Boyer Moore Algorithm

EXAMPLE BASED SUPPLEMENTARY MATERIAL

ANUJA DHARMARATNE

MONASH UNIVERSITY MALAYSIA

Pattern matching- naïve approach

- Assume you are searching for the pattern “bzaa” in the string “aabxaayaab”.

String= aabxaayaabb, Pattern= aabzaa

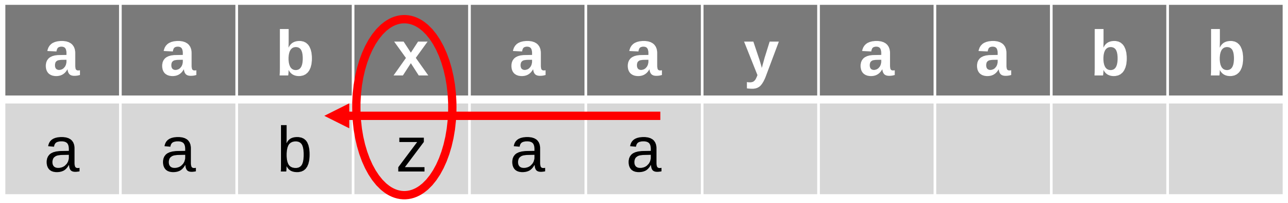
a	a	b	x	a	a	y	a	a	b	b
a	a	b	z	a	a					
	~~a~~	~~a~~	~~b~~	~~z~~	~~a~~	~~a~~				
		~~a~~	~~a~~	~~b~~	~~z~~	~~a~~	~~a~~			
			~~a~~	~~a~~	~~b~~	~~z~~	~~a~~	~~a~~		
				a	a	b	z	a	a	

SKIP

Since x doesn’t occur in the pattern, we can skip the next 3 comparisons.

Boyer Moore Algorithm

- Skips unnecessary comparisons
- Alignment from left to right
- But characters compared from right to left



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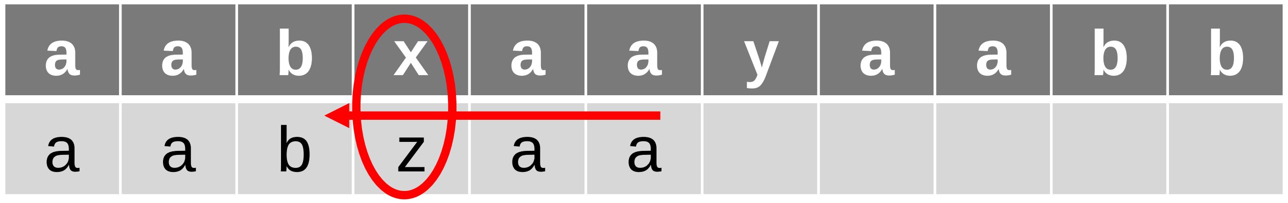

a	a	b	x	a	a	y	a	a	b	b
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- 2 main rules of Boyer Moore Algorithm:
  - Bad character shift rule
  - Good suffix shift rule

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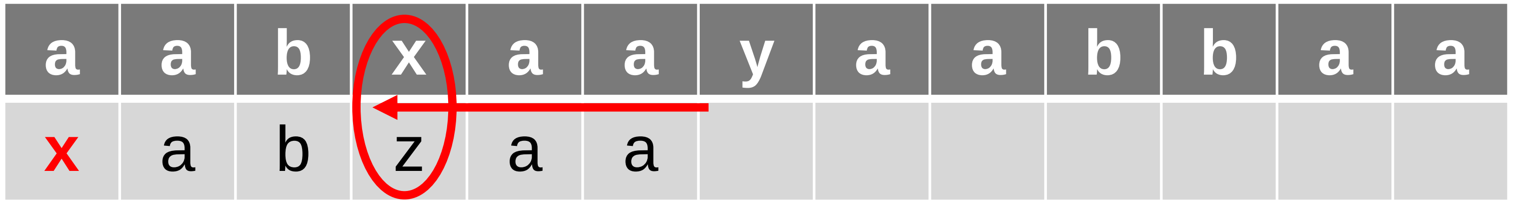
3. Alignments are done in one direction (i.e. left to right) and characters are compared in the opposite direction (i.e. right to left) **for longer skips**

1. Bad Character rule

When a mismatch happens, skip until:

a). the mismatch becomes a match, **OR**

b). the pattern moves past the mismatched character



An x is found in the pattern. So shift the pattern until they match (case a)).

