$T(n) = \begin{cases} 2T(n/2) + n, & n > 1 \\ 1, & n = 1 \end{cases}$

Input | Cost

$n = n/2^0$ ← n ← n — 0th level

$n/2 = n/2^1$ ← $n/2 \quad n/2$ — $n/2 + n/2 = n$ — 1st level

$n/4 = n/2^2$ ← $n/4 \quad n/4 \quad n/4 \quad n/4$ — $n/4 + n/4 + n/4 + n/4 = n$ — 2nd level

$n/8 = n/2^3$ ← $n/8 \quad n/8 \quad n/8 \quad n/8 \quad n/8 \quad n/8 \quad n/8 \quad n/8$ — $n = 8 \times n/8 = n$ — 3rd level

$\vdots$

$n/2^k$ ← kth step

Recursion tree

Recursion stops when input $= n/2^k = 1$ ← Question

$\Rightarrow n = 2^k$

$\Rightarrow \log_2 n = \log_2 2^k = k \times \log_2 2 =$

$\Rightarrow \log_2 n = k$

$\log_a a = 1$

Total cost -

$\underbrace{n + n + n + \dots + n}_{(1+k) \text{ terms}} =$

$= (k+1) \times n$

$= nk + n$

$= n \log_2 n + n$

$\approx n \log n$

$TC = \Theta(n \log n)$

Master Theorem

If the recurrence is of the form $T(n) = aT(\frac{n}{b}) + \Theta(n^k log^p n)$, where $a \geq 1, b > 1, k \geq 0$ and p is a real number, then:

1) If $a > b^k$, then $T(n) = \Theta\left(n^{log_b^a}\right)$
2) If $a = b^k$
   a. If $p > -1$, then $T(n) = \Theta\left(n^{log_b^a} log^{p+1} n\right)$
   b. If $p = -1$, then $T(n) = \Theta\left(n^{log_b^a} loglogn\right)$
   c. If $p < -1$, then $T(n) = \Theta\left(n^{log_b^a}\right)$
3) If $a < b^k$
   a. If $p \geq 0$, then $T(n) = \Theta(n^k log^p n)$

$T(n) = \underbrace{3T(n/2)} + n^2$

$aT(n/b) = 3T(n/2)$

$a = 3, \ b = 2$

$k, p \qquad n^2$

3) If $a < b^k$

a. If $p \geq 0$, then $T(n) = \Theta(n^k \log^p n)$

b. If $p < 0$, then $T(n) = O(n^k)$

$n^k \lg^p n = n^2$

$= n^2 \log^0 n$

$k = 2$, $p = 0$

Q. $T(n) = 4T(n/2) + n^2$

$a = 4$

$b = 2$

$p = 0$

$k = 2$

$b^k = 4$

$a = b^k$ (Cond^n 2nd)

Cond^n chk-

$a = 3$ $b - 2$ $k = 2$

$b^k = 4$

$a < b^k$ (Condition) 3rd wala

$p = 0$ (Condition a)

$T(n) = \Theta(n^k \cdot \log^p n) = \Theta(n^2 \log^0 n) = \Theta(n^2)$

$p = 0 > -1$ (Cond^n a)

$$T(n) = \Theta\left(n^{\log_b a} \cdot \log^{p+1} n\right) = n^{\log_2 4} \cdot \log^1 n$$

$$= n^{\log_2 2^2} \cdot \log n = n^{2 \log_2 2} \cdot \log n = n^2 \log n$$