

MAD76 Academy: D. Python

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1 Agenda

- What is Python? Why Python? (see Section 2)
- Math with Python (see Section 3)
- Procedural programming with Python (see Section 4)
- Object-oriented programming with Python (see Section 5)

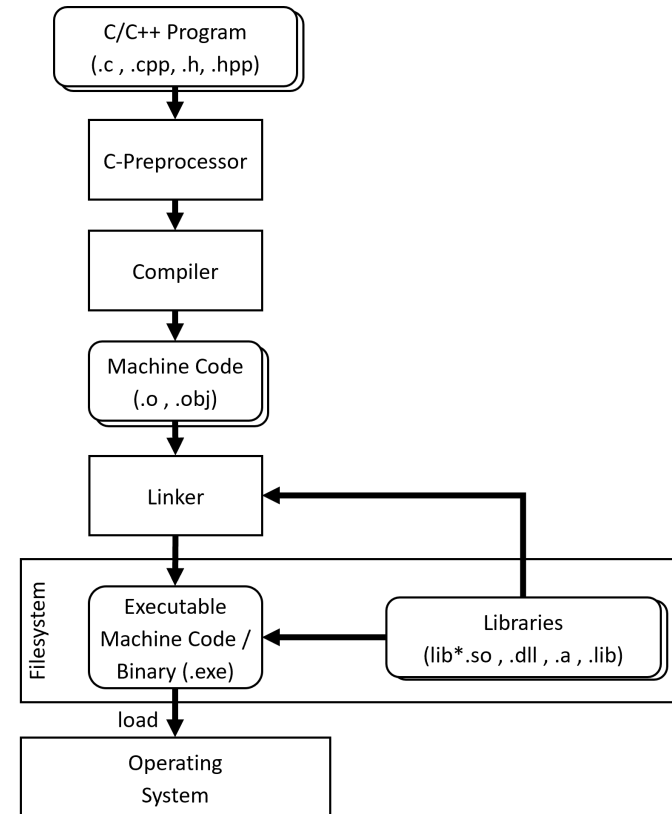
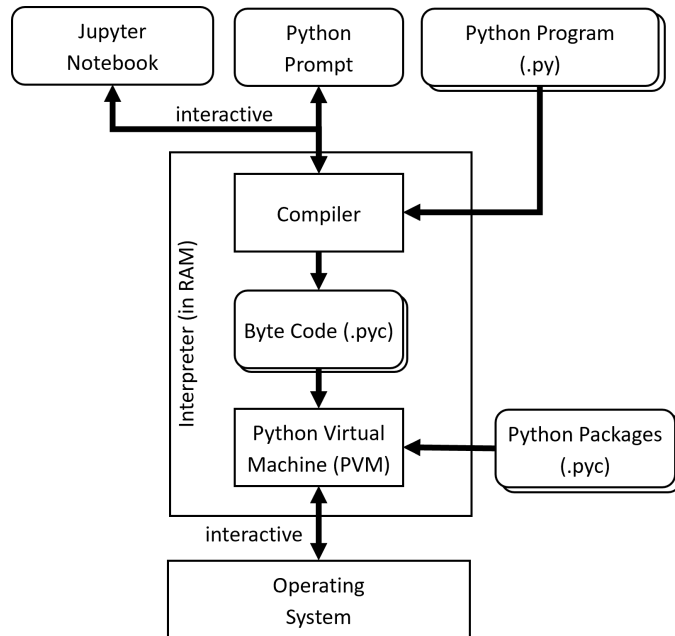
Teaching Objectives

- Understand Python as an interpreted programming language
- Learn Python syntax and semantics
- Learn how to use Python for mathematics at school
- Learn how to use Jupyter Notebooks for Python
- Learn procedural programming basics with Python
- Learn object-oriented programming basics with Python
- Learn how to use VS Code as an IDE for Python

