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06 April 2025 22:25

Matrix chain multiplication: A and B can be multiplied when number of row in A= number of column in B

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A\times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

No. of multiplications = 12 (By observation).

$= 2\times 3\times 2 = 12$ (by formulae).

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2.$

Minimum multiplication to find A1A2A3.

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2.$

Minimum multiplication to find A1A2A3.

$(A1 \cdot A2) \cdot A3$

$(A_1\ A_2) \cdot A_3$

Dimensions: $2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost: 24

Resulting Dimension: 2×4

Cost = $2\times 4\times 2 = 16$

$A1\ (A2 \cdot A3)$

$A_1 \cdot (A_2 \cdot A_3)$

Dimensions: $2\times 3 \quad 3\times 4 \quad 4\times 2$

$3\times 4\times 2 = 24$

Cost

Dimension: 3×2

Cost = $2\times 3\times 2 = 12$

Total cost = $24 + 12 = 36$

Resulting Dimension 2×4

Cost = 2×4×2 = 16

∴ Total cost operations = 40

Cost = 2×3×2 = 12

∴ Total cost = 24+12 = 36.

$A_1 = 2\times3$ $A_2 = 3\times4$ $A_3 = 4\times2$.

Minimum multiplication to find A1 A2 A3.

(A1 . A2) . A3

(A1 A2) . A3

Dimensions 2×3 3×4 4×2

Cost - 24

Resulting Dimension 2×4

Cost = 2×4×2 = 16

∴ Total cost operations = 40

A1 (A2 . A3).

A1 . (A2 . A3)

Dimensions 2×3 3×4 4×2

Cost 3×4×2 = 24

Dimension 3×2

Cost = 2×3×2 = 12

∴ Total cost = 24+12 = 36.

	1	2	3
1	0	24	
2		0	24
3			0

A1 A2

2×3 3×4

c[1,1] = 0

c[2,3] = c[2,2] + c[3,3] + 3×4×2

= 0 + 0 + 24

= 24

c[1,2] = (A1)(A2)

= c[1,1] + c[2,2] + 2×3×4

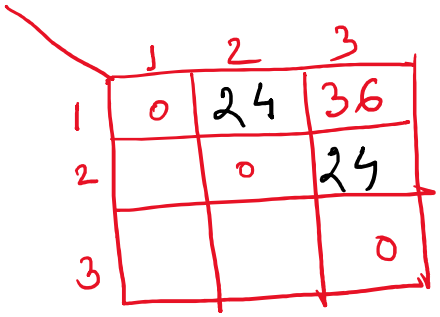
= 0 + 0 + 24 = 24

$A_1 = 2\times3$ $A_2 = 3\times4$ $A_3 = 4\times2$.

1 2 3

↔ (A1 A2) . (A3)

c[1,3] = c[1,2] + c[3,3] + 2×4×2



$c[1,3] = c[\quad] + c[\quad]$

$A_1 A_2 A_3$ — 2×4 — 4×2

$= 24 + 0 + 16$

$= 40.$

$(A_1)(A_2 \cdot A_3)$

2×3 — 3×2

$= c[1,1] + c[2,3] + 2\times3\times2$

$= 0 + 24 + 12$

$= 36.$

$A_1\ A_2\ A_3\ A_4$

$(A_1A_2)(A_3A_4)$

$A_1(A_2A_3A_4)$

$A_2(A_3A_4)$ $(A_2A_3)A_4$

$A_1 A_2 A_3$

$c[2,1]$ $A_2 A_1$