ABAIAJ

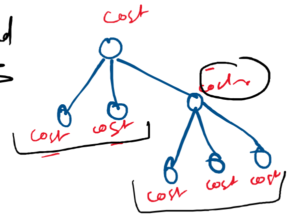
15 April 2025 10:05

The **Travelling Salesman Problem (TSP)** is a classic **optimization problem** in computer science and operations research. It's defined as:

Given: A list of cities and the distances between each pair of cities.

Goal: Find the shortest possible route that visits each city exactly once and returns to the starting city.

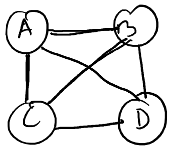
Branch & Bound



TSP appears in various real-world scenarios like Route planning (delivery trucks, sales routes)

Core Concepts

1. **Branching**: You build a **tree of subproblems**, where each node represents a partial tour (sequence of cities visited).
2. **Bounding**: At each node, you compute a **lower bound** (minimum possible cost to complete the tour from here).
3. **Pruning**: If a node's lower bound is worse than the best complete solution found so far, you discard (prune) that branch.



A B C D A.
A D C B A
A B D C A
B C D A B.
,
.
.
.

Steps to Solve TSP with Branch and Bound:

1. **Start with a cost matrix** of distances between all cities.
2. **Reduce the matrix**:
   - Subtract the smallest value in each row and each column (this gives a *lower bound*).
3. **Create a priority queue (min-heap)** to explore promising nodes first (ones with smaller bounds).
4. **At each node**:
   - Choose a city to visit next.
   - Update the matrix to reflect the path chosen (remove rows/columns).
   - Recalculate the reduced cost and total bound.
5. **Prune** paths with bounds higher than the best known solution.
6. Repeat until all promising paths are explored.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & 10 & 5 & 3 \\ 8 & \infty & 9 & 7 \\ 1 & 6 & \infty & 9 \\ 2 & 3 & 8 & \infty \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & 10 & 5 & 3 \\ 8 & \infty & 9 & 7 \\ 1 & 6 & \infty & 9 \\ 2 & 3 & 8 & \infty \end{bmatrix} \end{array}$$

Row Red^n, Col Red^n.

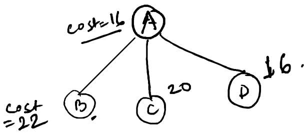
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & 10 & 5 & 3 \\ 8 & \infty & 9 & 7 \\ 1 & 6 & \infty & 9 \\ 2 & 3 & 8 & \infty \end{bmatrix} \end{array}\begin{array}{l} \\ \to 3 \\ \to 7 \\ \to 1 \\ \to 2 \\ \hline 13 \end{array} \qquad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & 7 & 2 & 0 \\ 1 & \infty & 2 & 0 \\ 0 & 5 & \infty & 8 \\ 0 & 1 & 6 & \infty \end{bmatrix} \\ \begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \end{array} \end{array} \qquad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & 6 & 0 & 0 \\ 1 & \infty & 0 & 0 \\ 0 & 4 & \infty & 8 \\ 0 & 0 & 4 & \infty \end{bmatrix} \end{array}$$

$0 + 1 + 2 + 0 = 3.$

$M_A =$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & 6 & 0 & 0 \\ 1 & \infty & 0 & 0 \\ 0 & 4 & \infty & 8 \\ 0 & 0 & 4 & \infty \end{bmatrix} \end{array}$$

cost(A) = 13 + 3 = 16.



$M_{AB} =$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 0 \\ 0 & \infty & \infty & 8 \\ 0 & \infty & 4 & \infty \end{bmatrix} \\ \begin{array}{cccc} 0 & \infty & 0 & 0 \end{array} \end{array}\begin{array}{l} \\ \to \infty \\ \to 0 \\ \to 0 \\ \to 0. \\ \\ \end{array}$$

cost(B) = cost(A) + Reduction + AB.

