Waveform transformations

Mike Boyle

May 19, 2015

Outline

- Motivation
  - Problems in the waveforms
  - Center-of-mass drifts
  - Cleaning up waveforms
- Asymptotic symmetries (BMS group)
  - Definition (with Penrose diagrams)
  - Requirements
  - Interpolation
  - Spin-weighted functions
- Conclusions

Motivation

A waveform mystery

SXS:BBH:0004

The center of mass

SXS:BBH:0004

Correcting the center of mass

$$\min_{\boldsymbol{x}_0, \boldsymbol{v}_0} \int_{t_i}^{t_f} \left| \boldsymbol{x}_{\text{CoM}}(t) - (\boldsymbol{x}_0 + \boldsymbol{v}_0\, t) \right|^2 dt$$

Correcting the center of mass

$$\min_{\boldsymbol{x}_0,\boldsymbol{v}_0} \int_{t_i}^{t_f} \left|\boldsymbol{x}_{\text{CoM}}(t) - (\boldsymbol{x}_0 + \boldsymbol{v}_0\, t)\right|^2 dt$$

$$\boldsymbol{x}_0 = \frac{4(t_f^2 + t_f t_i + t_i^2) \int \boldsymbol{x}_{\text{CoM}}(t)\, dt - 6(t_f + t_i) \int \boldsymbol{x}_{\text{CoM}}(t)\, t\, dt}{(t_f - t_i)^3}$$

$$\boldsymbol{v}_0 = \frac{12 \int \boldsymbol{x}_{\text{CoM}}(t)\, t\, dt - 6 \int \boldsymbol{x}_{\text{CoM}}(t)\, dt}{(t_f - t_i)^3}$$

Corrected waveform

SXS:BBH:0004′

Correcting the catalog

Correcting the initial data

Ask Sergei!

Asymptotic symmetries

Standard ambiguities

- Time translation
- Phase rotation

Standard ambiguities

- Time translation

- Phase rotation
- General rotation

Standard ambiguities

- Time translation
- Space translation
- Phase rotation
- General rotation

Standard ambiguities

- Time translation
- Space translation
- Phase rotation
- General rotation
- Boost

Standard ambiguities

- Time translation
- Space translation
- Phase rotation
- General rotation
- Boost

Poincaré group?

Standard ambiguities

- Time translation
- Space translation
- Phase rotation
- General rotation
- Boost

Poincaré group?
Diffeomorphism group?!

Isolated system

Asymptotic flatness

- Radial coordinate r
- "Regularity" of manifold as $r \to \infty$

Bondi coordinates

- Null coordinate u
- Angular coordinates $x^A = (\theta, \phi)$
- Metric:

$$ds^2 = \frac{V}{r} e^{2\beta} du^2 - 2e^{2\beta} du\, dr \\ + r^2 h_{AB} (dx^A - U^A du)(dx^B - U^B du)$$

