Corrigendum to "Mean action of periodic orbits of area-preserving annulus diffeomorphisms"

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1 Introduction

This corrigendum corrects several mathematical errors in [6]. Consequently, the main result [6, Thm. 1.9] must be modified to Theorem 1.1 by adding stronger hypotheses: that the annulus diffeomorphisms under consideration must be isotopic, relative to the boundary, to a rotation, and their Calabi invariants $\mathcal{V}(\tilde{\psi})$ must, for some lift with a positive action function, satisfy a stronger upper bound than originally claimed. The results of this corrigendum are weaker than those announced in the original paper. However, they are still new in the sense that Hutchings' earlier result [4, Thm. 1.2] does not include them. See Remark 1.6 below for further discussion.

We first state the new main theorem, discussing the stronger upper bound on $\mathcal{V}(\tilde{\psi})$ in Remark 1.3. We then prove the (very immediate) interpretation of the result as a "zero or infinity" statement in Corollary 1.4. Next, we explain in Remark 1.5 the reason for the new hypothesis on equal boundary rotation from the perspective of embedded contact homology (ECH). In Remark 1.6 we provide an example for which Theorem 1.1 is new. Finally, in §1.2, we list the errors in [6] before embarking upon their corrections in §2 and §3. Throughout this corrigendum we freely use notation set up in [6], as well as all results besides those indicated as erroneous in §1.2.

1.1 Main theorem

The new main theorem, replacing [6, Thm. 1.9], is

Theorem 1.1. *Let $y_0 \in \mathbb{R}$ and let ψ be an area-preserving diffeomorphism of (A, ω), with $\tilde{\psi}$ a lift of ψ to $\tilde{A}$ which is translation by $2\pi y_0$ near $\partial\tilde{A}$. Let F denote the flux of $\tilde{\psi}$. Assuming*

$$\mathcal{V}(\tilde{\psi}) < \frac{F \min\{y_0, -y_0 + F\}}{2 \max\{y_0, -y_0 + F\}} \tag{1.1}$$

and that $\tilde{\psi}$ has positive action function, or that there is some other lift with positive action function for which (1.1) holds, we have

$$\inf\left\{ \frac{\mathcal{A}(\gamma)}{\ell(\gamma)} \middle| \gamma \in \mathcal{P}(\psi) \right\} \leq \mathcal{V}(\tilde{\psi}).$$

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By replacing (ψ, y_0) with $(\psi^{-1}, -y_0)$, we update [6, Cor. 1.15] to

Corollary 1.2. *Let $y_0 \in \mathbb{R}$ and let ψ be an area-preserving diffeomorphism of (A, ω), with $\tilde{\psi}$ a lift of ψ to $\tilde{A}$ which is translation by $2\pi y_0$ near $\partial \tilde{A}$. Let F denote the flux of $\tilde{\psi}$. Assuming*

$$\mathcal{V}(\tilde{\psi}) > \frac{F \max\{y_0, -y_0 + F\}}{2 \min\{y_0, -y_0 + F\}} \tag{1.2}$$

and that $\tilde{\psi}$ has negative action function, or that there is some other lift with negative action function for which (1.2) holds, we have

$$\sup \left\{ \frac{\mathcal{A}(\gamma)}{\ell(\gamma)} \middle| \gamma \in \mathcal{P}(\psi) \right\} \geq \mathcal{V}(\tilde{\psi}).$$

Remark 1.3. (i) Note that

$$\frac{F \min\{y_0, -y_0 + F\}}{2 \max\{y_0, -y_0 + F\}} \leq \min\{y_0, -y_0 + F\}$$

as $F = \min\{y_0, -y_0 + F\} + \max\{y_0, -y_0 + F\}$. Thus we cannot prove the analogue of the case of rational boundary rotation from the initially claimed [6, Thm. 1.9]. Its proof in [6, Rmk. 1.10] was a tautology reinterpreting the original weaker upper bound on $\mathcal{V}(\tilde{\psi})$.

(ii) The new hypothesis (1.1) is only invariant of the choice of lift $\tilde{\psi}$ if $y_0 = -y_0 + F$ (i.e. $f(-1, y) = f(1, y)$), otherwise as y_0 increases, the right hand side increases more slowly than the left hand side does. However, the conclusion is invariant of the lift. Thus Theorem 1.1 and (respectively, Corollary 1.2) can hold if the lift satisfying (1.1) (resp. (1.2)) with positive (resp. negative) action function is different from $\tilde{\psi}$. Applying the proof as explained in this corrigendum to the different lift proves the result for the original lift.

We hope that in future work it will be possible to replace (1.1) by a weaker upper bound that is independent of the choice of lift $\tilde{\psi}$.

