Cognitive Modeling: Homework Assignment 2
Problem 4

March 18, 2024

All answers and solutions to non-programming questions should be submitted to LMS as a **legible** write-up (either fully digital or a scan). All code should be committed to and merged into the `main` branch of your team's GitHub repository.

Problem 4: Prior and Posterior Variance (4 points)

Show that the following identity holds

$$\text{Var}\left[\theta\right] = \mathbb{E}\left[\text{Var}\left[\theta \mid y\right]\right] + \text{Var}\left[\mathbb{E}\left[\theta \mid y\right]\right] \tag{1}$$

Clarification of terms:

1. $\text{Var}\left[\theta\right]$ – Prior variance.
2. $\mathbb{E}\left[\text{Var}\left[\theta \mid y\right]\right]$ – Expected posterior variance.
3. $\text{Var}\left[\mathbb{E}\left[\theta \mid y\right]\right]$ – Variance of posterior mean.

$$\mathbb{E}\left[\text{Var}\left[\theta \mid y\right]\right] + \text{Var}\left[\mathbb{E}\left[\theta \mid y\right]\right] \tag{2}$$

$$= \mathbb{E}\left[\mathbb{E}\left[\theta^2 \mid y\right] - \mathbb{E}\left[\theta \mid y\right]^2\right] + \mathbb{E}\left[\left(\mathbb{E}\left[\theta \mid y\right]\right)^2\right] - \left(\mathbb{E}\left[\mathbb{E}\left[\theta \mid y\right]\right]\right)^2 \tag{3}$$

$$= \mathbb{E}[\theta^2] - \mathbb{E}[\mathbb{E}(\theta \mid y)^2] + \mathbb{E}[\mathbb{E}(\theta \mid y)^2] - \mathbb{E}[\theta]^2 \tag{4}$$

$$= \mathbb{E}[\theta^2] - \mathbb{E}[\theta]^2 \tag{5}$$

$$= \text{Var}[\theta] \tag{6}$$