MATH F112
SOFTWARE BASED ASSIGNMENT

CR equations

Submitted to Dr. Priti Bajpai
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Acknowledgement

I would like to thank Dr. Priti Bajpai, professor, Dean Student Welfare Division, for this wonderful opportunity to learn the mathematical built-in functions in the software Python. This assignment has also provided knowledge on the steps taken to solve Cauchy Riemann equations.

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Aim-

1. To show that f'(z) does not exist at any point of f(z)=2x+i3y.
2. To show that f'(z) does not exist at any point of f(z)=$z - \overline{z}$.
3. To show that f'(z) does not exist at any point of f(z)=$e^{x}e^{-iy}$.

Theory of Cauchy-Riemann equations (CR equations)-

If a complex function f(z)=u(x, y)+ iv(x, y) is a complex differentiable, then,

$$\mathrm{f}(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

or

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

These 2 equations are known as Cauchy Riemann equations. They can also be written as,

$$u_x = v_y \text{ and } u_y = -v_x$$

Application of CR equations-

- It allows one to check whether a function is analytic or not.
- It helps in computing the complex derivative of a function which has a partial derivative.

Software used-

To solve the given problems, the programming language Python has been used. In python, the library Sympy, which contains functions applicable for symbolic mathematics, and a module cmath, which is used for solving complex numbers, has been called.

The purpose of the functions applied in the code are as follows-

- cr()- A user-defined function which will verify the given complex equation by applying Cauchy-Riemann equations.
- symbols()- It is used to define symbols. In this case, the function has been assigned to define variables.

- diff()- It has been used to find the partial derivatives for u_x, u_y, v_x and v_y.
- I- Itis the imaginary unit where I=$\sqrt{-1}$.
- im()- Returns imaginary part of expression.
- re()- Returns real part of expression.
- exp()- Calculates exponential value.
- conjugate()- Gives the conjugate of the complex number.

The selected problems aim to show whether the complex function exists in the complex plane or not via the conditions stated by CR equations.

The given 3 problems' complex functions has been assigned to the function cr() as a parameter. cr() then identifies the real and imaginary part of the complex function and calculates the first order partial derivatives. Using if-else statement cr() then compares the partial derivatives to see if they meet the conditions required in CR-equations. Finally, cr() prints a statement stating whether f'(z) exists in the complex plane or not.

For problem number (2), the conjugate of z has been calculated and substituted in the function before running the function to obtain the respective results.

Programming code-

```
# import everything from sympy module
from sympy import symbols, diff,I,im,re,exp,conjugate #a library for symbolic mathematics
import cmath #a module used to solve complex mathematics


x, y, z = symbols('x y z', real=True)#to define the variables x, y and z

def cr(f):
    u=re(f)#assigns the real part of the complex to u
    v=im(f)#assigns the imaginary part of the complex to u
    print('u=',u,'    ','v=',v)
    ux=diff(u, x)#diff calculates the 1st order partial derivatives
    uy=diff(u, y)
    vx=diff(v, x)
    vy=diff(v, y)
    print('Partial derivatives of u and v w.r.t x and y are \n','ux=',ux,'\tuy=',uy,'\tvx=',vx,'\tvy=',vy)
    if ux==vy and uy==(-vx):
        print('CR equations, ux=vy and uy=-vx, are satisfied and exist in the complex plane.')
    else:
        print("CR equations, ux=vy and uy=-vx, are not satisfied at any point on the plane,\nf('z) does not exist at any point in the complex plane.")

#-----------------------------------------------------------------------------------------------------------------------
z=(x+I*y)
f=2*x+3*y*I

f1=z-conjugate(z)
#conjugate() is a function used to find the conjugate of a complex in f(z)=z-z(bar)

f2=exp(x)*exp(-I*y)# exp gives the exponential format for the function f(z)=e^x+e^(-iy)
#print('\n\n',cr(f1))

print('To show that f’(z) does not exist at any point of f(z)=2x+i3y')
cr(f) #function cr() calculates whether the function exists at any point on the complex plane and gives the points where it exists if it does
print('\n\n\n')

print("To show that f’(z) does not exist at any point of f(z)=z-¯z")
cr(f1)
print('\n\n\n')

print('To show that f’(z) does not exist at any point of f(z)=e^x e^(-iy)')
cr(f2)
```

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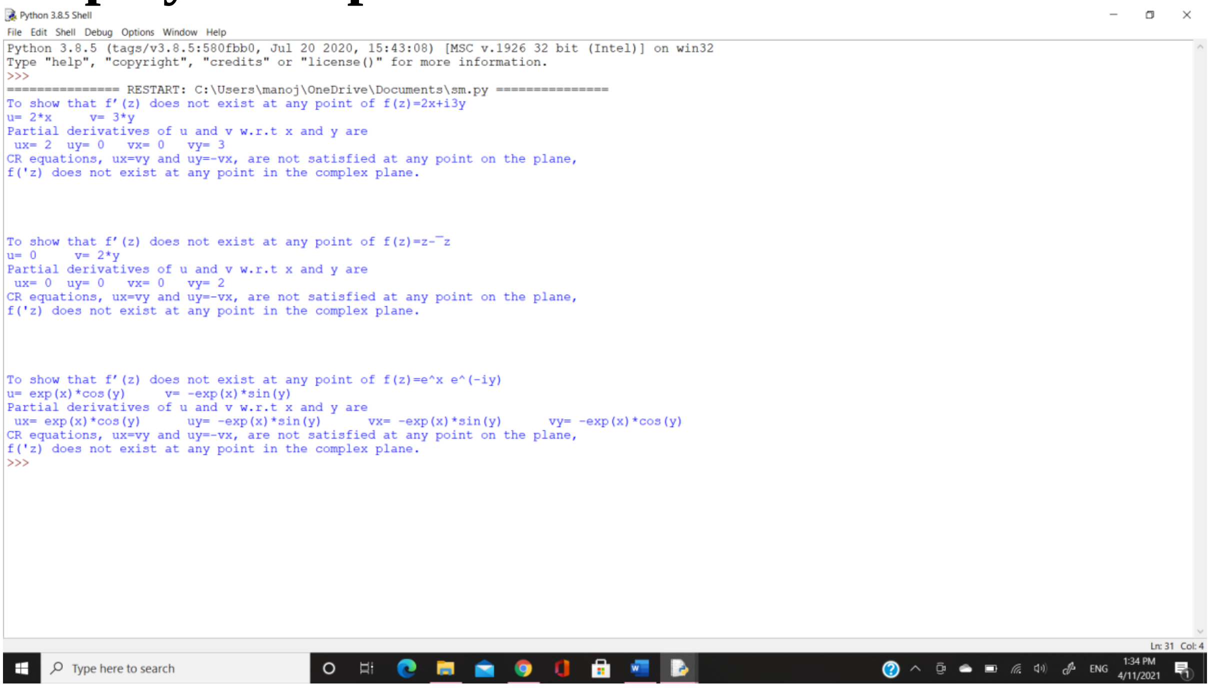
Displayed output-



Bibliography –

- https://www.sympy.org/en/index.html
- https://www.w3schools.com/python/module_cmath.asp
- https://en.wikipedia.org/wiki/Cauchy%E2%80%93Riemann_equations