

Matrix chain multiplication: A and B can be multiplied when number of row in A = number of column in B

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3 \times 2}$$

$$A \times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2 \times 2}$$

No. of multiplications = 12 (By observation).  
 $= 2 \times 3 \times 2 = 12$  (by formula).

$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$

Minimum multiplication to find  $A_1 A_2 A_3$ .

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Minimum multiplication to find  $A_1 A_2 A_3$ .

$(A_1 A_2) \cdot A_3$

$A_1 (A_2 A_3)$

Dimensions:  $(A_1 A_2) \cdot A_3$   
 $2 \times 3 \quad 3 \times 4 \quad 4 \times 2$   
 Cost:  $2 \times 3 \times 4 = 24$   
 Resulting Dimension:  $2 \times 4$   
 Cost:  $2 \times 4 \times 2 = 16$

Dimensions:  $A_1 \cdot (A_2 A_3)$   
 $2 \times 3 \quad 3 \times 4 \quad 4 \times 2$   
 Cost:  $3 \times 4 \times 2 = 24$   
 Dimension:  $3 \times 2$   
 Cost:  $2 \times 3 \times 2 = 12$

$\therefore$  Total cost operations =  $24 + 16 = 40$  .  $\therefore$  Total cost =  $24 + 12 = 36$ .

Min<sup>m</sup> multiplication opt =  $\min(\text{option 1, option 2})$   
 $= 36$

$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$

Minimum multiplication to find  $A_1 A_2 A_3$ .

$c[1,1] = A_1$

$c[1,2] = A_1 A_2$

$c[1,3] = A_1 A_2 A_3$

$c[2,3] = A_2 A_3$

$c[2,1] = A_2 A_1$

$c[1,2] =$

$A_1 A_2$

$c[1,1] + c[2,2] + \text{cost}$

$c[2,3] = A_2 A_3$

$c[2,2] + c[3,3] + 3 \times 4 \times 2$

$= 0 + 0 + 24 = 24$

|   | 1 | 2  | 3  |
|---|---|----|----|
| 1 | 0 | 24 | 36 |
| 2 |   | 0  | 24 |
| 3 |   |    | 0  |

$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2$

$c[1,3]$

$A_1 A_2 A_3$

$(A_1 A_2) A_3$

$A_1 (A_2 A_3)$

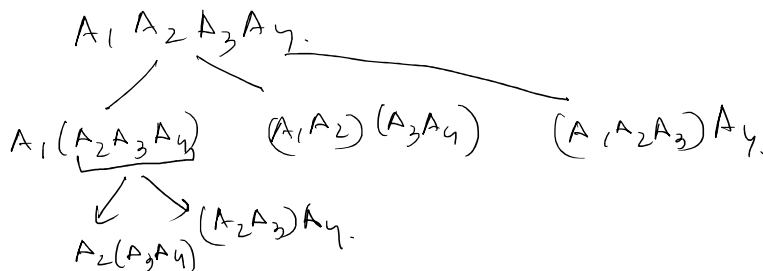
$(A_1 A_2) A_3$

$= c[1,2] + c[3,3] +$

cost

$\rightarrow 2 \times 4 \times 2$

$= 24 + 0 + 16 = 40$



$3 \times 2$   
/ \  
 $(2A_3)$ .

$$+ c[2,3] + 2 \cdot 3 \times 2$$

$$24 + 12 = 36.$$