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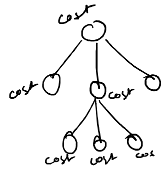
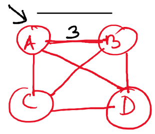
The **Travelling Salesman Problem (TSP)** is a classic **combinatorial optimization problem** in computer science and operations research. It's defined as:

Given: A list of cities and the distances between each pair of cities.

Goal: Find the shortest possible route that visits each city exactly once and returns to the starting city.

Branch && Bound.





$A - A = \infty$,

$A - B = 3$

$A - C$

Min^m distance

A B C D A

A C B D A

A C D B A,

TSP appears in various real-world scenarios like Route planning (delivery trucks, sales routes)

Core Concepts

1. **Branching**: You build a **tree of subproblems**, where each node represents a partial tour (sequence of cities visited).
2. **Bounding**: At each node, you compute a **lower bound** (minimum possible cost to complete the tour from here).
3. **Pruning**: If a node's lower bound is worse than the best complete solution found so far, you discard (prune) that branch.

Steps to Solve TSP with Branch and Bound:

1. **Start with a cost matrix** of distances between all cities.
2. **Reduce the matrix**:
   - Subtract the smallest value in each row and each column (this gives a *lower bound*).
3. **Create a priority queue (min-heap)** to explore promising nodes first (ones with smaller bounds).
4. **At each node**:
   - Choose a city to visit next.
   - Update the matrix to reflect the path chosen (remove rows/columns).
   - Recalculate the reduced cost and total bound.
5. **Prune** paths with bounds higher than the best known solution.
6. Repeat until all promising paths are explored.

$$
\begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} \infty & 10 & 5 & 3 \\ 8 & \infty & 9 & 7 \\ 1 & 6 & \infty & 9 \\ 2 & 3 & 8 & \infty \end{bmatrix}
\end{array}
$$

Row Reduction, Column Reduction.

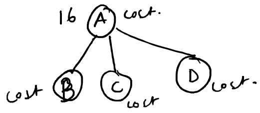
$$
\begin{array}{c c c}
 & \begin{array}{cccc} A & B & C & D \end{array} & \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} \infty & 10 & 5 & 3 \\ 8 & \infty & 9 & 7 \\ 1 & 6 & \infty & 9 \\ 2 & 3 & 8 & \infty \end{bmatrix} &
\begin{array}{l} \to 3 + \\ \to 7 + \\ \to 1 + \\ \to 2 \\ \hline 13 \end{array}
\end{array}
$$

$$
\begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} \infty & 7 & 2 & 0 \\ 1 & \infty & 2 & 0 \\ 0 & 5 & \infty & 8 \\ 0 & 1 & 6 & \infty \end{bmatrix} \\
 & \begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \end{array}
\end{array}
$$

$0 + 1 + 2 + 0 = 3$

$$
M_A = \begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} \infty & 6 & 0 & 0 \\ 1 & \infty & 0 & 0 \\ 0 & 4 & \infty & 8 \\ 0 & 0 & 4 & \infty \end{bmatrix}
\end{array}
$$

$cost(A) = 16$

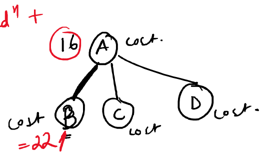


$$
M_{AB} = \begin{array}{c c c}
 & \begin{array}{cccc} A & B & C & D \end{array} & \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 0 \\ 0 & \infty & \infty & 8 \\ 0 & \infty & 4 & \infty \end{bmatrix} &
\begin{array}{l} \to \infty \\ \to 0 \\ \to 0 \\ \to 0 \end{array} \\
 & \begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & \infty & 0 & 0. \end{array} &
\end{array}
$$

$cost(B) = cost(A) + Red^n + AB$

$= 16 + 0 + 6$

$= 22$



$$
M_{AC} = \begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 \\ \infty & & \infty & \\ & & \infty & \end{bmatrix}
\end{array}
$$

$$
M_A = \begin{array}{c c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} \infty & 6 & 0 & 0 \\ 1 & \infty & 0 & 0 \\ 0 & 4 & \infty & 8 \\ 0 & 0 & 4 & \infty \end{bmatrix}
\end{array}
$$