# INTRO TO DATA SCIENCE CROSS VALIDATION AND NAÏVE BAYESIAN CLASSIFICATION

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#### **LAST TIME:**

I. WHAT IS MACHINE LEARNING?
II. MACHINE LEARNING PROBLEMS
III. CLASSIFICATION
IV. BUILDING EFFECTIVE CLASSIFIERS
V. K-NEAREST NEIGHBORS

**EXERCISES:** 

**VI. LAB: KNN CLASSIFICATION IN PYTHON** 

training set

model

test set

predictions

#### **QUESTIONS?**

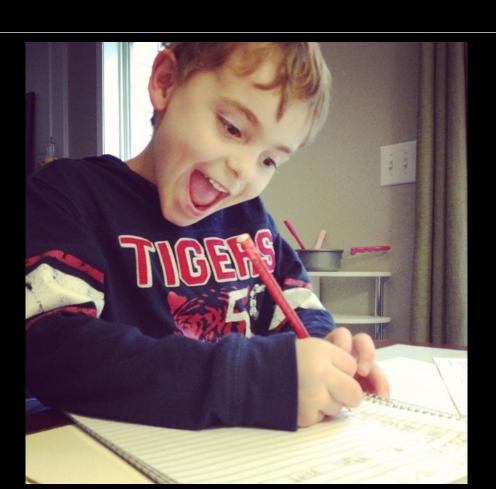
INTRO TO DATA SCIENCE

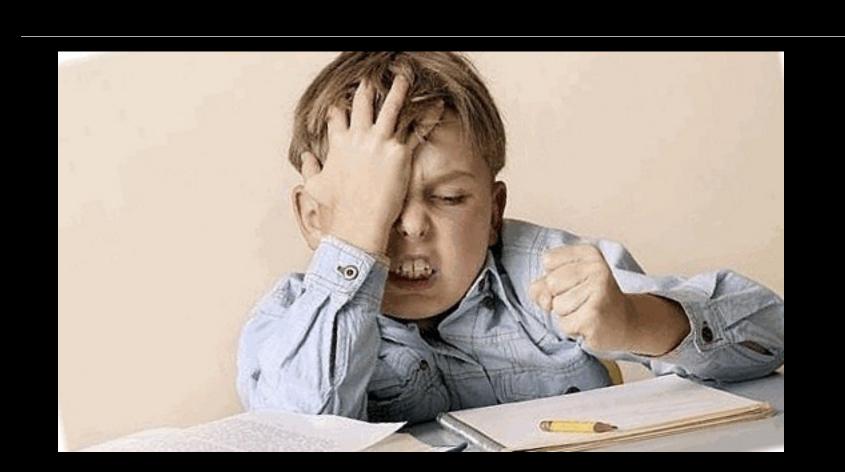
## QUESTIONS?

WHAT WAS THE MOST INTERESTING THING YOU LEARNT?

WHAT WAS THE HARDEST TO GRASP?

# HOW'S THE







I. CROSS VALIDATION
II. INTRO TO PROBABILITY
III. NAÏVE BAYESIAN CLASSIFICATION

### **EXERCISES:**

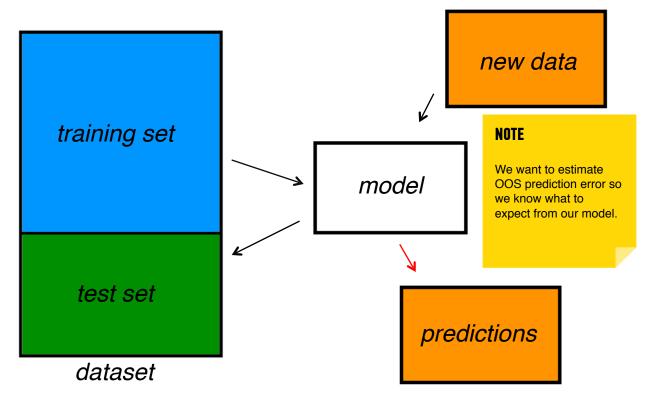
IV. NAÏVE BAYES CLASSIFICATION IN PYTHON

- UNDERSTAND HOW CROSS VALIDATION HELPS ESTIMATE THE OOS ERROR
- BE ABLE TO PERFORM CROSS VALIDATION
- UNDERSTAND PROBABILITY AND CONDITIONAL PROBABILITY
- UNDERSTAND WHY "NAÏVE" BAYES
- BE ABLE TO PERFORM CLASSIFICATION IN PYTHON

# CROSS VALIDATION

Q: What types of prediction error will we run into?

