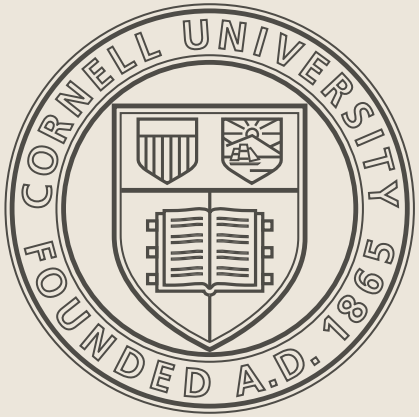
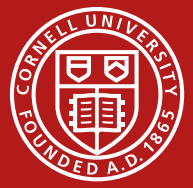
Implementing BMS transformations

Mike Boyle

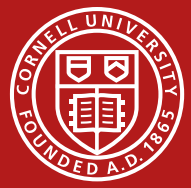
May 19, 2015



Outline



- Motivation
  - Problems in the waveforms
  - Center-of-mass drifts
  - Cleaning up waveforms
- Asymptotic symmetries (BMS group)
  - Definition (with Penrose diagrams)
  - Requirements
  - Interpolation
  - Spin-weighted functions
- Conclusions



Motivation

A waveform mystery

SXS:BBH:0004

The center of mass

SXS:BBH:0004

Correcting the center of mass

$$\min_{\boldsymbol{x}_0, \boldsymbol{v}_0} \int_{t_i}^{t_f} \left| \boldsymbol{x}_{\mathrm{CoM}}(t) - (\boldsymbol{x}_0 + \boldsymbol{v}_0\, t) \right|^2 dt$$

Correcting the center of mass

$$\min_{\boldsymbol{x}_0, \boldsymbol{v}_0} \int_{t_i}^{t_f} \left| \boldsymbol{x}_{\text{CoM}}(t) - (\boldsymbol{x}_0 + \boldsymbol{v}_0\, t) \right|^2 dt$$

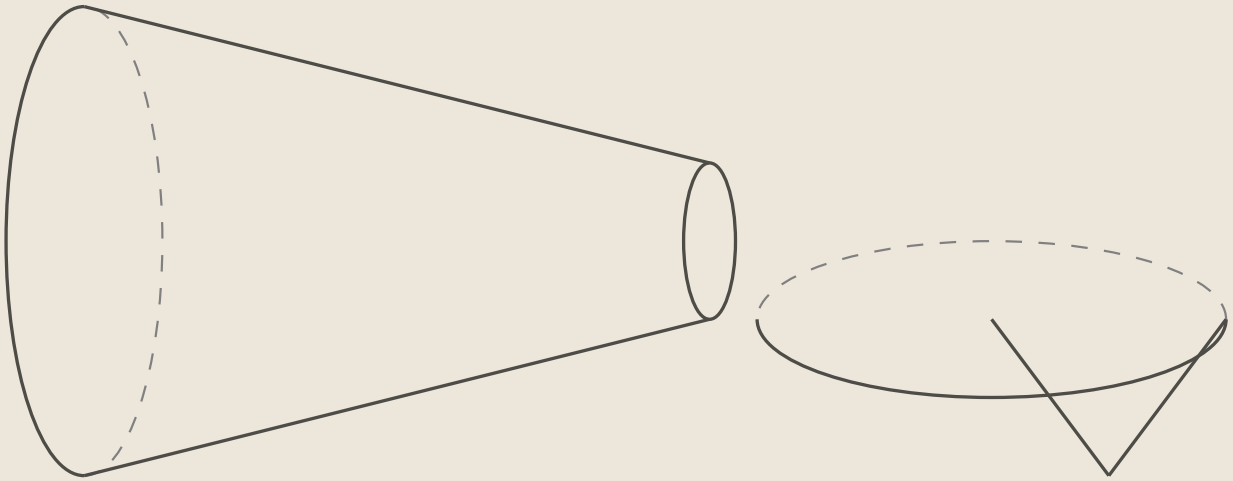
$$\boldsymbol{x}_0 = \frac{4(t_f^2 + t_f t_i + t_i^2) \int \boldsymbol{x}_{\text{CoM}}(t)\, dt - 6(t_f + t_i) \int \boldsymbol{x}_{\text{CoM}}(t)\, t\, dt}{(t_f - t_i)^3}$$

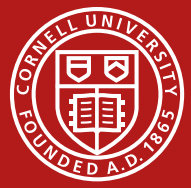
$$\boldsymbol{v}_0 = \frac{12 \int \boldsymbol{x}_{\text{CoM}}(t)\, t\, dt - 6 \int \boldsymbol{x}_{\text{CoM}}(t)\, dt}{(t_f - t_i)^3}$$

