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Matrix chain multiplication: A and B can be multiplied when number of row in A= number of column in B

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A\times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2\times 2}$$

No. of multiplications = 12 (By observation).

$= 2 * 3 * 2 = 12$. (by formulae).

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

Minimum multiplication to find A1A2A3.

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

Minimum multiplication to find A1A2A3.

$(A1 \cdot A2) \cdot A3$

$A1\ (A2 \cdot A3)$.

$(A_1\ A_2) \cdot A_3$

Dimensions $2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost $2*3*4 = 24$

Resulting Dimension 2×4

Cost $= 2*4*2 = 16$

$\therefore$ Total cost operations $= 24+16 = 40$.

$A_1 \cdot (A_2 \cdot A_3)$

Dimensions $2\times 3 \quad 3\times 4 \quad 4\times 2$

Cost $3*4*2 = 24$

Dimension 3×2

Cost $= 2*3*2 = 12$

$\therefore$ Total cost $= 24+12 = 36$.

Minm multiplication ops = min(option 1, option 2).

$= 36$

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

Minimum multiplication to find A1A2A3.

	1	2	3
1	0	24	36
2		0	24
3			0

$c[1,1] = A_1$

$c[1,2] = A_1A_2$

$c[1,3] = A_1A_2A_3$

$c[2,3] = A_2A_3$.

$c[2,1] = A_2A_1$.

$c[1,2] = A_1A_2$

$c[1,1] + c[2,2] + \text{cost}$ ($2*3*4$)

$c[2,3]$ A_2A_3

$c[2,2] + c[3,3] + 3*4*2$

$= 0+0+24 = 24$.

$A_1 = 2\times 3 \quad A_2 = 3\times 4 \quad A_3 = 4\times 2$.

$c[1,3]$

$A_1A_2A_3$.

$(A_1A_2)A_3$

$A_1(A_2A_3)$.

$(A_1A_2)A_3$

$= c[1,2] + c[3,3] +$ cost

cost $\rightarrow 2*4*2$

$= 24 + 0 + 16 = 40$

2×3. 3

$A_1(A$

$= c[1,1]$

$= 0+$

$A_1A_2A_3A_4$.

$A_1(A_2A_3A_4)$ $\quad (A_1A_2)(A_3A_4)$ $\quad (A_1A_2A_3)A_4$.

$A_2(A_3A_4)$ $\quad (A_2A_3)A_4$.

3×2

A_3).

$+ c[2,3] + 2 \cdot 3 \cdot 2$

$24 + 12 = 36$.