- 1) training error
- 2) generalization error
- 3) OOS error



We can do better than that.... as we will see in the next class....

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Let's recap briefly...

Suppose we do the train/test split.

Q: How well does generalization error predict OOS accuracy?

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Q: How well does generalization error predict OOS accuracy?

Thought experiment:

Suppose we had done a different train/test split.

Q: Would the generalization error remain the same?

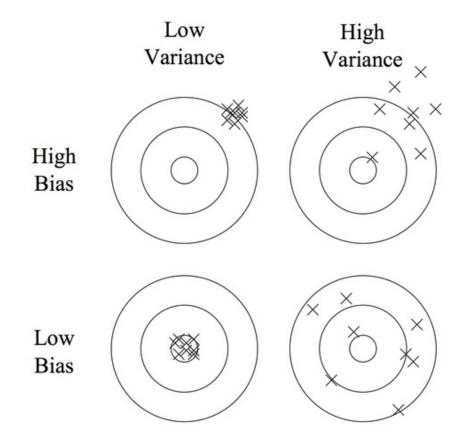
A: Of course not!

The generalization error gives a high-variance estimate of OOS accuracy.

NOTE

A: On its own, not very well.

#### **BIAS-VARIANCE**



Q: How can we do better?

Thought experiment:

Different train/test splits will give us different generalization errors.

Q: How can we do better?

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Different train/test splits will give us different generalization errors.

Q: What if we did a bunch of these and took the average?

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A: Now you're talking!

Q: How can we do better?

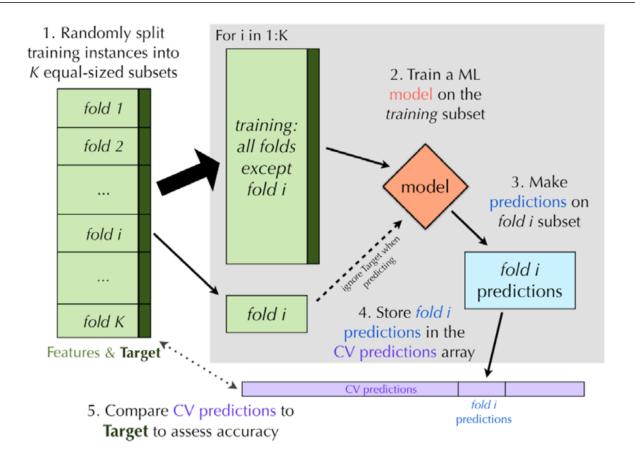
Thought experiment:

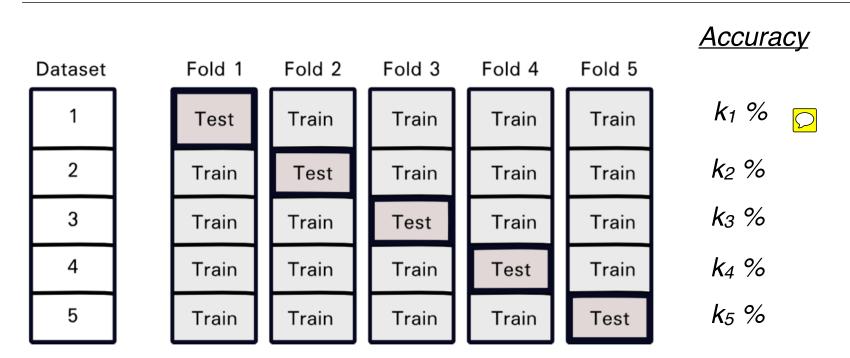
Different train/test splits will give us different generalization errors.

Q: What if we did a bunch of these and took the average?

A: Now you're talking!

