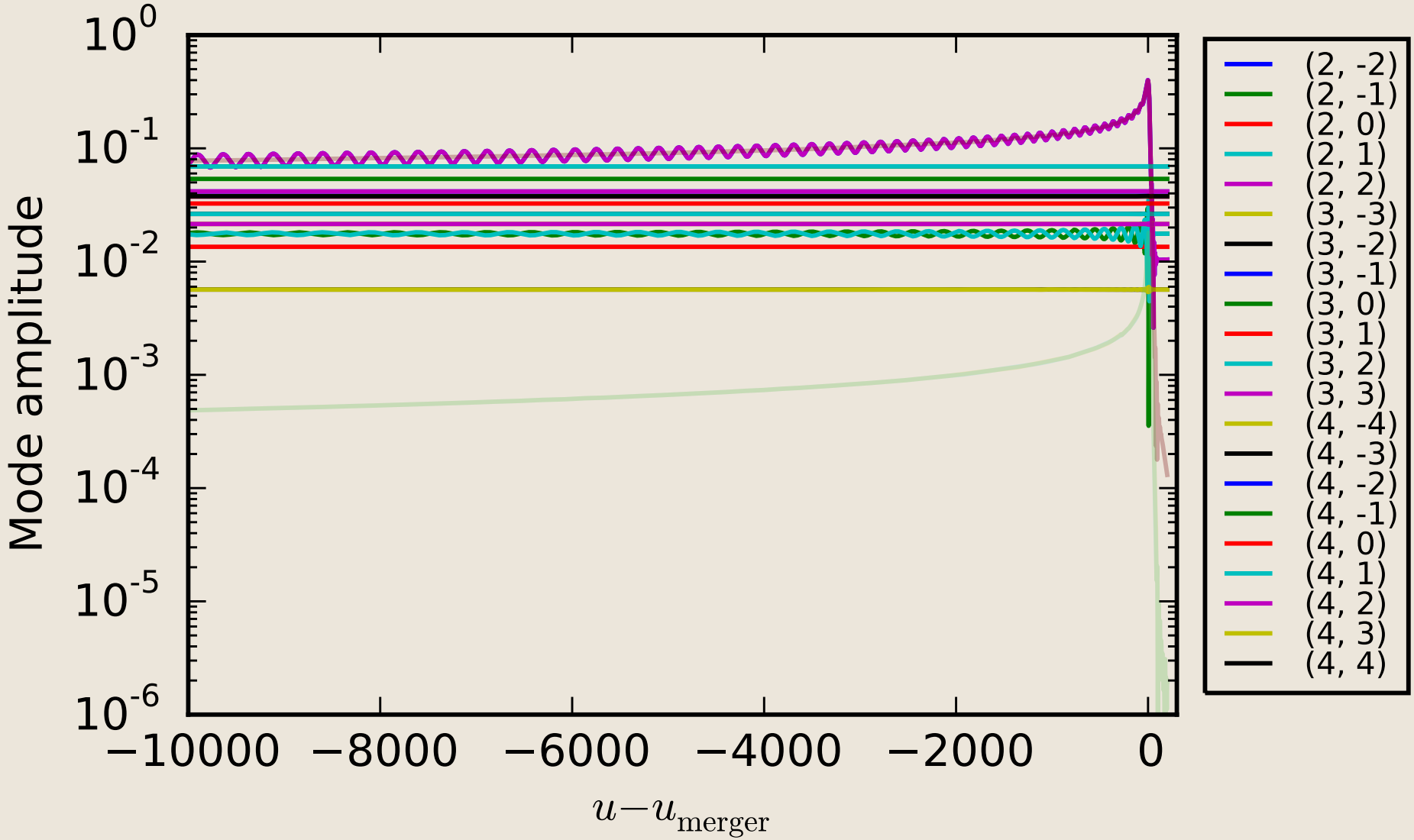
Effects on waveforms

Boost $\boldsymbol{v} = (10^{-4}, 0, 0)$



Effects on waveforms

Supertranslation $\sim 10^{-2}$



Summary

- Waveform comparisons
- Must understand ambiguities
- Ambiguities present in NR where we don't want them
  - We can remove them!
- Ambiguities absent in PN where we do want them
  - We can insert them!