A consequence of [6, Thm. 1.9, Cor. 1.15] is the following quantitative criterion for an annulus diffeomorphism to have periodic orbits:

Corollary 1.4. *If ψ is an area-preserving diffeomorphism of (A, ω) whose lift $\tilde{\psi}$ to $\tilde{A}$ is translation by $2\pi y_0$ near $\partial \tilde{A}$ and ψ does not have periodic orbits, then y_0 is irrational and*

$$\mathcal{V}(\tilde{\psi}) = y_0 = \frac{F}{2},$$

where F is the flux of $\tilde{\psi}$.

Proof. By [3, Thm. 3.3], if ψ does not have periodic orbits, then $y_0 = F/2$ and is irrational. Thus if the conclusion on $V(\tilde{\psi})$ does not hold, the map ψ satisfies the hypotheses of either Theorem 1.1 or Corollary 1.2. □

Corollary 1.4 follows in the same manner as it would using the original [6, Thm. 1.9, Cor. 1.15]. While not appearing in [6], we did explain the conclusions of [6] at the time it appeared by stating Corollary 1.4 in other places, so it is not new.

Remark 1.5. The addition of the hypothesis that ψ must rotate each boundary component by the same amount begs the question of whether or not Theorem 1.1 holds when the boundary rotation amounts are different.

We expect it is true, but that it would require more work on the ECH of toric lens spaces in order to prove, which would go far beyond the computations in [6]. The idea of the proof is to compute the knot filtration on the ECH chain complex using a model contact form with two Reeb orbits having the same rotation numbers as the one constructed from the annulus symplectomorphism (i.e., $1/y_+$ and $1/(-y_- + F)$, where $y_\pm$ are the boundary rotation numbers, taking the place of y_0 near each boundary component of A). When $y_+ \neq y_-$, it is not possible to devise a toric contact form (see [1] for inspiration about how to extend the ideas of toric domains to lens spaces) with these fixed boundary rotation numbers as a quotient of the boundary of an ellipsoid. Like ellipsoids, the ECH differential vanishes for the lens spaces studied in [6]; in the case of general $y_+ \neq y_-$, one would need to model the chain complex using the ideas of [5, 2].

Although we do believe the strategy outlined above could work, it would require delving deeper into the ECH moduli spaces than [6] does, and would rely on [8, 7] (written four years after [6] was) and [1] (which is not published). Therefore, we instead restrict ourselves to strengthening the hypotheses of Theorem 1.1, and save the more general case for future work.

Remark 1.6. There are many ψ and β for which Theorem 1.1 follows from [4, Thm. 1.2], the version for the disk. Previously, we found in [6, Prop. A.1] a criterion identifying ψ for which it was very difficult to see how [6, Thm. 1.9] could possibly follow from [4, Thm. 1.2]. Specifically, using

$$\kappa(x, y) = \left(\sqrt{\frac{x+1}{2}}, y\right); \quad \kappa^*\left(\frac{r}{\pi}\, dr \wedge d\theta\right) = \frac{1}{2}\, dx \wedge dy \tag{1.3}$$

and setting

$$\psi_\kappa(r, \theta) = \begin{cases} \kappa \circ \psi \circ \kappa^{-1}(r, \theta) & \text{if } r > 0 \\ (0, 0) & \text{if } r = 0, \end{cases}$$

we showed in [6, Prop. A.1] that if $\mathcal{V}(\tilde{\psi}) \geq F/2$, then the origin in $\mathbb{D}^2$ satisfied the conclusion of [4, Thm. 1.2] for ψ_κ. Because the origin is the image of the entire $x = -1$ boundary component of A, if the $x = -1$ boundary rotation number of ψ is irrational, the fact that the disk theorem identifies the origin as the periodic orbit of ψ_κ with bounded mean action will have no bearing on the mean action of any periodic orbits of ψ on A.

Unfortunately, the stronger hypothesis on $\mathcal{V}(\tilde{\psi})$ assumed by this corrigendum forces $\mathcal{V}(\tilde{\psi}) < F/2$, meaning that we need to find a new class of ψ and β for which Theorem 1.1 does not follow by applying [4, Thm. 1.2] to ψ_κ. For concreteness, we will use

$$\beta = \frac{x}{2\pi}\, dy \quad \text{and} \quad \beta_{\mathbb{D}^2} = \frac{r^2}{2\pi}\, d\theta.$$

Our strategy will be the following: instead of finding examples for which the conclusion of the disk theorem does not imply the conclusion of the annulus theorem, we will find an example for which the hypotheses of the annulus theorem for $\tilde{\psi}$ do not imply the hypotheses of the disk theorem for (ψ_κ, y_0). Let

$$\tilde{\psi}(x, y) = (x, y + 2\pi g(x)), \quad \text{where } g(x) = 1 - \sin(\pi x) - \frac{\cos(2\pi x)}{3}.$$