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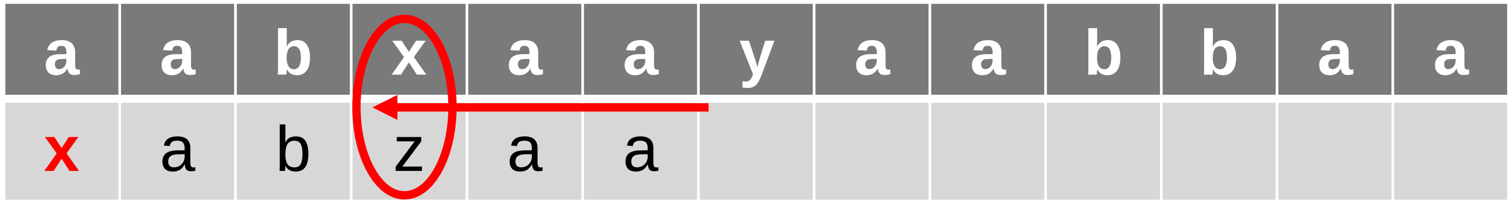

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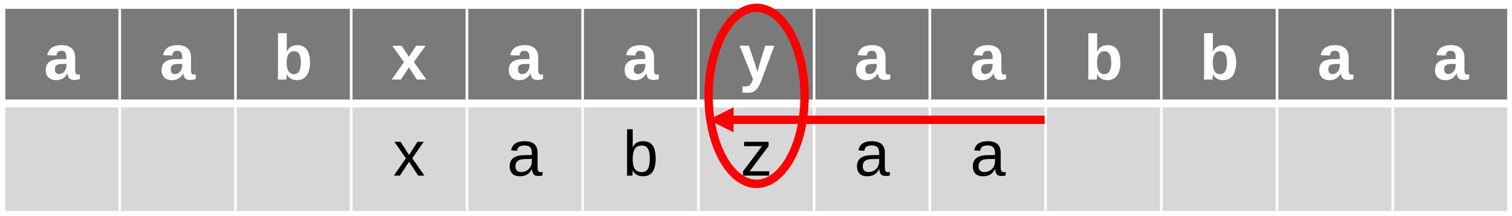

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Search for a letter "y" in the pattern. Is any there? NO. Thus shift until the mismatched character is past (case b)).

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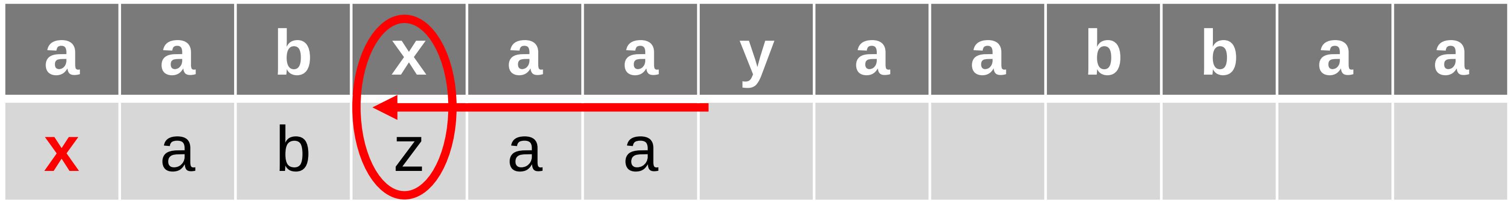

