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06 April 2025 22:25

Matrix chain multiplication: A and B can be multiplied when number of row in A= number of column in B

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A\times B=\begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

No. of multiplications = 12 (By observation).

$= 2\times 3\times 2 = 12$ (by formulae).

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

Minimum multiplication to find $A_1A_2A_3$.

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$(A_1 \cdot A_2) \cdot A_3$

Dimensions: $(A_1\ A_2)\cdot A_3$ — $2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost: 24

Resulting Dimension: 2×4

Cost = $2\times 4\times 2 = 16$

∴ Total cost operations = 40

$A_1 \cdot (A_2 \cdot A_3)$

Dimensions: $2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost: $3\times 4\times 2 = 24$

Dimension: 3×2

Cost = $2\times 3\times 2 = 12$

∴ Total cost = $24+12 = 36$.

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

Minimum multiplication to find $A_1A_2A_3$.

$(A_1 \cdot A_2) \cdot A_3$

Dimensions: $(A_1\ A_2)\cdot A_3$ — $2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost: 24

Resulting Dimension: 2×4

Cost = $2\times 4\times 2 = 16$

∴ Total cost operations = 40

$A_1 \cdot (A_2 \cdot A_3)$

Dimensions: $2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost: $3\times 4\times 2 = 24$

Dimension: 3×2

Cost = $2\times 3\times 2 = 12$

∴ Total cost = $24+12 = 36$.

	1	2	3
1	0	24	
2		0	24
3			0

A_1 A_2

2×3 3×4

$c[1,1] = 0$

$c[1,2] = (A_1)(A_2)$

$= c[1,1] + c[2,2] + 2\times 3\times 4$

$= 0 + 0 + 24 = 24$

$c[2,3] = c[2,2] + c[3,3] + 3\times 4\times 2$

$= 0 + 0 + 24$

$= 24$

$\leftrightarrow (A_1A_2)\cdot(A_3)$

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

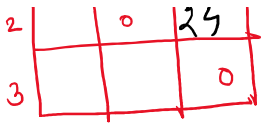
	1	2	3
1	0	24	36
2		0	24
3			0

$c[1,3] = c[1,2] + c[3,3] + 2\times 4\times 2$

$A_1A_2 \quad A_3$

$2\times 4 \quad 4\times 2$

$= 24 + 0 + 16$



$= 24 + 0 + 16$

$= 40.$

$(A_1)(A_2 \cdot A_3)$

$2 \times 3 \quad 3 \times 2$

$= c[1,1] + c[2,3] + 2 \times 3 \times 2$

$= 0 + 24 + 12$

$= 36.$

$A_1 A_2 A_3 A_4$

$(A_1 A_2)(A_3 A_4) \qquad A_1 (A_2 A_3 A_4)$

$A_2 (A_3 A_4) \quad (A_2 A_3) A_4$

$A_1 A_2 A_3$

$c[2,1] \qquad (A_2 A_1$