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FIT3155: Advanced Algorithms and Data Structures

Week 1: **Linear-time string pattern matching**

Faculty of Information Technology, Monash University

What is covered in this lecture?

Linear-time approaches to exact pattern matching problem on strings

- Gusfield's Z-algorithm
- Boyer-Moore algorithm

Source material and recommended reading

- Dan Gusfield, Algorithms on Strings, Trees and Sequences, Cambridge University Press. (Chapters 1-2).

Exact pattern matching: Introduction

The exact pattern matching problem

Given a reference text **txt**$[1...n]$ and a pattern **pat**$[1...m]$, find **ALL** occurrences, if any, of **pat** in **txt**.

Example:

	1	2	3	4	5	6	7	8	9	10	11	12
txt =	b	b	a	b	a	x	a	b	a	b	a	y

	1	2	3
pat =	a	b	a

matched positions of **pat** in **txt** are at positions 3, 7, and 9

- The practical importance of this problem should be plainly obvious to anyone who uses a computer.
- Problem arises in innumerable applications
    - Word processing – `grep` command in Unix
    - Search Engines – Google
    - Library catalogs
    - ⋮

Naive algorithm

```
n = |txt|
m = |pat|
for i from 1 to n-m+1  do
    for j from 1 to m do
         if txt[i+j-1] != pat[j] then
             break  // mismatch
         endif
    endfor;
    if (j == m+1) print i;
endfor
```

How many comparison of symbols does this approach perform in worst case?

Consider **txt**$[1...10] = \overset{1}{a}\ \overset{2}{a}\ \overset{3}{a}\ \overset{4}{a}\ \overset{5}{a}\ \overset{6}{a}\ \overset{7}{a}\ \overset{8}{a}\ \overset{9}{a}\ \overset{10}{a}$ and **pat**$[1...3] = \overset{1}{a}\ \overset{2}{a}\ \overset{3}{a}$.

Total number of comparisons, in the **worst case**, is $(n - m + 1) * m = 24$.

In general the number of comparison grows as $O(mn)$.

Early ideas for speeding up the naive method

- try to shift **`pat`** by > 1 character w.r.t. **`txt`** when mismatch occurs...
  - ...but never shift so far as to miss any occurrence of **`pat`** in **`txt`**;
  - if this can be achieved, we save unnecessary comparisons, and moves **`pat`** along **`txt`** more rapidly.
- Specifically, where possible, we would want to shift by skipping over parts of **`pat`** unrelated to **`txt`**.

Illustration of enumerated scenarios (prev. slide)

Naive approach makes too many unnecessary comparisons

	1	2	3	4	5	6	7	8	9	10	11	12	13
txt:	x	a	b	x	y	a	b	x	y	a	b	x	z
pat:	a	b	x	y	a	b	x	z					
	✗												
pat:		a	b	x	y	a	b	x	z				
		✓	✓	✓	✓	✓	✓	✓	✗				
pat:			a	b	x	y	a	b	x	z			
			✗										
pat:				a	b	x	y	a	b	x	z		
				✗									
pat:					a	b	x	y	a	b	x	z	
					✗								
pat:						a	b	x	y	a	b	x	z
						✓	✓	✓	✓	✓	✓	✓	✓

Overall 20 comparisons in the naive approach, on this example.

Illustration of enumerated scenarios (prev. slide)

Scenarios 1:

a smarter algorithm can gather that, after the ninth comparison, the next three comparisons of the naive algorithm will be mismatches.

	1	2	3	4	5	6	7	8	9	10	11	12	13
txt:	x	a	b	x	y	a	b	x	y	a	b	x	z
pat:	a	b	x	y	a	b	x	z					
	✗												
pat:		a	b	x	y	a	b	x	z				
		✓	✓	✓	✓	✓	✓	✓	✗				
pat:						a	b	x	y	a	b	x	z
						✓	✓	✓	✓	✓	✓	✓	✓

Overall, this ‘smarter’ algorithm saves 3 comparisons, on this example.

How does this algorithm achieve this?

After the ninth comparison, the algorithm knows that the first seven characters of **pat** match characters 2 through to 8 of **txt**. It can gather that the first character of **pat** (‘a’) does not occur until position 6 in **txt**. This is enough information to conclude that there are no possible matches in **txt** of **pat** to the left of position 6, allowing larger skips.

Illustration of enumerated scenarios (prev. slide)

Scenarios 2:

in fact, an ‘even smarter’ algorithm can gather more info after the ninth comparison, beyond scenario 1, and save 3 more comparisons.