Note that $\tilde{\psi}$ is not a translation near the boundary. For any $\delta > 0$, we can add a C^0-small smooth function $h(x)$ to g which is supported on $\{x < -1+\delta\} \cup \{x > 1-\delta\}$ with $|h'(x)| < 2\max\{|g'(-1)|, |g'(1)|\}$ and $g + h \equiv g(1)$ near ∂A. The change in F is controlled by $|h|$ and δ, while the changes in f and $\mathcal{V}$ are controlled by $|h'|$ and δ, so we may choose δ small enough for the hypotheses of Theorem 1.1 (equations (1.4) and (1.5) and positivity of f) to hold for $g+h$ so long as they hold for g. We now show the latter. We freely use the computations within and following the proof of [6, Prop. A.1].

- Both boundary rotation numbers are equal: $g(1) = g(-1) = 1 - 0 - 1/3 = 2/3$.
- The (annulus) action function depends only on x, and can be computed as follows:

$$\begin{aligned} f(x,y) &= \frac{2}{3} + \int_1^x t g'(t)\, dt \\ &= 1 - \frac{1}{\pi} - \frac{\cos(\pi x)}{\pi} - x\sin(\pi x) - \frac{x\cos(2\pi x)}{3} + \frac{\sin(2\pi x)}{6\pi}. \end{aligned}$$

  The action function is positive; plots of $g(x)$ and $f(x,y)$ for fixed y appear in Figure 1. (At its minimum, we have $f(-1/2, y) \approx 0.015 > 0$.)

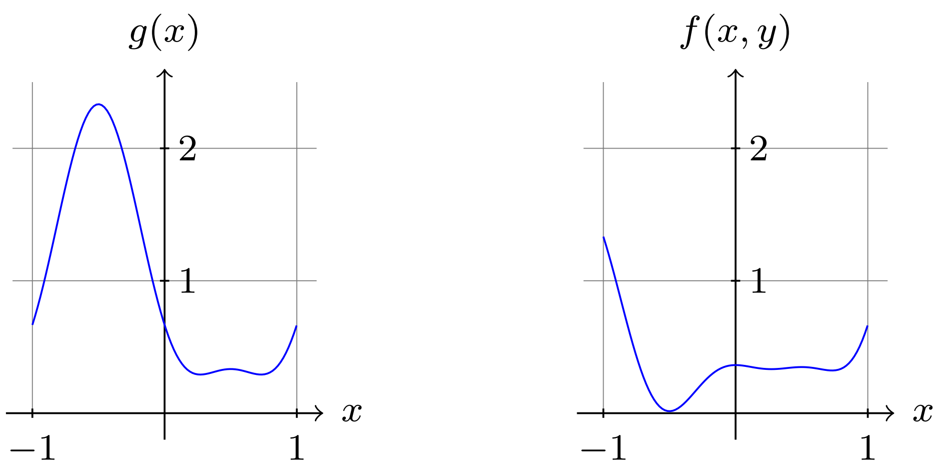

Figure 1: Graphs of the functions $g(x)$ and $x \mapsto f(x,y)$ for fixed y.

- The boundary values of the action function are $f(-1,y) = 4/3$ and $f(1,y) = 2/3$, and the flux is

$$F = \int_{-1}^{1} g(x)\, dx = 2.$$

  Therefore,

$$\frac{F \min\{f(-1,y), f(1,y)\}}{2\max\{f(-1,y), f(1,y)\}} = \frac{2 \cdot 2/3}{2 \cdot 4/3} = \frac{1}{2}.$$

- The Calabi invariant is

$$\mathcal{V}(\tilde{\psi}) = \frac{2\pi \int_{-1}^{1} f(x,y)\, dx}{2\pi \int_A \frac{1}{2\pi}\, dx \wedge dy} = 1 - \frac{2}{\pi} < \frac{1}{2}. \tag{1.4}$$

Thus the map ψ satisfies the hypotheses of Theorem 1.1, but ψ_κ does not satisfy the hypotheses of [4, Thm. 1.2], because

$$\mathcal{V}\left(\psi_\kappa, \frac{2}{3}\right) = \frac{\mathcal{V}(\tilde{\psi})}{2} + \frac{F}{4} = 1 - \frac{1}{\pi} > \frac{2}{3}. \tag{1.5}$$

Generalizing the above equation, Theorem 1.1 will be new so long as its hypotheses hold and

$$\frac{\mathcal{V}(\tilde{\psi})}{2} + \frac{\int_{-1}^{1} g(x)\,dx}{4} > g(1);$$

note that because $\mathcal{V}(\tilde{\psi}) < F/2$ (see Remark 1.3 (i)), this implies

$$\int_{-1}^{1} g(x)\,dx > 2g(1).$$

Finally, we analyze the effects of several natural adjustments we can make to $\tilde{\psi}$ or our choice of κ to ensure that we have not missed a simple way to prove Theorem 1.1 for $\tilde{\psi}$ from [4, Thm. 1.2].

