$$\mathrm{Var}[X] = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$$

$$= \mathbb{E}\left[X^2 - 2X\mathbb{E}[X] + \mathbb{E}[X]^2\right]$$

$$= \mathbb{E}[X^2] - \mathbb{E}\left[2X\mathbb{E}[X]\right] + \mathbb{E}[X]^2$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2$$

$$= \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

2- $\underline{Var[\alpha X + \beta] = \alpha^2 Var[X]}$ prove

$$\mathbb{E}\left[(\alpha X + \beta - \mathbb{E}[\alpha X + \beta])^2\right]$$

$$= \mathbb{E}\Big[\alpha^2 X^2 + 2\alpha\beta X + \beta^2 - 2(\alpha X + \beta)(\mathbb{E}[\alpha X + \beta]) + \mathbb{E}[\alpha X + \beta]^2\Big]$$

$$= \mathbb{E}[\alpha^2 X^2] + \mathbb{E}[2\alpha\beta X] + \mathbb{E}[\beta]^2 - \mathbb{E}[2\alpha X \mathbb{E}[\alpha X + \beta]] - \mathbb{E}[2\beta \mathbb{E}(\alpha X + \beta)]$$

$\mathbb{E}[2\alpha X(\alpha \mathbb{E}[X] + \beta)]$ $\quad$ $2\alpha^2 X \mathbb{E}[X] + \beta$

$+ \mathbb{E}[\alpha X + \beta]$ $\quad$ $2\alpha^2 \mathbb{E}[X]^2 + \beta$

$$= \alpha^2 \mathbb{E}[X^2] + 2\alpha\beta \mathbb{E}[X] + \beta^2$$

$-$

$Var[\alpha X + \beta] = \alpha^2 Var[X]$ prove

$$E[((\alpha X + \beta) - E(\alpha X + \beta))^2] =$$

$$E[(\alpha^2 X^2 + 2\alpha\beta X + \beta^2) - (2(\alpha X + \beta)E(\alpha X + \beta)) + (E(\alpha X + \beta))^2]$$

$$-2(\alpha X E[\alpha X + \beta] + \beta E[\alpha X + \beta)$$

$$= \alpha^2 X^2 + 2\alpha\beta X + \beta^2 - 2\alpha X E[\alpha X + \beta] - 2\beta E[\alpha X + \beta]$$

$$+ \alpha^2 E[X]^2 + \beta^2 \quad (2\alpha^2 X E[X] - 2\alpha\beta X - 2\alpha\beta E[X] - 2\beta^2$$

$$= E[\alpha^2 X^2 + \cancel{2\alpha\beta X} + \cancel{\beta^2} - 2\alpha^2 X E[X] - \cancel{2\alpha\beta X} - \cancel{2\alpha\beta E[X]} - \cancel{2\beta^2}$$

$$+ \alpha^2 E[X]^2 + \cancel{2\alpha\beta E[X]} + \cancel{\beta^2}$$

$$E[\alpha^2 X^2 - 2\alpha^2 X E[X] + \alpha^2 E[X]^2]$$

$$= E(\alpha^2 [X^2 - 2X E[X] + E[X]^2])$$

$$= \alpha^2 E[(X - E[X])^2]$$

$$= \alpha^2 Var(X)$$

Add 3 & ← shades different
multiply by 5 ← scales
different