	1	2	3	4	5	6	7	8	9	10	11	12	13
txt:	x	a	b	x	y	a	b	x	y	a	b	x	z
pat:	a	b	x	y	a	b	x	z					
	✗												
pat:		a	b	x	y	a	b	x	z				
		✓	✓	✓	✓	✓	✓	✓	✗				
pat:						a	b	x	y	a	b	x	z
									✓	✓	✓	✓	✓

Overall, this ‘even smarter’ algorithm saves 6 comparisons over naive.

How does this algorithm achieve this?

An even smarter algorithm can preprocess **pat**, and from it know that **pat**$[1..3]$ (i.e ‘abx’) appears again at **pat**$[5..7]$. So, after the ninth comparison, the algorithm realizes **pat**[5..7]=**txt**[6..8]. But since **pat**[1..3]=**pat**[6..8], after shift, when **pat**$[1..]$ is being compared with **txt**[6...], we already know **pat**$[1..3]$ = **txt**$[6..8]$, saving us 3 unnecessary comparisons.

Take home message from these illustrated examples

- These illustrate the kind of ideas that allow some comparisons to be skipped.
    - Although, we haven't yet processed how an algorithm can **`efficiently`** implement these ideas.
- Some algorithms permit their efficient realization.
- We will study 2 algorithms that can be implemented to run in linear ($O(n+m)$) time.
    - Gusfield's Z-algorithm (guaranteed linear-time)
    - Boyer-Moore's algorithm (linear time with some caveats!)

1. Gusfield's Z-algorithm

Gusfield's Z-algorithm – Defining Z_i

Definition of Z_i:

For a string **str**$[1...n]$, define Z_i (for each position $i > 1$ in **str**) as the **length** of the **longest substring** starting at **position** i of **str** that **matches its prefix** (i.e., **str**$[i...i+Z_i\text{-}1]$ $=$ **str**$[1...Z_i]$).

Example

	1	2	3	4	5	6	7	8	9	10	11
str=	a	a	b	c	a	a	b	x	a	a	z

$$\begin{array}{ll} Z_2 = 1 & Z_7 = 0 \\ Z_3 = 0 & Z_8 = 0 \\ Z_4 = 0 & Z_9 = 2 \\ Z_5 = 3 & Z_{10} = 1 \\ Z_6 = 1 & Z_{11} = 0 \end{array}$$

Gusfield's Z-algorithm – Defining Z_i-box

Definition of Z-box:

For a string **`str`**$[1..n]$, **and for any** $i > 1$ **such that** $Z_i > 0$, a Z_i-box is defined as the interval $[i...i+Z_i\text{-}1]$ of **`str`**.

Example

	1	2	3	4	5	6	7	8	9	10	11
`str`=	a	a	b	c	a	a	b	x	a	a	z

$Z_2 = 1$ Z_2-box $= [2..2]$

$Z_3 = 0$ undefined

$Z_4 = 0$ undefined

$Z_5 = 3$ Z_5-box $= [5..7]$

$Z_6 = 1$ Z_6-box $= [6..6]$

$Z_7 = 0$ undefined

$Z_8 = 0$ undefined

$Z_9 = 2$ Z_9-box $= [9..10]$

$Z_{10} = 1$ Z_{10}-box $= [10..10]$

$Z_{11} = 0$ undefined

Gusfield's Z-algorithm – Defining r_i

Definition of r_i:

For a string **`str`**$[1..n]$, and for all $i > 1$, r_i is the **`right-most endpoint`** of all Z-boxes that begin at or before position i.

Alternately, r_i is the largest value of j+Z_j-1 over all $1 < j \leq i$, such that $Z_j > 0$.

Example

$$\begin{array}{lccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \textbf{str}= & \mathtt{a} & \mathtt{a} & \mathtt{b} & \mathtt{c} & \mathtt{a} & \mathtt{a} & \mathtt{b} & \mathtt{x} & \mathtt{a} & \mathtt{a} & \mathtt{z} \end{array}$$

$$\begin{array}{ll||c||ll} Z_2 = 1 & Z_7 = 0 & Z_2\text{-box} = [2..2] & r_2 = 2 & r_7 = 7 \\ Z_3 = 0 & Z_8 = 0 & Z_5\text{-box} = [5..7] & r_3 = 2 & r_8 = 7 \\ Z_4 = 0 & Z_9 = 2 & Z_6\text{-box} = [6..6] & r_4 = 2 & r_9 = 10 \\ Z_5 = 3 & Z_{10} = 1 & Z_9\text{-box} = [9..10] & r_5 = 7 & r_{10} = 10 \\ Z_6 = 1 & Z_{11} = 0 & Z_{10}\text{-box} = [10..10] & r_6 = 7 & r_{11} = 10 \end{array}$$

Gusfield's Z-algorithm – Defining l_i

Definition of l_i:

For a string $\texttt{str}[1..n]$, and for all $i > 1$, l_i is the **`left end`** of the Z-box that ends at r_i.