- Adding a constant to the y-component of the lift changes all quantities involved by the addition of that constant (with the exception of the upper bound on $\mathcal{V}(\tilde{\psi})$, see Remark 1.3). Taking powers changes all quantities by multiplying by that power. Neither operation will cause ψ_κ to satisfy the hypotheses of [4, Thm. 1.2].

- The reflection $(x, y) \mapsto \psi(-x, y)$ does not satisfy the hypotheses of Theorem 1.1, nor does its corresponding disk map satisfy the hypotheses of [4, Thm. 1.2].

- Instead of κ, we could map A into a narrower annulus in $\mathbb{D}^2$, still centered at the origin but bounded between two circles of radius strictly between zero and one. The obvious extension of $\kappa \circ \psi \circ \kappa^{-1}$ to $\mathbb{D}^2$ rotates $\mathbb{D}^2 \setminus \kappa(A)$ by y_0 (we use this extension to avoid creating periodic orbits of ψ_κ not corresponding to any of ψ). The effect on the action function is to contract the region on which $f'_\kappa \neq 0$ into the smaller annulus $\kappa(A)$ and increase the regions on which the action function is constant at $f_\kappa(0, 0)$ and $f_\kappa(1, \theta)$. The value of $f_\kappa(1, \theta)$ is y_0; the value of $f_\kappa(0, 0)$ is closer to y_0 than its original value of $F/2$ (one can see this by generalizing the calculation of $f_\kappa(0, 0)$ in the proof of [6, Prop. A.1]; the difference from $f_\kappa(1, 0)$ will be multiplied by the square of the width of the annulus $\kappa(A)$).

  In our example, the relevant values of the original disk action function are 1 and 2/3, so this method can only provide us with a Calabi invariant closer to, yet still larger than, 2/3.

Of course, given ψ, y_0, and β, there might be some very nonobvious κ for which Theorem 1.1 follows from [4, Thm. 1.2], but we expect it would likely have to be rather unusual and specifically tailored to ψ and β.

1.2 Corrections to the proof

Certain sections of [6] (§3 and §5, and parts of §6) require modification, with the most significant changes in §5 and the beginning of §6. The two significant errors are the following:

1. The statement and proof of [6, Prop. 3.1] are incorrect; in particular, in Step 3 of the proof, the contact manifold is misidentified as $L(y_+ - y_- + F, y_+ - y_- + F - 1)$, when in fact it is $L(F, F-1)$. While we believe the rest of the paper (barring the errors below) is entirely correct when $y_+ = y_-$, several sections (§5.2, §5.3, §6.1, and §6.2) require adjustments to indicate they only apply in this special case.

2. In the proof of [6, Prop. 6.3] we need to show that a Reeb orbit set satisfying certain action and intersection number inequalities is nonempty. We accomplish this by showing that its intersection number with a page, equation (6.12), is positive. However, the original argument is incorrect, as it relies on a function C of N, defined in (6.14), to be uniformly bounded below one as N goes to infinity (the parameter $N \in \mathbb{Z}$ corresponds to the choice $\tilde{\psi}$ of lift of ψ to $\tilde{A}$). This is not the case.

   We have fixed this error by changing our hypothesis on $V(\tilde{\psi})$. See Remark 1.6 and the discussion following Remark 3.2.

We have also identified two less impactful errors:

3. The statement of [6, Prop. 6.1] applies to all contact forms, while the proof only accounts for nondegenerate forms. We simply note here that the extension to the case of degenerate forms follows exactly as in Step 2 of the proof of [4, Prop. 3.1].

4. The original paper used the word "flux" in a nonstandard way, referring to the flux of the map $\tilde{\psi}$ rather than the map ψ. We correct this here. Moreover, our new hypothesis on $\mathcal{V}(\tilde{\psi})$ allows us to decrease our dependence on the relationship between $\tilde{\psi}$ and ψ in the proof of Proposition 3.1.

We correct the first error in §2 and the second error in §3. Throughout this corrigendum we use the notation of [6].

1.3 Acknowledgements

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2 Changes to §3, §5.2, §5.3, and §6.1

In this section we make the adjustments necessary only due to error #1.

2.1 Correction to [6, Prop. 3.1] and its proof

The correct version of [6, Prop. 3.1], in which we constructed a contact manifold from the mapping torus of the annulus symplectomorphism ψ, is given in Proposition 2.1. First we explain the idea, which highlights in more detail why the original construction was incorrect. Note that in this subsection we are not necessarily assuming $y_+ = y_-$.

