

Matrix chain multiplication: A and B can be multiplied when number of row in B = number of column in A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{3 \times 2}$$

$$A \times B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{bmatrix}_{2 \times 2}$$

No. of multiplications = 12 (By observation).

$$= 2 \times 3 \times 2 = 12. \quad \text{(by formulae).}$$

$$A_1 = \underline{2 \times 3} \quad A_2 = \underline{3 \times 4} \quad A_3 = \underline{4 \times 2}.$$

Minimum multiplication to find $A_1 A_2 A_3$.

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2.$$

... .. multiplication to find $A_1 A_2 A_3$.

Minimum multiplication to find $A_1 A_2 A_3$.

$$(A_1 \cdot A_2) \cdot A_3$$

$$A_1 (A_2 \cdot A_3)$$

$$\begin{array}{l} \text{Dimensions} \quad \overbrace{2 \times 3}^{A_1} \quad \overbrace{3 \times 4}^{A_2} \quad \overbrace{4 \times 2}^{A_3} \\ \text{Cost} \quad 2 \times 3 \times 4 = 24 \\ \text{Resulting Dimension} \quad 2 \times 4 \\ \text{Cost} = 2 \times 4 \times 2 = 16 \end{array}$$

$$\therefore \text{Total cost operations} = 24 + 16 = 40.$$

$$\begin{array}{l} \text{Dimensions} \quad \overbrace{2 \times 3}^{A_1} \quad \overbrace{3 \times 4}^{A_2} \quad \overbrace{4 \times 2}^{A_3} \\ \text{Cost} \quad 3 \times 4 \times 2 = 24 \\ \text{Dimensions} \quad 2 \times 3 \quad 3 \times 2 \\ \text{Cost} = 2 \times 3 \times 2 = 12 \end{array}$$

$$\therefore \text{Total cost} = 24 + 12 = 36.$$

$$A_1 = 2 \times 3 \quad A_2 = 3 \times 4 \quad A_3 = 4 \times 2.$$

Minimum multiplication to find $A_1 A_2 A_3$.

$$c[1,2] = c[1,1] + c[2,2] + \text{some cost}.$$

$$A_1 A_2 = 0 + 0 + 2 \times 3 \times 4 = 24$$

$$\begin{array}{l} A_1 A_2 \quad A_3 \\ 2 \times 4 \quad 4 \times 2 \end{array}$$

	1	2	3
1	0	24	36
2		0	24

$$\begin{array}{l} \overbrace{(A_1 A_2)} \quad \overbrace{A_3} \\ \Downarrow \\ c[1,3] = c[1,2] + c[3,3] + \\ 2 \times 4 \times 2 \\ = 24 + 0 + 16 = 40. \end{array}$$

$$A_1 (A_2 A_3)$$

$A_1 A_2$
 $\downarrow \quad \downarrow$
 $A_1 \quad A_2$

$$= 0 + 0 + 2 \times 3 \times 4 = 24$$

$$c[2,3] = c[2,2] + c[3,3] + \underline{3 \times 4 \times 2}$$

$$= 0 + 0 + 24 = 24$$

$$A_1 \Rightarrow 2 \times 3$$

$$(A_2 A_3) \Rightarrow 3 \times 2$$

2	0	24
3		0

$$A_1 (A_2 A_3)$$

$$c[1,3] = c[1,1] + c[2,3]$$

$$+ \underline{2 \times 3 \times 2}$$

$$= 0 + 24 + 12$$

$$= 36$$