In case there is more than one Z-box ending at r_i, then l_i can be chosen to be the left end of **any** of those Z-boxes.

Example

$$\begin{array}{lccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \texttt{str}= & \texttt{a} & \texttt{a} & \texttt{b} & \texttt{c} & \texttt{a} & \texttt{a} & \texttt{b} & \texttt{x} & \texttt{a} & \texttt{a} & \texttt{z} \end{array}$$

$$\begin{array}{ll||c||ll} Z_2 = 1 & Z_7 = 0 & Z_2\text{-box} = [2..2] & l_2 = 2 & l_7 = 5 \\ Z_3 = 0 & Z_8 = 0 & Z_5\text{-box} = [5..7] & l_3 = 2 & l_8 = 5 \\ Z_4 = 0 & Z_9 = 2 & Z_6\text{-box} = [6..6] & l_4 = 2 & l_9 = 9 \\ Z_5 = 3 & Z_{10} = 1 & Z_9\text{-box} = [9..10] & l_5 = 5 & l_{10} = 9 \\ Z_6 = 1 & Z_{11} = 0 & Z_{10}\text{-box} = [10..10] & l_6 = 5 & l_{11} = 9 \end{array}$$

Another worked out example: calculating Z_i, Z_i-box, and (l_i,r_i) values

Example

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
str=	a	a	b	a	a	b	c	a	x	a	a	b	a	a	b	c	y

$Z_2 = 1$
$Z_3 = 0$
$Z_4 = 3$
$Z_5 = 1$
$Z_6 = 0$
$Z_7 = 0$
$Z_8 = 1$
$Z_9 = 0$

$Z_{10} = 7$
$Z_{11} = 1$
$Z_{12} = 0$
$Z_{13} = 3$
$Z_{14} = 1$
$Z_{15} = 0$
$Z_{16} = 0$
$Z_{17} = 0$

Z_2-box = [2..2]
Z_4-box = [4..6]
Z_5-box = [5..5]
Z_8-box = [8..8]
Z_{10}-box = [10..16]
Z_{11}-box = [11..11]
Z_{13}-box = [13..15]
Z_{14}-box = [14..14]

$(l_2, r_2) = (2, 2)$
$(l_3, r_3) = (2, 2)$
$(l_4, r_4) = (4, 6)$
$(l_5, r_5) = (4, 6)$
$(l_6, r_6) = (4, 6)$
$(l_7, r_7) = (4, 6)$
$(l_8, r_8) = (8, 8)$
$(l_9, r_9) = (8, 8)$

$(l_{10}, r_{10}) = (10, 16)$
$(l_{11}, r_{11}) = (10, 16)$
$(l_{12}, r_{12}) = (10, 16)$
$(l_{13}, r_{13}) = (10, 16)$
$(l_{14}, r_{14}) = (10, 16)$
$(l_{15}, r_{15}) = (10, 16)$
$(l_{16}, r_{16}) = (10, 16)$
$(l_{17}, r_{17}) = (10, 16)$

Main point of Gusfield's Z-algorithm!

- In the previous slides, for any given string, we have defined: $\{Z_i$, Z_i-box, l_i, $r_i\}$.
- The fundamental preprocessing task of Gusfield's Z-algorithm relies on computing these values, given some string, in **`linear`** time.
- That is, for a string **`str`**$[1..n]$, we would like to compute
  $\{Z_i$, Z_i-box, l_i, $r_i\}$
  for each position $i > 1$ in **`str`** in $O(n)$-time.

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Plan ahead

Once we convince ourselves of the linear time preprocessing, we can then use this for linear-time exact pattern matching.

Overview of the linear-time preprocessing

- In this preprocessing phase, we compute $\{Z_i$, Z_i-box, l_i, $r_i\}$ values **`for each successive position`** i, starting from $i = 2$.
- All successively computed Z_i values are remembered.
  - Note: Each Z_i-box interval can be computed from its corresponding Z_i value in $O(1)$ time
- At each iteration, to compute (l_i,r_i), this preprocessing only needs values of (l_j, r_j) for $j = i - 1$.
  - Note: no earlier (l_j, r_j) values are needed...
  - ...so, temporary variables (l, r) can be used to keep track of the most recently computed (l_{i-1}, r_{i-1}) values to update (l_i,r_i).