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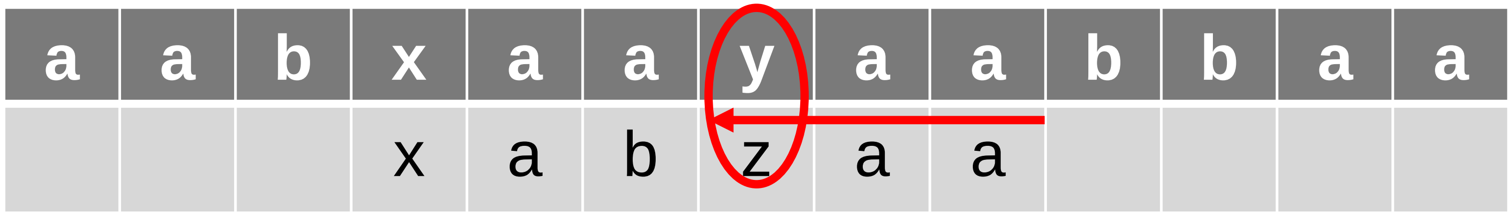

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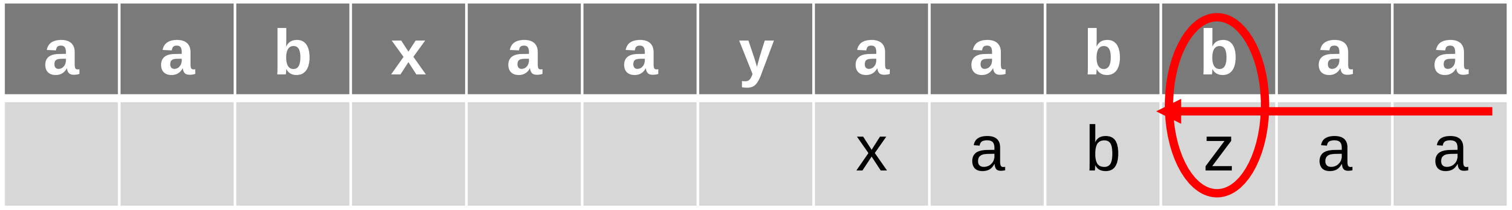

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- <u>**Example**</u>: P= “potato”

```
                  1 2 3 4 5 6
```

  R(‘a’)= 4, R(‘o’)= 6, R(‘p’)= 1, R(‘t’)= 5.
- Let’s see how we can use these R values calculated in the preprocessing step in our algorithm.

Preprocessing contd

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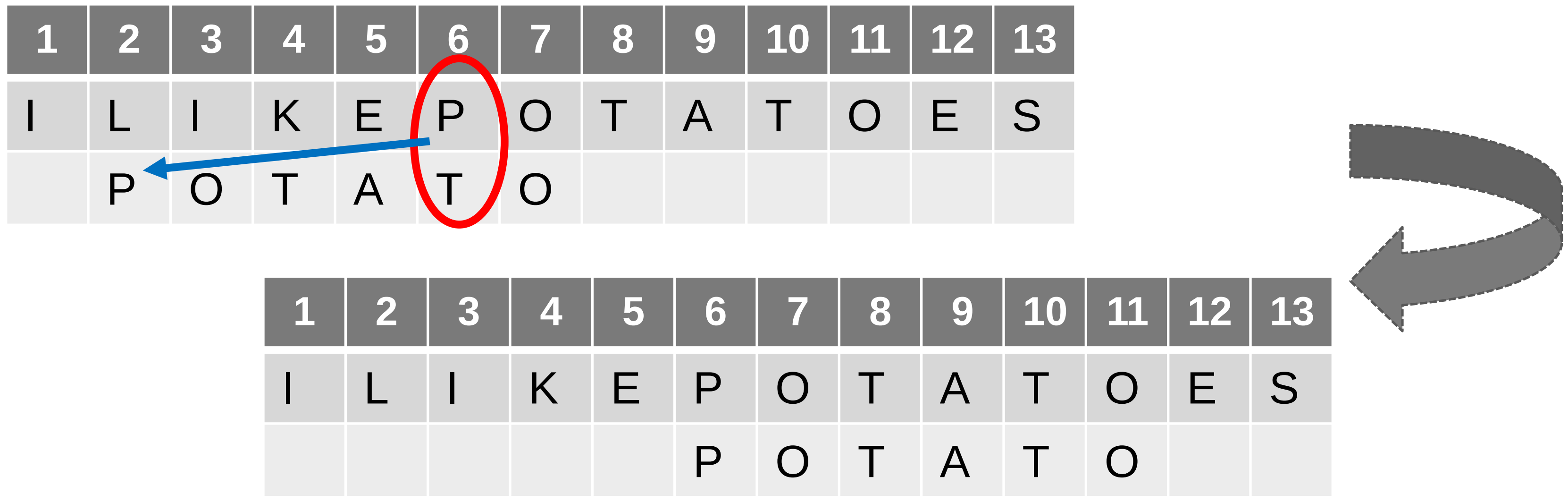
1	2	3	4	5	6	7	8	9	10	11	12	13
I	L	I	K	E	P	O	T	A	T	O	E	S
	P	O	T	A	T	O						