Corrected waveform

SXS:BBH:0004′

Correcting the catalog



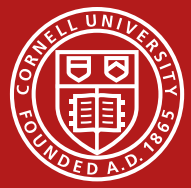


Asymptotic symmetries

Standard ambiguities

- Time translation
- Phase rotation

Symmetry transformations



- Time translation
- Space translation
- Supertranslation
- Rotation
- Boost

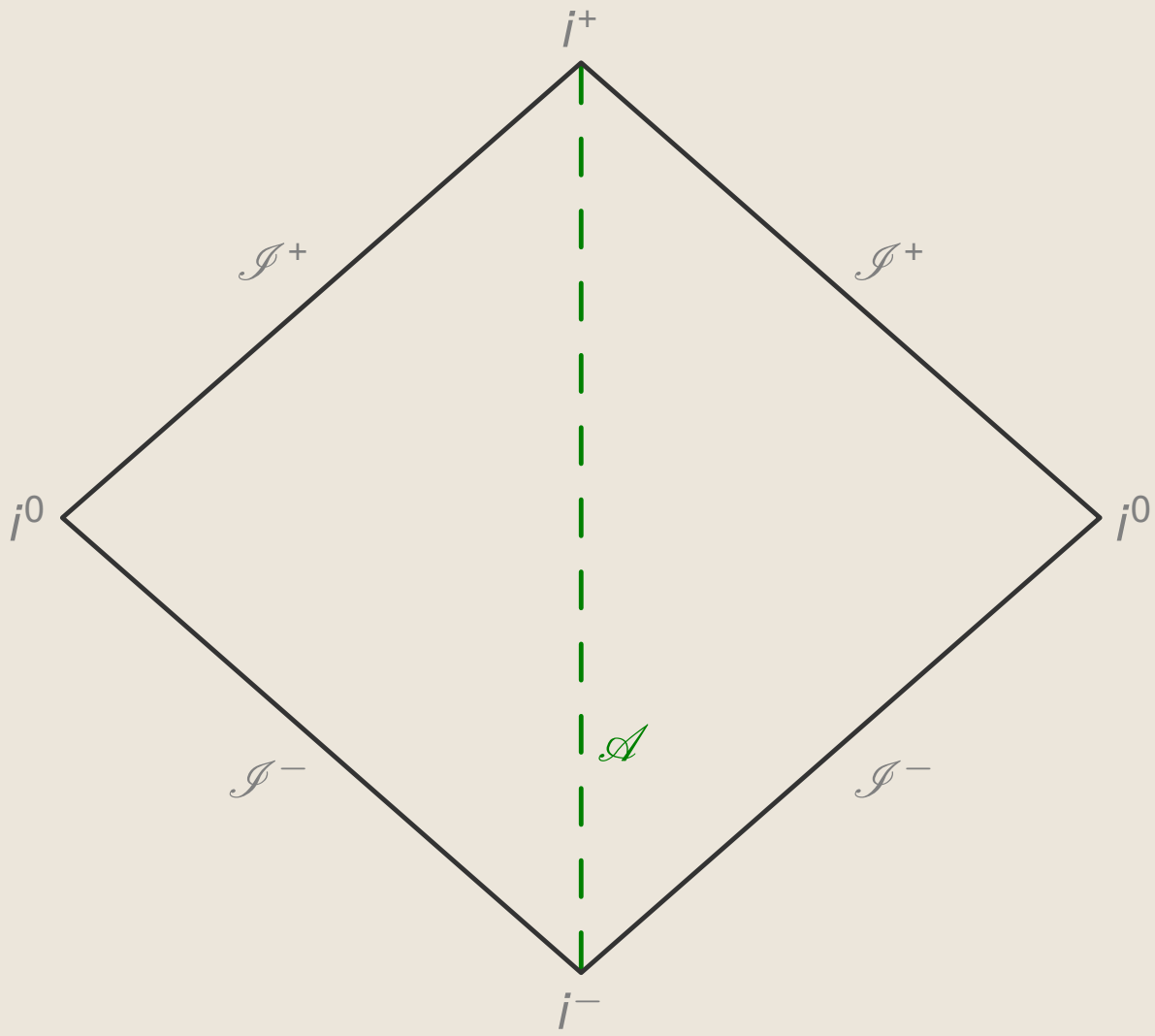
Symmetry transformations

- Time translation
- Space translation
- Supertranslation
- Rotation
- Boost

Bondi-Metzner-Sachs (BMS) group

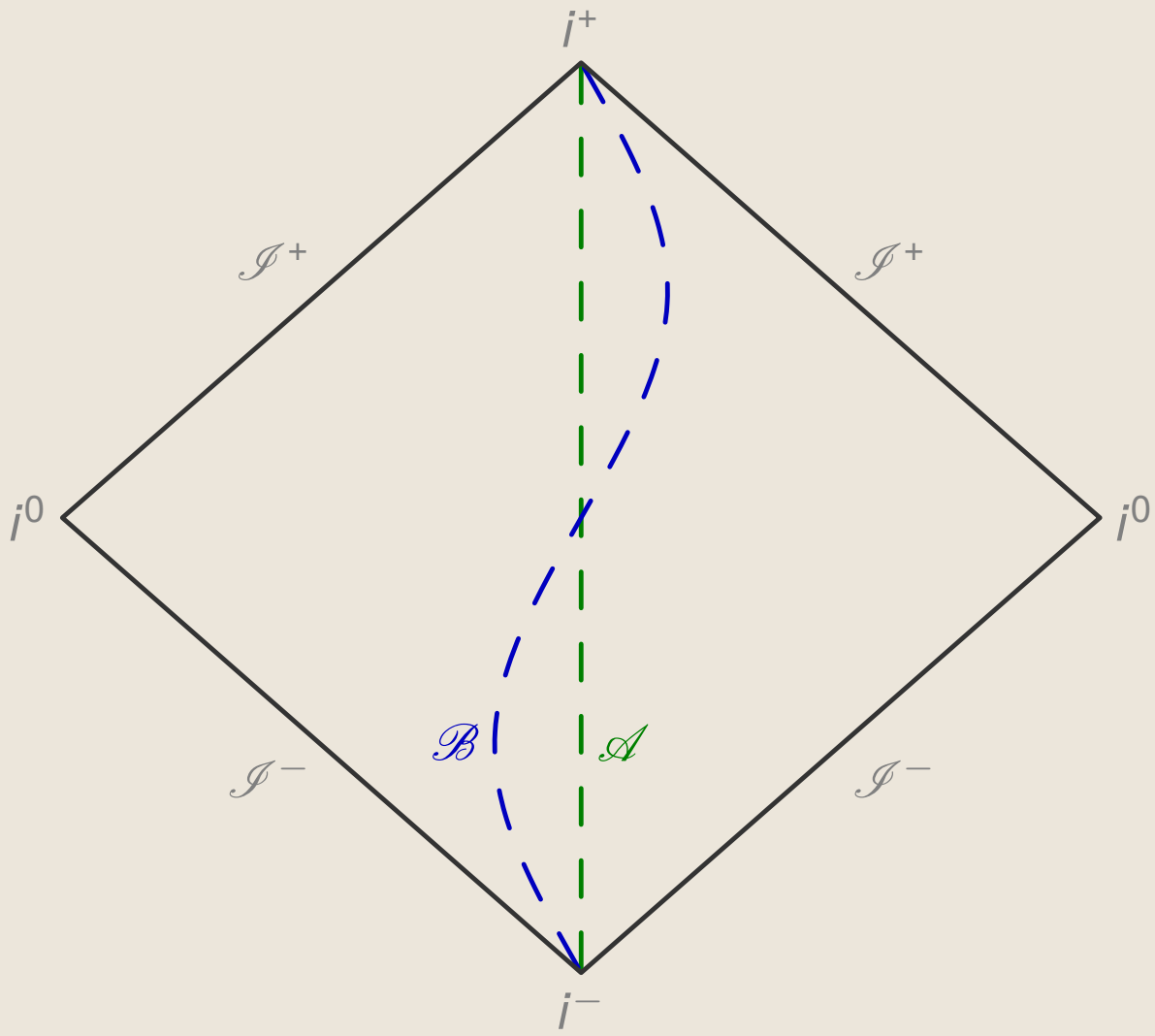
Coordinates on $\mathscr{I}^+$

Penrose diagram