The goal is to construct a contact manifold (Y, λ) for which

- the annulus A is a global surface of section for the Reeb flow, with return map ψ,

- the rotation numbers of the binding orbits are the reciprocals of the values of the action function on the corresponding boundary components of A,
- the return time is the action function, and
- the contact volume is the Calabi invariant times the symplectic area of A.

These properties are listed as the conclusions of Proposition 2.1, with more precision.

We build Y from the mapping torus of ψ together with a contact form constructed so that its Reeb vector field equals the $[0,1]$ direction of the mapping torus and the last two conditions above (on return time and contact volume) hold. The next step is to glue solid tori to a neighborhood of the boundary of the mapping torus so that the condition on rotation numbers holds. This is where we see the most significant difference from the situation in [4, Prop. 2.1]. Along the $x = -1$ boundary, the action function no longer equals the boundary rotation number, but involves an extra flux term.

When identifying the monodromy of the open book supporting $\ker\lambda$, we need to compute the return map of a vector field which points in the meridional direction near the binding. This return map will differ from that of the Reeb vector field near a binding component by a twist by the value of action function on the corresponding boundary component of A. Thus near the $x = +1$ boundary component the monodromy simply "untwists" the return map, while near the $x = -1$ boundary component the monodromy untwists the return map but overshoots by the difference between the value of the action function at $x = -1$ and the amount by which ψ rotates along $x = -1$; this difference is F.

In the gluing step of the original proof we introduced coordinates $\hat{y}$ and $\check{y}$; we believe these coordinates complicated the original proof unnecessarily in the annulus setting, leading to our confusion on the computation of the monodromy map. We have removed them in the updated proof below.

Proposition 2.1. *Let ψ be an area-preserving diffeomorphism of (A, ω) which is rotation by $2\pi y_\pm$ near $\partial_\pm A$, whose flux is $F \in \mathbb{Z}$, for which both y_+ and $-y_- + F$ are irrational, and whose action function f is positive. Then there is a contact form $\lambda_{\tilde{\psi}}$ on $L(F, F-1)$ for which:*

1. *An open book decomposition (B_F, P_F) of $L(F, F-1)$ with abstract open book (A, D_F) is adapted to $\lambda_{\tilde{\psi}}$. Let A_0 denote the closure of the zero page. The return time of the Reeb flow from A_0 to A_0 is given by the action function f, and ψ is the return map of $(\lambda_{\tilde{\psi}}, B_F, P_F)$.*

2. *The binding orbits have action one, are elliptic, and have rotation numbers $\frac{1}{y_+}$ and $\frac{1}{-y_-+F}$ in the trivializations which have linking number zero with their component of B_F with respect to A_0.*

3. *Let $\{|B_F|\}$ denote the set of components of B_F. There is a bijection $\mathcal{P}(\psi) \cup \{|B_F|\} \to \mathcal{P}(\lambda_{\tilde{\psi}})$. The symplectic action of the Reeb orbit corresponding to $\gamma \in \mathcal{P}(\psi)$ is $\mathcal{A}(\gamma)$, and its intersection number with the page A_0 is $\ell(\gamma)$.*

4. *The contact volume satisfies $\mathrm{vol}(L(F, F-1), \lambda_{\tilde{\psi}}) = 2\mathcal{V}(\psi)$.*

Proof. Step 1 holds without change, and Steps 4-5 can be replaced with exact analogues. Replace Steps 2-3 with the following:

Step 2: The closed manifold

Consider the oriented coordinates (ρ_+, μ_+, t_+) and (ρ_-, t_-, μ_-) on the solid tori $\mathbb{T}_\pm = \mathbb{D}^2(\epsilon_\pm) \times (\mathbb{R}/2\pi\mathbb{Z})$, where $\rho_\pm \in [0, \epsilon_\pm]$ and $\mu_\pm \in \mathbb{R}/2\pi\mathbb{Z}$ are coordinates on $\mathbb{D}^2(\epsilon_\pm)$ and the coordinate on $\mathbb{R}/2\pi\mathbb{Z}$ is $t_\pm \in \mathbb{R}/2\pi\mathbb{Z}$. Let $g_\pm : \mathring{M}_\psi \to \mathbb{T}_\pm$ be given by

$$\begin{aligned} g_+(x, y, \theta) &= \left(\sqrt{1-x}, 2\pi\theta, y + 2\pi\theta y_+\right) \\ g_-(x, \theta, y) &= \left(\sqrt{x+1}, y + 2\pi\theta(y_- - F), 2\pi\theta\right), \end{aligned}$$

in oriented coordinates on both the domain and target. Because $F \in \mathbb{Z}$, the map g_- is well-defined.

Let Y_ψ denote the union of $\mathring{M}_\psi$ with the $\mathbb{T}_\pm$s via the $g_\pm$s.

