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Matrix chain multiplication: **A and B can be multiplied when number of row in B= number of column in A**

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A\times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

No. of multiplications = 12 (By observation).

$= 2\times 3\times 2 = 12$ (by formulae).

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2.$

Minimum multiplication to find A1 A2 A3.

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2.$

Minimum multiplication to find A1 A2 A3.

$\underbrace{(A1 . A2)} . \underbrace{A3}$

$(A_1 \; A_2) . A_3$

Dimensions $\underbrace{2\times 3 \quad 3\times 4} \quad 4\times 2$

Cost - 24

Resulting Dimension 2×4

Cost = $2\times 4\times 2 = 16$

$\underbrace{A1} \; \underbrace{(A2 . A3)}.$

$A_1 . (A_2 . A_3)$

Dimensions $2\times 3 \quad \underbrace{3\times 4 \quad 4\times 2}$

Cost $3\times 4\times 2 = 24$

Dimension 3×2

Cost = $2\times 3\times 2 = 12$

Total cost = 24+12 = 36

Resulting Dimension 2×4

Cost = $2\times4\times2 = 16$

∴ Total cost operations = 40

Cost = $2\times3\times2 = 12$

∴ Total cost = $24+12=36.$

$A_1 = 2\times3$ $\quad A_2 = 3\times4$ $\quad A_3 = 4\times2.$

Minimum multiplication to find A1 A2 A3.

$\underbrace{(A1.A2)}\,.\,\underbrace{A3}$

$(A_1\ A_2)\ .\ A_3$

Dimensions $\underbrace{2\times3 \quad 3\times4} \quad 4\times2$

Cost. 24

Resulting Dimension 2×4

Cost = $2\times4\times2 = 16$

∴ Total cost operations = 40

$\underbrace{A1}\ \underbrace{(A2.A3)}.$

$A_1\ .\ (A_2\ .\ A_3)$

Dimensions 2×3 $\quad \underbrace{3\times4 \quad 4\times2}$

Cost $3\times4\times2 = 24$

Dimension 3×2

Cost = $2\times3\times2 = 12$

∴ Total cost = $24+12=36.$

	1	2	3
1	0	24	
2		0	24
3			0

$c[1,1] = 0$

$\underset{2\times3}{A_1} \quad \underset{3\times4}{A_2}$

$c[1,2] = (A_1)(A_2)$

$= c[1,1] + c[2,2] + 2\times3\times4$

$= 0+0+24 = 24$

~~c[1,3]~~ $c[2,3] = c[2,2] + c[3,3]$ (with A_2 above $c[2,2]$, A_3 above $c[3,3]$)

$+\ 3\times4\times2$

$=\ 0+0+24$

$=24$

$A_1 = 2\times3$ $\quad A_2 = 3\times4$ $\quad A_3 = 4\times2.$

1 2 3

$\longleftrightarrow (A_1 A_2).(A_3)$

$c[1,3] = c[1,2] + c[3,3] + 2\times4\times2$

	1	2	3
1	0	24	36
2		0	24
3			0

$c[1,3] = c[1,2] + c[$

$\underbrace{A_1 A_2}\underbrace{A_3}$ $\quad 2\times4 \quad 4\times2$

$= 24 + 0 + 16$

$= 40.$

$\rightarrow (A_1)(A_2 \cdot A_3)$

$2\times3 \quad 3\times2$

$= c[1,1] + c[2,3] + \underline{2\times3}\times2$

$= 0 + 24 + 12$

$= 36.$

$A_1 \; A_2 \; A_3 \; A_4$

$(A_1 A_2)(A_3 A_4) \qquad A_1(A_2 A_3 A_4)$

$A_2(A_3 A_4) \quad (A_2 A_3)A_4$

$A_1 A_2 A_3$

$c[2,1]$ $A_2 A_1$