Let's see how this all works in practice.

preprocessing in practice – base case

- To begin, compute Z_2 by explicit **`left-to-right`** comparison of characters **`str`**[2...] with **`str`**[1...] until a mismatch is found.
    - Note: Z_2 is the length of the **`matching`** substring.
- If $Z_2 > 0$
    - set r (i.e., r_2) to $Z_2 + 1$
    - set l (i.e., l_2) to 2
- else (i.e., if $Z_2 == 0$)
    - set r (i.e., r_2) to 0
    - set l (i.e., l_2) to 0

preprocessing in practice – general case

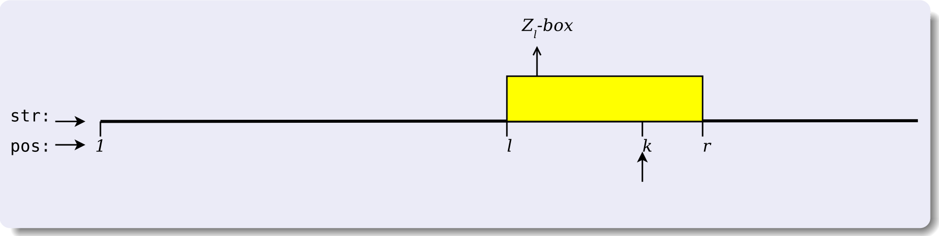
Assume inductively...

- ...we have correctly computed the values Z_2 through to Z_{k-1}.
- ...that r currently holds r_{k-1},
- ...that l currently holds l_{k-1}.

For computing Z_k at position k, these two scenarios arise

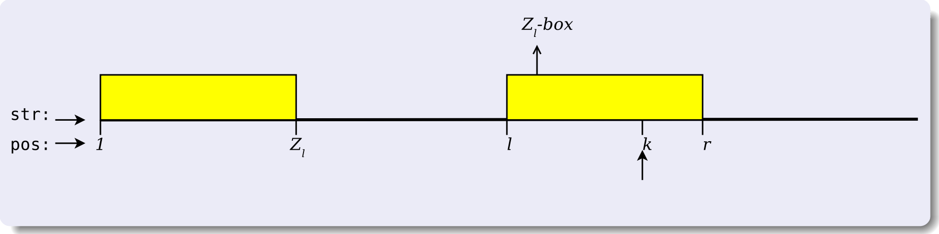
- CASE 1, if $k > r$:
  - Compute Z_k by explicitly comparing characters **str**[k...] with **str**[1...] until mismatch is found.
  - If $Z_k > 0$:
    - set r (i.e., r_k) to $k + Z_k - 1$.
    - set l (i.e., l_k) to k.

preprocessing in practice – general case (continued)



- CASE 2, if $k \leq r$:
    - The character **str**$[k]$ lies in the substring **str**$[l...r]$ (i.e., within Z_l-box).

preprocessing in practice – general case (continued)



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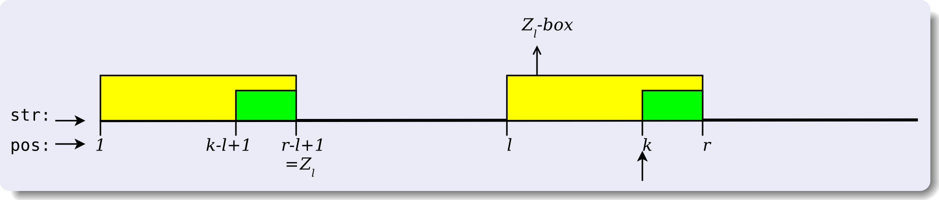
preprocessing in practice – general case (continued)



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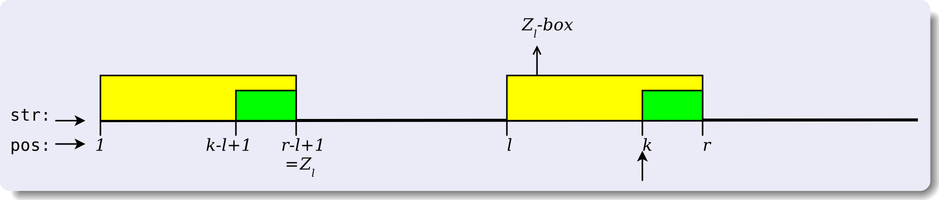
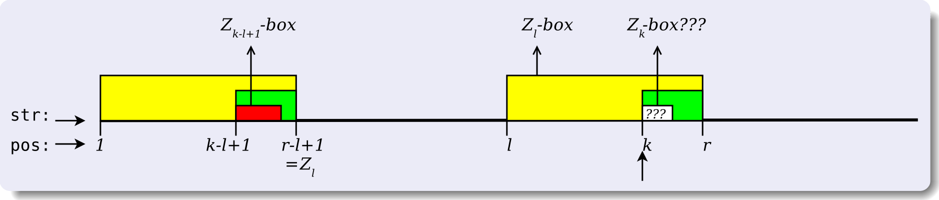
  - This implies the character **str**$[k]$ is identical to **str**$[k - l + 1]$

preprocessing in practice – general case (continued)



