Asymptotic Notation —

$$TC = 2n^2 + 5n + 10$$

quadr — linear — Const.

$\approx c \times n^2$

Dominant term $= n^2$

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < 2^n < 3^n < n^n$

→ Increasing TC → Increasing Dominance

$\underbrace{\text{for large values of } n.}$

$n < n\log n.$

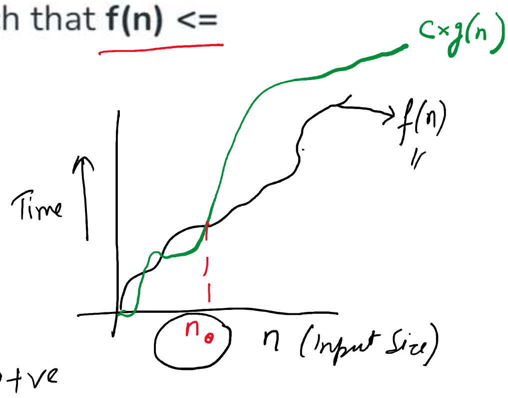
$n=1, \quad 1 < 1\log(1)$

$1 < 0$

3 notations —

1. Big O → Worst Case
2. Big Omega → Best Case
3. Theta → Average.

Given two functions **f(n)** and **g(n)**, we say that **f(n)** is **O(g(n))** if there exist constants **c > 0** and **n_0** >= 0 such that **f(n) <=** **c*g(n)** for all **n >= n_0**.



Eg- Find upper bound of $f(n) = 2n+3$.

$f(n) = O(g(n))$

$\Rightarrow f(n) <= c \times g(n)$ → $g(n) = ?$, $c = ?$, $n_0 = ?$

Cond^n: $c, n_0 \to +ve$; $n >= n_0$

$2n+3 <= c \times g(n)$

$\boxed{g(n) = n}$

$2n+3 <= c \times n.$

Trial & Error method.

$c=1, \quad 2n+3 <= n$

$c=2, \quad 2n+3 <= 2n \Rightarrow 3 <= 0$ ✗

$c=3, \quad 2n+3 <= 3n$

Trial && Error method

$c=1$, $2n+3 <= n$

$\Rightarrow n <= -3 \Rightarrow n <= -3$

$c=2$, $2n+3 <= 2n$

$\Rightarrow 3 <= 0$ ✗

$2n+3 <= 3n$

$\Rightarrow 3 <= n$

$n >= 3$ $\quad n_0 = 3$

$c=5$, $2n+3 <= 5n$

$\Rightarrow 3 <= 3n$

$\Rightarrow 1 <= n$

$\Rightarrow n >= 1$

$c = 5$ ✓

$n >= n_0 \rightarrow n >= 1$ ✓

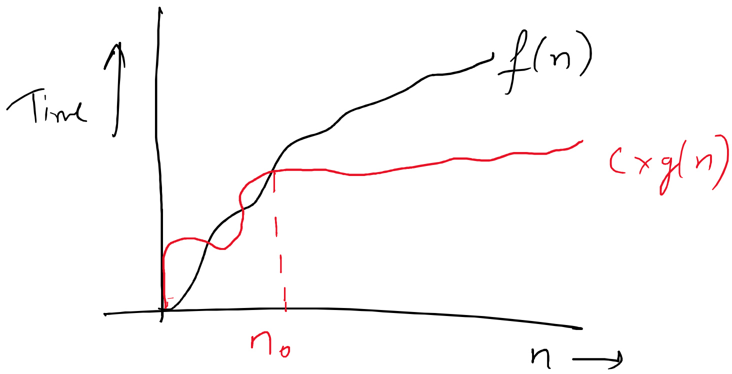
$n_0 = 1 = +ve$ ✓

Big Omega – $f(n) = \Omega(g(n))$

$c \cdot g(n) <= f(n)$

$f(n) = O(g(n))$

$f(n) <= c \cdot g(n)$



Find the lower bound of $f(n) = 10n^2 + 5$.

Ans $f(n) = \Omega(g(n))$

$\Rightarrow \cancel{f}\; c \cdot g(n) <= f(n)$.

$\Rightarrow c \cdot g(n) <= 10n^2 + 5$.

$\Rightarrow c \cdot n^2 <= 10n^2 + 5$

$n^2 <= 10n^2$

$c=1$, $\underbrace{n^2}_{+ve} <= \underbrace{10n^2}_{+ve} + \underbrace{5}_{+ve}$

$\Rightarrow c = 1$, $n >= 1$

$c=2$,

$n^2 <= 10n^2 + 5$ $\quad c = (1, \dots, 10)$

$c=2,$

$c=10,$ $10n^2 <= 10n^2 + 5$ $c = (1, \ldots, 10)$

$n >= 1$

$n_0 = 1$ $c = +ve$

Theta Notation $f(n) = \Theta(g(n))$

$c_1 \cdot g(n) <= f(n) <= c_2 \cdot g(n)$