  - In this example, mismatching (bad) character is P.
  - R(P)= 2, k= 6, thus max(1, k-R(P))= 4 ➔ shift by 4 characters

1	2	3	4	5	6	7	8	9	10	11	12	13
I	L	I	K	E	P	O	T	A	T	O	E	S
					P	O	T	A	T	O		

Extended Bad character rule



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I	L	I	K	E	P	O	T	A	T	O	E	S
	P	O	T	A	T	O						

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- What does this mean? Look at the above example.
  - When such a mismatch occurs, then shift the pattern to the right so that the closest P in the pattern that is to the left of position k (in this case k=6) is now below the (previously mismatched) P in the string.

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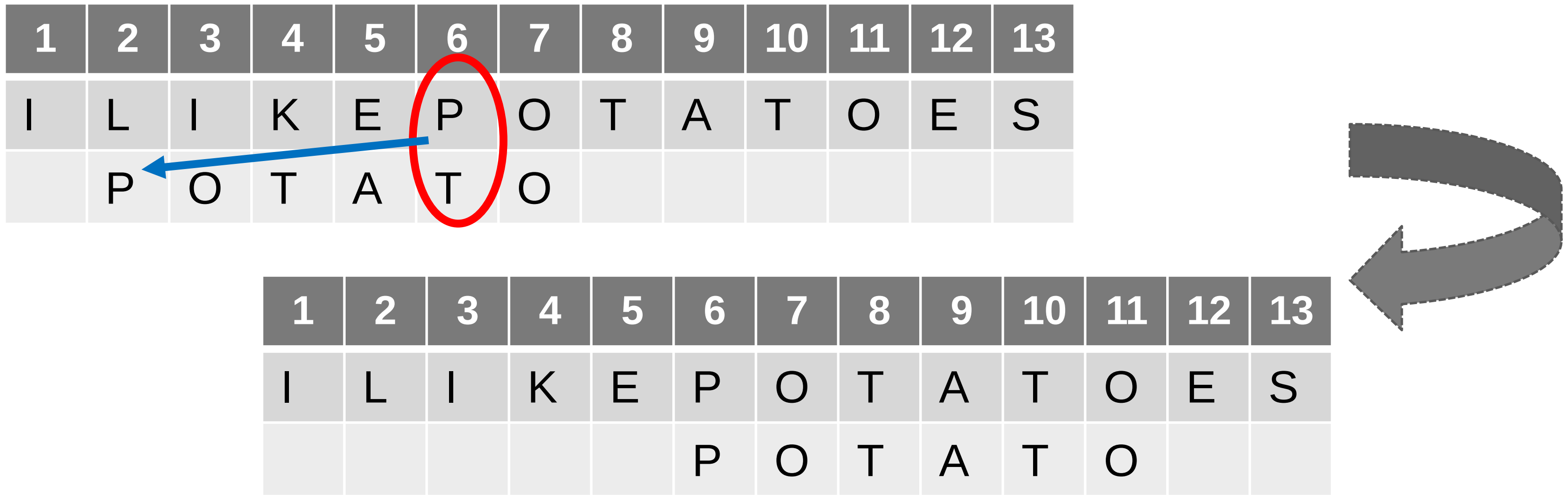