#### A: Cross-validation!





5-Fold Generalization Error =  $(k_1 + k_2 + k_3 + k_4 + k_5) / 5$ 

#### MORE ACCURATE

Cross validation is a **more accurate** estimate of Out Of Sample (OOS) prediction error.

**MORE EFFICIENT** 

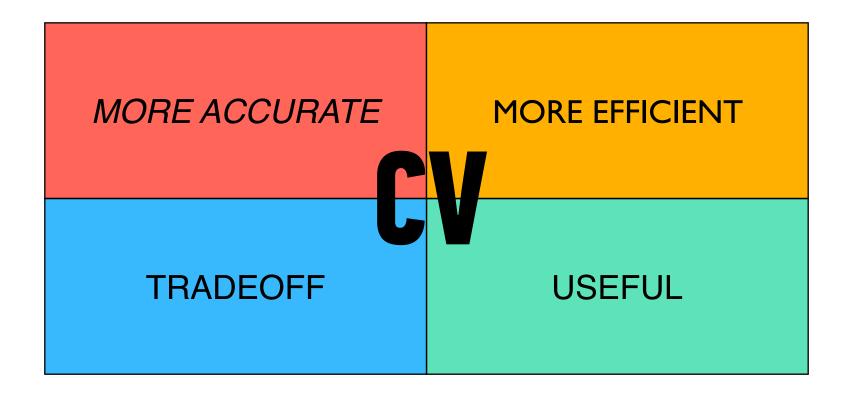
Each record is used for both training and testing

10-fold CV is 10x more computationally expensive than a single train/test split

**TRADEOFF** 

## Cross validation average score can be used for model selection

**USEFUL** 



# Can you think of some situations where running a cross validation could be problematic?

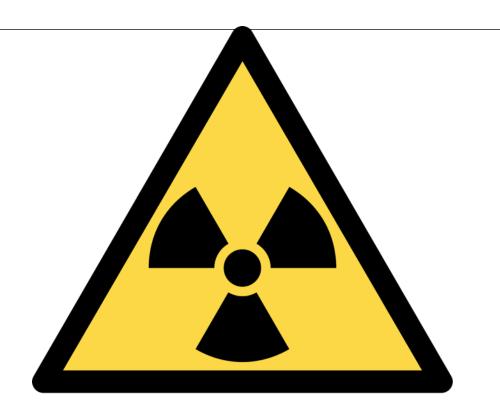
THINK - PAIR - SHARE

#### INTRO TO DATA SCIENCE

# QUESTIONS?

# LAB: CROSS VALIDATION

# INTRO TO PROBABILITY

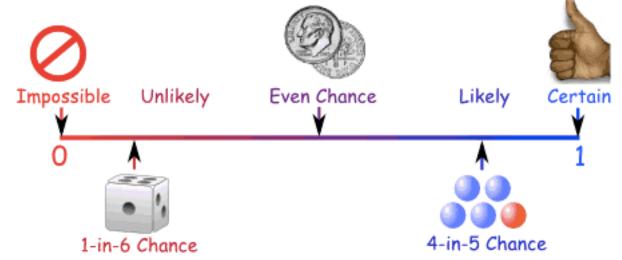


**BEWARE: EQUATIONS AHEAD!** 

## Q: What is a probability?

#### YOU TELL ME

The probability p(A) for some event A is number between 0 and 1 that characterizes the **likelihood** the event A will occur.





