Matrix chain multiplication: A and B can be multiplied when number of row in B= number of column in A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A\times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

No. of multiplications = 12 (By observation).

$$= 2\times 3\times 2 = 12 \quad \text{(by formulae)}.$$

$A_1 = 2\times 3 \qquad A_2 = 3\times 4 \qquad A_3 = 4\times 2.$

Minimum multiplication to find A1 A2 A3.

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Minimum multiplication to find A1 A2 A3.

$(A1 . A2) . A3$ 　　　　　　　　$A1 (A2 . A3).$

	$(A_1 \quad A_2) . A_3$
Dimensions	$2\times 3 \quad 3\times 4 \quad 4\times 2$
Cost -	24
Resulting Dimension	2×4
Cost =	$2\times 4\times 2 = 16$

∴ Total cost operations = 40

$A_1 \cdot (A_2 \cdot A_3)$

Dimensions $\quad 2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost $\qquad\qquad\qquad 3\times 4\times 2 = 24$

Dimension $\qquad\qquad\qquad\qquad 3\times 2$

Cost = $\quad 2\times 3\times 2 = 12$

∴ Total cost = 24+12 = 36.

$A_1 = 2\times 3 \qquad A_2 = 3\times 4 \qquad A_3 = 4\times 2.$

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∴ Total cost = 24+12 = 36.

	1	2	3
1	0	24	
2		0	24
3			0

$c[1,1] = 0$ 　　　　$\boxed{A_1 \quad A_2}$　$2\times 3 \quad 3\times 4$

$c[1,2] = (A_1)(A_2)$

$= c[1,1] + c[2,2] + 2\times 3\times 4$

$= 0 + 0 + 24 = 24$

$\quad\quad\quad\quad A_2 \quad\quad A_3$

$c[2,3] = c[2,2] + c[3,3]$

$\qquad\qquad + 3\times 4\times 2$

$\qquad\qquad = 0 + 0 + 24$

$\qquad\qquad = 24$

$\longrightarrow (A_1 A_2) . (A_3)$

$c[1,3] = c[1,2] + c[3,3] + 2\times 4\times 2$

$\quad\quad\quad\quad A_1 A_2 A_3 \qquad 2\times 4 \quad 4\times 2$

$\qquad\qquad = 24 + 0 + 16$

$A_1 = 2\times 3 \qquad A_2 = 3\times 4 \qquad A_3 = 4\times 2.$

	1	2	3
1	0	24	36
2		0	24
3			0

	0	24
3		0

$= 24 + 0 + 16$

$= 40.$

$\rightarrow (A_1)(A_2 \cdot A_3)$

2×3 3×2

$= c[1,1] + c[2,3] + 2\times3\times2$

$= 0 + 24 + 12$

$= 36.$

$A_1 \, A_2 \, A_3 \, A_4$

$(A_1 A_2)(A_3 A_4)$ $A_1(A_2 A_3 A_4)$

$A_2(A_3 A_4) \quad (A_2 A_3)A_4$

$A_1 \, A_2 \, A_3$

$c[2,1]$ $(A_2 A_1$