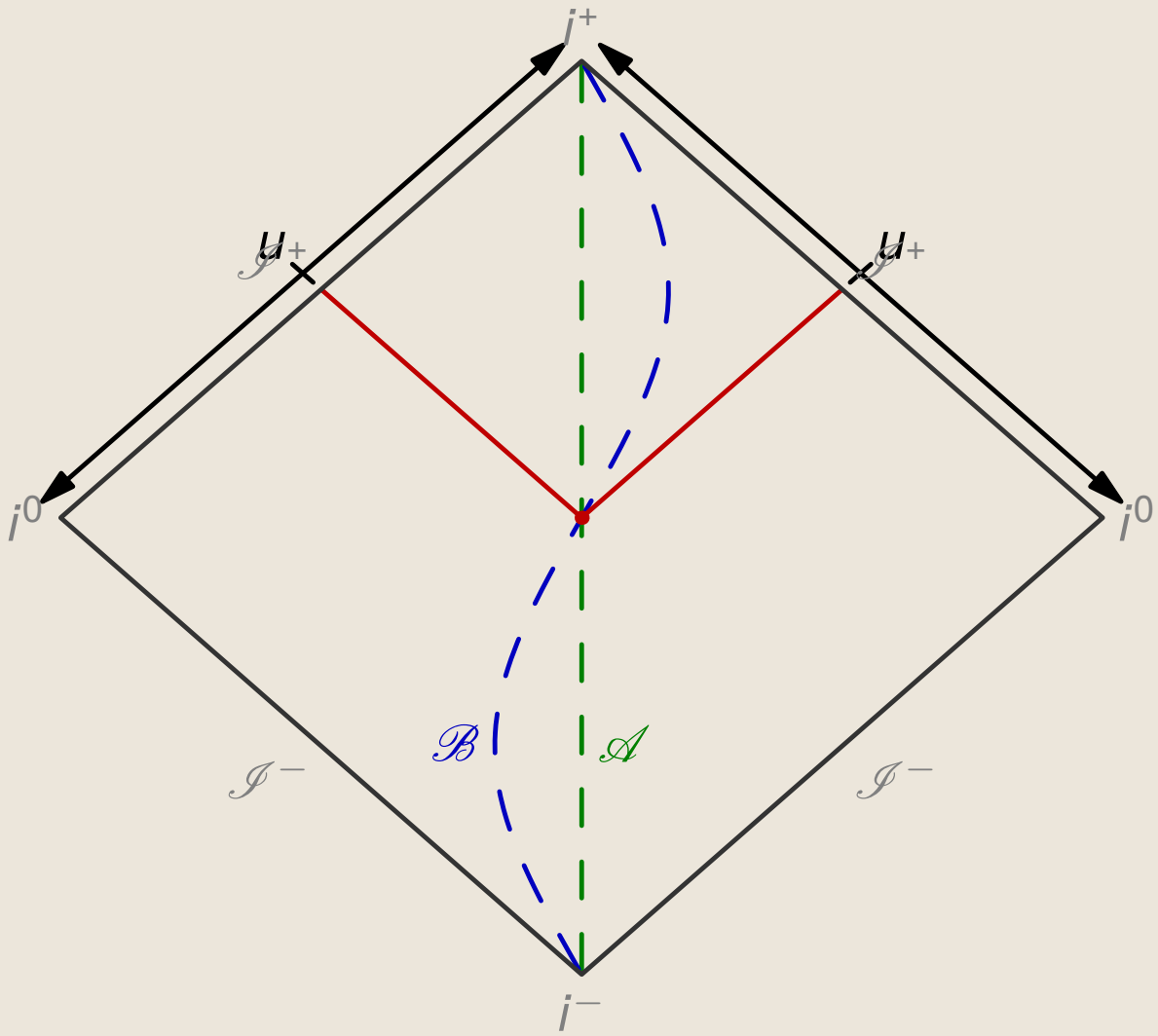
Coordinates on $\mathscr{I}^+$

Two observers



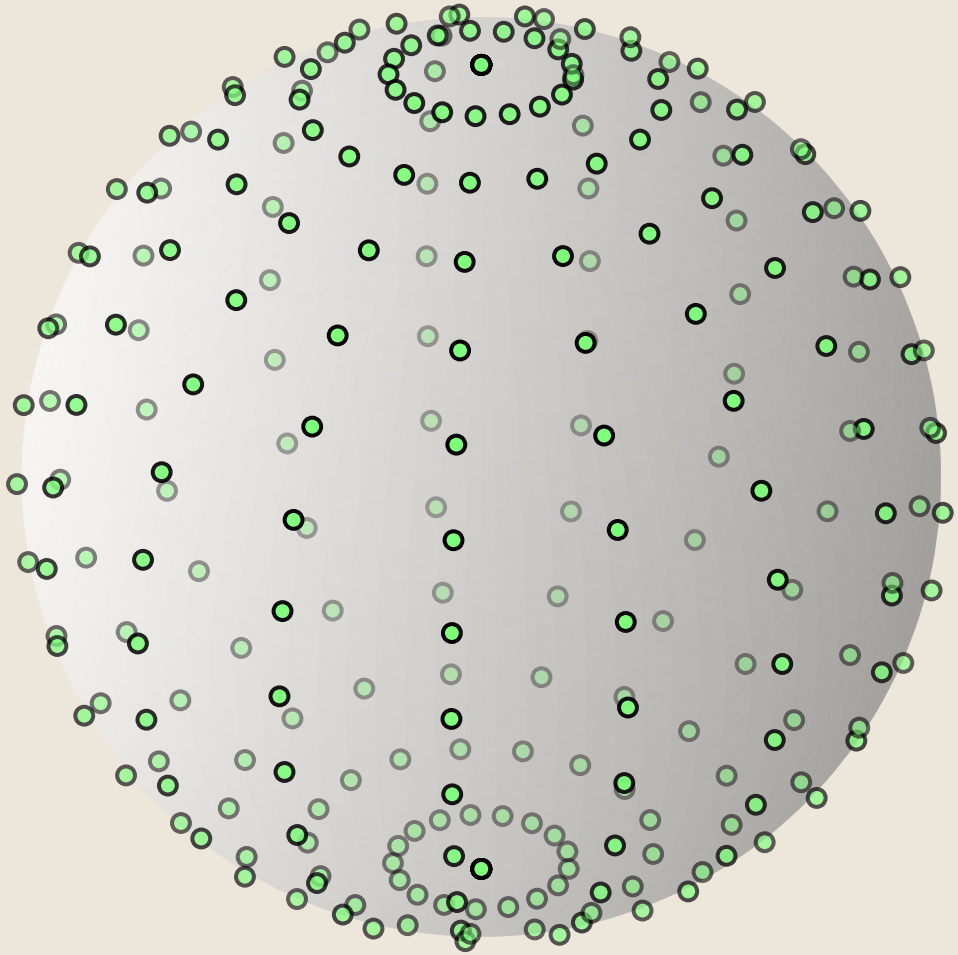
Coordinates on $\mathscr{I}^+$

Null rays



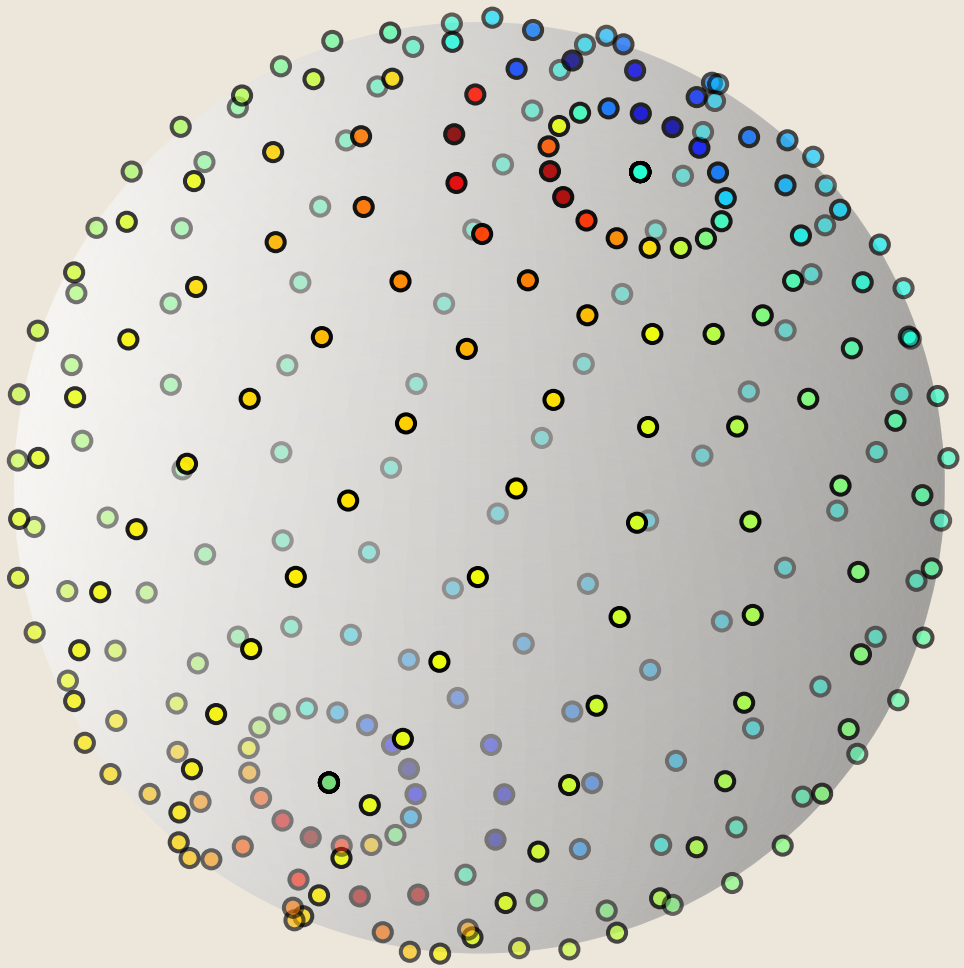
Coordinates on $\mathscr{I}^+$

Sphere