1	2	3	4	5	6	7	8	9	10	11	12	13
I	L	I	K	E	P	O	T	A	T	O	E	S
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  - When such a mismatch occurs, then shift the pattern to the right so that the closest P in the pattern that is to the left of position k (in this case k=6) is now below the (previously mismatched) P in the string.
- How can you achieve this?
  - Preprocess the pattern so that for each position k in the pattern and for each character x, the position of the closest occurrence of x to the left of each position k can be efficiently looked up. **(THINK!)**

Quiz

What if the bad character does not occur in the pattern? i.e. the mismatching character (say, x) is not present in the pattern, which means R(x)=0.

Quiz- Solution

What if the bad character does not occur in the pattern? i.e. the mismatching character (say, x) is not present in the pattern, which means R(x)=0.

The entire pattern can be shifted one position past the point of mismatch in the string.

In this example, we don't have letter M in the pattern.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
I	L	O	V	E	T	O	M	A	T	O	E	S	A	L	O	T
					P	O	T	A	T	O						

Therefore, we shift the whole pattern until it past the M.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
I	L	O	V	E	T	O	M	A	T	O	E	S	A	L	O	T
								P	O	T	A	T	O			

2. Good Suffix rule

If a matching substring ***t*** between the string and the pattern exists, skip when a mismatch happens, until:

a). there are no mismatches between pattern & ***t***, **OR**

b). a prefix of the pattern matches a suffix of ***t***, **OR**

c). the pattern moves past ***t*, whichever happens first.**

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a	**a**	**a**	**x**	**a**	**a**	**y**	**a**	**a**	**b**	**b**	**a**	**a**
x	a	a	z	a	a							

Now, we are going to check whether there are any matches between the pattern and substring ***t*****. Can you find any? YES**

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a	**a**	**a**	**x**	**a**	**a**	**y**	**a**	**a**	**b**	**b**	**a**	**a**
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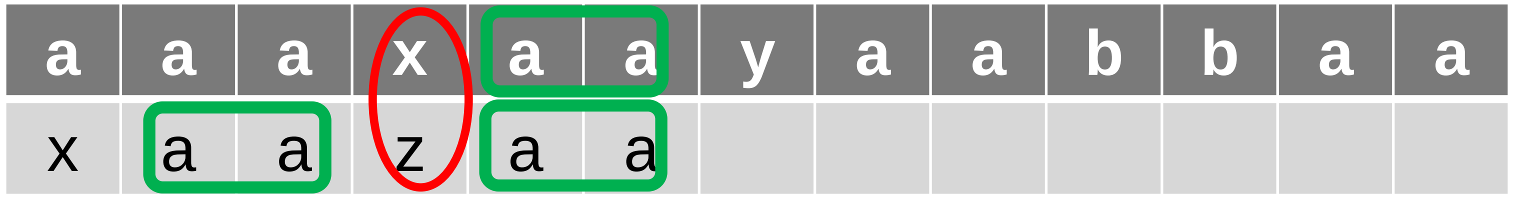
Now, we can skip until the first ‘aa’ overlaps the second.