2 What is Python

- Python is a high-level programming language [1]
 - for procedural and object-oriented programming
- Python is an *interpreter*
 1. Python prompts for input
 2. User enters Python command
 3. Python replies to command right away
- Python is the most popular programming language in the world (<https://www.tiobe.com/tiobe-index/>)
- Python is THE programming language in Artificial Intelligence (AI)
- Python is strong for scripting as a high-level alternative to Bash
- Python is one of the programming languages of ROS2 and MAD76
- Python is straight-forward to use for beginners
 - which is not the case for C++ or Rust
- Python is strong in numerics (mathematics on computers)
 - not as strong as MATLAB, though, but free to use
- But:
 - Python is comparably slow and resource-intensive [2]
 - This leads to high energy consumption \rightsquigarrow high CO_2 emissions of AI Foundation Models training
 - Python is not reliable and non-realtime \rightsquigarrow not suitable for safety-critical applications (in cars, aeroplanes)

Interpreter versus Compiler



3 Math with Python

Agenda

- Use Jupyter with Python as a calculator (see Section 3.1)
- Solve quadratic equations (see Section 3.2)
- Solve linear equation systems (see Section 3.3)
- Function plotting (see Section 3.4)
- Python data types (see Section 3.5)

3.1 Python as Calculator

- Jupyter is an interactive user interface
- Jupyter notebooks are *living documents*
- Jupyter notebooks contain both
 - Python code
 - and documentation (in Markdown)
 - including math formulas (in \LaTeX)
- Jupyter mainly supports Python, but also other programming languages
- Jupyter is available both locally in VS Code and online (see <https://jupyter.org/>)

1. Create a new directory and start VS Code

```
cd  
mkdir -p src/pythonmath  
cd src/pythonmath  
code .
```

(a) by hitting File – New File...

(b) selecting Jupyter Notebook in the dropdown menu

2. Save the notebook as file `math.ipynb` by hitting `Ctrl+S`

3. Enter the following code in the cell:

```
1 + 2
```

- Note that `1 + 2` is a valid Python expression

4. Hit `Ctrl+Enter` to execute the cell

5. Select `Python Environments ...` and then `Python 3.x.x /usr/bin/python` (or similar)

6. Jupyter then displays the result of the calculation

7. Try some more calculations by hitting `Alt+Enter` for adding new cells and `Ctrl+Enter` for executing cells

```
3 * 4 / 2 * 3
3 * 4 / (2 * 3)
2 ** -1
2 ** -3 * 8
```

8. Import Python package `numpy` for numerical calculations

```
import numpy as np
```

9. Use numpy to calculate the square root of a number

```
np.sqrt(2)
```

10. and for trigonometry

```
np.pi  
np.pi * 0.5  
np.pi * 5e-1  
np.sin(np.pi * .5)  
np.cos(np.pi)  
np.cos(np.deg2rad(180))  
np.sin(101)**2 + np.cos(101)**2  
np.tan(np.pi * .5)
```

11. Use variables

```
x = 2  
y = 3  
z = x + y
```

12. Loops

```
for i in range(5):  
    print(i, i**2)
```

3.2 Solve Quadratic Equations

- Solve quadratic equations of the form $ax^2 + bx + c = 0$
 - Use the quadratic formula $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
1. Create a new Markdown cell for documentation by hitting + Markdown

```
# Quadratic Equations
* equation: $a x^2 + b x + c = 0$
* solutions: $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 a c}}{2 a}$
```

Note: \$ is used to define inline math formulas using \LaTeX

2. Create a new Code cell

```
a = 4
b = -20
c = -200
d = b**2 - 4*a*c
x1 = (-b + np.sqrt(d)) / (2*a)
x2 = (-b - np.sqrt(d)) / (2*a)
x1 , x2
```

3. Easier: Use roots function of numpy that solves polynomial equations $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$ of any degree $n \geq 1$

```
np.roots([a, b, c])
```

3.2.1 Exercises

D.3.2.1 Solve the quadratic equation $x^2 + 2x + 1 = 0$. Required results are:

- Python code
- Results x_1 and x_2

D.3.2.2 Solve the cubic equation $x^3 + 3x^2 + 3x + 1 = 0$. Required results are:

- Python code
- Results x_1 , x_2 and x_3

3.3 Solve Linear Equation Systems

- Solve linear equation systems of order $n \geq 1$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

- This equation system can be formulated as a matrix equation

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$$

- with

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

- Example with $m = 2$ and $n = 2$:

$$\begin{aligned} x_1 + 2x_2 &= 5 \\ 3x_1 + 4x_2 &= 6 \end{aligned}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

- Use the `numpy.linalg.solve` function

1. Create a new Markdown cell

```
# Solve Linear Equation System
```

2. Create a new Code cell

```
A = np.array([[1, 2], [3, 4]])  
b = np.array([5, 6])  
x = np.linalg.solve(A, b)  
x
```

3.3.1 Exercises

D.3.3.1 Solve the linear equation system of order $n = 2$

$$\begin{aligned} 2x_1 + 4x_2 &= 2 \\ x_1 + 2x_2 &= 1 \end{aligned}$$

- Required results are:
 - Python code
 - Results x_1 and x_2

D.3.3.2 Solve the linear equation system of order $n = 3$

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= 10 \\4x_1 + 5x_2 + 6x_3 &= 11 \\7x_1 + 8x_2 + 9x_3 &= 12\end{aligned}$$

- Required results are:
 - Python code
 - Results x_1 , x_2 and x_3

3.4 Function Plotting

1. Create a new Markdown cell

```
# Function Plotting
```

2. Create a new Code cell to import Python package matplotlib.pyplot for plotting

```
import matplotlib.pyplot as plt
```

3. Define 1000 sampling points in the range $[-100, 100]$

```
x = np.linspace(-100, 100, 1000)
```

4. Compute the values at the sampling points of the Sinc function $y = f(x) = \frac{\sin(x)}{x}$

```
y = np.sin(x) / x
```

5. Plot the function using matplotlib

```
plt.plot(x, y)
plt.title("Sinc Function")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.grid()
plt.show()
```

3.4.1 Exercises

D.3.4.1 Plot a cosine function with frequency 50Hz and amplitude 230V. Required results are:

- Python code
- Function plot of y over $x = t$ with time t in the range $[0, 0.1\text{s}]$

3.5 Python Data Types

- Python variables are *dynamically typed*
 - The data type is determined at runtime
 - The data type is determined by assigning values to a variable
 - A variable can hold values of different types at different times
- The data type can be determined using the `type` function

```
type(-1.0)
type(2)
type(np.pi)
type(A)
type(x)
```

Numeric Types

Data type	Description	Example values
int	integer with no limits	1, -1
float	64bit floating point on most CPUs	3.14, -0.001, .5, 10e-1
np.float32	32bit floating point	np.float32(3.14)
np.float64	64bit floating point	np.float64(3.14)
complex	complex numbers	1+2j, -3-4j
bool	boolean values	True, False

Text Type

string | string with UTF-8 characters | "Max-Planck-Str. 39", 'Heilbronn'

Aggregate Types

Data type	Description	Example values
list	mutable list with different data types	<code>[1, -1, 3.14, "text"]</code>
list	empty list	<code>[]</code>
np.array	mutable array with one data type	<code>np.array([1, -1, 3.14])</code>
tuple	immutable list with different data types	<code>(1, -1, 3.14, "text")</code>
tuple	empty tuple	<code>()</code>
range	immutable sequence of numbers (10 integers 0, 1, ..., 9) (9 integers 1, 2, ..., 9)	<code>range(10)</code> <code>range(1, 10)</code>
set	unordered collection of unique elements	<code>set([1, 2, 3, 4, 5])</code>
dict	dictionary of key-value pairs	<code>{"lastname": "Mouse", "firstname": "Mickey"}</code>

Element Access

- Define list and arrays

```
l = [ 1, -1, 3.14, "text" ]  
A = np.array([[11, 12, 13], [21, 22, 23], [31, 32, 33]])  
x = np.array([1, 2, 3, 4, 5])
```

