

Matrix chain multiplication: A and B can be multiplied when number of row in B = number of column in A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3 \times 2}$$

$$A \times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2 \times 2}$$

No. of multiplications = 12 (By observation).
 $= 2 \times 3 \times 2 = 12$ (by formula).

$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$

Minimum multiplication to find $A_1 A_2 A_3$.

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Minimum multiplication to find $A_1 A_2 A_3$.

$(A_1 A_2) \cdot A_3$

$A_1 (A_2 A_3)$

Dimensions: $(A_1 A_2) \cdot A_3$
 $2 \times 3 \quad 3 \times 4 \quad 4 \times 2$
 Cost: $2 \times 3 \times 4 = 24$
 Resulting Dimension: 2×4
 Cost: $2 \times 4 \times 2 = 16$

Dimensions: $A_1 \cdot (A_2 A_3)$
 $2 \times 3 \quad 3 \times 4 \quad 4 \times 2$
 Cost: $3 \times 4 \times 2 = 24$
 Dimension: 3×2
 Cost: $2 \times 3 \times 2 = 12$

\therefore Total cost operations = $24 + 16 = 40$. \therefore Total cost = $24 + 12 = 36$.

Min^m multiplication opt = $\min(\text{option 1, option 2})$
 $= 36$

$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$

Minimum multiplication to find $A_1 A_2 A_3$.

$c[1,1] = A_1$

$c[1,2] = A_1 A_2$

$c[1,3] = A_1 A_2 A_3$

$c[2,3] = A_2 A_3$

$c[2,1] = A_2 A_1$

$c[1,2] =$

$A_1 A_2$

$c[1,1] + c[2,2] + \text{cost}$

$c[2,3] = A_2 A_3$

$c[2,2] + c[3,3] + 3 \times 4 \times 2$
 $= 0 + 0 + 24 = 24$

	1	2	3
1	0	24	36
2		0	24
3			0

$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$

$c[1,3]$

$A_1 A_2 A_3$

$(A_1 A_2) A_3$

$A_1 (A_2 A_3)$

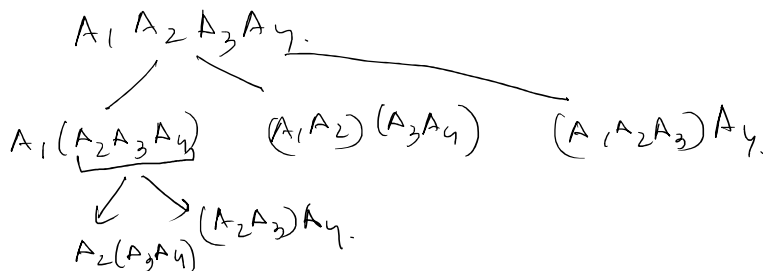
$(A_1 A_2) A_3$

$= c[1,2] + c[3,3] +$

cost

$\rightarrow 2 \times 4 \times 2$

$= 24 + 0 + 16 = 40$



3×2
/ \
 $(2A_3)$.

$$+ c[2,3] + 2 \cdot 3 \times 2$$

$$24 + 12 = 36.$$