Unit 2: Conditioning and Independence

MITx Statistics and Data Science

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Note

Bayes' rule is the foundation for the field of inference. It guides on how to process data and make inferences about unobserved quantities or phenomena

[1] Conditioning and Bayes'Rule

Conditional Probabilities

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$, defined only when $P(B) > 0$.
- $P(A \cap B|A) = \frac{P(A \cap B)}{P(A)}$
- Properties:
  - $P(A|B) \geq P(B)$, assuming $P(B) > 0$
  - If $A \cap B =$, then $P(A \cup C|B) = P(A|B) + P(C|B)$

Three important tools

- Multiplication rule
  $P(A \cap B) = P(B)P(A|B) + P(A)P(B|A)$
  $P(A^c \cap B \cap C^c) = P(A^c)P(B|A^c)P(C^c|A^c \cap B)$
  $P(A_1 \cap A_2 \cap A_3) = P(A_1) \prod_{i=2}^{n} P(A_i|A_1 \cap ... \cap A_{i_1})$
- Total probability theorem
  Mutually exclusive $(B_i \cap B_j = \oslash)$, $\{B_1, B_2, ..., B_k\}$
  $P(B) = \sum_{i=1}^{k} P(A_i)P(B|A_i)$, which is a weighted average of $P(B|A)$. Note that $\sum_i P(A_i) = 1$
- Bayes' rule
  $P(A_i|B) = \frac{A_i \cap B}{P(B)} = \frac{P(A_i)P(B|A_i)}{\sum_{j=1}^{k} P(A_j)P(B|A_j)} = \frac{P(A_i)P(B|A_i)}{P(B)}$
  Inference
  - initial belief $P(A_i)$ on possible causes of an observed event B
    $A_i \xrightarrow{\text{model } \mathbf{P}(B|A_i)} B$
  - draw conclusions about causes
    $B \xrightarrow{\text{inference } \mathbf{P}(A_i|B)} A_i$