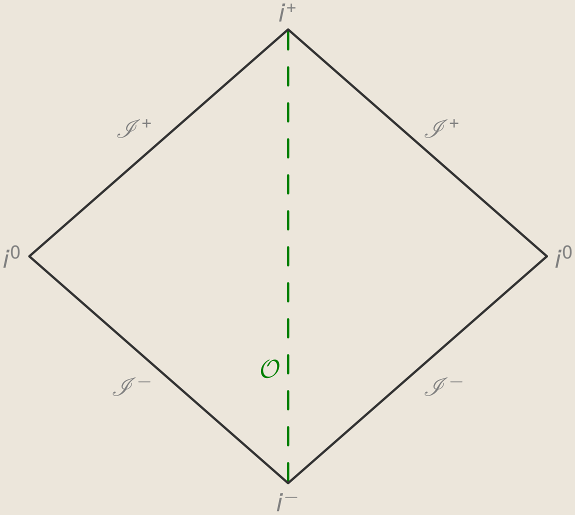
General symmetry group

- Boost
- General rotation
- Supertranslation

Bondi-Metzner-Sachs (BMS) group

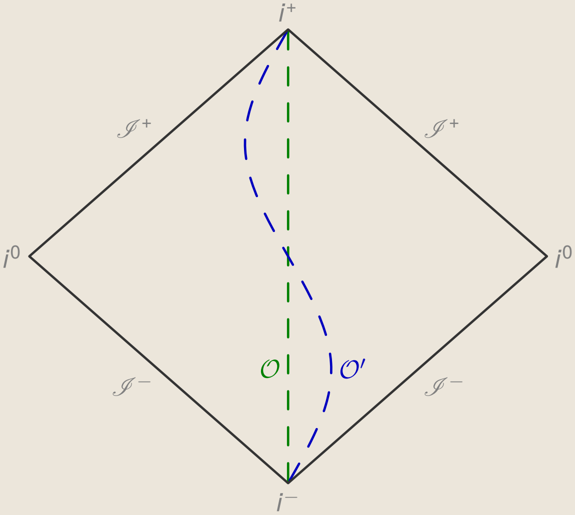
Coordinates on $\mathscr{I}^+$

Penrose diagram



Coordinates on $\mathscr{I}^+$

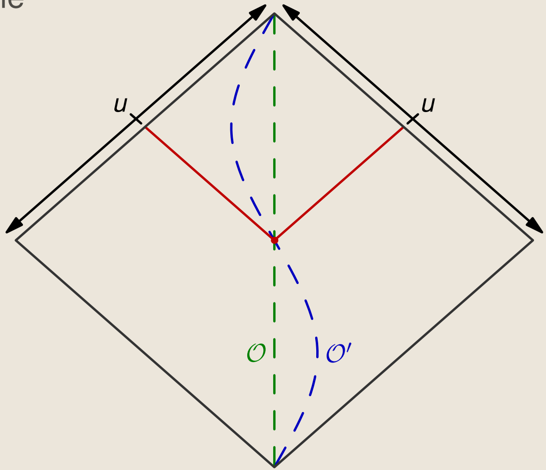
Two observers



Coordinates on $\mathscr{I}^+$

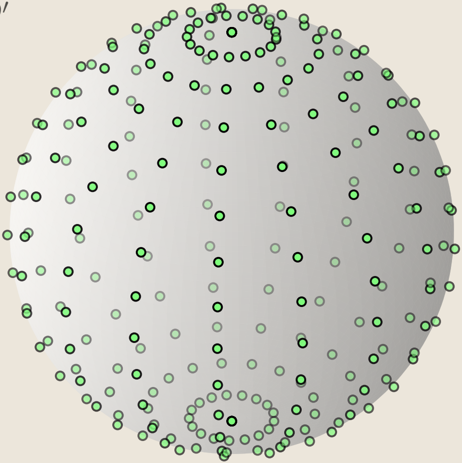
Local time $\rightarrow$ retarded time

$u = t - r$



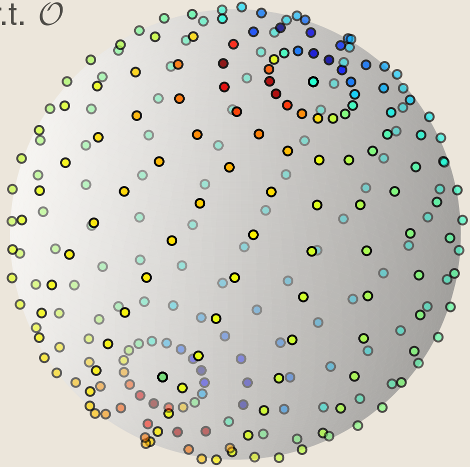
Coordinates on $\mathscr{I}^+$

Equiangular sphere in $\mathscr{O}'$



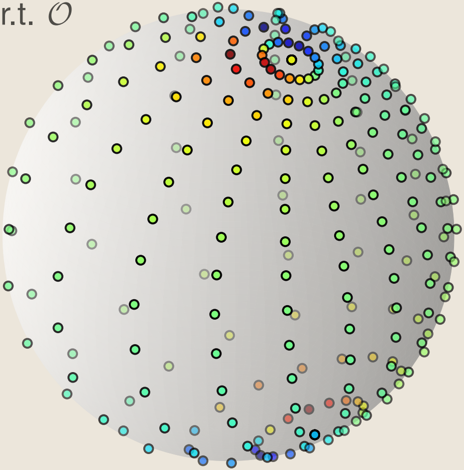
Coordinates on $\mathscr{I}^+$

