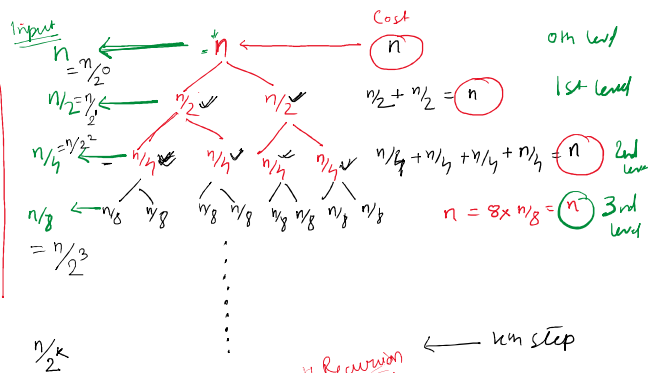


$$T(n) = \begin{cases} 2T(n/2) + n, & n > 1 \\ 1, & n = 1 \end{cases}$$



Total cost -

$$\begin{aligned} n + n + n + \dots + n &= \\ (1+k) \text{ terms} &= \\ = (k+1) \times n &= \\ = n \log_2 n + n &= \\ \approx n \log_2 n &= \\ TC = O(n \log_2 n) \end{aligned}$$

Master Theorem

If the recurrence is of the form $T(n) = aT(\frac{n}{b}) + \Theta(n^k \log^p n)$, where $a \geq 1, b > 1, k \geq 0$ and p is a real number, then:

- 1) If $a > b^k$, then $T(n) = \Theta(n^{\log_b a})$
- 2) If $a = b^k$
 - a. If $p > -1$, then $T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$
 - b. If $p = -1$, then $T(n) = \Theta(n^{\log_b a} \log \log n)$
 - c. If $p < -1$, then $T(n) = \Theta(n^{\log_b a})$
- 3) If $a < b^k$

$$T(n) = 3T(n/2) + n^2$$

$$aT(n/b) = 3T(n/2)$$

$$a = 3, b = 2$$

$$k, p = 2$$

3) If $a < b^k$

a. If $p \geq 0$, then $T(n) = \Theta(n^k \log^p n)$

b. If $p < 0$, then $T(n) = O(n^k)$

$$n^k \log^p n = n^2 \\ = n^2 \log^0 n$$

$$k=2, p=0$$

$$T(n) = 4T(n/2) + n^2$$

~~a = 4~~ $a = 4$

$$b = 2$$

$$p = 0$$

$$k = 2$$

$$b^k = 4$$

$$a = b^k \text{ (Cond}^n \text{ 2nd)}$$

Condⁿ 1st-
 $b^k = 4$

$a < b^k$ (condition) 3rd case
 $p \geq 0$ (condition a)

$$T(n) = \Theta(n^k \log^p n) = \Theta(n^2 \log^0 n) = \Theta(n^2)$$

$$p = 0 > -1 \quad (\text{cond}^n a)$$

$$T(n) = O(n^{\log_b a} \cdot \log^{p+1} n) = n^{\log_2 4} \cdot \log^3 n$$

$$= n^{\log_2 2^2} \cdot \log n = n^{2 \log_2 2} \cdot \log n = n^2 \log n$$

