19 January 2025 21:39

Recursion - A function which calls itself.

```
A(n){                                   → 1 time
    if(n>1){
        return A(n/2) + A(n/2)          → T(n/2)  T(n/2);
    }
    else return 1;                      → 1 time.
}
```

F^n call	Time
$A(n)$	$T(n)$
$A(n/2)$	$T(n/2)$

$$\boxed{T(n) = \begin{cases} 1 + 2T(n/2), & n > 1 \\ 2, & n \le 1 \end{cases}}$$

← Recurrence Relation

Recursion TC ??

① Recurrence Relation Find

② Recurrence Relat^n Solve → Substitution

(1) Recurrence Relation ...

(2) Recurrence Relation Solve

- Substitution
- Recursion tree
- Master Theorem

Substitution Method —

$$T(n) = \begin{cases} 1 + T(n-1), & n > 1 \\ 1, & n = 1 \end{cases}$$

$T(n) = 1 + T(n-1)$

$T(n-1) = 1 + T(n-1-1) = 1 + T(n-2)$

$T(n-2) = 1 + T(n-3)$

$T(n-3) = 1 + T(n-4)$

$T(n-4) = 1 + T(n-5)$

$\vdots$

$T(4) = 1 + T(3)$

$T(3) = 1 + T(3-1) = 1 + T(2)$

$T(2) = 1 + T(1)$

$T(1) = 1$

$$T(n) + \cancel{T(n-1)} + \cancel{T(n-2)} + \cancel{T(n-3)} + \cdots + \cancel{T(3)} + \cancel{T(2)} + \cancel{T(1)} = 1 + \cancel{T(n-1)} + 1 + \cancel{T(n-2)} + 1 + \cancel{T(n-3)} + 1 + \cancel{T(n-4)} + \cdots + 1 + \cancel{T(2)} + 1 + \cancel{T(1)} + 1$$

$$\Rightarrow T(n) = 1 + 1 + 1 + \cdots + 1 = n * 1 = n = O(n)$$

$$T(n) = \begin{cases} n + T(n-1) & , n > 1 \\ 1 & , n = 1. \end{cases}$$

$$T(n) = n + T(n-1)$$

$$T(n-1) = (n-1) + T(n-1-1) = (n-1) + T(n-2)$$

$$T(n-2) = (n-2) + T(n-3)$$

$$T(n-3) = (n-3) + T(n-4)$$

$\vdots$

$T(n-3) = (n-3) + T(n-4)$

$T(4) = 4 + T(3) =$

$T(3) = 3 + T(2)$

$T(2) = 2 + T(1)$

$T(1) = 1 .$

$$T(n) + \cancel{T(n-1)} + \cancel{T(n-2)} + \cdots + \cancel{T(3)} + \cancel{T(2)} + \cancel{T(1)} = \begin{array}{l} n + \cancel{T(n-1)} + \\ n-1 + \cancel{T(n-2)} + \\ n-2 + \cancel{T(n-3)} + \\ \vdots \\ + \\ 3 + \cancel{T(2)} + \\ 2 + \cancel{T(1)} + \\ 1 \end{array}$$

$$T(n) = n + (n-1) + (n-2) + \cdots + 3 + 2 + 1 = \frac{n \times (n+1)}{2} = \frac{n^2 + n}{2} \approx n^2$$

$$TC = O(n^2)$$