2. Good Suffix rule- contd..



a	a	a	x	a	a	y	a	a	b	b	a	a
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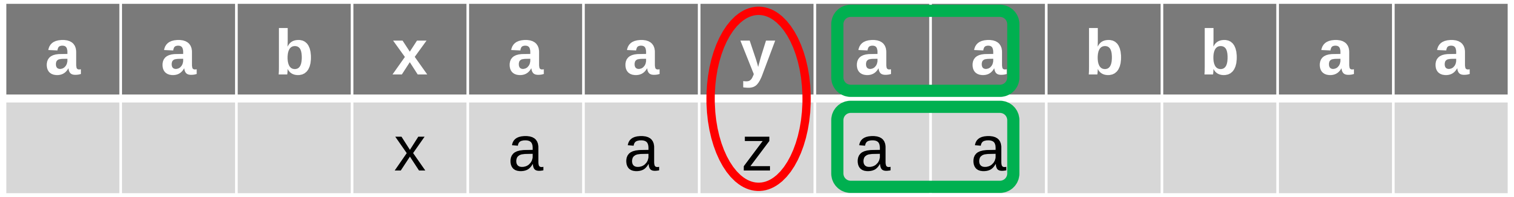
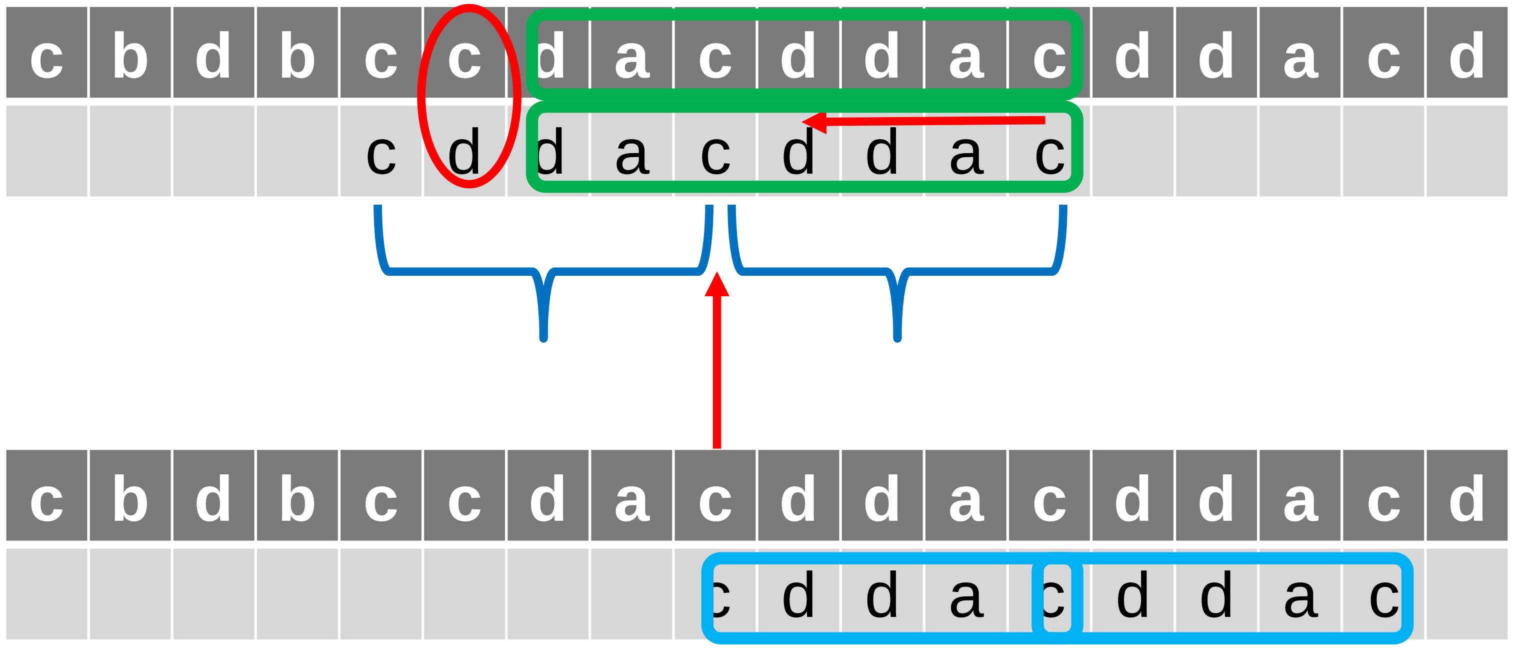


a	a	b	x	a	a	y	a	a	b	b	a	a
			x	a	a	z	a	a				

This shows us the case a).
i.e. skip when a mismatch happens, until:
there are no mismatches between the pattern & ***t***

