

Matrix chain multiplication: A and B can be multiplied when number of row in A = number of column in B

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3 \times 2}$$

$$A \times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2 \times 2}$$

No. of multiplications = 12 (By observation).

$$= 2 \times 3 \times 2 = 12 \text{ (by formulae).}$$

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$$

Minimum multiplication to find $A_1 A_2 A_3$.

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Minimum multiplication to find $A_1 A_2 A_3$.

$$(A_1 \cdot A_2) \cdot A_3$$

$$A_1 (A_2 \cdot A_3)$$

Dimensions $(A_1 A_2) \cdot A_3$
 $2 \times 3 \quad 3 \times 4 \quad 4 \times 2$
 Cost $2 \times 3 \times 4 = 24$
 Resulting Dimension 2×4
 Cost $2 \times 4 \times 2 = 16$

Dimensions $A_1 \cdot (A_2 \cdot A_3)$
 $2 \times 3 \quad 3 \times 4 \quad 4 \times 2$
 Cost $3 \times 4 \times 2 = 24$
 Resulting Dimension 3×2
 Cost $2 \times 3 \times 2 = 12$

$$\therefore \text{Total cost operations} = 24 + 16 = 40 \quad \therefore \text{Total cost} = 24 + 12 = 36$$

$$\text{Min multiplication ops} = \min(\text{option 1, option 2}) = 36$$

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$$

Minimum multiplication to find $A_1 A_2 A_3$.

$$C[1,1] = A_1$$

$$C[1,2] = A_1 A_2$$

	1	2	3
1	0	24	36
2		0	24
3			0

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$$

$$C[1,1]$$

$$A_1 A_2 A_3$$

$$(A_1 A_2) A_3$$

$$A_1 (A_2 A_3)$$

$$(A_1 A_2) A_3$$

$$= C[1,2] + C[3,3] +$$

cost

$$\rightarrow 2 \times 4 \times 2$$

$$= 24 + 0 + 16 = 40$$

$$2 \times 3 \quad 3 \times 2$$

$$A_1 (A_2 A_3)$$

$$= C[1,1] + C[2,3] + 2 \times 3 \times 2$$

$$= 0 + 24 + 12 = 36$$

$$c[1,2] = A_1 A_2$$

$$c[1,3] = A_1 A_2 A_3$$

$$c[2,3] = A_2 A_3$$

$$c[2,1] = A_2 A_1$$

$$c[1,2] =$$

$$A_1 A_2$$

$$c[1,1] + c[2,2] + \text{cost}$$

3			0

$$c[2,3] \quad A_2 \quad A_3$$

$$c[2,2] + c[3,3] + 3 \times 4 \times 2 = 0 + 0 + 24 = 24$$

$$(A_1 A_2) A_3 \quad A_1 (A_2 A_3)$$

$$\rightarrow 2 \times 4 \times 2$$

$$= 24 + 0 + 16 = 40$$