Step 3: Open book decomposition

Denote by B_F the subset of Y_ψ where $\{\rho_\pm = 0\}$. Let $P_F : Y_\psi - B_F \to S^1$ be given by $(t, z) \mapsto t$. The preimages $P_F^{-1}(t)$ are diffeomorphic to $\mathring{A}$. We claim that P_F is a projection map for an open book decomposition with page A.

The meridional direction near the component of B_F corresponding to $\partial_\pm A$ is given by $\partial_{\mu_\pm}$, which extends to $\mathring{M}_\psi$ as $-y_+\partial_y + \frac{1}{2\pi}\partial_\theta$ near $\partial_+ A$ and $(-y_- + F)\partial_y + \frac{1}{2\pi}\partial_\theta$ near $\partial_- A$. The direction ∂_θ is transverse to the fibers of P_F. Choose smooth monotone interpolations

- $\delta_+ : [-1, 1] \to [-y_+, 0]$ with $\delta_+|_{[-1, 1-\epsilon_+^2]} = 0$ and $\delta_+(1) = -y_+$,
- $\delta_- : [-1, 1] \to [0, -y_- + F]$ with $\delta_-|_{[\epsilon_-^2 - 1, 1]} = 0$ and $\delta_-(-1) = -y_- + F$.

Let V be the vector field

$$V = (\delta_+(x) + \delta_-(x))\partial_y + \frac{1}{2\pi}\partial_\theta,$$

which is transverse to the pages of P_F and equals $\partial_{\mu_\pm}$ near B_F.

We claim that the return map of the flow of V from $P_F^{-1}(0)$ to itself is homotopic (relative to ∂A) to the F-fold right-handed Dehn twist D_F. Because the coefficient of ∂_θ in V is $\frac{1}{2\pi}$, it takes at least time 2π to send $P^{-1}(0)$ to itself. The return map of the time 2π flow of V near the $\partial_+ A$ component of $P^{-1}(0)$ is

$$(x, y, 0) \mapsto (x, y - 2\pi y_+, 1) \sim (x, y, 0),$$

while near the $\partial_- A$ component, the return map is

$$(x, 0, y) \mapsto (x, 1, y + 2\pi(-y_- + F)) \sim (x, 0, y + 2\pi F),$$

where we do not make the simplification $y + 2\pi F \sim y \in \mathbb{R}/2\pi\mathbb{Z}$ to emphasize the F-fold right-handed Dehn twist. □

Throughout the paper, $\tilde{p}$ should be replaced with F; below, we discuss only changes to notation, results, and proofs, and leave it to the reader to make the necessary changes to the connecting text.

2.2 Corrections to §5.2

From here on out, we assume $y_+ = y_- = y_0$. The correct version of [6, Lem. 5.5] is the following:

Lemma 2.2. *The rotation numbers of $e_\pm^F$ in the trivializations of $\ker \lambda_{\tilde{\psi}}$ which have linking number zero with $e_\pm^F$ with respect to their Seifert surfaces are $\frac{F}{y_0} - 1$ and $\frac{F}{-y_0 + F} - 1$.*

Proof. Replace $\tilde{p}$ with F in the proof of [6, Lem. 5.5]. □

The model contact forms constructed in [6, Prop. 5.4] and used later to compute the knot filtration only have the correct binding rotation numbers for both binding components when $y_+ = y_-$. In general, we can only expect one of the rotation numbers of $e_\pm$ to agree with those of Proposition 2.1. The corrected version is as follows:

Proposition 2.3. *If $\frac{F}{y_0} - 1, \frac{F}{-y_0+F} - 1 \in \mathbb{R} \setminus \mathbb{Q}$, there is a nondegenerate contact form $\lambda^F_{y_0}$ on $L(F, F-1)$ satisfying*

1. $\ker \lambda^F_{y_0}$ *and* $\ker \lambda_{\tilde{\psi}}$ *are contactomorphic.*
2. *Under the diffeomorphism of 1., the orbits $e_\pm$ of $\lambda_{\tilde{\psi}}$ are both also simple nondegenerate elliptic Reeb orbits for $\lambda^F_{y_0}$, and $\lambda^F_{y_0}$ has no other simple Reeb orbits.*
3. (a) *The nullhomologous cover e^F_+ of e_+ has rotation number $\frac{F}{y_0} - 1$ and as a Reeb orbit of λ^F_+ when computed in the trivialization of $\ker \lambda^F_+$ which has linking number zero with e^F_+ with respect to its Seifert surface S_+.*
   (b) *The nullhomologous cover e^F_- of e_- has rotation number $\frac{F}{-y_0+F} - 1$ as a Reeb orbit of λ^F_- when computed in the trivialization of $\ker \lambda^F_-$ which has linking number zero with e^F_- with respect to its Seifert surface S_-.*

Proof. The proof is identical to that of [6, Prop. 5.4], except we define

$$\mathfrak{q}_F^* \lambda^F_{y_0} = \lambda_{(1,b_0)},$$

where

$$b_0 := \frac{y_0}{-y_0 + F}.$$

□

The connecting text in the rest of §5.2 can be read as-is, replacing $\tilde{p}$ with F. We thus obtain a combinatorial chain complex for $ECC_*(L(F, F-1), \lambda^F_{y_0}, J)$, which we describe in the following way.

