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Matrix chain multiplication: A and B can be multiplied when number of row in A= number of column in B

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A\times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

No. of multiplications = 12 (By observation).

$= 2*3*2 = 12$. (by formulae).

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

Minimum multiplication to find A1A2A3.

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Minimum multiplication to find A1A2A3.

(A1.A2).A3

A1 (A2.A3).

$(A_1 \; A_2) \cdot A_3$

Dimensions 2×3 3×4 4×2

Cost - 2*3*4 = 24

Resulting Dimension 2×4

Cost = 2*4*2 = 16

∴ Total cost operations = 24+16 = 40.

$A_1 \cdot (A_2 \cdot A_3)$

Dimensions 2×3 3×4 4×2

Cost 3*4*2 = 24

Dimension 3×2.

Cost = 2*3*2 = 12.

∴ Total cost = 24+12 = 36.

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$

Minimum multiplication to find A1A2A3.

c[1,1] = A_1 c[2,2] = A_2

c[1,2] = A_1A_2

c[2,3] = A_2A_3.

c[1,3] = $A_1A_2A_3$.

	1	2	3
1	0	24	36
2			24
3			0

$c[1,3] = A_1 A_2 A_3$

$c[1,1] = A_1$ =

$c[2,1] = A_2 A_1$

$c[2,3] = A_2 A_3$

$c[2,2] + c[3,3] + cost$

$0 \quad 0 \quad 24$

$= 24$

$c[1,2] = A_1 A_2$ 24

$c[1,1] + c[2,2] + cost.$ $2*3*4$

$0 \quad 0$

$= 24$

$A_1 = 2\times3 \quad A_2 = 3\times4 \quad A_3 = 4\times2$

$c[1,3]$

$A_1 A_2 A_3$

$(A_1A_2)A_3 \quad A_1(A_2A_3)$

$(A_1 A_2) A_3$ 2×4

$= c[1,2] + c[3,3] +$

$cost$ $2*4*2$

$= 24 + 0 + 16$

$= 40.$

$A_1 (A_2 A_3)$ 2×3 3×2

$= c[1,1] + c[2,3] + cost$

$0 \quad 24 \quad 2*3*2$

$12.$

$= 36.$