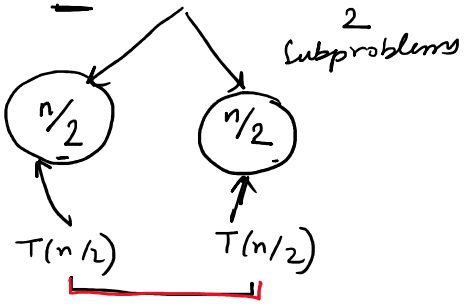
18 February 2025 08:37

Merge Sort Analysis -

$\rightarrow T(n) = 2T(n/2) + n.$ (2: Divide; n: Merge / Conquer)

$n \rightarrow$ Size of problem. $\leftarrow T(n)$



$O(n) \rightarrow$ Merge 2 sorted arrays into a single sorted array.

$T(n) = 2T(n/2) + n.$ $\qquad T(n) = aT(n/b) + n^k \log^p n$

$\Downarrow$ By Master Th^m →

$a = 2, b = 2, k = 1, p = 0$

$a \; b^k$

$2 = 2^1$ $\qquad$ Case 2a → $T(n) = O\left(n^{\log_b a} \times \log^{p+1} n\right)$

Property $\log_a a = 1$

$= O\left(n^{\log_2 2} \cdot \log^{0+1} n\right)$

$= O(n^1 \cdot \log n) = n \log n.$

Space complexity - $O(n) \rightarrow$ Merge 2 sorted arrays into 1 sorted array then it uses

Space → O(n) → Merge ... into 1 sorted array then it uses a temporary array of size = left arr size + right arr size.

Temp arr ka max size = n

Quick Sort Analysis → TC, SC.

SC → O(1) → No XTRA space used.

TC → Best Case → ① Pivot element ka original position Hamesha middle hoga.

n

Recurrence Relation = $T(n) = 2T(n/2) + n$ → Swapping perform whole array traverse

Master Thm

$T(n) = O(n \log n)$.

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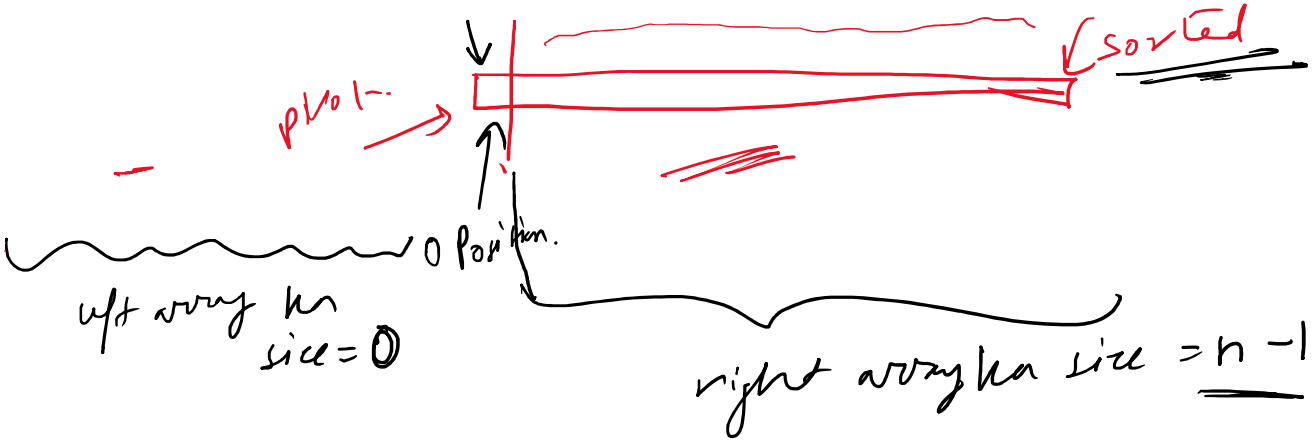
Worst Case - ① Input array is already sorted and you r considering 1st element as pivot.



RR ⇒ $T(n) = T(n-1) + n$ ← Swapping $T(n) = aT(n/b) + t(n^h \log^p n)$

⇓ Substitution method.

