

Asymptotic Notation. — Time Complexity represent in form of mathematical function.

$$1 < \lg n < \sqrt{n} < n < n \lg n < n^2 < n^3 < 2^n < 3^n < n^n \}} \}} \}}$$

This is true for large n .

- ① Worst Case — Big O
- ② Best Case — Big Omega
- ③ Average Case — Theta

Big O — Given two functions $f(n)$ and $g(n)$, we say that $f(n)$ is $O(g(n))$ if there exist constants $c > 0$ and $n_0 \geq 0$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$.

eg- $TC = f(n) = (2n^2 + 4n + 5) \approx c \times n^2$

$TC \rightarrow \underline{f(n)} = 2n + 3$, Upper bound.

Ans- $f(n) = O(g(n))$

$\Rightarrow f(n) \leq c \cdot g(n)$ ← $g(n) = ?$
 $c = ?$
 $n_0 = ?$

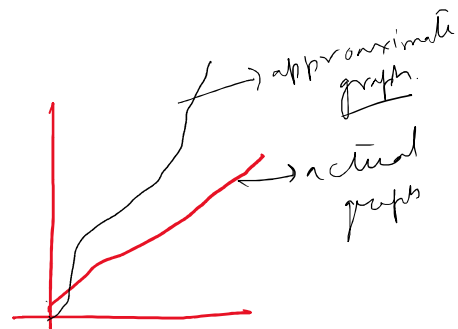
$2n + 3 \leq c \cdot g(n)$ ← From Defⁿ.

$\Rightarrow 2n + 3 \leq c \cdot n$

$\Rightarrow 2n + 3 \leq 5n$

$\Rightarrow 3 \leq 3n$

$\Rightarrow n \geq 1$ $n \geq n_0$



$g(n) = n$

$c = 5$

$n_0 = 1$

$f(n) = 2n + n^2 \leq c \cdot g(n) \rightarrow$

$2n + n^2 \leq 5 \cdot n^2$

$2n \leq 4n^2$

$\Rightarrow n \leq n^2 \Rightarrow n^2 \geq n$

$\Rightarrow n \geq 1$

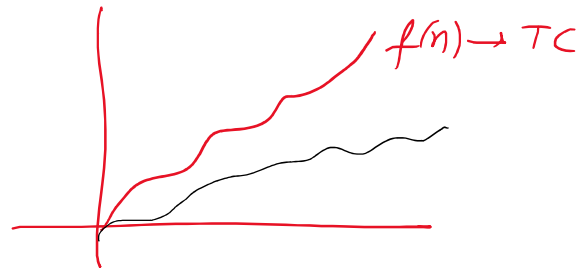
① Big $\Omega \rightarrow$ Best Case

$$f(n) = \Omega(g(n))$$

$$\Rightarrow f(n) \leq c \cdot g(n)$$

$$f(n) = \Omega(g(n))$$

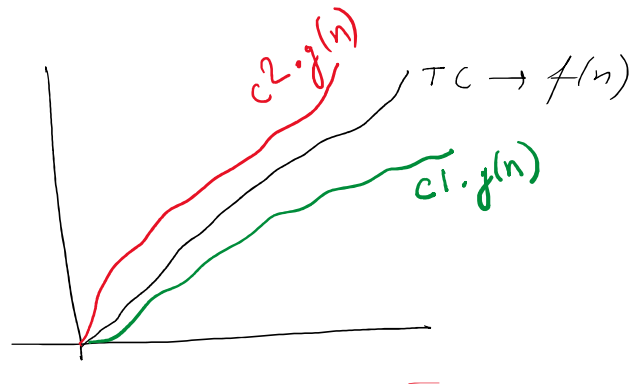
$$c \cdot g(n) \leq f(n)$$



③ Theta notation -

$$f(n) = \Theta(g(n))$$

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$



Q - Find ~~lower~~ bound of

$$f(n) = 100n + 5$$

$$f(n) = \Omega(g(n))$$

$$\Rightarrow c \cdot g(n) \leq f(n)$$

$$\Rightarrow c \cdot g(n) \leq 100n + 5$$

$$\Rightarrow c \cdot n \leq 100n + 5$$

$$105n - 100n \leq 5$$

$$5n \leq 5$$

$$n \leq 1$$

c	n ₀
105	1
210	2
10	

$$c > 0, n_0 > 0$$

$$n \geq n_0$$