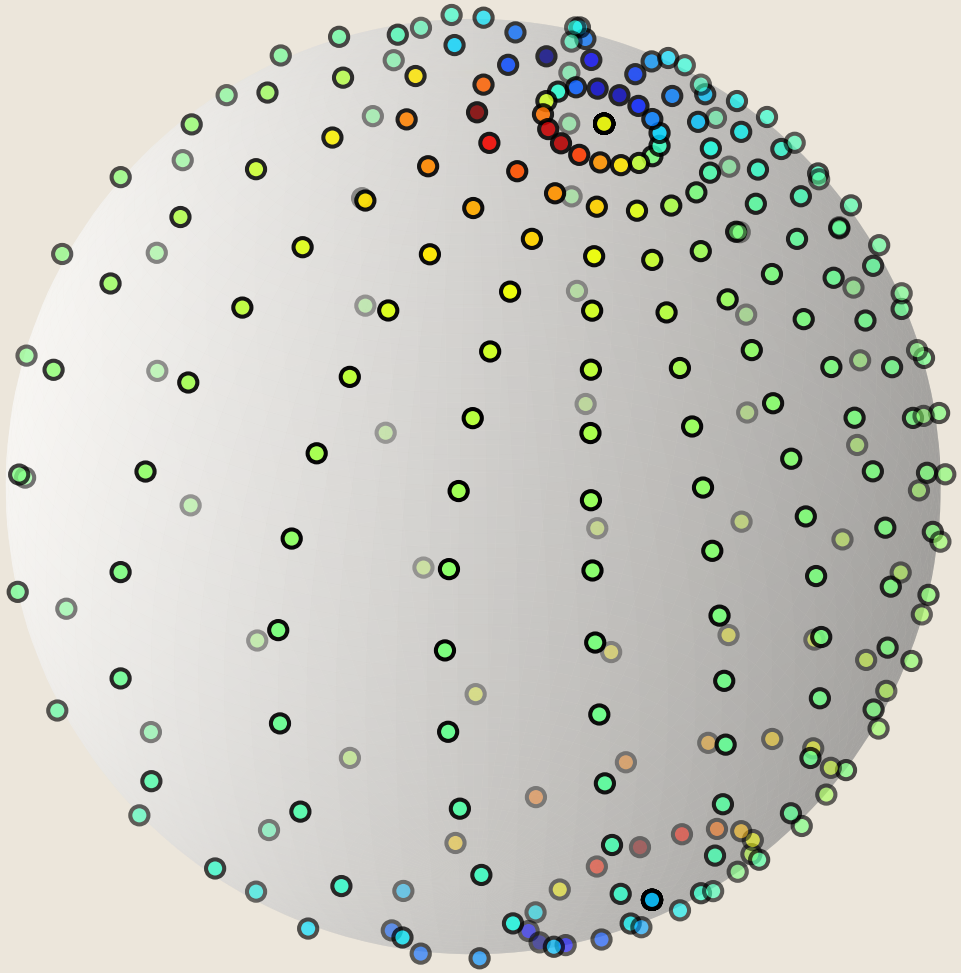
Coordinates on $\mathscr{I}^+$

Rotated sphere



Coordinates on $\mathscr{I}^+$

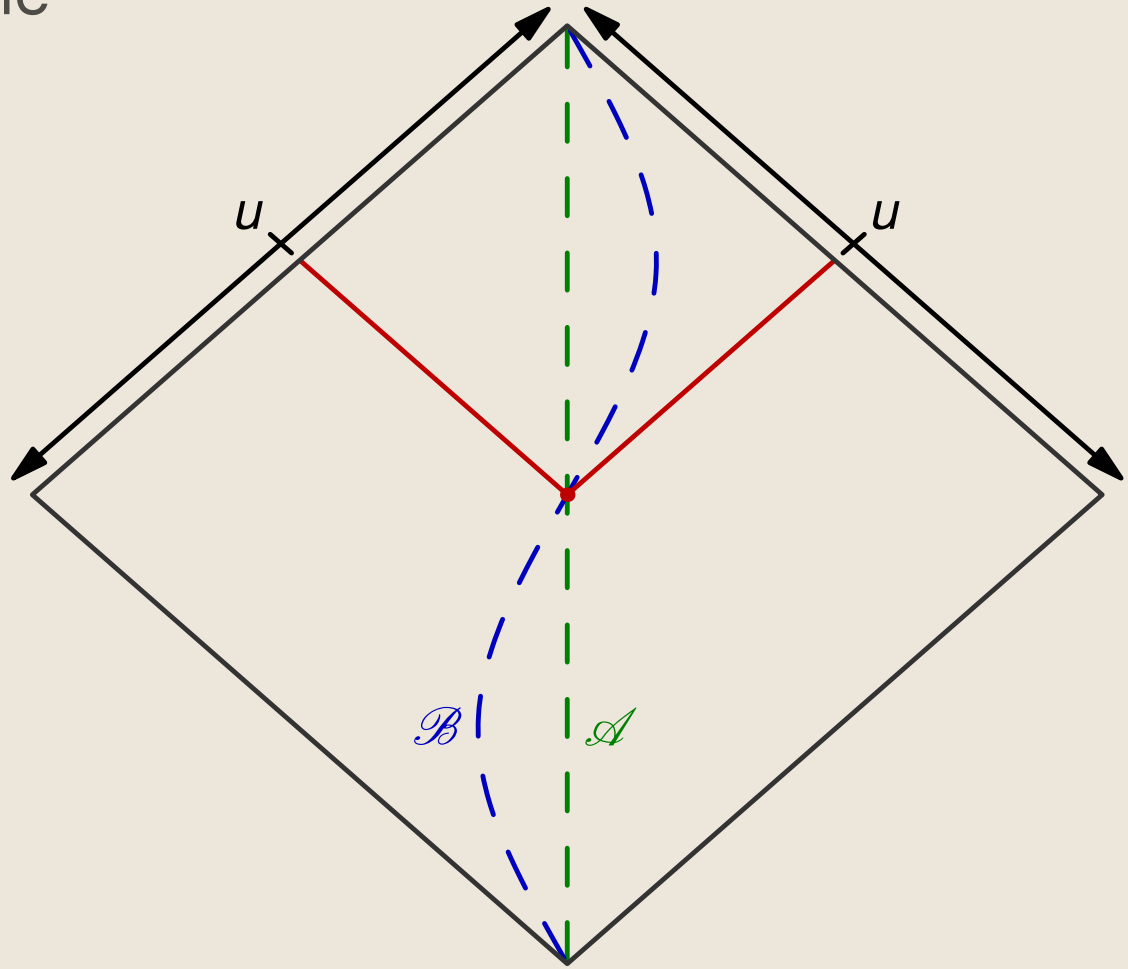
Boosted sphere



Coordinates on $\mathscr{I}^+$

Local time $\rightarrow$ retarded time

$u = t - r$

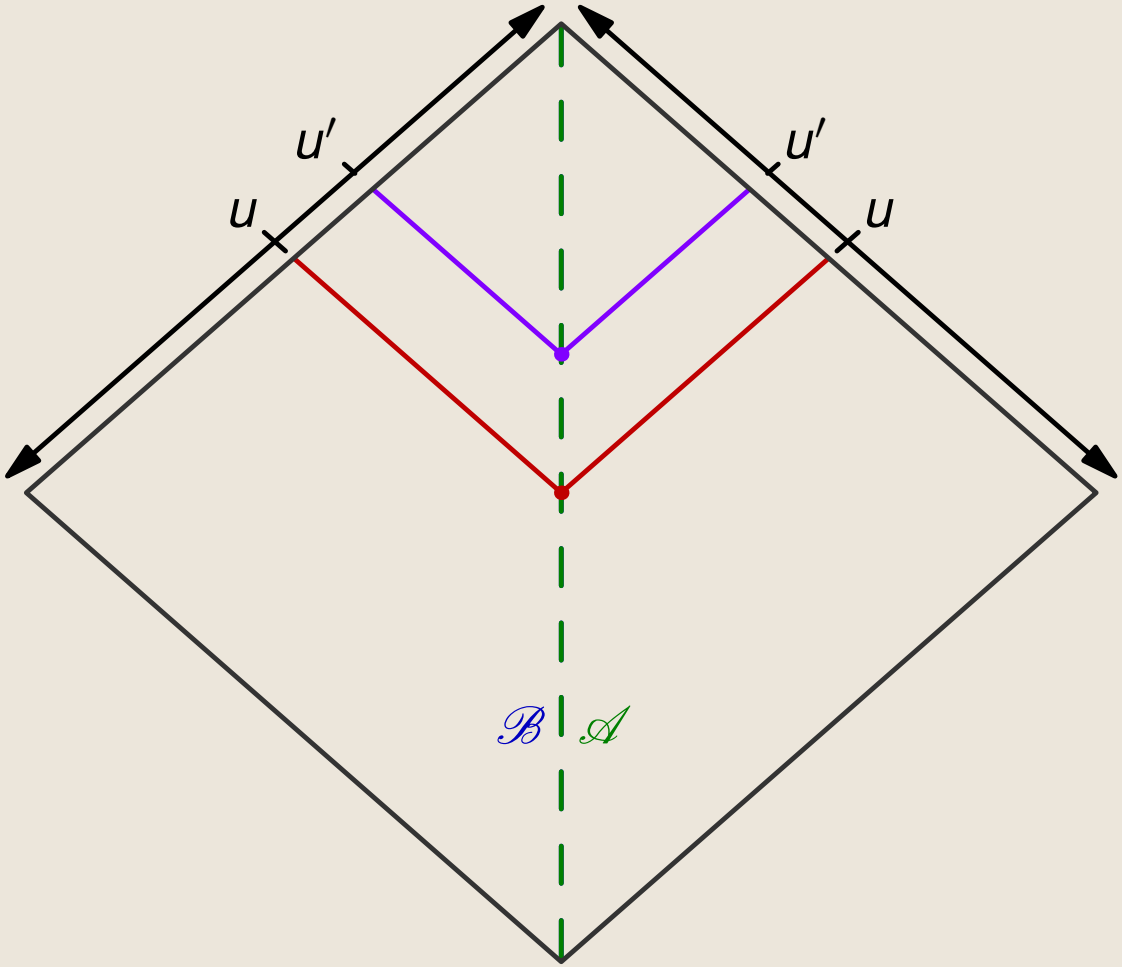


