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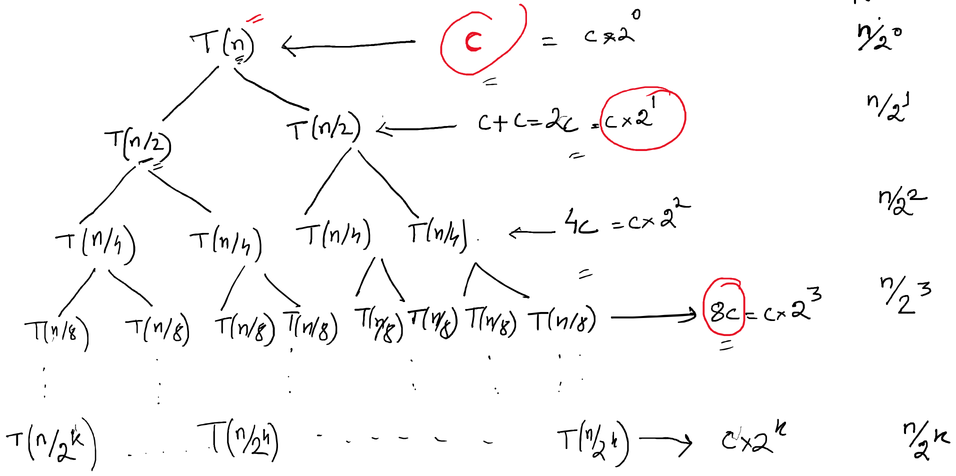
Recursion tree method

$$T(n) = \begin{cases} 2T(n/2) + c, & n > 1 \\ c, & n = 1 \end{cases}$$



Recursion stop → $\frac{n}{2^k} = 1$

$\Rightarrow n = 2^k$

$\Rightarrow \log_2 n = \log_2 (2^k)$

$\Rightarrow \log_2 n = k \underline{\log_2 2}$

$\Rightarrow k = \log_2 n$

GP sum $= \frac{a(r^n - 1)}{r-1}$

Total cost

$c + 2c + 4c + 8c + \cdots\cdots + 2^k \times c$

$= c2^0 + c2^1 + c2^2 + c2^3 + \cdots\, c2^k$

$= c\left(2^0 + 2^1 + \cdots + 2^k\right)$

$= \frac{c \times 1 \times (2^{k+1} - 1)}{2-1}$

$= c \times 2^k \cdot 2 - c$

$= c \cdot 2^k - c$

$\approx 2^k$

$= 2^{\log_2 n}$

$= n^{\log_2 2}$

$= n$

$\Rightarrow TC = O(n)$.

$$T(n) = \begin{cases} 2T(n/2) + n, & n > 1 \\ 1, & n = 1 \end{cases}$$

$T(n) \longrightarrow n \longrightarrow$ 0th step

$T(n/2) \quad T(n/2) \longrightarrow n/2 + n/2 = n \longrightarrow$ 1st step

$T(n/4) \quad T(n/4) \quad T(n/4) \quad T(n/4) \longrightarrow n/4 + n/4 + n/4 + n/4 = n$ — 2^{nd}

$T(n/8) \quad T(n/8) \quad T(n/8) \quad T(n/8) \quad T(n/8) \quad T(n/8) \quad T(n/8) \quad T(n/8) = 8 \times n/8 = n$ — 3^{rd}

$\vdots$

$T(n/2^k) \quad T(n/2^k)$

$n \longrightarrow k^{th}$ step

n
$n/2^1$
$n/2^2$
$n/2^3$
$n/2^k$

Recursion stop, $n/2^k = 1$

$\Rightarrow n = 2^k$

$\Rightarrow \log_2 n = \log_2 (2^k)$

$\Rightarrow \log_2 n = k \times \log_2 2$

Total cost $= n \times k = n \times \log_2 n$

$T.C = O(n \log_2 n)$

$\Rightarrow k = \log_2 n$

Master Theorem -

If the recurrence is of the form $T(n) = aT(\frac{n}{b}) + \Theta(n^k \log^p n)$, where $a \geq 1, b > 1, k \geq 0$ and p is a real number, then:

1) If $a > b^k$, then $T(n) = \Theta(n^{\log_b^a})$
2) If $a = b^k$
   a. If $p > -1$, then $T(n) = \Theta(n^{\log_b^a} \log^{p+1} n)$ $\rightarrow n^{\log_b a}$
   b. If $p = -1$, then $T(n) = \Theta(n^{\log_b^a} \log\log n)$ $n^{\log_b a}$
   c. If $p < -1$, then $T(n) = \Theta(n^{\log_b^a})$
3) If $a < b^k$
   a. If $p \geq 0$, then $T(n) = \Theta(n^k \log^p n)$
   b. If $p < 0$, then $T(n) = O(n^k)$

Q. $T(n) = 3T(n/2) + n^2$ $a = 3, b = 2, k = 2, p = 0$

$a = 3, b^k = 2^2 = 4, a < b^k \Rightarrow p = 0,$

$Cond^n \to 3a.$ $\quad T(n) = \Theta(n^k \log^p n) = n^2 \log^0 n = n^2$

2 $\quad T(n) = 4T(n/2) + n^2,$ $\quad a=4,\ b=2,\ k=2,\ p=0$

$b^k = 2^2 = 4$

$a = b^k$

$Cond^n - 2, a) -$ $\quad T(n) = n^{\log_b a} \log^{p+1} n = \underbrace{n^{\log_2 4} \cdot \log^{0+1} n}_{} = n^2 \cdot \log n$

$\log_2 4 \to 2^2$

$n^{\log_2 4} = n^{\log_2 2^2} = n^{2 \log_2 2} = n^2$

H/W

$T(n) = 16T(n/4) + n$ $\quad$ An $- n^2$

$T(n) = 2T(n/2) + \frac{n}{\log n}$, $\quad$ $A_2 -$ $n \log \log n.$