- CASE 2, if $k \leq r$:
  - The character **str**$[k]$ lies in the substring **str**$[l \ldots r]$ (i.e., within Z_l-box).
  - By the definition of Z_l-box, **str**$[l \ldots r]$=**str**$[1 \ldots Z_l]$.
  - This implies the character **str**$[k]$ is identical to **str**$[k-l+1]$
  - By extending this logic, it also implies that the substring **str**$[k \ldots r]$ is identical to **str**$[k-l+1 \ldots Z_l]$.

preprocessing in practice – general case (continued)



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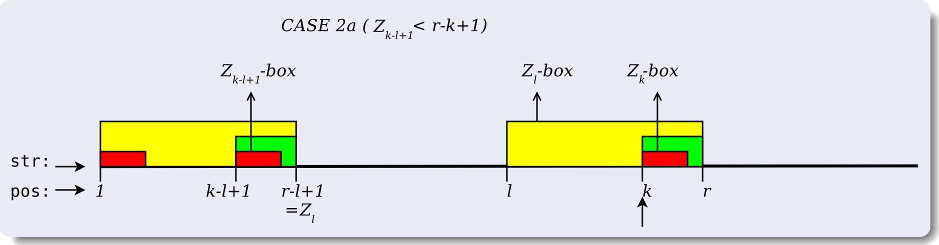
  - By extending this logic, it also implies that the substring **str**$[k \ldots r]$ is identical to **str**$[k - l + 1 \ldots Z_l]$.
  - But, in previous iterations, we already have computed Z_{k-l+1} value.
    - ★ can the value of Z_{k-l+1} inform the computation of Z_k?

preprocessing – case 2 (continued)

In the previous slide, we asked "*can the value of Z_{k-l+1} inform the computation of Z_k?*". The answer is **yes**, and this can be handled by two **sub**-cases" CASES 2a and 2b (described below):

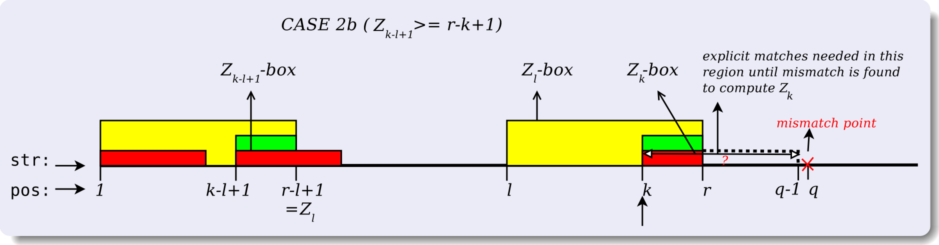
preprocessing – case 2 (continued)

- CASE 2a, if $Z_{k-l+1} < r - k + 1$:
  - set Z_k to Z_{k-l+1}.
  - r and l remain unchanged.



preprocessing – case 2 (continued)

- CASE 2b, if $Z_{k-l+1} \geq r-k+1$:
  - Z_k must also be $\geq r-k+1$
  - So, start explicitly comparing **`str`**$[r+1]$ with **`str`**$[r-k+1]$ and so on until mismatch occurs.
  - Say the mismatch occurred at position $q \geq r+1$, then:
    - ★ set Z_k to $q-k$,
    - ★ set r to $q-1$.
    - ★ set l to k.



Preprocessing (of Z_i values) for $\texttt{str}[1..n]$ takes $O(n)$ time

- Total time is directly proportional to the SUM of:

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Preprocessing (of Z_i values) for **str**$[1..n]$ takes $O(n)$ time

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- This preprocessing has $n - 1$ iterations.

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- The number of **`matches`** is also at most n, because:

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  - $r_k \geq r_{k-1}$ (for all iterations)

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  - update to r_k is of the form $r_k = r_{k-1} + \delta$ (involving $\delta \geq 0$ **`matches`**).

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  - update to r_k is of the form $r_k = r_{k-1} + \delta$ (involving $\delta \geq 0$ **matches**).
  - But $r_k \leq n$.

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  - $r_k \geq r_{k-1}$ (for all iterations)
  - update to r_k is of the form $r_k = r_{k-1} + \delta$ (involving $\delta \geq 0$ **`matches`**).
  - But $r_k \leq n$.
  - Thus, there are at most n **`matches`**.

Using Z-algorithm for linear-time exact pattern matching

Recall the exact pattern matching problem

Given a reference text **`txt`**$[1...n]$ and a pattern **`pat`**$[1...m]$, find **ALL** occurrences, if any, of **`pat`** in **`txt`**.

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Realizing a linear-time solution using Gusfield's Z-algorithm/preprocessing

- Construct a new string **str** by concatenation as follows:
  $\texttt{str} = \texttt{pat}[1...m] + \$ + \texttt{txt}[1...n]$.
  Note, $|\texttt{str}| = m + 1 + n$.

