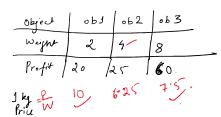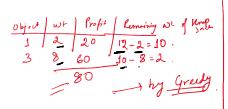In the 0-1 Knapsack Problem, we are given a Knapsack or a Bag that can hold weight up to a certain value. We have various items that have different weights and values associated with them. Now we have to fill the knapsack in such a way so that the sum of the total weights of the filled items does not exceed the maximum capacity of the knapsack and the sum of the values of the filled items is maximum.

Given a Knapsack with maximum weight limit as W and two arrays value[] and weight[]. You have to fill the knapsack in such a way so that the total weight of the filled items is less than or equal to W and the sum of the values of the filled items is maximum. value[i] and weight[i] will store the value and weight associated with ith item. You can not partially fill an item in the knapsack.
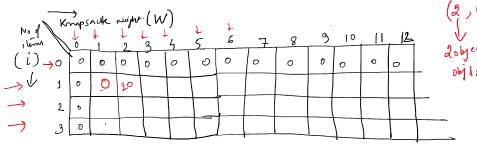
0 — Don't take   1 — Take.

Object	ob 1	ob 2	ob 3
Weight	2	4	8
Profit	20	25	60

Knapsack capacity = 12 kg

$\frac{1 \text{ kg}}{\text{Price}} = \frac{P}{W}$ 10   6.25   7.5

Pumpkin = 2 kg

Fractional

Knapsack = 1kg   Knapsack = 1 kg

Object	wt	Profit	Remaining wt of Knapsack
1	2	20	12 - 2 = 10
3	8	60	10 - 8 = 2
		= 80	

→ by Greedy

By observation —
obj 2 + obj 3
4 + 8 = 12 kg
25 + 60 = 85

Not Taking —

No. of items   Current capacity of Knapsack

$V[i, w] = MAX \begin{cases} V[i-1, w], & \rightarrow \text{Not Taken} \\ V[i-1, w - weight[i]] + profit[i] & \rightarrow \text{Taken} \end{cases}$

0-1 Knapsack.

Object	ob 1	ob 2	ob 3
Weight	2	4	8
Profit	20	25	60

Knapsack Capacity = W = 12 kg

$(2, 6)$.
2 objects
obj 1, obj 2 → 6 kg.

Knapsack weight (W)

No of items (i)

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	20										
2	0												
3	0												

$V[i, W] = Max \left(V[i-1, w], \rightarrow \geq 0. \right.$
$\left. V[i-1, w - weight[i]] + \right.$
profit [i])

$V[1,1] = Max \left(\begin{array}{l} V[0, 1] = 0, \\ V[0, 1-2] \end{array} \right.$

$V[1,2] = Max \left(\begin{array}{l} V[0, 2] = 0, \end{array} \right.$

$$V[i-1, \boxed{w - weight[i]}] + profit[i])$$

$$V[1,2] = Max \begin{cases} V[0, 1-2 \\ V[0,2] = 0, \\ V[0, 2-2] + profit[1]) \\ = 0 + 20 = 20 \end{cases}$$

i\w	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	20	20	20	20	20	20	20	20	20	20	20
2	0	0	20	20	25	25	45	45	45	45	45	45	45
3	0	0	20	20	25	25	45	45	60	60	65	65	85

object.

Take   Not Taken.

$W >= weight[object]$ False

Take   Cannot Take-