

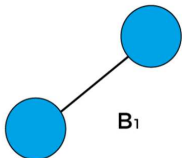
A Binomial Tree B_k is an ordered tree defined recursively, where k represents the order of the binomial tree.

- If the binomial tree is of order 0 (B_0), it consists of a single node.
- In general, a binomial tree of order k (B_k) consists of two binomial trees of order $k - 1$, where one is linked as the left subtree of the other.

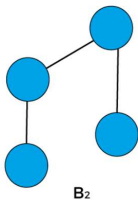
If B_0 , where k is 0, there would exist only one node in the tree.



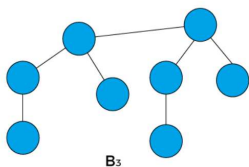
If B_1 , where k is 1. Therefore, there would be two binomial trees of B_0 in which one B_0 becomes the left subtree of another B_0 .



If B_2 , where k is 2. Therefore, there would be two binomial trees of B_1 in which one B_1 becomes the left subtree of another B_1 .

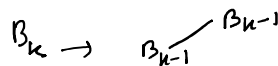
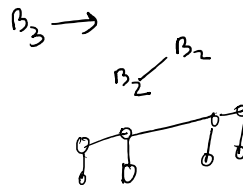
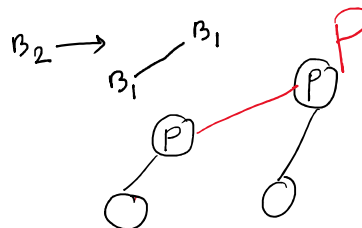
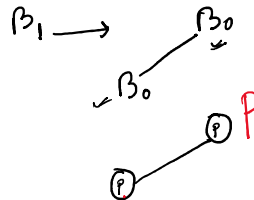


If B_3 , where k is 3. Therefore, there would be two binomial trees of B_2 in which one B_2 becomes the left subtree of another B_2 .



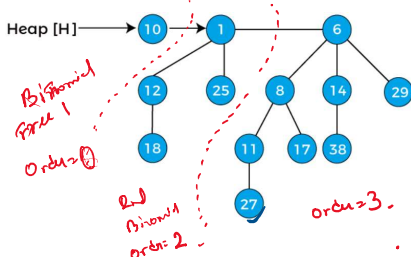
B_k .

$(P) \rightarrow$ Binomial Tree
Order = 0.



A binomial heap is a collection of binomial trees that satisfies the following binomial heap properties:

1. No two binomial trees in the collection have the same order.
2. Every binomial tree in the heap must follow the min-heap property, i.e., the value of a child node is greater than parent node.



Binomial Heap Union Operation

To perform the union of two binomial heaps, we have to consider the below cases -

Case 1: If $\text{degree}[x]$ is not equal to $\text{degree}[\text{next } x]$, then move pointer ahead.

Case 2: if $\text{degree}[x] = \text{degree}[\text{next } x] = \text{degree}[\text{sibling}(\text{next } x)]$ then,

Move the pointer ahead.

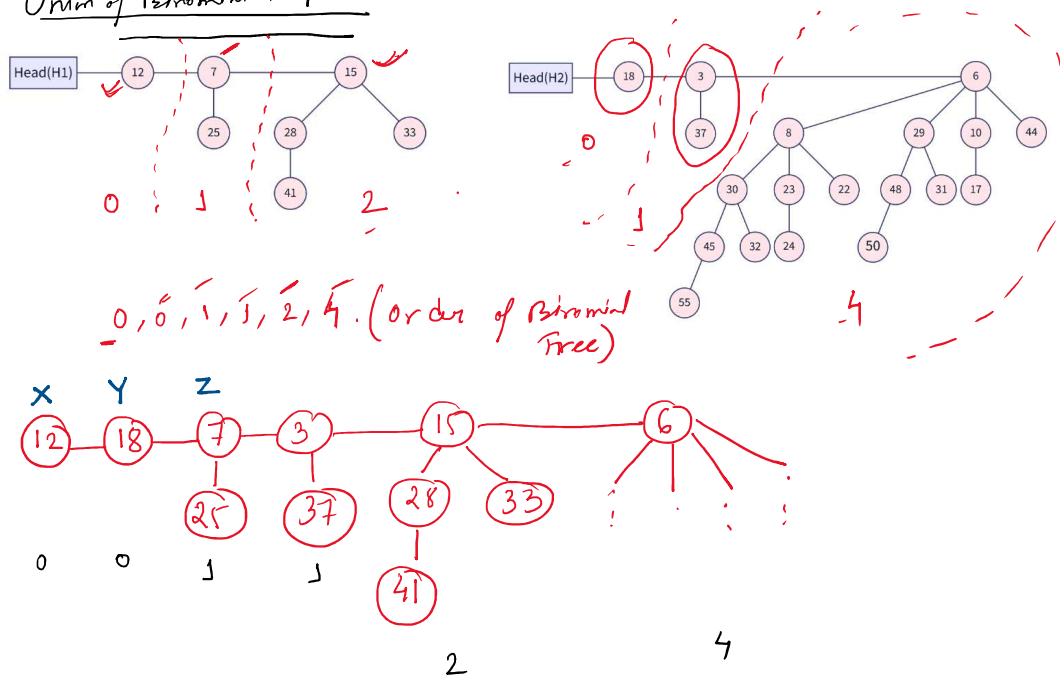
Case 3: If $\text{degree}[x] = \text{degree}[\text{next } x]$ but not equal to $\text{degree}[\text{sibling}(\text{next } x)]$

and $\text{key}[x] < \text{key}[\text{next } x]$ then remove $[\text{next } x]$ from root and attached to x .

