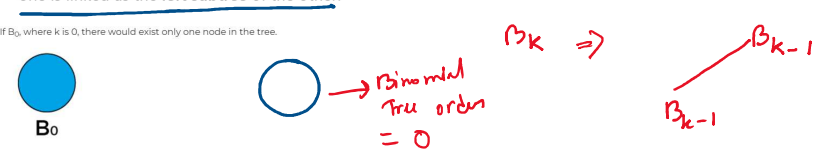
Binomial heap S section

13 February 2025 10:19

A **Binomial Tree** B_k is an ordered tree defined recursively, where k represents the order of the binomial tree.

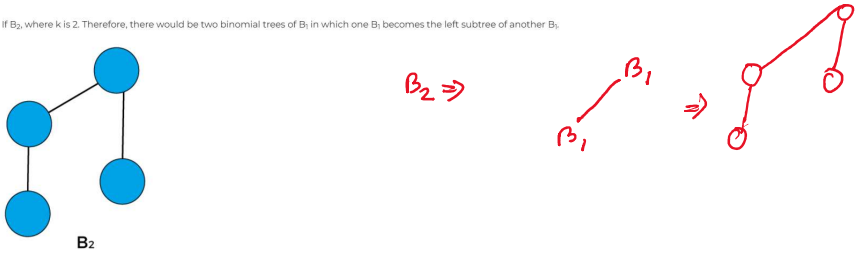
- If the binomial tree is of order **0** (B_0), it consists of a single node.
- In general, a binomial tree of order **k** (B_k) consists of **two binomial trees** of order $k-1$, where one is linked as the **left subtree** of the other.

If B_0, where k is 0, there would exist only one node in the tree.

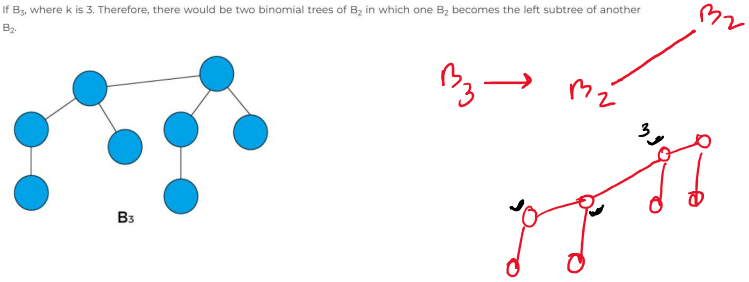


If B_1, where k is 1. Therefore, there would be two binomial trees of B_0 in which one B_0 becomes the left subtree of another B_0.

If B_2, where k is 2. Therefore, there would be two binomial trees of B_1 in which one B_1 becomes the left subtree of another B_1.

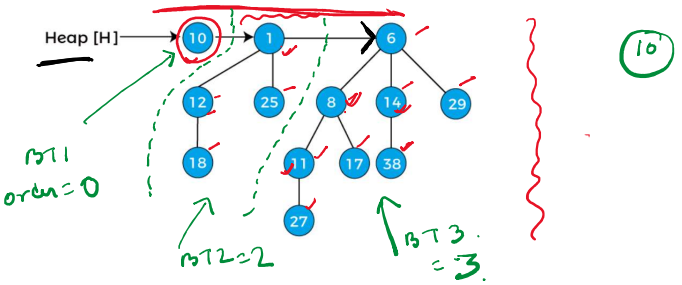


If B_3, where k is 3. Therefore, there would be two binomial trees of B_2 in which one B_2 becomes the left subtree of another B_2.



A binomial heap is a collection of binomial trees that satisfies the following binomial heap properties:

1. No two binomial trees in the collection have the same order.
2. Every binomial tree in the heap must follow the min-heap property, i.e., the value of a child node is greater than parent node.



Binomial Heap Union Operation

To perform the union of two binomial heaps, we have to consider the below cases -

Case 1: If degree[x] is not equal to degree[next x], then move pointer ahead.

Case 2: if degree[x] = degree[next x] = degree[sibling(next x)] then,

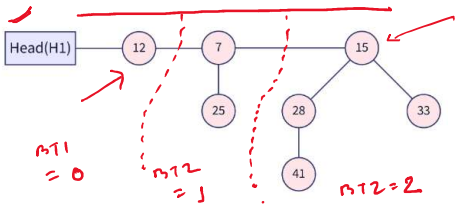
Move the pointer ahead.

