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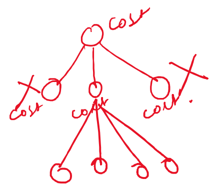
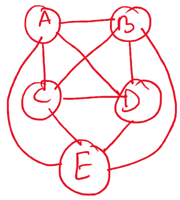
The **Travelling Salesman Problem (TSP)** is a classic **problem** in computer science and operations research. It's defined as:

Given: A list of cities and the distances between each pair of cities.

Goal: Find the shortest possible route that visits each city exactly once and returns to the starting city.

Branch & Bound -





A B C D E A
A D C E B A.

TSP appears in various real-world scenarios like Route planning (delivery trucks, sales routes)

Core Concepts

1. **Branching**: You build a **tree of subproblems**, where each node represents a partial tour (sequence of cities visited).
2. **Bounding**: At each node, you compute a **lower bound** (minimum possible cost to complete the tour from here).
3. **Pruning**: If a node's lower bound is worse than the best complete solution found so far, you discard (prune) that branch.

minimum length of route that covers all the cities.

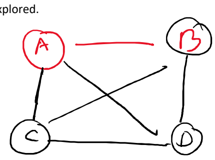
AB
BC
AC

Steps to Solve TSP with Branch and Bound:

1. **Start with a cost matrix** of distances between all cities.
2. **Reduce the matrix**:
   - Subtract the smallest value in each row and each column (this gives a *lower bound*).
3. **Create a priority queue (min-heap)** to explore promising nodes first (ones with smaller bounds).
4. **At each node**:
   - Choose a city to visit next.
   - Update the matrix to reflect the path chosen (remove rows/columns).
   - Recalculate the reduced cost and total bound.
5. **Prune** paths with bounds higher than the best known solution.
6. Repeat until all promising paths are explored.

$A \to A = 0$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 10 & 5 & 3 \\ B & 8 & \infty & 9 & 7 \\ C & 1 & 6 & \infty & 9 \\ D & 2 & 3 & 8 & \infty \end{array}$$

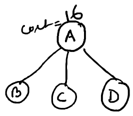


$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & 10 & 5 & 3 & -3 \\ B & 8 & \infty & 9 & 7 & -7 \\ C & 1 & 6 & \infty & 9 & -1 \\ D & 2 & 3 & 8 & \infty & -2 \\ \hline & & & & & 13 \end{array}$$

Row reduction && column Reduction.

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 7 & 2 & 0 \\ B & 1 & \infty & 2 & 0 \\ C & 0 & 5 & \infty & 8 \\ D & 0 & 1 & 6 & \infty \end{array} \Rightarrow M_A = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$

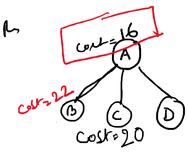
$0 + 1 + 2 + 0 = 3$



Red^n value $= 13 + 3 = 16$

$$M_A = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array} \rightarrow \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & \infty & \infty & 0 & 0 & - 0 \\ C & 0 & \infty & \infty & 8 & - 0 \\ D & 0 & \infty & 4 & \infty & - 0 \\ \hline & 0 & \infty & 0 & 0 & \end{array}$$

Row Red^n Col^n



$$M_{AB} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & \infty & \infty & 0 & 0 \\ C & 0 & \infty & \infty & 8 \\ D & 0 & \infty & 4 & \infty \end{array}$$

$cost(B) = cost(A) + Red^n + AB$

$= 16 + 0 + 6 = 22$

$$M_{AC} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & 1 & \infty & \infty & 0 & \to 0 \\ C & \infty & 0 & \infty & 4 & \to 4 \\ D & 0 & 0 & \infty & \infty & \to 0 \\ \hline & 0 & 0 & \infty & 0 & \end{array}$$

$Cost(C) = cost(A) + Red^n + AC$

$= 16 + 4 + 0$

$= 20$

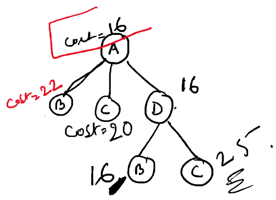
$$M_A = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$

$$M_{AD} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \leftarrow \infty \\ B & 1 & \infty & 0 & \infty & 0 \\ C & 0 & 4 & \infty & \infty & 0 \\ D & \infty & 0 & 4 & \infty & 0 \\ \hline & 0 & 0 & 0 & \infty & \end{array}$$