Case 4: If $\text{degree}[x] = \text{degree}[\text{next } x]$ but not equal to $\text{degree}[\text{sibling}(\text{next } x)]$

and $\text{key}[x] > \text{key}[\text{next } x]$ then remove x from root and attached to $[\text{next } x]$.

Union of Binomial Heaps.



Algo - ① $\text{deg}(x)$, $\text{deg}(y)$, $\text{deg}(z)$.

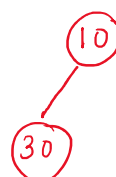
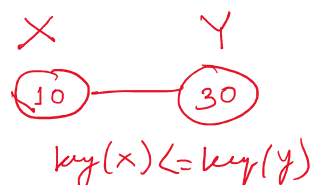
① $\text{deg}(x) = \text{deg}(y) = \text{deg}(z) \rightarrow$ Move x, y, z to right direction by 1 place.

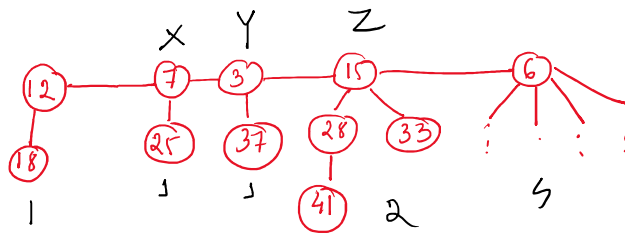
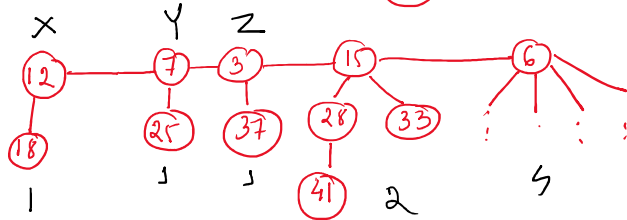
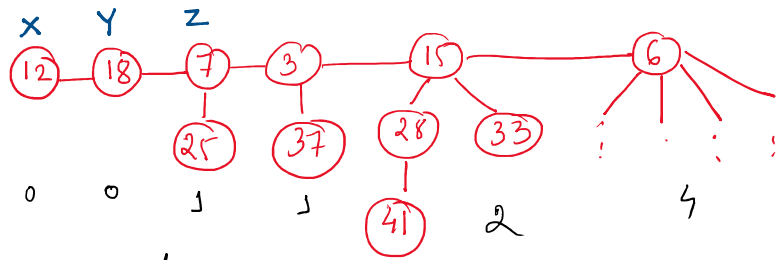
② $\text{deg}(x) \neq \text{deg}(y) \rightarrow$ Move x, y, z to right direction by 1 place.

③ $\text{deg}(x) = \text{deg}(y) \neq \text{deg}(z)$

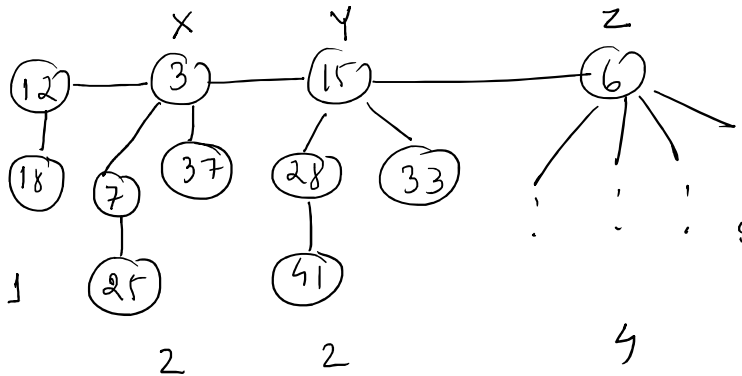
$\rightarrow \text{key}(x) \leq \text{key}(y)$
 $\underline{X \rightarrow \text{left} = Y.}$

$\text{key}(x) > \text{key}(y)$
 $\underline{Y \rightarrow \text{left} = X}$

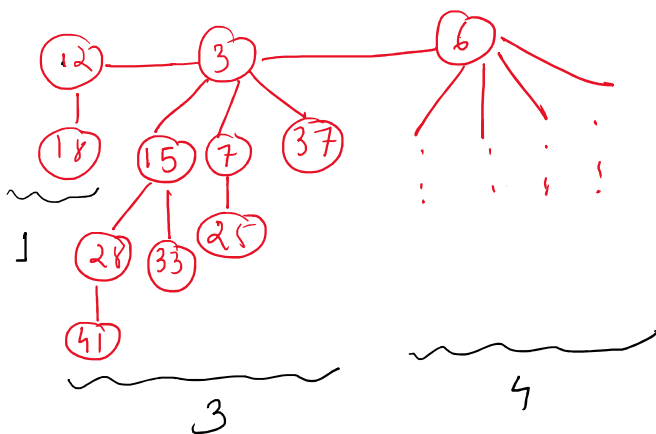




$key(x)$ $key(y)$



$key(x)$ $key(y)$



Topic to be covered —

- Shell Sort
- Bucket Sort
- n - relation.

- Bucket Sort
- B Tree Deletion.