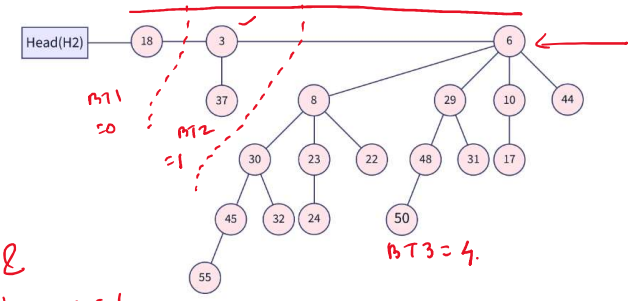
Case 3: If degree[x] = degree[next x] but not equal to degree[sibling[next x]]

and key[x] < key[next x] then remove [next x] from root and attached to x.

Case 4: If degree[x] = degree[next x] but not equal to degree[sibling[next x]]

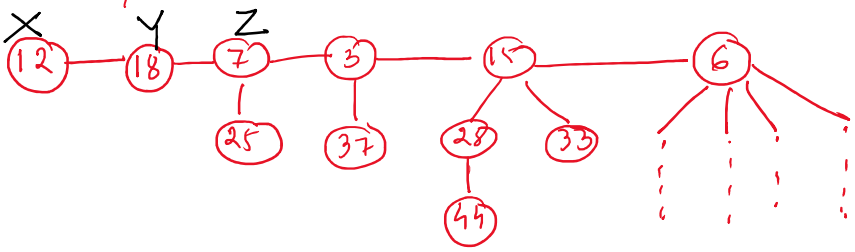
and key[x] > key[next x] then remove x from root and attached to [next x].





0, 1, 2
0, 1, 4 ⇒ Merge the orders & write in increasing order.

0, 0, 1, 1, 2, 4.



Algo – ① Take 3 pointers X, Y, Z

① deg(X), deg(Y), deg(Z).

① deg(x) = deg(Y) = deg(Z) → Move X, Y, Z to right direction by 1 place.

② deg(X) ≠ deg(Y) → Move X, Y, Z to right direction by 1 place.

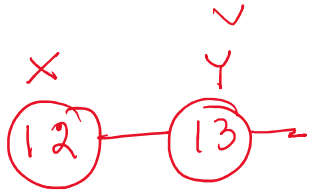
③ deg(X) = deg(Y) ≠ deg(Z)

→ key(X) <= key(y)

X → left = Y

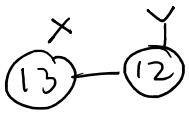
→ key(x) > key(y)

Y → left = X



key(x) key(y)

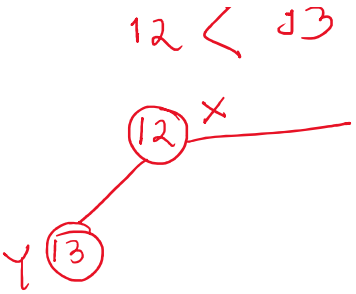
12 < 13

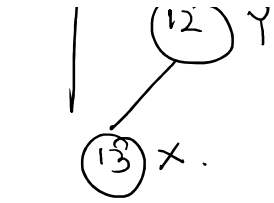


key(x) > key(y)

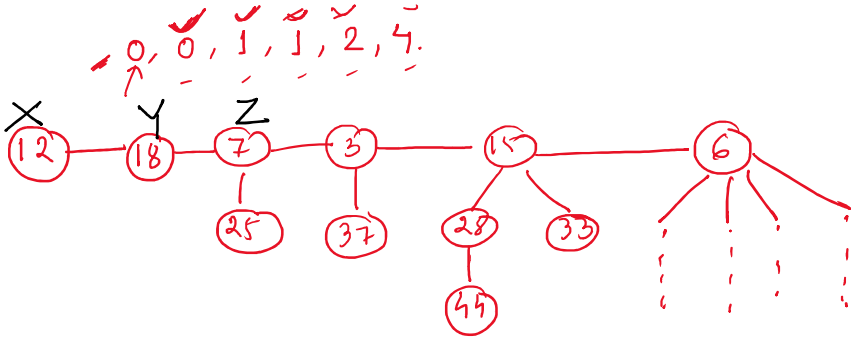
$y \rightarrow left = x$ }.

$12 < 13$



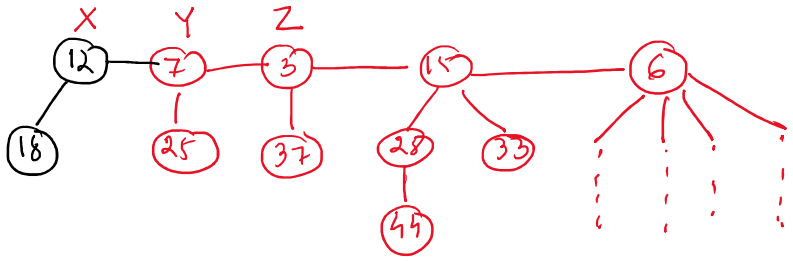


0, 0, 1, 1, 2, 4.