Coordinates on $\mathscr{I}^+$

Time translation

$t \mapsto t + \delta t$

$u \mapsto u + \delta t$

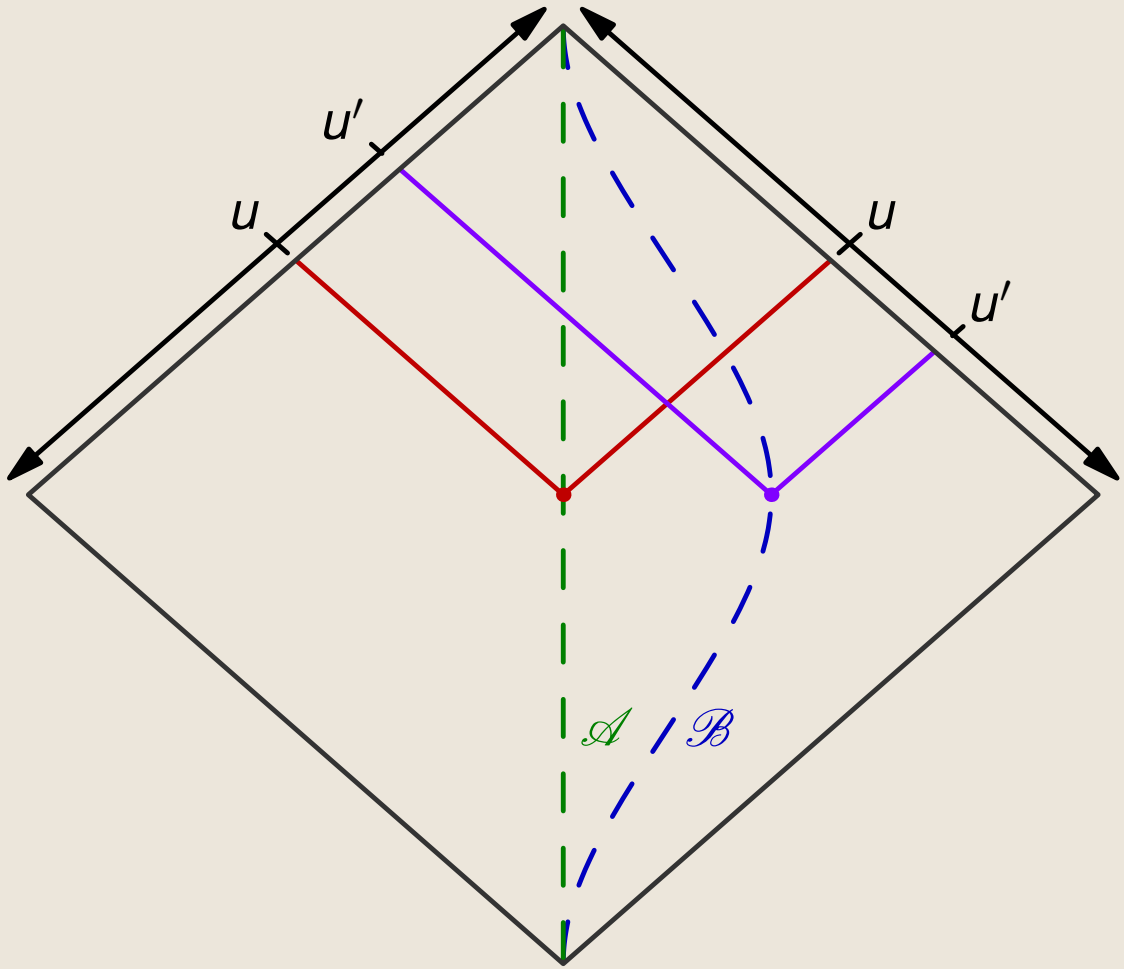


Coordinates on $\mathscr{I}^+$

Space translation

$\boldsymbol{x} \mapsto \boldsymbol{x} + \delta\boldsymbol{x}$