$Cost(D) = cost(A) + Red^n + AD$

$= 16 + 0 + 0 = 16$

$$M_{AD} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & 1 & \infty & 0 & \infty \\ C & 0 & 4 & \infty & \infty \\ D & \infty & 0 & 4 & \infty \end{array}$$



$M_{ADB} = ??$ $M_{ADC} = ???$

A B C D

$$\begin{array}{c|cccc} & & & & \\ A & \infty & \infty & \infty & \infty \\ B & \infty & \infty & 0 & \infty \\ C & 0 & \infty & \infty & \infty \\ D & \infty & \infty & \infty & \infty \end{array}$$

$cost(B) = cost(D) + Red^n + DB.$
$= 16 + 0 + 0 = 16.$

$M_{AD} =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & 1 & \infty & 0 & \infty \\ C & 0 & 4 & \infty & \infty \\ D & \infty & 0 & 4 & \infty \end{array}$$

$cost(C) = cost(D) + DC + Red^n$
$= 16 + 4 + 5$
$= 25.$

$M_{ADC} =$

$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & 0 & \infty & \infty & \infty & \to 1 \\ C & \infty & 0 & \infty & \infty & \to 4 \\ D & \infty & \infty & \infty & \infty & \to \infty \\ \hline & 1 & 0 & \infty & \infty & \\ & 0 & & & & \end{array}$$

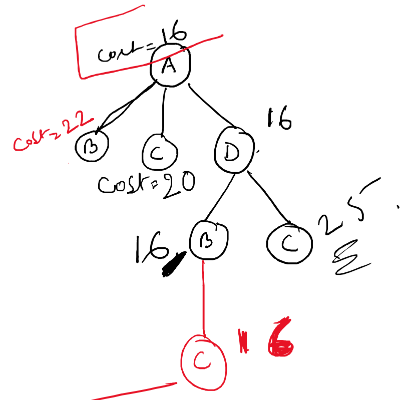
$M_{ADB} =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & \infty & \infty & 0 & \infty \\ C & 0 & \infty & \infty & \infty \\ D & \infty & \infty & \infty & \infty \end{array}$$

$M_{ADBC} =$

$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & \infty & \infty & \infty & \infty & \\ C & \infty & \infty & \infty & \infty & \\ D & \infty & \infty & \infty & \infty & \end{array}$$

$M_{ADBC} = = ???!$



$cost(C) = cost(B) + Red^n + BC.$
$= 16 + 0 + 0 = 16$

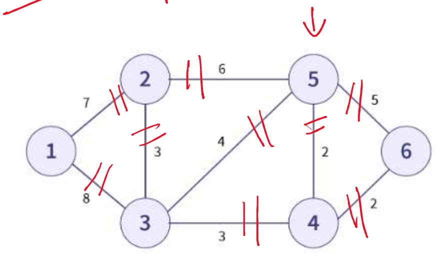
$Path = \underbrace{A \to D}_{3} \to B \to C \to A = 16$, with $\overbrace{D \to B}^{3}$, $\underbrace{B \to C}_{9}$, $\overbrace{C \to A}^{1}$

$3 + 3 + 9 + 1 = 16$

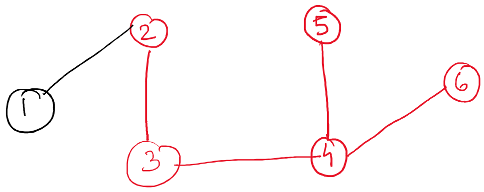
$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 10 & 5 & 3 \\ B & 8 & \infty & 9 & 7 \\ C & 1 & 6 & \infty & 9 \\ D & 2 & 3 & 8 & \infty \end{array}$$

Prim's.

Find MST



Graph G



Node from	Node T	Wt
5	2	6 × Loop.
5	3	4 × Loop.
5	6	5 × Loop.
5	4	2 ✓
4	3	3 ✓
4	6	2 ✓
3	1	8
3	2	3 ✓
2	1	7 ✓
		17