Those points rotated w.r.t. $\mathcal{O}$



Coordinates on $\mathscr{I}^+$

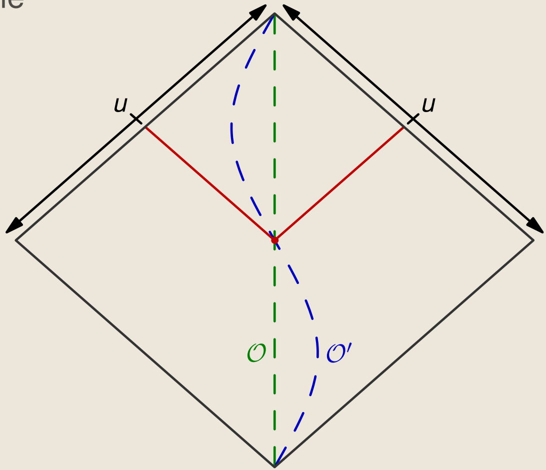
Those points boosted w.r.t. $\mathcal{O}$



Coordinates on $\mathscr{I}^+$

Local time $\rightarrow$ retarded time

$u = t - r$

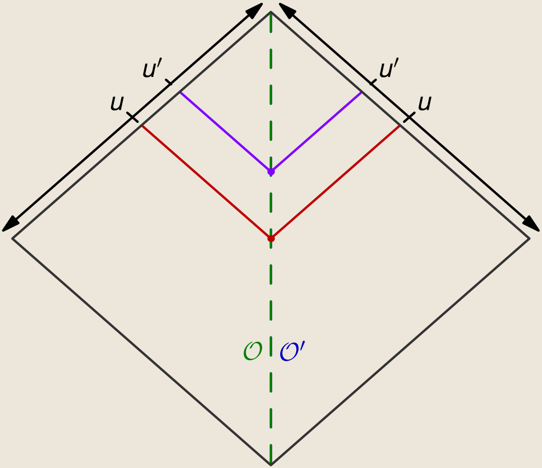


Coordinates on $\mathscr{I}^+$

Time translation

$t \mapsto t + \delta t$

$u \mapsto u + \delta t$

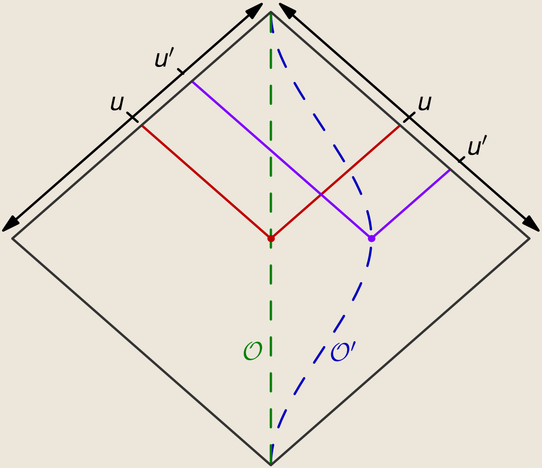


Coordinates on $\mathscr{I}^+$

Space translation

$\boldsymbol{x} \mapsto \boldsymbol{x} + \delta\boldsymbol{x}$

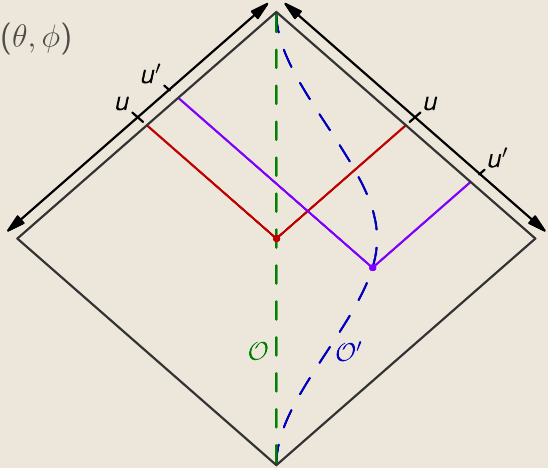
$u \mapsto u - \delta\boldsymbol{x} \cdot \hat{\boldsymbol{r}}$



