

**Matrix chain multiplication:** A and B can be multiplied when number of row in A = number of column in B

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3 \times 2}$$

$$A \times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2 \times 2}$$

No. of multiplications = 12 (By observation).  
 $= 2 \times 3 \times 2 = 12$ . (by formulae).

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$$

Minimum multiplication to find  $A_1 A_2 A_3$ .

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$$

Minimum multiplication to find  $A_1 A_2 A_3$ .

$$(A_1 A_2) \cdot A_3$$

Dimensions  $(A_1 A_2) \cdot A_3$   
 $2 \times 4 \quad 4 \times 2$   
 Cost =  $2 \times 3 \times 4 = 24$   
 Resulting Dimension  $2 \times 4$   
 Cost =  $2 \times 4 \times 2 = 16$   
 $\therefore$  Total cost operations =  $24 + 16 = 40$

$$A_1 (A_2 A_3)$$

Dimensions  $A_1 \cdot (A_2 A_3)$   
 $2 \times 3 \quad 3 \times 2$   
 Cost =  $3 \times 4 \times 2 = 24$   
 Dimensions  $2 \times 3$   
 Cost =  $2 \times 3 \times 2 = 12$   
 $\therefore$  Total cost =  $24 + 12 = 36$

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$$

Minimum multiplication to find  $A_1 A_2 A_3$ .

$$c[1,1] = 0 \quad c[2,2] = c[3,3] = 0$$

$A_1 \quad A_2 \quad A_3$

$$c[1,2] = c[1,1] + c[2,2] + 2 \times 3 \times 4$$

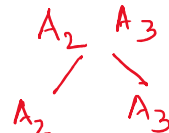
$$A_1 A_2 = 0 + 0 + 24 = 24$$



	1	2	3
1	0	24	
2		0	24
3			0

$$c[2,3] = c[2,2] + c[3,3] + 3 \times 4 \times 2$$

$$= 24$$



	1	2	3
1	0	24	36
2		0	24
3			0

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$$

Minimum multiplication to find  $A_1 A_2 A_3$ .

$$c[1,3] = c[1,1] + c[2,3] + 2 \times 3 \times 2 = 0 + 24 + 12 = 36$$

$A_1 A_2 A_3$

1	2	3
3		0

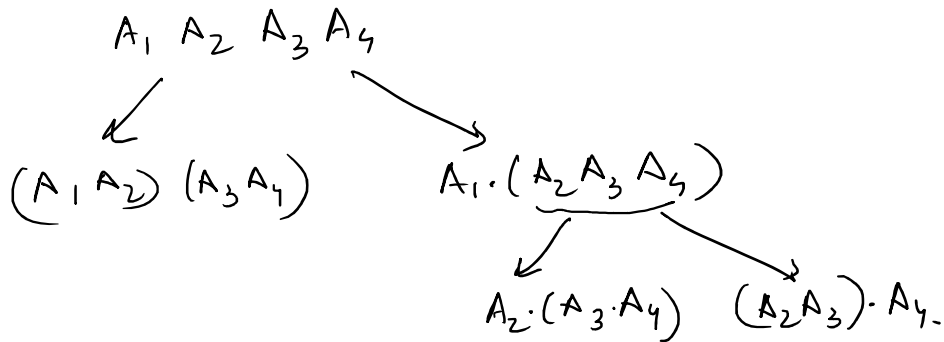
$$c[1,3] = c[1,1] + c[1,2] + \dots = 36.$$

$$A_1 A_2 A_3$$

$$\begin{aligned} & \swarrow \searrow \\ & (A_1 \cdot (A_2 A_3)) \quad ((A_1 A_2) \cdot A_3) \\ & \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ & 2 \times 3 \quad 3 \times 2 \quad 2 \times 4 \quad 4 \times 2 \end{aligned}$$

$$= c[1,2] + c[3,3] + 2 \times 4 \times 2$$

$$= 24 + 0 + 16 = 40.$$



Topic - Short Notes -

What ?? why ?? How ???

~~Short~~ Algorithm.

TC SC

Differences

Greedy & DP

DP & Divide & Conquer.

Difference between 2 sorting algos.

Prims vs Kruskal.

Dijkstra vs Floyd Warshall  
vs Bellman Ford.

LCS  $\rightarrow$   $\times$   
 $\Downarrow$   
Prepare.

Prims

Kruskal.