- First element (Python has zero-based indexing)

```
l[0]  
A[0, 0]  
x[0]
```

- Last element

```
l[-1]  
A[-1, -1]  
x[-1]
```

- Sizes

```
len(l)  
A.shape  
x.size
```

- Slicing

```
l[1:3]  
A[1, 1:3]  
A[0, 1:]  
x[1:]
```

- Slicing with step

```
l[0:-1:2]  
A[0, ::2]  
x[::2]
```

4 Procedural Programming

- Python is imperative: programs are sequences of statements
 - which are executed in order, step-by-step
- Python is procedural
 - Sequences of statements can be grouped into functions \rightsquigarrow *reusability*
 - Functions can be called with parameters (arguments)
 - Functions can return values
 - Functions can be called from different places in the program
- Python is object-oriented (see Section 5)

Agenda

- Simple hello world program (see Section 4.1)
- Hello world program with functions (see Section 4.2)
- Function for solving quadratic equations (see Section 4.3)

4.1 Hello World

1. Create a Python program `helloworld.py` in VS Code
 - (a) by hitting File - New File...
 - (b) selecting Python File in the dropdown menu
2. Enter the following code in the text editor

```
#!/usr/bin/env python3  
  
print("Hello, World!")
```

- `#!/usr/bin/env python3` is a *shebang* line that tells Linux to use the Python interpreter
 - `print` is a built-in function in Python to generate output in the terminal
 - `"Hello, World!"` is a string of UTF-8 characters
 - `()` are parentheses used to call functions and to enclose function arguments
3. Save the program to a file `helloworld.py` by hitting `Ctrl+S`
 4. Run the program by hitting `F5`
 5. Select Python Debugger and then Python File from the dropdown menu

6. In the Terminal window, the output should be displayed as Hello, World!

Run program from terminal

1. open a terminal window
2. navigate to the directory where the file is saved
3. run the Python interpreter and start the program

```
python helloworld.py
```

4. Or make the file helloworld.py executable and run it directly

```
chmod +x helloworld.py  
./helloworld.py
```

Run program from Python prompt

1. open a terminal window
2. navigate to the directory where the file is saved
3. run the Python interpreter python

```
python
```

which gives you a Python prompt

4. load and run the program at the Python prompt

```
import helloworld
```

5. Exit Python

```
quit()
```

or hit Ctrl+D

Run program from Jupyter notebook

1. create and execute a new cell

```
import helloworld
```

4.2 Hello World with Functions

1. Create a new Python program `helloworld_function.py` in VS Code
2. Enter the following code in the text editor

```
#!/usr/bin/env python3

def hello_world():
    print("Hello world")

def hello(name):
    print("Hi", name, "!")
    print("How are you today?")

if __name__ == "__main__":
    hello_world()
    hello("Asterix")
    hello("Obelix")
```

- `def` is a keyword in Python to define a function
- `def <functionname> (<argument1>, <argument2>, ...):` defines a function where
 - `<functionname>` is the name of the function

- `<argument1>`, `<argument2>`, ... is a list of arguments passed to the function
- The function body is indented by the TAB key
- The function body contains the statements to be executed when the function is called
- `hello_world` is a function that prints a greeting message
- `hello` is a function that takes a name as an argument and prints a personalized greeting
- The `if __name__ == "__main__":` block ensures that the functions are only called when the script is run directly
 - The functions are only called if the program is run by hitting F5 in VS Code
 - or by running the program from the terminal with `python helloworld_function.py`
 - but not when the program is imported with the `import` statement

4.2.1 Exercises

D.4.2.1 Extend the function `hello` by adding the additional message "And how is Idefix?" if the function argument `name` is equal to "Obelix". Required results are:

