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The **Travelling Salesman Problem (TSP)** is a classic **combinatorial optimization problem** in computer science and operations research. It's defined as:

Given: A list of cities and the distances between each pair of cities.

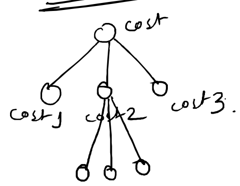
Goal: Find the shortest possible route that visits each city exactly once and returns to the starting city.

TSP appears in various real-world scenarios like Route planning (delivery trucks, sales routes)

Core Concepts

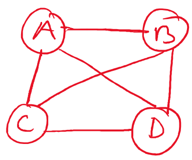
1. **Branching**: You build a **tree of subproblems**, where each node represents a partial tour (sequence of cities visited).
2. **Bounding**: At each node, you compute a **lower bound** (minimum possible cost to complete the tour from here).
3. **Pruning**: If a node's lower bound is worse than the best complete solution found so far, you discard (prune) that branch.

Branch & Bound-



Steps to Solve TSP with Branch and Bound:

1. **Start with a cost matrix** of distances between all cities.
2. **Reduce the matrix**:
   - Subtract the smallest value in each row and each column (this gives a *lower bound*).
3. **Create a priority queue (min-heap)** to explore promising nodes first (ones with smaller bounds).
4. **At each node**:
   - Choose a city to visit next.
   - Update the matrix to reflect the path chosen (remove rows/columns).
   - Recalculate the reduced cost and total bound.
5. **Prune** paths with bounds higher than the best known solution.
6. Repeat until all promising paths are explored.



A B C D A   A B C B D A.

A C B D A.

A D C B A.

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 10 & 5 & 3 \\ B & 8 & \infty & 9 & 7 \\ C & 1 & 6 & \infty & 9 \\ D & 2 & 3 & 8 & \infty \end{array}$$

$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & 10 & 5 & 3 & -3 \\ B & 8 & \infty & 9 & 7 & -7 \\ C & 1 & 6 & \infty & 9 & -1 \\ D & 2 & 3 & 8 & \infty & -2 \\ \hline & & & & & 13 \end{array}$$

Row Red^n, Col red^n

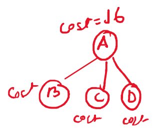
$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & 7 & 2 & 0 & -3 \\ B & 1 & \infty & 2 & 0 & -7 \\ C & 0 & 5 & \infty & 8 & -1 \\ D & 0 & 1 & 6 & \infty & -2 \\ \hline & \downarrow & \downarrow & \downarrow & \downarrow & \\ & 0 & 1 & 2 & 0 & \end{array}$$

$0 + 1 + 2 + 0 = 3.$

Cost = Row Red^n + Col Red^n = 3 + 13 = 16.

$M_A =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$



$M_A =$

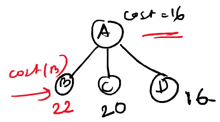
$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$

$M_{AB} =$

$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & \infty & \infty & 0 & 0 & \to 0 \\ C & 0 & \infty & \infty & 8 & \to 0 \\ D & 0 & \infty & 4 & \infty & \to 0 \\ \hline & \downarrow & \downarrow & \downarrow & \downarrow & \\ & 0 & \infty & 0 & 0 & \end{array}$$

cost(B) = cost(A) + Reduction + AB

= 16 + 0 + 6 = 22



$M_A =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$

$M_{AC} =$

$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \leftarrow \infty \\ B & 1 & \infty & \infty & 0 & \leftarrow 0 \\ C & \infty & 0 & \infty & 4 & \leftarrow 4. \\ D & 0 & 0 & \infty & \infty & \leftarrow 0 \\ \hline & \downarrow & & & & \\ & 0 & 0 & \infty & 0 & \end{array}$$

cost(C) =

cost(A) + Reduction

+ AC

= 16 + 4 + 0 = 20

$M_A =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & 6 & 0 & 0 \\ B & 1 & \infty & 0 & 0 \\ C & 0 & 4 & \infty & 8 \\ D & 0 & 0 & 4 & \infty \end{array}$$