the **SAMPLE SPACE** is the set of all possible events



**EXAMPLES?** 



the **SAMPLE SPACE** is the set of all possible events



$$p(\Omega) = ?$$

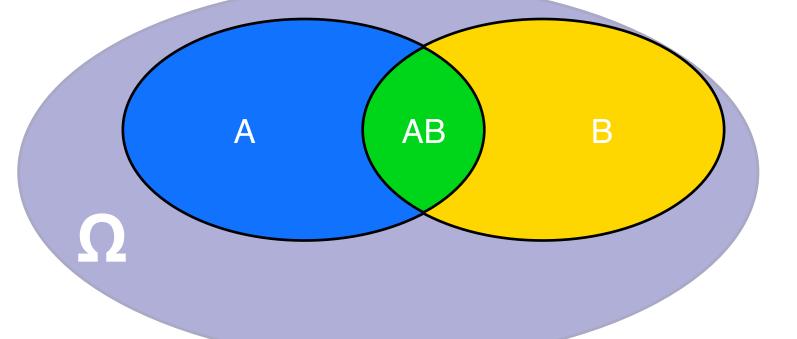


the **SAMPLE SPACE** is the set of all possible events

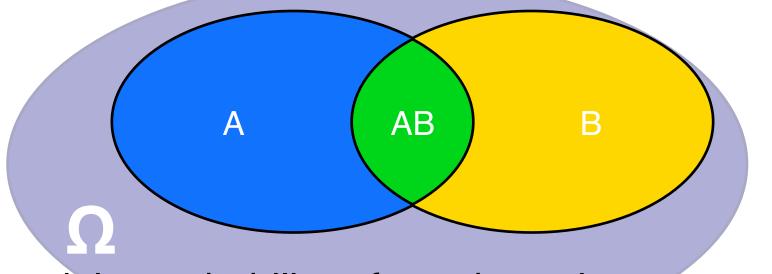


$$p(\Omega) = 1$$

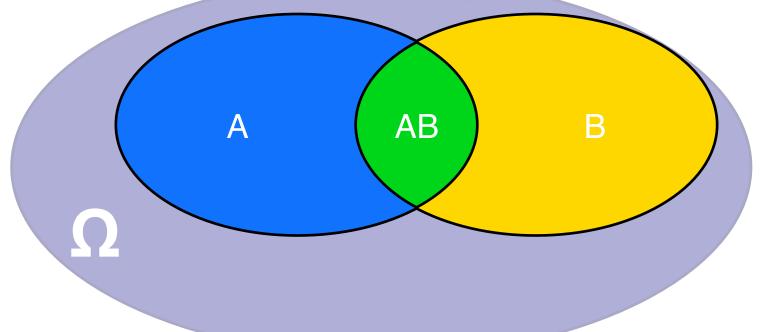
Q: Consider two events A & B. How can we characterize the intersection of these events?

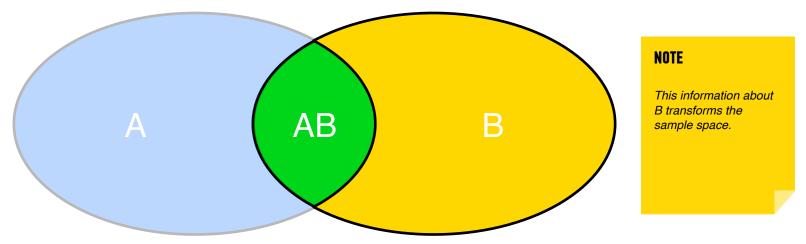


Q: Consider two events A & B. How can we characterize the intersection of these events?



joint probability of A and B, written P(AB).





The intersection of A & B divided by region B.

## This is called the conditional probability of A given B

Written P(A|B) = P(AB) / P(B).

This is called the conditional probability of A given B

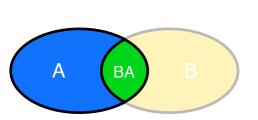
A AB B

*Written* 
$$P(A|B) = P(AB) / P(B)$$

or P(A|B) P(B) = P(AB)...

# Now let's ask the converse question: what is P(B|A)?

This is called the conditional probability of B given A



*Written* 
$$P(B|A) = P(BA) / P(A)$$

$$P(AB) = P(A|B) * P(B)$$
 conditional probability of A given B

$$P(AB) = P(A|B) * P(B)$$
 conditional probability of A given B  
 $P(BA) = P(B|A) * P(A)$  by substitution

$$P(AB) = P(A|B) * P(B)$$
  

$$P(BA) = P(B|A) * P(A)$$

But P(AB) = P(BA)

conditional probability of A given B by substitution

 $since\ event\ AB = event\ BA$ 

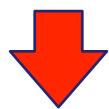
$$P(AB) = P(A|B) * P(B)$$

$$P(BA) = P(B|A) * P(A)$$

conditional probability of A given B by substitution

$$But P(AB) = P(BA)$$

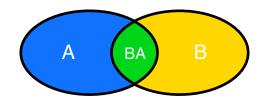
 $since\ event\ AB = event\ BA$ 



$$P(A|B) * P(B) = P(B|A) * P(A)$$

### This result is called Bayes' theorem

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$



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B = positive mammogram

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A = breast cancerB = positive mammogram

