13 January 2025 12:05

Asymptotic Notation -

TC $\rightarrow f(n) = 2n^2 + 5n + 21$

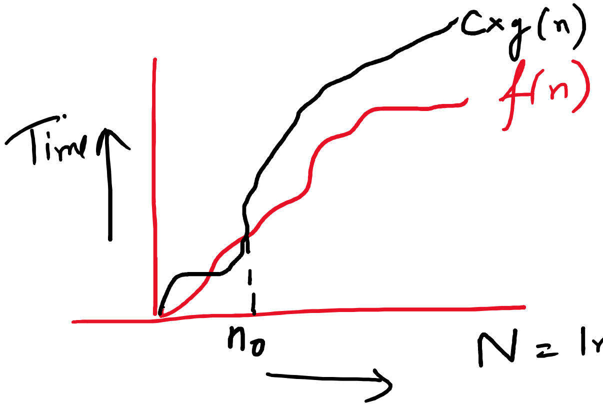
$\approx c \times n^2$

Big OH -

$f(n) = O(g(n))$

$f(n) <= c \cdot g(n) \rightarrow$ from Defⁿ.

c = const > 0



Q Find upper bound of $f(n) = 2n + 3$

$f(n) = O(g(n))$ $g(n)$ = Dominant term

Big O defⁿ -

$f(n) = O(g(n))$

$\Rightarrow f(n) <= c \cdot g(n)$

$\Rightarrow 2n+3 <= c \cdot n.$

$g(n) =$ Dominant term

$c = ??$

Trial & Error.

$f(n) = O(g(n))$

$f(n) <= c \cdot g(n)$

$c > 0$

$n >= n_0 \rightarrow n_0 >= 0$

$c = 1, \quad 2n+3 <= n$

$\Rightarrow 2n - n <= -3$

$\Rightarrow n <= -3$

$c = 2, \quad 2n+3 <= 2n$

$\Rightarrow 3 <= 0$ ✗

$c = 3,$

$2n+3 <= 3n$

$\Rightarrow 3 <= n$

$\Rightarrow n >= 3$

$n_0 = 3$

$c = 5,$

$2n+3 <= 5n$

$\Rightarrow 3 <= 3n$

$\Rightarrow 1 <= n$

$\Rightarrow n >= 1$

Big Omega –

$f(n) = \Omega(g(n))$

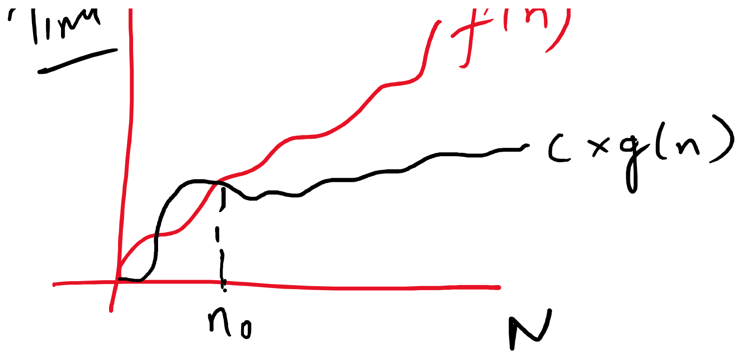
$c \times g(n) <= f(n)$

$f = O(g)$

$f(n) <= c \cdot g(n)$

Time

$f(n)$



Find lower bound of $f(n) = 10n^2 + 5$

Ans

$$f(n) = \Omega(g(n))$$

$$\Rightarrow \quad c \cdot g(n) <= f(n)$$

$$c \cdot n^2 <= 10n^2 + 5$$

$c = 10,$

$10n^2 <= 10n^2 + 5$

$\Rightarrow 0 <= 5$ $\Rightarrow n >= 0$

$n_0 = 0$

$c = 1,$

$n^2 <= 10n^2 + 5$

$n >= 0$

- Transitivity: $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n)) \Rightarrow f(n) = \Theta(h(n))$. Valid for O and Ω as well.
- Reflexivity: $f(n) = \Theta(f(n))$. Valid for O and Ω.
- Symmetry: $f(n) = \Theta(g(n))$ if and only if $g(n) = \Theta(f(n))$.
- Transpose symmetry: $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.
- If $f(n)$ is in $O(kg(n))$ for any constant $k > 0$, then $f(n)$ is in $O(g(n))$.
- If $f_1(n)$ is in $O(g_1(n))$ and $f_2(n)$ is in $O(g_2(n))$, then $(f_1 + f_2)(n)$ is in $O(\max(g_1(n)), (g_1(n)))$.
- If $f_1(n)$ is in $O(g_1(n))$ and $f_2(n)$ is in $O(g_2(n))$ then $f_1(n)\, f_2(n)$ is in $O(g_1(n)\, g_1(n))$.

Prove all the above theorems. Might get ask in exam to prove