$M_{AD} =$

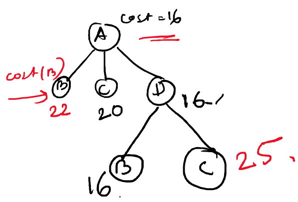
$$\begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & 1 & \infty & 0 & \infty & \to 0 \\ C & 0 & 4 & \infty & \infty & \to 0 \\ D & \infty & 0 & 4 & \infty & \to 0 \\ \hline & \downarrow & \downarrow & \downarrow & \downarrow & \\ & 0 & 0 & 0 & \infty & \end{array}$$

cost (D) = cost(A) + Red^n +

AD

= 16 + 0 + 0

= 16.



$M_{AD} =$

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & \infty & \infty & \infty & \infty \\ B & 1 & \infty & 0 & \infty \\ C & 0 & 4 & \infty & \infty \\ D & \infty & 0 & 4 & \infty \end{array}$$

M_{ADB}

$$= \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & \infty & \infty & 0 & \infty & \to 0 \\ C & 0 & \infty & \infty & \infty & \to 0 \\ D & \infty & \infty & \infty & \infty & \to \infty \\ \hline & 0 & \infty & 0 & \infty & \end{array}$$

cost(B) = cost(D) +

Reduction +

DB.

= 16 + 0 + 0

= 16

$$M_{ADC} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \hline A & \infty & \infty & \infty & \infty & \to \infty \\ B & 0 & \infty & \infty & \infty & \to 1 \end{array}$$

cost(C) = cost(D) + Red^n + DC.

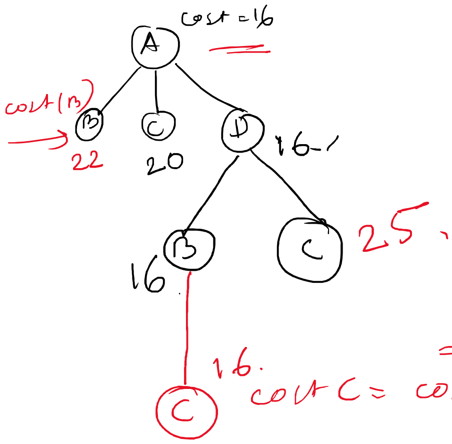
$$
M_{ADC} = \begin{array}{c|cccc|l}
 & A & B & C & D & \\
\hline
A & \infty & \infty & \infty & \infty & \rightarrow \infty \\
B & 0 & \infty & \infty & \infty & \rightarrow 1 \\
C & \infty & 0 & \infty & \infty & \rightarrow 4 \\
D & \infty & \infty & \infty & \infty & \rightarrow \infty \\
\hline
 & \downarrow 0 & \downarrow 0 & \downarrow \infty & \downarrow \infty &
\end{array}
$$

$\underline{5}$

$$\text{cost}(C) = \text{cost}(D) + \text{Red}^n + DC.$$

$$= 16 + 5 + 4 = 25.$$

$$
M_{ADBC} = \begin{array}{c|cccc|}
 & A & B & C & D \\
\hline
A & \infty & \infty & \infty & \infty \\
B & \infty & \infty & \infty & \infty \\
C & \infty & \infty & \infty & \infty \\
D & \infty & \infty & \infty & \infty
\end{array}
$$



16. cost C = cost(B) + Red^n + BC = 16

Shortest Path = $\overset{3}{\overbrace{A\,D}}\overset{1}{\overbrace{\;B\;C\;A}}$ with $\underbrace{A\,D}_{3}\,\underbrace{B\,C}_{9}$ $\longrightarrow$ $\underline{16}$

$$
\begin{array}{c|cccc|}
 & A & B & C & D \\
\hline
A & \infty & 10 & 5 & \underline{3} \\
B & 8 & \infty & \underline{9} & 7 \\
C & \boxed{1} & 6 & \infty & 9 \\
D & 2 & \underline{3} & 8 & \infty
\end{array}
$$