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- Construct a new string **`str`** by concatenation as follows:
  `str` $=$ **`pat`**$[1...m] + \$ +$ **`txt`**$[1...n]$.
  Note, $|$**`str`**$| = m + 1 + n$.
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Using Z-algorithm for linear-time exact pattern matching

Recall the exact pattern matching problem

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- Thus, this pattern matching algorithm takes $O(m + n)$ time. **`QED`**

2. Boyer-Moore Algorithm

Boyer-Moore Algorithm – Introduction

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Boyer-Moore Algorithm – Introduction

- Boyer-Moore algorithm incorporate three clever ideas:
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  2. bad character shift rule
  3. good suffix shift rule

Boyer-Moore Algorithm – right-to-left scan

For any comparison of **pat**$[1...m]$ against **txt**$[i...i+m-1]$, the Boyer-Moore algorithm checks/scans for matched characters right to left (instead of the normal left to right scan, as in the naive algorithm).

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- **pat**[4] with **txt**[6] – match.
- **pat**[3] with **txt**[5] – mismatch

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After mismatch, to avoid naïvely shifting **pat** rightwards by **1 position**, BM algorithm employs two additional ideas/tricks discussed below.

Boyer-Moore Algorithm – Bad character shift rule

Example

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
txt:	x	p	b	c	t	b	x	a	b	p	q	x	c	t	b	p	q
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- Scanning right-to-left, we found a mismatch comparing **pat**$[3] \equiv$ a with **txt**$[5] \equiv$ t.
- But the rightmost occurrence in the entire **pat** of the mismatched character in **txt** (i.e. **txt**$[5] \equiv$ t) is at position 1 of **pat** (i.e., **pat**$[1] \equiv$ t).

Boyer-Moore Algorithm – Bad character shift rule

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- Scanning right-to-left, we found a mismatch comparing **pat**$[3] \equiv$ a with **txt**$[5] \equiv$ t.
- But the rightmost occurrence in the entire **pat** of the mismatched character in **txt** (i.e. **txt**$[5] \equiv$ t) is at position 1 of **pat** (i.e., **pat**$[1] \equiv$ t).
- So, in this case, one case safely shift **pat** by **two places** to the right (instead of naively shifting by only one place) so as to match characters **pat**$[1] \equiv$ t and **txt**$[5] \equiv$ t.

Boyer-Moore Algorithm – Formalizing Bad character shift rule

- Let **pat**$[1...m]$ and **txt**$[1...n]$ be strings from the alphabet $\aleph$.

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- Let **pat**$[1...m]$ and **txt**$[1...n]$ be strings from the alphabet $\aleph$.
- Preprocess **pat**, and store for each character $x \in \aleph$, the rightmost position of occurrence of character x in **pat**. (Call this position, $R(x)$. Note, $R(x) = 0$ when x does not occur in **pat**.)

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- These preprocessed $R(x)$ values will be used (for > 1 position shifts of **pat** under **txt**, were possible) as follows:

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Before that, note, storing $R(x)$ values for **pat** requires at most $O(|\aleph|)$ space, and one table lookup per mismatch.

Boyer-Moore Algorithm – Formalizing Bad character shift rule (continued)

- In some iteration, say **txt**$[j...j+m-1]$ and **pat**$[1...m]$ are being compared via right-to-left scan.

Boyer-Moore Algorithm – Formalizing Bad character shift rule (continued)

- In some iteration, say **txt**$[j...j+m-1]$ and **pat**$[1...m]$ are being compared via right-to-left scan.
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- **RULE:** Then, the bad-character shift rule asks us to to shift rightwards **pat** along **txt** by $\max\{1, k - R(\mathbf{x})\}$ **positions**.

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- **RULE:** Then, the bad-character shift rule asks us to to shift rightwards **pat** along **txt** by $\max\{1, k-R(\mathbf{x})\}$ **positions**.
- Further, if **x** does not occur in **pat**$[1..m]$ $(R(\mathbf{x})=0)$, then the entire **pat** can be shifted one position past the point of mismatch in **txt**.