$T(n) = T(n-1) + n$.

$T(n-1) = T(n-2) + (n-1)$

$T(n-2) = T(n-3) + (n-2)$

$T(2) = T(1) + 2.$

$T(1) = 1.$

$T(n) + T(n-1) + T(n-2) + \ldots + T(2) + T(1) = T(n-1) + n + T(n-2) + (n-1) + T(n-3) + (n-2) + \ldots + T(1) + 2 + 1.$

$\Rightarrow T(n) = n + (n-1) + (n-2) + \ldots + 2 + 1$

$= \frac{n \times (n+1)}{2} = \frac{n^2 + n}{2} \approx O(n^2).$

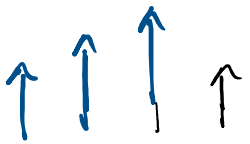
Avg Case — TC $\rightarrow O(n \log n).$

3, 3, 3, 3, 3

SC $\rightarrow O(1)$ — ∵ No x TRA space

5

Insertion sort- 3 11 1 19.



Eg

① 3 | 11 1 19

Sorted

② 3 11 | 1 10

sorted unsorted.

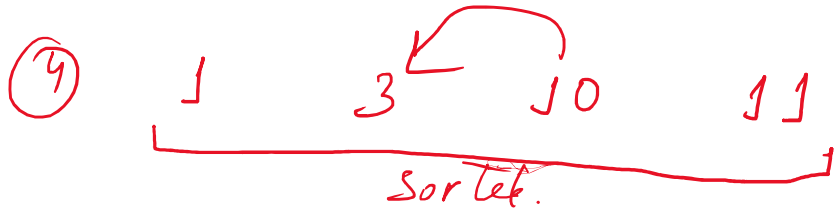
③ 3 11 1 | 10

sorted unsorted

3 1 11 10

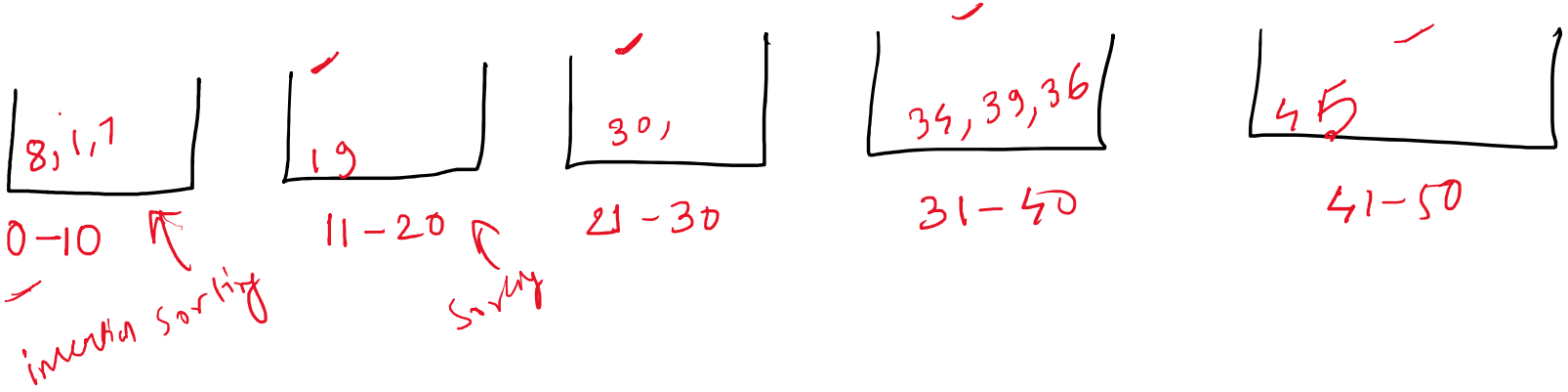
1 3 11 | 10

sorted unsorted..



Bucket sort:

Q: No. of buckets = 10.



1, 7, 8, 19, 30, 34, 36, 39, 45, 82, 89 ← sorted