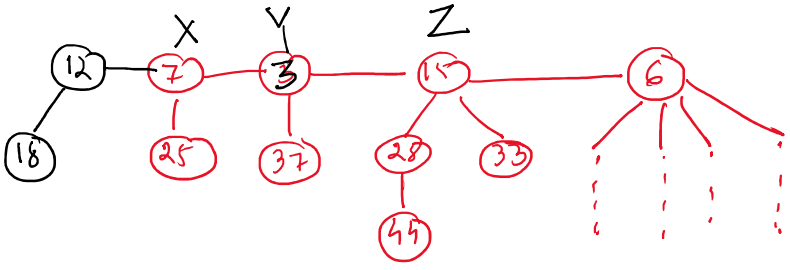


$deg(x) = 0 \quad deg(y) = 0 \quad deg(z) = 1. \quad 0 = 0 \neq 1$

$key(x) = 12 \quad key(y) = 18 \quad key(x) < key(y). \quad x \rightarrow left = y.$



1 1 1 2 4



1 1 1 2 4

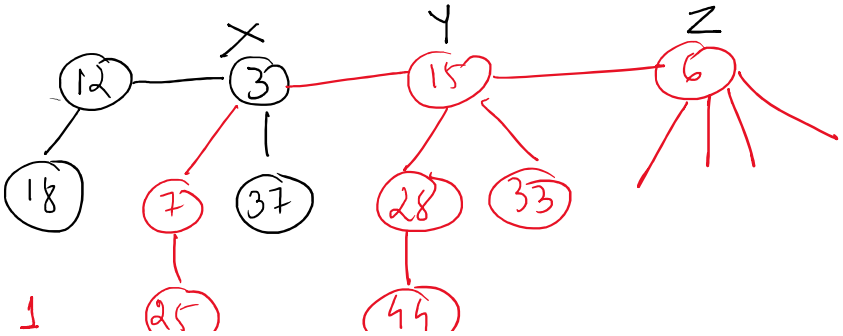
$1 = 1 \neq 2$

$key(x) = 7$

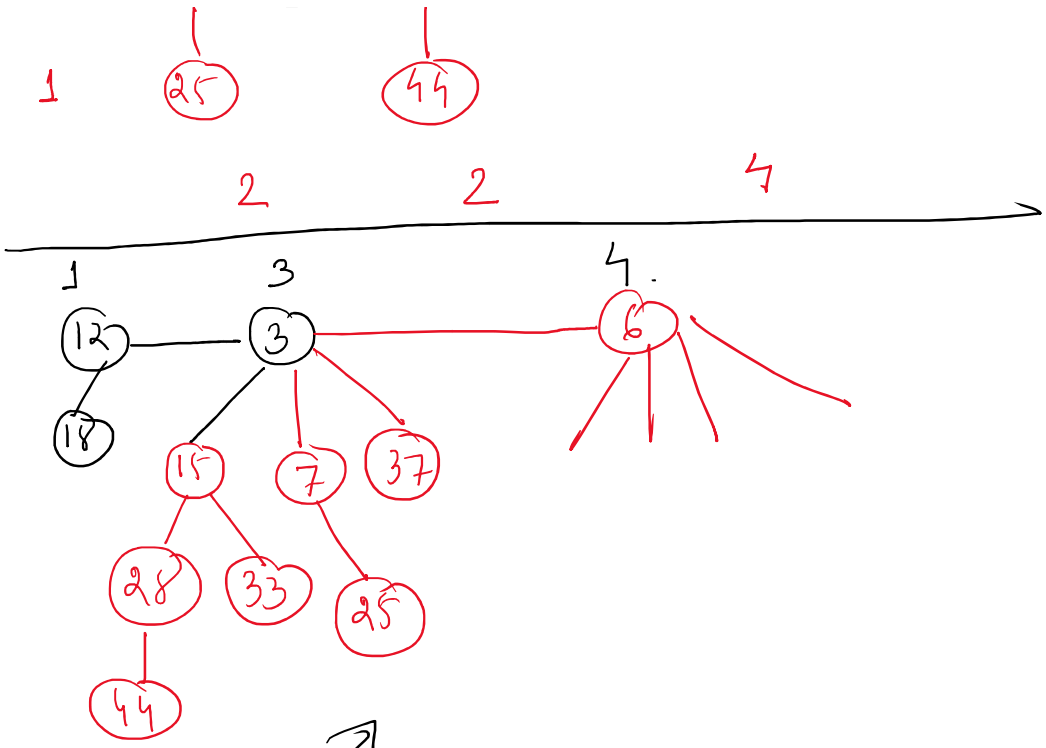
$key(y) = 3.$

$key(x) > key(y)$

$y \rightarrow left = x.$



1



$2 = 2 \neq 4$

key(x) < key(y)

3 15

Binomial heap.

H/W

Bucket Sort — Imp

Shell sort

B-tree deletion