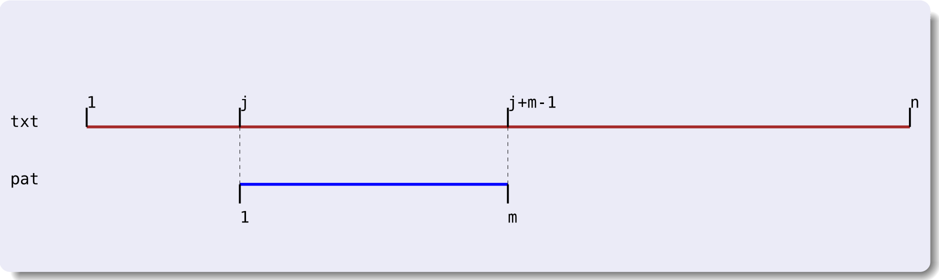
An extension to the bad-character shift rule

Extended Bad-Character Rule

When a mismatch occurs at some position k in **pat**$[1...m]$, and the corresponding mismatched character is **x** $= $ **txt**$[j + k - 1]$, then **shift** **pat**$[1..m]$ to the right so that **the closest x in pat that is to the left of position** k is now below the (previously mismatched) **x** in **txt**.

- To achieve this, preprocess **pat**$[1...m]$ so that, for each position $1 \leq k \leq m$ in **pat**, and for each character $x \in \aleph$, the position of the closest occurrence of x to the left of each position k can be efficiently looked up.
- A 2D array (**shift/jump table**) of size $m \times |\aleph|$ can store this information. (Think how this can be implemented more space-efficiently?)

Boyer-Moore Algorithm – Good suffix rule



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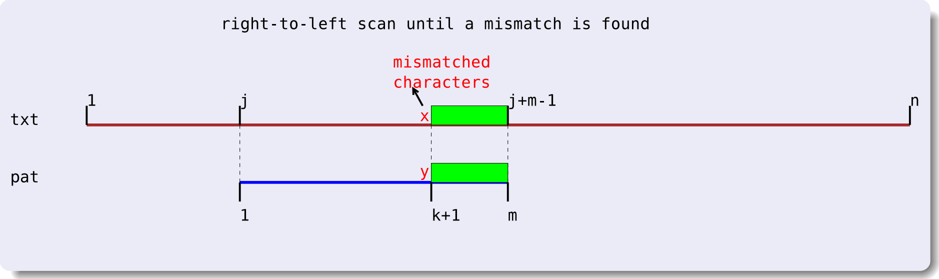

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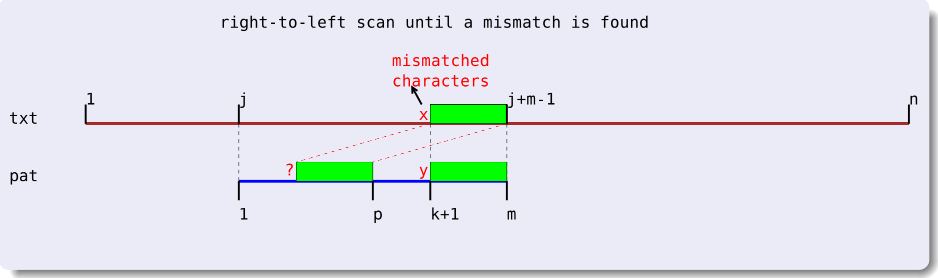

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- If we knew that $p < m$ is the rightmost position in **pat** where the longest substring (of length $>= 1$) ending at position p matches its suffix, that is:
    - **pat**$[p-m+k+1...p] \equiv$ **pat**$[k+1....m]$.
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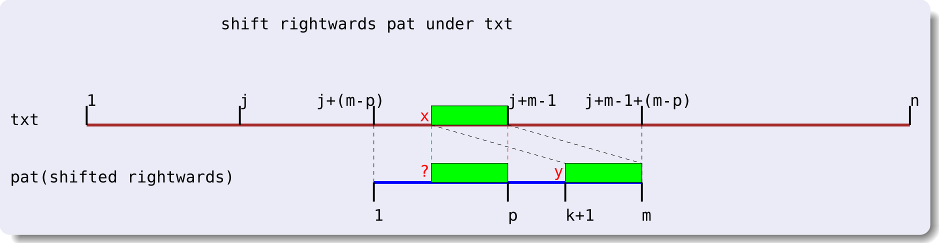

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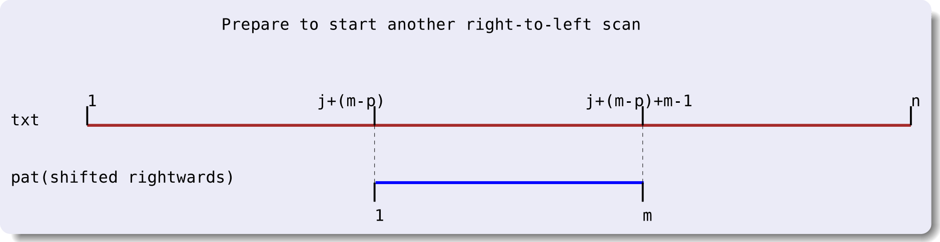

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- then, **pat** can be safely shifted right by $m-p+1$ positions,
- and a new iteration can be restarted.

