Matrix chain multiplication: A and B can be multiplied when number of row in B= number of column in A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \xrightarrow{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3\times 2}$$

$$A \times B = \begin{bmatrix} a_{11} & b_{11} + a_{12} & b_{21} + a_{13} & b_{31} \\ a_{21} & b_{11} + a_{22} & b_{21} + a_{23} & b_{31} \end{bmatrix} \xrightarrow{a_{11} b_{12} + a_{12} b_{22} + a_{23} b_{32}} \xrightarrow{a_{21} b_{12} + a_{23} b_{32}}$$

$$N_0 = \begin{bmatrix} a_{11} & a_{12} & b_{21} + a_{23} & b_{31} \\ a_{21} & b_{11} + a_{22} & b_{21} + a_{23} & b_{31} \end{bmatrix} \xrightarrow{a_{21} b_{12} + a_{22} b_{22} + a_{23} b_{32}} \xrightarrow{2\times 2}$$

$$N_0 = \begin{bmatrix} a_{11} & a_{12} & a_{23} & b_{31} \\ a_{21} & b_{11} + a_{22} & b_{21} + a_{23} & b_{31} \end{bmatrix} \xrightarrow{a_{21} b_{12} + a_{22} b_{22} + a_{23} b_{32}} \xrightarrow{2\times 2}$$

$$N_0 = \begin{bmatrix} a_{11} & a_{12} & a_{23} & b_{31} \\ a_{21} & b_{21} & a_{22} & b_{22} + a_{23} & b_{32} \end{bmatrix} \xrightarrow{2\times 2}$$

$$N_0 = \begin{bmatrix} a_{11} & a_{12} & a_{23} & b_{31} \\ a_{21} & b_{21} & a_{22} & b_{22} + a_{23} & b_{32} \end{bmatrix} \xrightarrow{2\times 2}$$

$$= 2 \times 3 \times 2 = 12 \quad \text{(by formulas)}.$$

 $A_1 = 2 \times 3$ $A_2 = 3 \times 4$ $A_3 = 4 \times 2$. Ninimum multiplication to find AIA2A3.

$$A_1 = 2 \times 3$$
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Ninimum multiplication to find AIA2A3.

Dimensions
$$(A_1 A_2) \cdot A_3$$

 $2 \times 3 \cdot 3 \times 4 \cdot 4 \times 2$
 $Cost$ 24
 $Cost = 2 \times 4 \times 2 = 16$
 $Told cost operation = 40$

(A1.A2) · A3

A1 (A2 · A3).

A1 · (A2 · A3)

Dimensions
$$2 \times 3$$
 $2 \times 4 \times 2$,

 $3 \times 4 \times 2 = 24$

Dimensions

 $3 \times 4 \times 2 = 24$

Dimensions

 3×2
 $2 \times 4 \times 2 = 12$

Total ask = $24 \times 12 = 36$.

$$A_1 = 2 \times 3$$
 $A_2 = 3 \times 4$ $A_3 = 4 \times 2$.
Minimum multiplication to find AIA2A3.

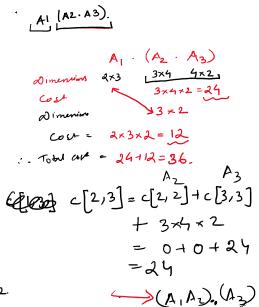
(A1.A2). A3

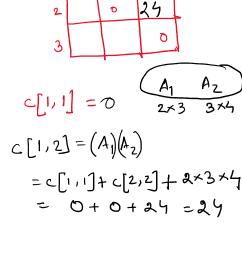
Dimensions (A1.A2). A3

Existing Dimension 2×4

Cost = 2×4×2=16

Told cost operations = 40





24

$$A_{1} = 3 \times 3 \qquad A_{2} = 3 \times 4 \qquad A_{3} = 4 \times 2.$$

$$1 \qquad 0 \qquad 2 \qquad 4 \qquad 3 \qquad 6$$

$$2 \qquad 0 \qquad 2 \qquad 4$$

$$= 24$$

$$(A_1A_2).(A_3)$$

$$c[1/3] = c[1/2] + c[3/3] + 2x4 \times 2$$

$$A_1A_2A_3 \qquad 2x4 \qquad 4x2$$

$$= 24 + 0 + 16$$

$$\longrightarrow (A_1)(A_2 \cdot A_3)$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad$$

AIAZ A3 A4 $(A_1A_2)(A_3A_4) \qquad A_1(A_2A_3A_4)$ $A_2(A_3A_4)(A_2A_3)A_4$

A, Az A3

[2,1] A2A,