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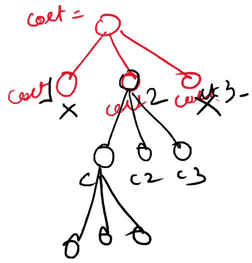
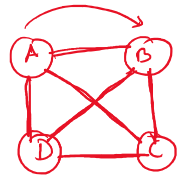
The **Travelling Salesman Problem (TSP)** is a classic **combinatorial optimization problem** in computer science and operations research. It's defined as:

- **Given**: A list of cities and the distances between each pair of cities.
- **Goal**: Find the shortest possible route that visits each city exactly once and returns to the starting city.




Minimum length of route

A B C D A

A D C B A

C B A D C

A C B D A

TSP appears in various real-world scenarios like Route planning (delivery trucks, sales routes)

**Core Concepts**

1. **Branching**: You build a **tree of subproblems**, where each node represents a partial tour (sequence of cities visited).
2. **Bounding**: At each node, you compute a **lower bound** (minimum possible cost to complete the tour from here).
3. **Pruning**: If a node's lower bound is worse than the best complete solution found so far, you discard (prune) that branch.

Branch && Bound

**Steps to Solve TSP with Branch and Bound:**

1. **Start with a cost matrix** of distances between all cities.
2. **Reduce the matrix**:
   - Subtract the smallest value in each row and each column (this gives a *lower bound*).
3. **Create a priority queue (min-heap)** to explore promising nodes first (ones with smaller bounds).
4. **At each node**:
   - Choose a city to visit next.
   - Update the matrix to reflect the path chosen (remove rows/columns).
   - Recalculate the reduced cost and total bound.
5. **Prune** paths with bounds higher than the best known solution.
6. Repeat until all promising paths are explored.

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 10 & 5 & 3 \\ B & 8 & \infty & 9 & 7 \\ C & 1 & 6 & \infty & 9 \\ D & 2 & 3 & 8 & \infty \end{array}$$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 10 & 5 & 3 \\ B & 8 & \infty & 9 & 7 \\ C & 1 & 6 & \infty & 9 \\ D & 2 & 3 & 8 & \infty \end{array}$$

Row Reduction → Column Reduction.

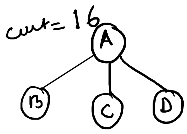
$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & 10 & 5 & 3 & -3 \\ B & 8 & \infty & 9 & 7 & -7 \\ C & 1 & 6 & \infty & 9 & -1 \\ D & 2 & 3 & 8 & \infty & -2 \end{array}$$

$3 + 7 + 1 + 2 = 13$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 7 & 2 & 0 \\ B & 1 & \infty & 2 & 0 \\ C & 0 & 5 & \infty & 8 \\ D & 0 & 1 & 6 & \infty \end{array}$$

$0 + 1 + 2 + 0 = 3$

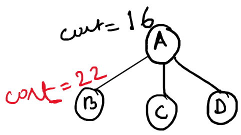
$$M_A = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$




$M_{AB} = ??$

$$M_A = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$

$$M_{AB} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & \infty & \infty & 0 & 0 & \to 0 \\ C & 0 & \infty & \infty & 8 & \to 0 \\ D & 0 & \infty & 4 & \infty & \to 0 \\ \hline & 0 & \infty & 0 & 0 & \end{array}$$




$cost(B) = cost(A) + Reduction + AB$

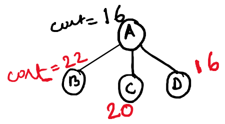
$= 16 + 0 + 6 = 22$

$M_A =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$

$M_{AC} = ???$

$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & -\infty \\ B & 1 & \infty & \infty & 0 & -0 \\ C & \infty & 0 & \infty & 4 & -4 \\ D & 0 & 0 & \infty & \infty & -0 \\ \hline & 0 & 0 & \infty & 0 & + \ 4 \end{array}$$




cost (C) = cost (A) + Reduction + AC

= 16 + 4 + 0 = 20

$M_{AD} = ???$

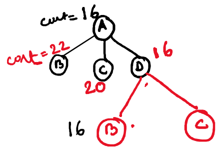
$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & -\infty \\ B & 1 & \infty & 0 & \infty & -0 \\ C & 0 & 4 & \infty & \infty & -0 \\ D & \infty & 0 & 4 & \infty & -0 \\ \hline & 0 & 0 & 0 & \infty & \end{array}$$

cost (D) = cost (A) + Reduct + AD

= 16 + 0 + 0 = 16.

$M_{AD} =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & 1 & \infty & 0 & \infty \\ C & 0 & 4 & \infty & \infty \\ D & \infty & 0 & 4 & \infty \end{array}$$




$M_{ADB} =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & \infty & \infty & 0 & \infty \\ C & 0 & \infty & \infty & \infty \\ D & \infty & \infty & \infty & \infty \end{array}$$

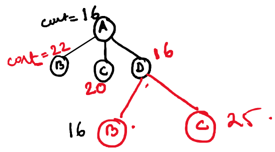
c (B) = c (D) + Redⁿ + DB.

= 16 + 0 + 0

= 16

$M_{AD} =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & 1 & \infty & 0 & \infty \\ C & 0 & 4 & \infty & \infty \\ D & \infty & 0 & 4 & \infty \end{array}$$



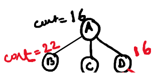

c(C) = c(D) + Redⁿ + DC.

= 16 + 5 + C = 25

$M_{ADC} =$

$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \infty \\ B & 0 & \infty & \infty & \infty & 1 \\ C & \infty & 0 & \infty & \infty & 4 \\ D & \infty & \infty & \infty & \infty & \infty \\ \hline & 0 & 0 & \infty & \infty & 5 \end{array}$$

A B C D



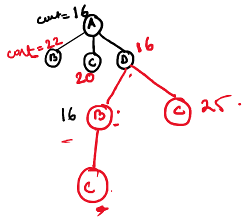

A B

$$M_{ADB} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & \infty & \infty & 0 & \infty \\ C & 0 & \infty & \infty & \infty \\ D & \infty & \infty & \infty & \infty \end{array}$$

$$M_{ADBC} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \infty \\ B & \infty & \infty & \infty & \infty & \infty \\ C & 0 & \infty & \infty & \infty & 0 \\ D & \infty & \infty & \infty & \infty & \infty \\ \hline & 0 & \infty & \infty & \infty & \end{array}$$

$Red^n = \infty$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 10 & 5 & 3 \\ B & 8 & \infty & 9 & 7 \\ C & 1 & 6 & \infty & 9 \\ D & 2 & 3 & 8 & \infty \end{array}$$




A B

$c(C) = c(B) + BC + Red^n.$

$= 16 + 0 + 0$

$= 16.$

Path = $A \to D \to B \to C \to A.$

$3 + 3 + 9 + 1$

$= 16$

ABCDA

ACBDA.