= 16 + 0 + 6 = 22.

$$M_{AC} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 \\ \infty & 0 & \infty & 4 \\ 0 & 0 & \infty & \infty \end{bmatrix} \\ \begin{array}{cccc} 0 & 0 & \infty & 0 \end{array} \end{array}\begin{array}{l} \\ \to \infty \\ \to 0 \\ \to 4 \\ \to 0 \\ \hline 4 \end{array}$$

cost(C) = cost(A) + Red^n + AC.

= 16 + 4 + 0 = 20.

$M_A =$

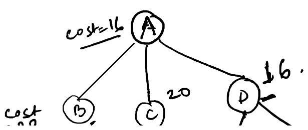
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & 6 & 0 & 0 \\ 1 & \infty & 0 & 0 \\ 0 & 4 & \infty & 8 \\ 0 & 0 & 4 & \infty \end{bmatrix} \end{array}$$

$$M_{AD} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty \\ 0 & 4 & \infty & \infty \\ \infty & 0 & 4 & \infty \end{bmatrix} \\ \begin{array}{cccc} 0 & 0 & 0 & \infty \end{array} \end{array}\begin{array}{l} \\ \infty \\ 0 \\ 0 \\ 0 \\ \hline +0 \end{array}$$

Cost(D) = cost(A) + Red^n + AD

= 16 + 0 + 0

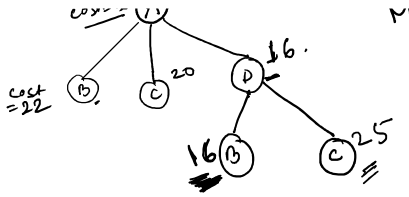
= 16.



$$M_{ADB} = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty \end{bmatrix} \end{array}\begin{array}{l} \\ - \\ - \end{array}$$

cost(B) = cost(D) + Red^n + DB

= 16 + 0 + 0



$$M_{ADB} = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty \\ 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$$

DB

$= 16 + 0 + 0$

$= 16.$

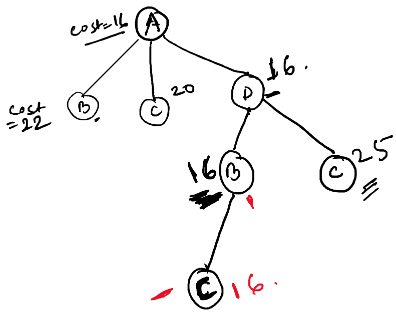
$$M_{AD} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & 1 & \infty & 0 & \infty \\ C & 0 & 4 & \infty & \infty \\ D & \infty & 0 & 4 & \infty \end{array}$$

$$M_{ADC} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & - \\ B & 0 & \infty & \infty & \infty & -1 \\ C & \infty & 0 & \infty & \infty & -4 \\ D & \infty & \infty & \infty & \infty & \\ \hline & & & & & 5 \end{array}$$

$cost(C) = cost(D) + Red + DC$

$= 16 + 5 + 4$

$= 25.$



$M_{ADBC} = ?? \longrightarrow M_{ADB}$

$$M_{ADB} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & - \\ B & \infty & \infty & 0 & \infty & - \\ C & 0 & \infty & \infty & \infty & - \\ D & \infty & \infty & \infty & \infty & - \end{array}$$

$$M_{ADBC} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & \infty & \infty & \infty & \infty \\ C & \infty & \infty & \infty & \infty \\ D & \infty & \infty & \infty & \infty \end{array}$$

$Cost(C) = cost(B) + Red + BC$

$= 16 + 0 + 0 = 16.$ Route has Minimum length.

Path $\longrightarrow A \xrightarrow{3} D \xrightarrow{3} B \xrightarrow{9} C \xrightarrow{1} A.$

$= 16$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 10 & 5 & 3 \\ B & 8 & \infty & 9 & 7 \\ C & 1 & 6 & \infty & 9 \\ D & 2 & 3 & 8 & \infty \end{array}$$