Proposition 2.4. 1. *The generators of $ECC_*(L(F, F-1), \lambda^F_{y_0}, J)$ correspond to points (d, m_+) in the second skew quadrant determined by the x-axis and the line $y = Fx$:*

$$(d, m_+) \leftrightarrow e_+^{m_+} e_-^{m_-}, \text{ where } \frac{m_+ - m_-}{F} =: d.$$

2. *There is a bijection between generators and $2\mathbb{Z}_{\geq 0}$ given by the order in which a line of slope y_0 moving northwest passes through the points in the second skew quadrant in 1.*

2.3 Corrections to §5.3

Note that by simple geometry, the y-coordinate of the y-intercept of the line through (d, m_+) of slope y_0 equals $f_\pm \mathcal{F}_{e_\pm}(e^{m_+} e^{m_-})$, where $f_+ = y_0$ and $f_- = -y_0 + F$, the values of the action function on $\partial_\pm A$. As in [6, §5.3], this can be used to prove the computation of the link filtration on the ECH of $L(F, F-1)$, which was proved but not stated in [6, Prop. 5.9] in the special case $y_+ = y_- = y_0$.

Proposition 2.5.

$$ECH_{2k}^{\mathcal{F}_{e_+} + \mathcal{F}_{e_-} \leq \ell}\Big(L(F, F-1), \xi_{\tilde{\psi}}, e_+, e_-, \mathrm{rot}(e_+), \mathrm{rot}(e_-)\Big) = \begin{cases} \mathbb{Z}/2\mathbb{Z} & \text{if } \ell \geq N_{w(k)}\left(\frac{1}{y_0}, \frac{1}{-y_0+F}\right) \\ 0 & \text{else} \end{cases}.$$

2.4 Corrections to §6.1

The identification of a Reeb orbit of $\lambda_{\tilde{\psi}}$ satisfying the necessary suite of numerical properties in [6, Prop. 6.1] must be corrected to the following.

Proposition 2.6. *Let λ be a contact form on $L(F, F-1)$ contactomorphic to the contact form λ_F from [6, Lem. 2.6]. Suppose that both binding components $b_\pm$ of the open book decomposition (H_F, Π_F) are elliptic and their nullhomologous covers $b_\pm^F$ have rotation numbers equal to those of $e_\pm^F$ as in Lemma 2.2. Then, for all $\epsilon > 0$, for all sufficiently large integers k there is an orbit set α_k not including either $b_\pm$ and nonnegative integers $m_{k,\pm}$ for which*

$$I(b_+^{m_{k,+}}\alpha_k b_-^{m_{k,-}}) = 2k$$

$$\mathcal{A}(\alpha_k) \leq \sqrt{2k(\mathrm{vol}(L(F,F-1),\lambda)+\epsilon)} - m_{k,+}\mathcal{A}(b_+) - m_{k,-}\mathcal{A}(b_-) \tag{2.1}$$

$$\alpha_k \cdot A_0 \geq N_{w(k)}\left(\mathrm{rot}(b_+) + \frac{1}{F}, \mathrm{rot}(b_-) + \frac{1}{F}\right) - m_{k,+}\,\mathrm{rot}(b_+) - m_{k,-}\,\mathrm{rot}(b_-). \tag{2.2}$$

Proof. The proof is very similar to that of [6, Prop. 6.1]. We outline the differences here.

The first step, which invokes the approximation of the contact volume by ECH capacities, is identical. Thus we can assume there is some k for which:

- there exists a cycle $x_k \in ECC_{2k}(L(F, F-1), \lambda, J)$ representing the generator of the group $ECH_{2k}(L(F, F-1), \ker \lambda_F)$,
- we may write $x_k = \sum_i x_{k_i}$, where each x_{k_i} is an admissible orbit set and the sum is finite,
- and for all i, the action is bounded: $\mathcal{A}(x_{k_i}) \leq \sqrt{2k\,\mathrm{vol}(L(F,F-1),\lambda)+\epsilon}$.

Writing $x_{k_i} = b_+^{m_{k_i,+}}\alpha b_-^{m_{k_i,-}}$, where α is an admissible orbit set not including either $b_\pm$, gives us (2.1) for each i.