2. Good Suffix rule- contd..

Look at the following example for case b).
i.e. skip when a mismatch happens, until:
a prefix of the pattern matches a suffix of ***t***


c b d b c c d a c d d a c d d a c d
c d d a c d d a c
c b d b c c d a c d d a c d d a c d
c d d a c d d a c

2. Good Suffix rule- contd..

Now, the last case c), which is much simpler.
i.e. skip when a mismatch happens, until:
the pattern moves past ***t***

c	b	d	b	c	c	d	a	c	d	d	a	c
				b	d	d	a					

c	b	d	b	c	c	d	a	c	d	d	a	c
								b	d	d	a	

Efficiently implementing Good Suffix Rule

- Use Gusfield's Z-algorithm
  - Given a pattern of length 1…m, define a suffix , Z_i^{suffix} (for each position i < m) as the length of the longest substring ending at position i of the pattern that matches its suffix.
- Computation of Z_i^{suffix} values on the pattern is just the same as computation of Z_i values on the reversed pattern.
- Therefore, Z_i^{suffix} values can be computed in O(m) time.

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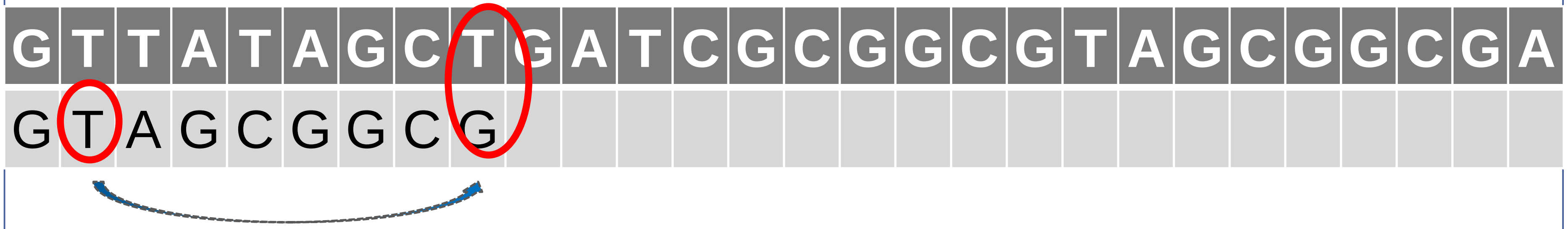

Bad character rule shifts 6 characters, but good suffix rule doesn't apply, thus 0 shifts. Therefore, **apply bad character rule.**

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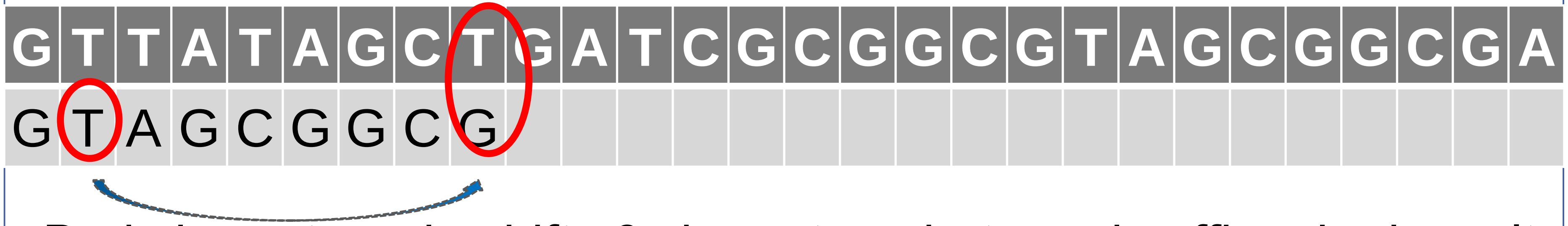