$$P(A) = 0.01$$
  
 $P(B|A) = 0.80$ 

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

B = positive mammogram

$$P(A) = 0.01$$
  
 $P(B|A) = 0.80$   
 $P(B|\sim A) = 0.096$ 

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$$P(A|B) = ?$$

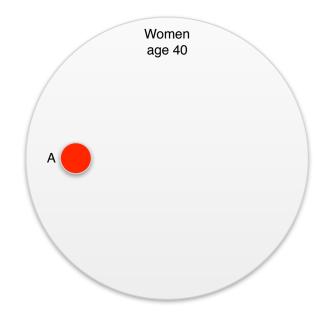
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$$P(A|B) = ?$$

1% of women at age forty who participate in routine screening have breast cancer. 80% of women with breast cancer will get positive mammograms. 9.6% of women without breast cancer will also get positive mammograms. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?



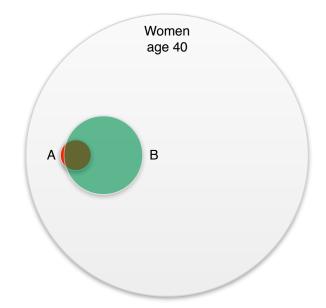
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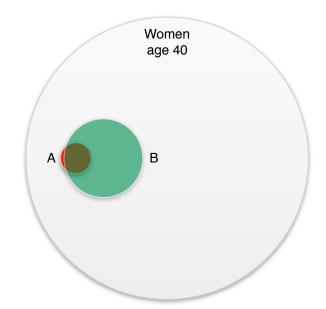
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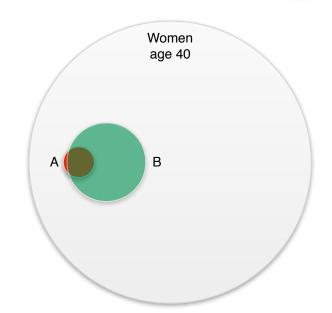
$$P(B|A) = 0.80$$

$$P(B|\sim A) = 0.096$$

$$P(B) = P(B|A)*P(A) + P(B|\sim A)*P(\sim A)$$

$$= 0.80*0.01 + 0.096*0.99 = 0.10304$$

$$P(A|B) = ?$$

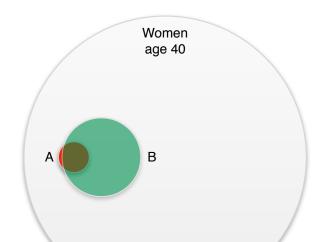


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$$\begin{split} & P(A) = 0.01 \\ & P(B \mid A) = 0.80 \\ & P(B \mid \sim A) = 0.096 \\ & P(B) = P(B \mid A) * P(A) + P(B \mid \sim A) * P(\sim A) \\ & = 0.80 * 0.01 + 0.096 * 0.99 = 0.10304 \\ & P(A \mid B) = \frac{P(B \mid A) * P(A)}{P(B)} = \frac{0.8 * 0.01}{0.10304} = 0.0776 \end{split}$$



What is the probability that she actually has breast cancer? About 7.8% chance of actually having cancer!!!!!

## INTERPRETATIONS OF PROBABILITY

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The **frequentist** interpretation regards an event's probability as its limiting frequency across a very large number of trials.

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The **frequentist** interpretation regards an event's probability as its limiting frequency across a very large number of trials.

The **Bayesian** interpretation regards an event's probability as a "degree of belief," which can apply even to events that have not yet occurred.

## INDEPENDENT EVENTS

### Q: When are 2 events independent?

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A: Information about one does not affect the probability of the other.