- Extended `helloworld_function.py`

4.3 Function for Solving Quadratic Equations

1. Create a new Python program `quadratic.py` in VS Code
2. Enter the following code in the text editor

```
#!/usr/bin/env python3

import numpy as np

def solve_quadeqn(a, b, c):
    """
    Solves the quadratic equation  $ax^2 + bx + c = 0$  for real roots.

    Parameters:
        a (float): Coefficient of  $x^2$ 
        b (float): Coefficient of  $x$ 
        c (float): Constant term

    Returns:
        tuple: A tuple containing the two real roots (x1, x2)
    """
    d = b ** 2 - 4.0*a*c
    x1 = (-b + np.sqrt(d)) / (2.0 * a)
    x2 = (-b - np.sqrt(d)) / (2.0 * a)
    return (x1, x2)
```

```
if __name__ == "__main__":  
    a = 1.0  
    b = -3.0  
    c = 2.0  
    roots = solve_quadeqn(a, b, c)  
    print("The roots are:", roots)
```

3. Run the code by hitting F5
4. Change b to 1.0 and rerun the code

4.3.1 Exercises

D.4.3.1 Modify the code to check for a negative discriminant and return an empty tuple in that case. Required results are:

- Extended quadratic.py

5 Object-Oriented Programming

- Python is object-oriented
 - Data and functions can be grouped into *classes*
 - Classes can model real-world entities
 - Classes represent the *properties* as well as *behaviors* of entity types
 - * *attributes* = properties = data
 - * *methods* = behavior = functions
 - A class can be instantiated multiple times to create *objects* (*class instances*)
 - * Each object represents an individual entity and has its own set of attribute values
 - Classes are encapsulated
 - * Implementation details are hidden from the user
 - * Users interact with objects through well-defined interfaces (subsets of the methods and attributes)
 - Classes can inherit from other classes and thus model sub-typing relationships between entities
 - * A *quadratic equation* is a sub-type of a *polynomial equation*, which in turn is a sub-type of an *equation*

5.1 Class for Quadratic Equations

- The following code defines a class `QuadraticEquation` for solving quadratic equations of the form $ax^2 + bx + c = 0$
- Class `QuadraticEquation` represents a quadratic equation and stores the equation coefficients as attributes
- Its behavior is defined by the following methods

Method	Description
<code>__init__(self, a, b, c)</code>	Constructor to initialize the equation coefficients a , b , and c . This constructor is called when instantiating the class with <code>QuadraticEquation(a, b, c)</code>
<code>solve(self)</code>	Solves the quadratic equation and returns the real roots as a tuple
<code>__str__(self)</code>	Returns a string representation of the quadratic equation. <code>__str__(self)</code> is internally called by <code>print(eq)</code> .

- Within the class definition, `self` refers to the current instance of the class
- Upon instantiation, the variable `eq` holds a reference to the newly created `QuadraticEquation` object
- With `eq.solve()`, the `solve` method is called for object `eq`

```
#!/usr/bin/env python3

import numpy as np

class QuadraticEquation:
    def __init__(self, a, b, c):
        self.a = a
        self.b = b
        self.c = c

    def solve(self):
        """
        Solves the quadratic equation  $ax^2 + bx + c = 0$  for real roots.

        Returns:
            tuple: A tuple containing the two real roots (x1, x2)
        """
        d = self.b ** 2 - 4.0 * self.a * self.c
        if d < 0.0:
            return None # No real roots
        x1 = (-self.b + np.sqrt(d)) / (2.0 * self.a)
        x2 = (-self.b - np.sqrt(d)) / (2.0 * self.a)
        return (x1, x2)

    def __str__(self):
```

```
        return f"QuadraticEquation({self.a}*x^2 + {self.b}*x + {self.c} = 0)"

if __name__ == "__main__":
    eq = QuadraticEquation(1, -3, 2)
    print(eq)
    roots = eq.solve()
    print("The roots are:", roots)
```

References

- [1] E. Matthes. *Python Crash Course: A Hands-On, Project-Based Introduction to Programming*. 3rd. No Starch Press, 2021. ISBN: 978-1718502703.
- [2] Rui Pereira et al. “Ranking programming languages by energy efficiency”. In: *Science of Computer Programming* 205 (2021), p. 102609.