Because the contact structures $\ker \lambda_\pm^F$ and $\ker \lambda$ are contactomorphic (all being contactomorphic to the model $\ker \lambda_F$), and $\mathrm{rot}(b_\pm^F) = \mathrm{rot}(e_\pm^F)$, Proposition 2.5 shows that there must be some i for which

$$\left(\mathcal{F}_{b_+} + \mathcal{F}_{b_-}\right)(x_{k_i}) \geq N_{w(k)}\left(\frac{1}{y_0}, \frac{1}{-y_0+F}\right),$$

from which (2.2) follows as in the original proof. □

3 Corrections to §6.2

In this section we correct error #2 explained in §1.2, keeping in mind the changes put in place by the corrections in the previous section.

The key argument in [6, Prop. 6.3], which transforms the Reeb orbit existence shown in Proposition 2.6 into an annulus periodic orbit existence result with an estimate involving $\mathcal{V}(\tilde{\psi})$, must be corrected to the following:

Proposition 3.1. *Let ψ be an area-preserving diffeomorphism of (A, ω) which is rotation by $2\pi y_0$ near ∂A, whose flux applied to the class of the $(x, 0)$ curve in $\tilde{A}$ is $F \in \mathbb{Z}$, whose action function f is positive, and where y_0 and $-y_0 + F$ are irrational. Further assume*

$$\mathcal{V}(\tilde{\psi}) < \frac{F\min\{y_0, -y_0+F\}}{2\max\{y_0, -y_0+F\}}.$$

Then

$$\inf\left\{\frac{\mathcal{A}(\gamma)}{\ell(\gamma)}\middle|\gamma\in\mathcal{P}(\psi)\right\}\le\sqrt{\mathrm{hm}(y_0,-y_0+F)(\mathcal{V}(\tilde{\psi}))}. \tag{3.1}$$

Proof. Note that the hypotheses imply that also $\frac{F}{y_0}-1$ and $\frac{F}{-y_0+F}-1$ are irrational, so we can apply Proposition 2.6.

The proof is the same until the line above [6, (6.13)]. Our goal remains [6, (6.15)]:

$$\frac{m_{k,+}}{y_0}+\frac{m_{k,-}}{-y_0+F}<\sqrt{\frac{2kF}{y_0(-y_0+F)}-c_1k^{1/2}+c_2}. \tag{3.2}$$

When k is large enough, we may ignore the contribution from $-c_1k^{1/2}+c_2$. Let

$$m=\min\{y_0,-y_0+F\}\quad\text{and}\quad M=\max\{y_0,-y_0+F\}.$$

By (2.1) and the fourth conclusion of Proposition 2.1, we have

$$m_{k,+}+m_{k,-}\le\sqrt{2k(2\mathcal{V}(\tilde{\psi})+\epsilon)}$$

as the action $\mathcal{A}(\alpha_k)$ is nonnegative. If we choose ϵ small enough, we obtain

$$m_{k,+}+m_{k,-}\le\sqrt{\frac{2kFm}{M}}\iff\frac{m_{k,+}}{m}+\frac{m_{k,-}}{m}\le\sqrt{\frac{2kF}{mM}},$$

which is stronger than our goal (3.2). The rest of the proof proceeds as before. □

Remark 3.2. (i) Note that although we no longer rely on [6, Lem. 6.4] to apply [6, Prop. 6.3] to prove Theorem 1.1, we still need it to apply Proposition 2.1.

(ii) One might wonder why we need to assume

$$\mathcal{V}(\tilde{\psi})<\frac{F\min\{y_0,-y_0+F\}}{2\max\{y_0,-y_0+F\}}$$

in light of [6, Lem. 6.4] when the left hand side of (3.2) is evidently asymptotic to $1/N$ while the right hand side is asymptotic to $1/\sqrt{N}$ and so eventually larger (here $N\in\mathbb{Z}$ indicates the difference in choices of lift $\tilde{\psi}$ of ψ to $\tilde{A}$). The problem is that when N increases, the three-manifold and its contact form from Proposition 2.1 both change. Because the actions of the Reeb orbits do not scale uniformly (the binding orbits always have action one while the actions of the other orbits increase by N), we have no control on how the $m_{k,\pm}$ might change with N.

When we use the action bound to replace the $m_{k,\pm}$ with a function of $\mathcal{V}(\tilde{\psi})$, we now have quantities that all vary similarly with N, but they are asymptotic to each other, hence the change in the hypothesis on $\mathcal{V}(\tilde{\psi})$.

The arguments in the rest of the paper may now be applied as written, with Cases 1(b), 2(a)(iii), 2(a)(iv), and 2(b) removed. Note that we also no longer have to lift the requirement $y_+-y_-\in\mathbb{Z}$, as that is not a hypothesis of Proposition 2.1 (though it is of its analogue [6, Prop. 3.1]).

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