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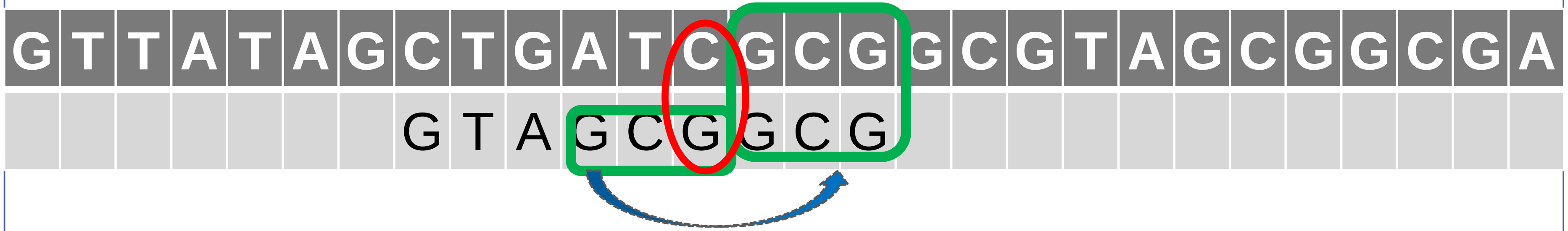
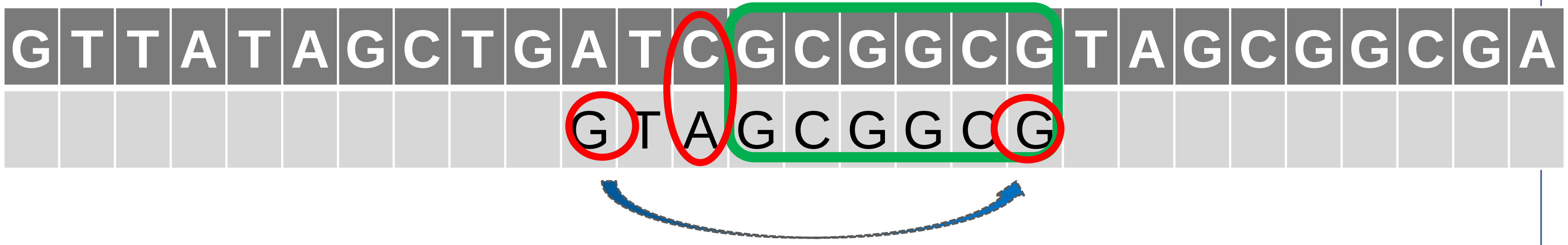
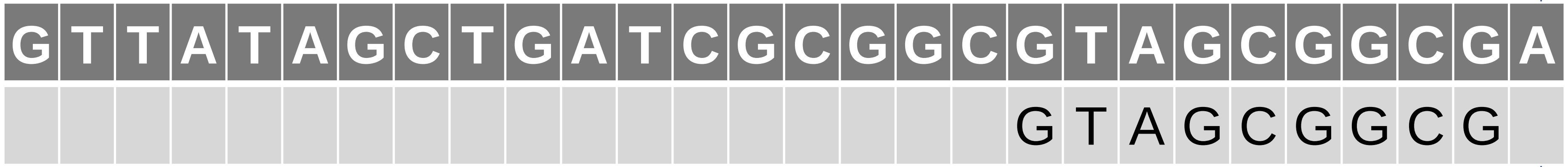


Bad character rule doesn't apply, but good suffix rule can shift 3 characters. Therefore, **apply Good Suffix rule**.

Putting both rules together- contd…



No matching character “C” found in the pattern. Thus shift until the matching component is past. i.e. good suffix rule is applied since it shifts more characters (7) rather than just 2 shifts by bad character rule.



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- Use the **good suffix rule** to find how many places to the right PAT should be shifted under TXT. Call this amount $\boldsymbol{n_{GS}}$.
- Shift PAT to the right under TXT by max($\boldsymbol{n_{BC}}$, $\boldsymbol{n_{GS}}$) places.
- The Boyer Moore algorithm has the worst-case time-complexity of ***O(m + n).***