13 January 2025 12:05

Asymptotic Notation.

$TC = 2n^2 + 2n + 5.$

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < 2^n < 3^n < n^n$

⟶ Increasing TC / Dominant Term ⟶

for large value of n.

$2n^2 + 2n + 5 \approx c \times n^2$

① Big OH ↓ Upper bound ↓ Worst Case

② Big - Omega ↓ Lower bound ↓ Best Case

③ Theta. ↓ Average Case.

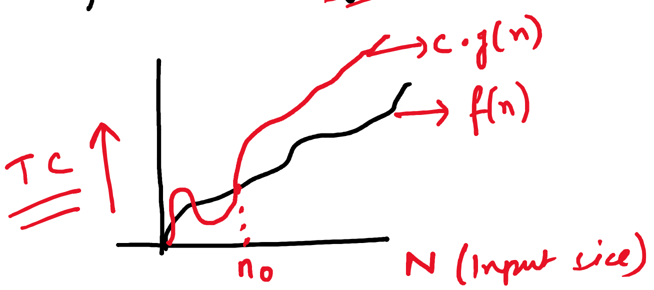
Big OH

Given two functions **f(n)** and **g(n)**, we say that **f(n)** is **O(g(n))** if there exist constants **c > 0** and $\mathbf{n_0} >= 0$ such that **f(n) <= c*g(n)** for all $\mathbf{n >= n_0}$.

$f(n) \rightarrow TC$ $\quad g(n)$ — From the list of functions

$f(n) = O(g(n))$

$\Rightarrow f(n) <= c.g(n)$, $c > 0$, $g(n)$, $n >= n_0$



$f(n) = 2n+3$. Find upper bound of $f(n)$.

Ans $\underline{f(n) = O(g(n))}$ $g(n) = ?$, $c > 0$, $n_0 > 0$, $n \geq n_0$

$\Rightarrow f(n) <= c \cdot g(n)$. (from Definition)

$\Rightarrow 2n+3 <= c \cdot g(n)$.

↓

Dominant term

$g(n) = n$

$\Rightarrow 2n+3 <= c \cdot n$.

$n >= n_0$ → +ve const.

$\underline{c = 1}$, $2n+3 <= n$

$n <= -3$

$\underline{c = 2}$, $2n+3 \leq 2n$

$\Rightarrow 0 \geq 3$ ✗

$\underline{c = 3}$, $2n+3 \leq 3n$

$3 \leq 3n - 2n$

$3 \leq n$

$n \geq 3$

$n_0 = 3$

$c = 5$,

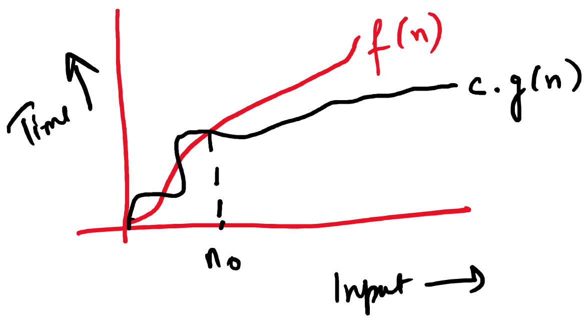
$2n+3 \leq 5n$

$\Rightarrow 3 \leq 3n$

$\Rightarrow n \geq 1 \rightarrow n_0 = 1$

Big Omega - $c \cdot g(n) \leq f(n)$

Big oh. $f(n) \leq c \cdot g(n)$.



① Find lower bound of $f(n) = 10n^2 + 5$.

Ans $f(n) = \Omega(g(n))$.

$\Rightarrow c \cdot g(n) \leq f(n)$ (Def^n).

$f(n) = \Omega(g(n))$.

$\Rightarrow c \cdot g(n) <= f(n)$ (Defⁿ)

$\Rightarrow c \cdot g(n) <= 10n^2 + 5$.

$g(n) = n^2$ (Dominant Term)

$c \cdot n^2 <= 10n^2 + 5$.

$C = 10$ $\quad 10n^2 <= 10n^2 + 5$

$\Rightarrow 0 <= 5$ ✓

$n >= n_0 \longrightarrow n_0 =$ +ve const

$n_0 = 1$

$n >= n_0$, where $n_0 = 1$

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