Matrix chain multiplication: A and B can be multiplied when number of row in B= number of column in A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A\times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

No. of multiplications = 12 (By observation).

$$= 2*3*2 = 12.$$ (by formulae).

$$A_1 = 2\times 3 \qquad A_2 = 3\times 4 \qquad A_3 = 4\times 2.$$

Minimum multiplication to find A1 A2 A3.

$$A_1 = 2\times 3 \qquad A_2 = 3\times 4 \qquad A_3 = 4\times 2.$$

Minimum multiplication to find A1 A2 A3.

$(A1 \cdot A2) \cdot A3$

$A1 (A2 \cdot A3).$

$(A_1 \ A_2) \cdot A_3$

| | | |
Dimensions 2×3 3×4 4×2
Cost- $2*3*4 = 24$
Revulting Dimension 2×4
  Cost = $2*4*2 = 16$
∴ Total cost operations $= 24 + 16 = 40.$

$A_1 \cdot (A_2 \cdot A_3)$

Dimensions 2×3 3×4 4×2
Cost $3*4*2 = 24$
Dimension $3\times 2.$
Cost = $2*3*2 = 12.$
∴ Total cost $= 24 + 12 = 36.$

$$A_1 = 2\times 3 \qquad A_2 = 3\times 4 \qquad A_3 = 4\times 2.$$

Minimum multiplication to find A1 A2 A3.

$$c[1,1] = A_1 \qquad c[2,2] = A_2$$

$$c[1,2] = A_1 A_2$$

$$c[2,3] = A_2 A_3.$$

$$c[1,3] = A_1 A_2 A_3.$$

	1	2	3
1	0	24	36
2		⬤	24
3			0

$c[1,3] = A_1 A_2 A_3.$

$c[1,1] = A_1 \qquad =$

$c[2,1] = A_2 A_1$

$c[2,3] = A_2 A_3$

$\underbrace{c[2,2]}_{0} + \underbrace{c[3,3]}_{0} + \underbrace{cost}_{24}$

$= 24$

$c[1,2] = A_1 A_2 \qquad \underbrace{24}_{2*3*4}$

$\underbrace{c[1,1]}_{0} + \underbrace{c[2,2]}_{0} + cost.$

$= 24$

$A = 2 \times 3 \qquad A_2 = 3 \times 4 \qquad A_3 = 4 \times 2$

$c[1,3]$

$A_1 A_2 A_3$

$(A_1 A_2) A_3 \qquad A_1 (A_2 A_3)$

$\overset{2 \times 4}{(A_1 A_2)} A_3$

$= c[1,2] + c[3,3] +$

$\underbrace{cost}_{2*4*2}$

$= 24 + 0 + 16$

$= 40.$

$\overset{2\times3 \quad 3\times2}{A_1 (A_2 A_3)} = c[1,1] + c[2,3] + \underbrace{cost}_{\substack{2*3*2. \\ 12.}}$

$\qquad\qquad 0 \qquad 24$

$= \boxed{36}.$