Coordinates on $\mathscr{I}^+$

Spacetime translation

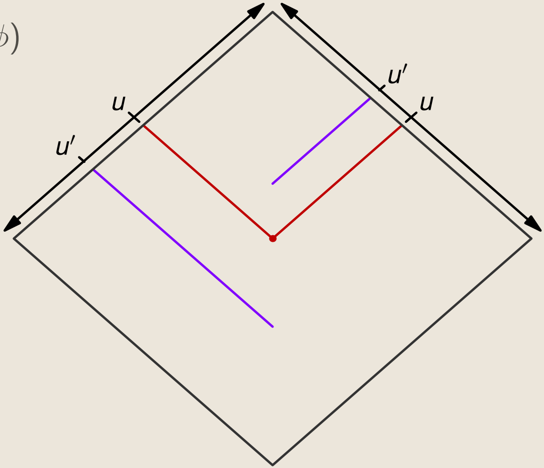
$u \mapsto u + \sum_{\ell=0,1;m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



Coordinates on $\mathscr{I}^+$

Supertranslation

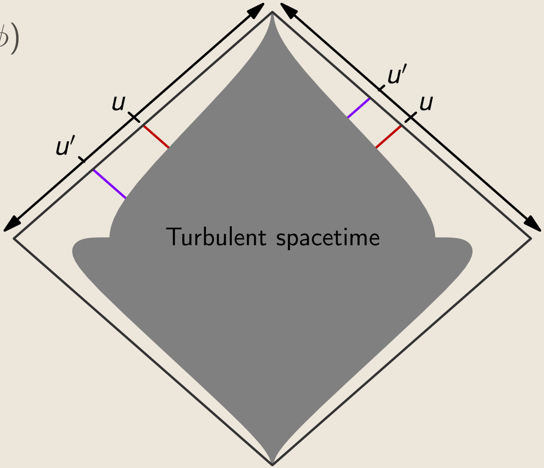
$u \mapsto u + \sum_{\ell,m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



Coordinates on $\mathscr{I}^+$

Supertranslation

$u \mapsto u + \sum_{\ell,m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



BMS transformations

$$\theta \mapsto \theta'$$
$$\phi \mapsto \phi'$$
$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

BMS transformations

$$\theta \mapsto \theta'$$
$$\phi \mapsto \phi'$$
$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

$$u' \approx \gamma\ [u + u\,\boldsymbol{v} \cdot \boldsymbol{n} - \alpha(\theta', \phi')]$$

Waveform transformations

$$r\,h \mapsto \frac{e^{-2i\lambda}}{K^2}\left[r\,h - \bar{\eth}^2\alpha\right]$$

$$r^2\,\sigma \mapsto \frac{e^{2i\lambda}}{K^3}\left[r^2\,\sigma - \eth^2\alpha\right]$$

$$c \mapsto \frac{e^{2i\lambda}}{K}\left[c - \eth^2\alpha\right]$$

$$\text{News} \sim \frac{\partial c}{\partial u} \mapsto \frac{e^{2i\lambda}}{K^2}\left[\frac{\partial c}{\partial u}\right]$$

$$r\,\Psi_4 \mapsto \frac{e^{-2i\lambda}}{K^4}\left[r\,\Psi_4'\right]$$

BMS transformations

$$r\,h \mapsto \frac{e^{-2i\lambda}}{K^2}\left[r\,h - \bar{\eth}^2\alpha\right]$$

BMS transformations

$$r\,h(u,\theta,\phi) \mapsto \frac{e^{-2i\lambda(\theta',\phi')}}{K^2(\theta',\phi')}\left[r\,h(u',\theta',\phi') - \bar{\eth}^2\alpha(\theta',\phi')\right]$$

Spin-weighted functions

Weighted functions are contractions between tensors and tetrad elements, as functions of position on the sphere.

Weighted functions are *not* just functions on the sphere; you need to specify a position on the sphere, as well as a choice of the tetrad.

For spin-weighted functions, this is just an alignment of the vectors in the tangent space — equivalent to choice of unit vector at each point.

But SW functions are not functions from $S^2 \times S^1$. We can choose a section of that bundle. But the Hairy-ball theorem forbids continuous choice of such a thing for our purposes.

Instead, it's the Hopf bundle

Rotor of a boost

$$\Theta' = \arccos\left[\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{r}}\right]$$

$$\Theta = 2\arctan\left[\sqrt{\frac{1-\beta}{1+\beta}}\tan\frac{\Theta'}{2}\right]$$

$$R_B = \exp\left[\frac{\Theta - \Theta'}{2}\,\frac{\boldsymbol{r}\times\boldsymbol{v}}{|\boldsymbol{r}\times\boldsymbol{v}|}\right]$$

Conclusions

- No such thing as invariant waveform
- No preferred reference frame
- When comparing waveforms, frames must agree