Boyer-Moore Algorithm – Ideas to implement the ‘good suffix’ rule efficiently

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To efficiently implement this 'good suffix' rule, we take 'inspiration' from the computation of Z_i values in Gusfield's algorithm (refer slide 11), and define Z_i^{suffix} (specifically on **`pat`**) as follows:

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Definition of Z_i^{suffix}

Given a **pat**$[1...m]$, define Z_i^{suffix} (for each position $i < m$) as the **length** of the **longest substring ending at position** i of **pat** that matches its suffix (i.e., **pat**[i-Z_i^{suffix}+1...i] = **pat**[m-Z_i^{suffix}+1...m]).

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- Note, computation of Z_i^{suffix} values on **pat** corresponds to the computation of Z_i values on reverse(**pat**).
- Thus, Z_i^{suffix} values can be computed in $O(m)$ time for **pat**$[1...m]$.

Boyer-Moore Algorithm – Ideas to implement 'good suffix' rule efficiently (continued)

In fact, for each **suffix** starting at position j in **pat**, we want to store the rightmost position p in **pat** such that:

- $\mathbf{pat}[j..m] \equiv \mathbf{pat}[p - Z_p^{\text{suffix}} + 1...p]$.
- $\mathbf{pat}[j-1] \neq \mathbf{pat}[p - Z_p^{\text{suffix}}]$.

Boyer-Moore Algorithm – Ideas to implement 'good suffix' rule efficiently (continued)

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- $\texttt{pat}[j..m] \equiv \texttt{pat}[p - Z_p^{\text{suffix}} + 1...p]$.
- $\texttt{pat}[j-1] \neq \texttt{pat}[p - Z_p^{\text{suffix}}]$.

Store these rightmost positions as $\texttt{goodsuffix}(j) = p$. These can be computed as:

```
m := |pat|
for j from 1 to m  do
    goodsuffix(j) := 0
endfor

for p from 1 to m-1  do
    j := m - Z^{suffix}_p + 1
    goodsuffix(j) := p
endfor
```

Boyer-Moore Algorithm – Using ‘good suffix’ rule during search

In any iteration, the following cases have to handled:

Case 1a: if a mismatch occurs at some **pat**$[k]$, and **goodsuffix**$(k+1)>0$ then

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  - **matchedprefix**$(k+1)$ denotes the length of the **largest** suffix of **pat**$[k+1..m]$ that is identical to the prefix of **pat**$[1..m-k]$.

Boyer-Moore Algorithm – Using 'good suffix' rule during search

In any iteration, the following cases have to handled:

Case 1a: if a mismatch occurs at some **pat**$[k]$, and **goodsuffix**$(k+1)>0$ then

- shift **pat** by $m -$ **goodsuffix**$(k+1)$ places.

Case 1b: if a mismatch occurs at some **pat**$[k]$, and **goodsuffix**$(k+1)=0$ then

- shift **pat** by $m -$ **matchedprefix**$(k+1)$ places
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    - Note, **matchedprefix**$(.)$ values for **pat** can be precomputed, just as we did with **goodsuffix**$(.)$ values – how?

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Boyer-Moore Algorithm – Using 'good suffix' rule during search

In any iteration, the following cases have to handled:

Case 1a: if a mismatch occurs at some **pat**$[k]$, and **goodsuffix**$(k+1)>0$ then

- shift **pat** by $m-$ **goodsuffix**$(k+1)$ places.

Case 1b: if a mismatch occurs at some **pat**$[k]$, and **goodsuffix**$(k+1)=0$ then

- shift **pat** by $m-$ **matchedprefix**$(k+1)$ places
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Case 2: when **pat**$[1...m]$ fully matches **txt**$[j...j+m-1]$

- shift **pat** by $m-$ **matchedprefix**(2) places

Boyer-Moore Algorithm – Bringing all pieces together

- Preprocess **`pat`** ...

Boyer-Moore Algorithm – Bringing all pieces together

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  - ...for jump tables (eg. $R(\cdot)$ values) needed for 'bad-character' shifts (see slides 27-30)

Boyer-Moore Algorithm – Bringing all pieces together

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- The **Boyer Moore algorithm** has the *worst-case time-complexity* of $O(m+n)$ (with some caveats; to be discussed in lecture!)

Lecture Summary

- Naive algorithm takes $O(m * n)$-time.
- Gusfield's Z algorithm guaranteed in $O(n + m)$-time, worst case
- Boyer-Moore's algorithm (as we discussed above) takes
  - $O(n + m)$-time (with some caveats)...
  - ...but $O(\frac{n}{m})$-time (sublinear) on 'realworld' settings .

In the next lecture...

Linear time suffix trees (Ukkonen's algorithm) and suffix array (Karkkainen's algorithm) construction

-=o0o=-
END
-=o0o=-