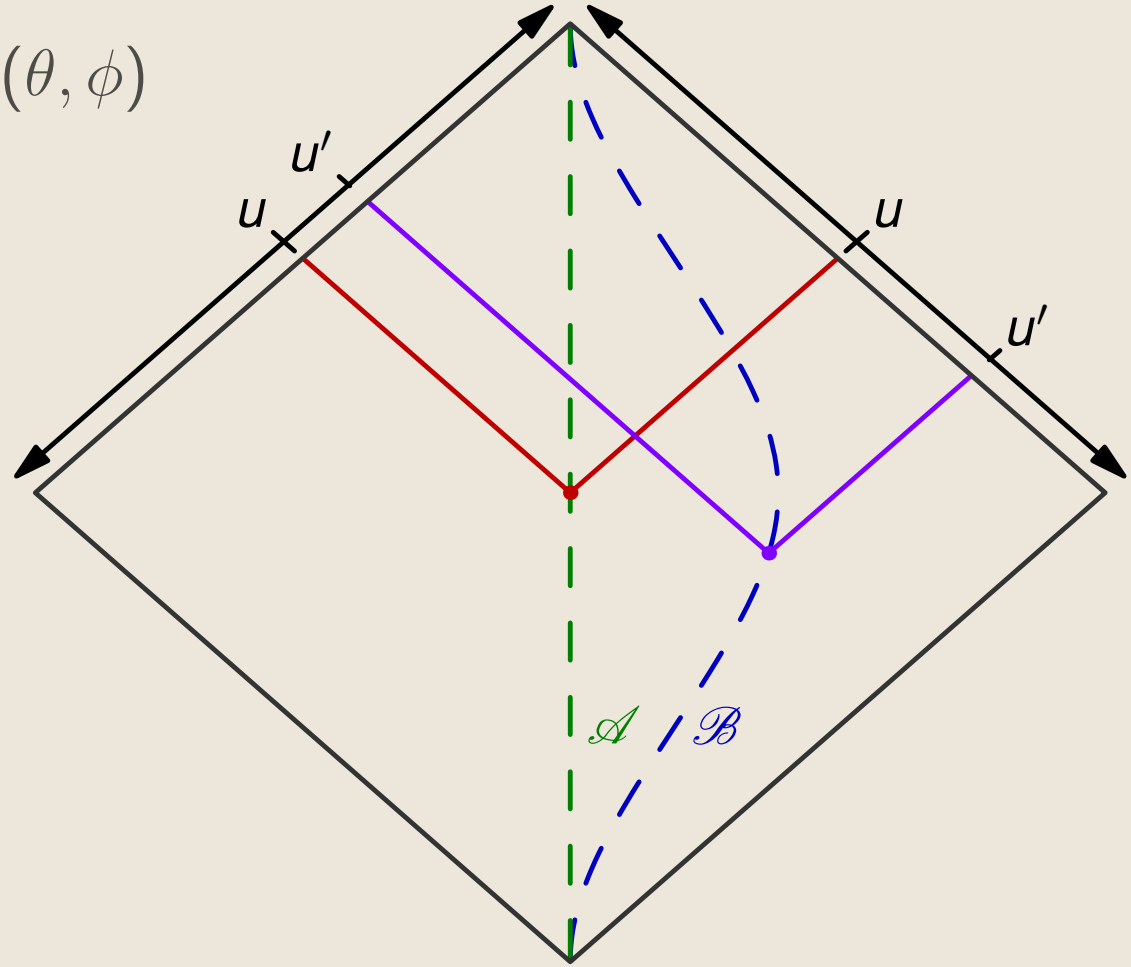
$u \mapsto u - \delta\boldsymbol{x} \cdot \hat{n}$



Coordinates on $\mathscr{I}^+$

Spacetime translation

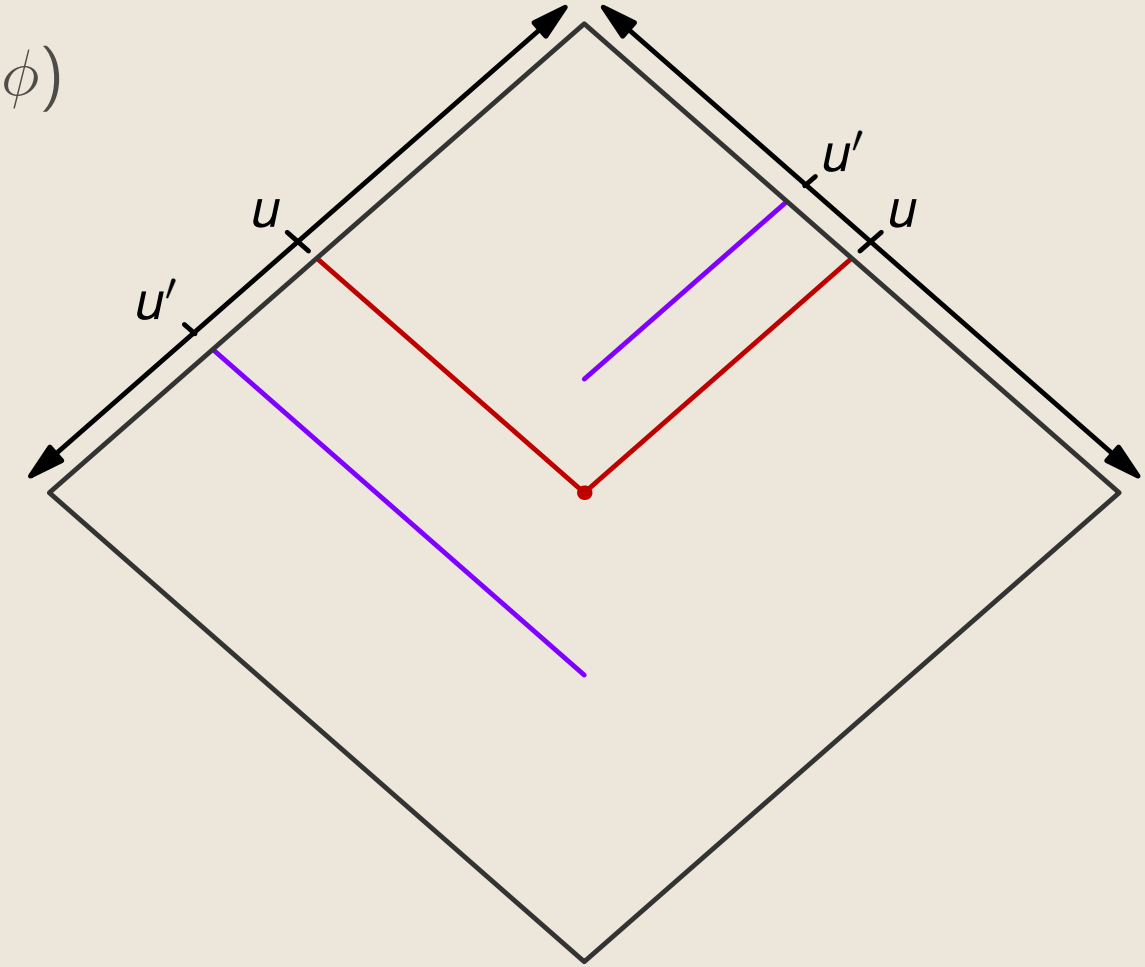
$u \mapsto u + \sum_{\ell=0,1;m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