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$$P(A|B) = P(A)$$

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A: Information about one does not affect the probability of the other.

$$P(A|B) = P(A)$$

using the definition of conditional probability:

$$P(A|B) = P(AB) / P(B) = P(A) \rightarrow P(AB) = P(A) * P(B)$$

#### **ADDITIONAL RESOURCES**

http://www.yudkowsky.net/rational/bayes

https://en.wikipedia.org/wiki/Bayes%27\_theorem

http://betterexplained.com/articles/an-intuitive-and-short-explanation-of-bayes-theorem/

http://alexanderetz.com/2015/08/09/understanding-bayes-visualization-of-bf/

http://jakevdp.github.io/blog/2015/08/07/frequentism-and-bayesianism-5-model-selection/

#### **EXPLAIN IN YOUR OWN WORDS**

What's the difference between frequentist and Bayesian interpretations of probability?

What does Bayes Theorem allow us to do?

# NAÏVE BAYESIAN CLASSIFICATION

Suppose we have a dataset with features  $x_1, ..., x_n$  and a class label C. What can we say about classification using Bayes' theorem?

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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe. Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the likelihood function. It represents the joint probability of observing features  $\{x_i\}$  given that that record belongs to class C.

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$$P(\operatorname{class} C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \operatorname{class} C) \cdot P(\operatorname{class} C)}{P(\{x_i\})}$$

We can observe the value of the likelihood function from the training data. This term is the prior probability of C. It represents the probability of a record belonging to class C before the data is taken into account.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The value of the prior is also observed from the data.

This term is the normalization constant. It doesn't depend on C, and is generally ignored until the end of the computation.

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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The normalization constant doesn't tell us much.

This term is the posterior probability of C. It represents the probability of a record belonging to class C after the data is taken into account.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

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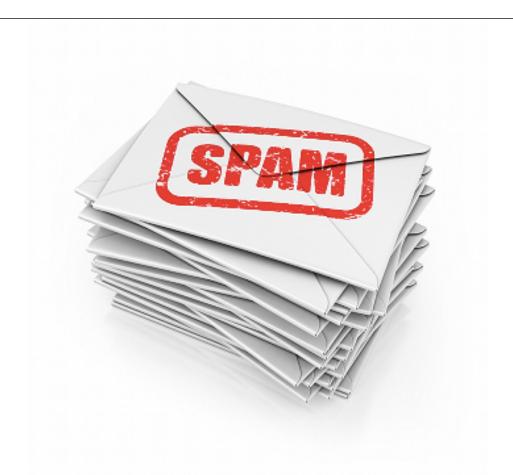
$$P(\operatorname{class} C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \operatorname{class} C) \cdot P(\operatorname{class} C)}{P(\{x_i\})}$$

The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to update our beliefs about the distribution of C using the data ("evidence") at our disposal.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.



C: IS SPAM ? {1,0}

xi: how many times is word i present in email {0, Inf}

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

 $P(\{xi\}|C)$  = count of emails with words frequencies  $\{xi\}$  in the SPAM subset P(C) = ratio of SPAM emails  $P(\{xi\})$  = normalization constant



Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

#### Remember the likelihood function?

$$P({x_i} | C) = P({x_1, x_2, ..., x_n}) | C)$$

Remember the likelihood function?

$$P({x_i} | C) = P({x_1, x_2, ..., x_n}) | C)$$

Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

A: Estimating the full likelihood function.

A: Make a simplifying assumption. In particular, we assume that the features  $x_i$  are conditionally independent from each other:

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$$P(\{x_i\} | C) = P(x_1, x_2, ..., x_n | C) \approx P(x_1 | C) * P(x_2 | C) * ... * P(x_n | C)$$

A: Make a simplifying assumption. In particular, we assume that the features  $x_i$  are conditionally independent from each other:

$$P(\{x_i\} | C) = P(x_1, x_2, ..., x_n) | C) \approx P(x_1 | C) * P(x_2 | C) * ... * P(x_n | C)$$

This "naïve" assumption simplifies the likelihood function to make it tractable.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

the training phase of the model involves computing the <u>likelihood</u> <u>function</u>, which is the conditional probability of each feature given each class.

the prediction phase of the model involves computing the posterior probability of each class given the observed features, and choosing the class with the highest probability.

## Advantages:

- Fast to train (single scan). Fast to classify
- Not sensitive to irrelevant features
- Handles real and discrete data
- Handles streaming data well

## Disadvantages:

- Assumes independence of features

## LAB IV. NAIVE BAYESIAN CLASSIFICATION