Coordinates on $\mathscr{I}^+$

Supertranslation

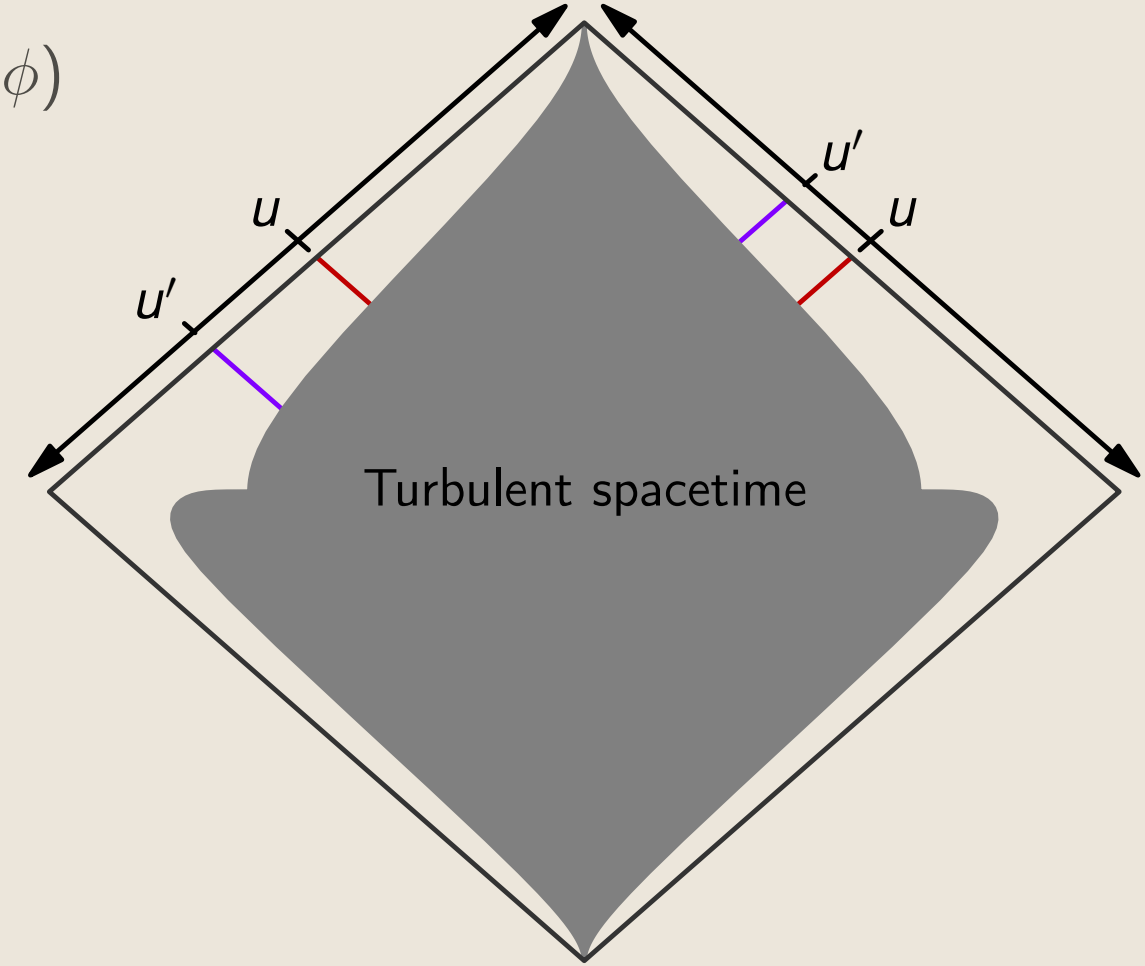
$u \mapsto u + \sum_{\ell,m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



Coordinates on $\mathscr{I}^+$

Supertranslation

$u \mapsto u + \sum_{\ell,m} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)$



BMS transformations

$$\theta \mapsto \theta'$$

$$\phi \mapsto \phi'$$

$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

BMS transformations

$$\theta \mapsto \theta'$$
$$\phi \mapsto \phi'$$
$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

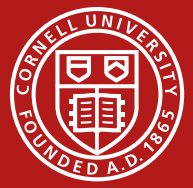
$$h \mapsto \frac{e^{-2i\lambda}}{K^b}\left[h - \bar{\eth}^2\alpha\right]$$

BMS transformations

$$\theta \mapsto \theta'$$
$$\phi \mapsto \phi'$$
$$u \mapsto K(\theta', \phi')\,[u - \alpha(\theta', \phi')]$$

$$h(u, \theta, \phi) \mapsto \frac{e^{-2i\lambda(\theta', \phi')}}{K^b(\theta', \phi')}\left[h(u', \theta', \phi') - \bar{\eth}^2\alpha(\theta', \phi')\right]$$

Summary



- Waveform comparisons
- Must understand ambiguities
- Ambiguities present in NR where we don't want them
  - We can remove them!
- Ambiguities absent in PN where we do want them
  - We can insert them!