# Powertrain Torsional Vibration System Model Development in Modelica for NVH Studies

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#### **Abstract**

For developing high-quality and cost-efficient products, it is important to evaluate and compare system level performance for different configurations early in the development process. This paper will present the development of a vehicle system model in Modelica that is used to study the overall vehicle power-train torsional vibrations that impact Noise, Vibration & Harshness (NVH) characteristics of the vehicle. In this study, a detailed crank-angle resolved, multi-cylinder engine model is constructed, which includes intake/exhaust dynamics, combustion, heat transfer, engine friction and rotational dynamics of piston-crank mechanism. The engine model accurately reproduced real-world engine torque and acceleration fluctuations. The lumped parameter powertrain system model which includes clutch (and associated vibration isolation components), transmission, driveline and chassis is developed and used with the engine model to predict torsional vibrations. This system model is used to understand the powertrain torsional vibration characteristics in different operating regions such as idling, driving and coasting conditions.

To demonstrate the applicability of the developed models, results of unit tests for independent components, especially the engine torque variation and the clutch torsion characteristic, and the system-level quantitative validation with test data are presented. A special model that does fast Fourier transform of the signal on the fly is presented and its role in the analysis discussed. We present a comparison of rattle noise between two compliant clutch (isolator) designs and discuss the rattle metrics used in the analysis. Generic considerations for the deployment of such system level models are also discussed in this paper.

Keywords: gear rattle; engine model; fast Fourier transform

#### 1 Introduction

The noise induced by vibrations of gear-pairs is of great interest to powertrain developers. "Gear rattle", as this phenomenon is called, is caused by the torsional vibrations of the crank shaft due to engine dynamics (primarily combustion). These cyclic angular accelerations are transmitted from engine to transmission gear pairs and result in undesirable rattle noise. Theoretical and numerical studies are required to improve our understanding of this phenomenon and to enable us to predict such phenomenon so as to improve the underlying design.

Various theoretical and numerical studies on this problem can be found in the literature [1, 2, 3, and 4]. Numerical studies are useful in the automotive industry as they enable a quick analysis of the influence of various parameters and design factors on the dynamic behavior of powertrain. Two numerical methods are usually employed for dynamic analysis of gear rattle. In the "Uncoupled" method, the torsionally loaded path is separated from unloaded gear pairs and is first analyzed. Later, un-loaded gear pairs are modeled as single degree-of-freedom systems where the excitation is that which was obtained in the baseline torsional study. On the other hand, a completely "coupled" model considers contemporary interactions between loaded and unloaded gears. Coupled models are preferred in that they account for all the factors that might have an effect on the rattle noise.

Crowther et al. [5] concluded that metrics measured after impact correlate well, and the relative acceleration between impacting bodies and their relative kinetic energy determine the severity of impact. Several researchers have studied rattle phenomenon in a single maneuver like idling or driving. There are even instances where Modelica® has been used for similar studies [10].

The objective of this modeling effort is to develop an overall generic system model that can be used to study gear rattle phenomenon in various man-

euvers. A procedure for development and validation of various powertrain component models is outlined. These validated component models are used to build a complete vehicle model. The models were developed in Modelica using the Dymola® environment. The condition of vehicle coasting with idle speed control is studied in detail using the developed models. For this maneuver, rattle metrics obtained from the simulation model are shown to correlate well with experimental noise measurement. Further based on the understanding provided by the model, an improvement in the isolator design is introduced to reduce the rattle noise during this condition. Simulation and test results that confirm the reduction in rattle noise as a result of this design change are presented.

### 2 Vehicle Architecture

The vehicle system model is comprised of engine, compliant clutch (isolator), transmission, driveline and chassis components connected in series with a provision to mount the engine and transmission through bearings (Figure 1). The detailed crankangle resolved engine model produces a fluctuating torque that is required to carry out NVH studies.

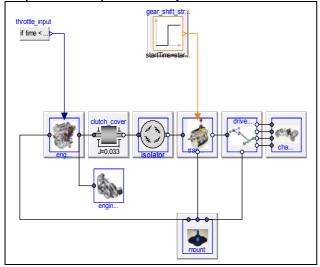


Figure 1 Vehicle Architecture

The non-linear, multi-stage isolator model isolates the engine fluctuations before they get transmitted downstream. The transmission is modeled with certain gears fixed on the input shaft and others on the output shaft. The simple chassis model consists of a lumped vehicle mass and road loads connected by a kinematic tire model. The effects due to tire and suspension compliance are neglected. In the driveline model, a front-wheel-drive system is modeled. The front half shafts are modeled as non-symmetric

compliances connecting the transmission to the wheels.

# 3 Component Models

### 3.1 Engine Model

A reasonable approach for powertrain torsional vibration analysis is to use experimental engine cylinder pressure data for the various drive conditions as inputs to the vehicle system model. However, especially in the early vehicle development phase, it is difficult to generate pressure traces for engines that are either in development, or for which the available data is limited. Hence it is important to have a detailed physical engine model to predict engine torque fluctuation [8, 9]. This is especially important during transient operation such as cranking, when predicting cylinder pressures would be difficult without a model. Such models can even be used to study the effect of engine manufacturing variation on the engine performance much before the engine goes into production [11]. Another important consideration is that by integrating a detailed transient engine model in the vehicle system model, the drive conditions that involve interactions with the engine control strategy such as idle speed control can be studied. Finally, for high speed operation, pressure data alone is not sufficient since the inertial forces of the piston become significant. The detailed engine model would account for the interaction between the combustion force and the piston inertial force.

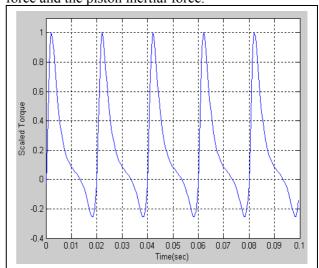


Figure 2 Sample crankshaft torque simulation.

The engine model used in this work was developed in-house to simulate both spark-ignited and diesel engines. The model is parameterized such that relevant engine design parameters can be easily entered into the model from a customized GUI. Fig-

ure 2 shows a typical crankshaft torque prediction using the engine simulation model.

This model includes crank angle-resolved, multicylinder modeling of the main engine thermodynamics and rotational mechanics including:

- Intake and exhaust breathing
- Combustion
- Heat transfer
- Rotational dynamics of piston, crank-slider mechanism, and crankshaft
- Engine friction (look-up table based)

### 3.1.1 Model Structure

Figure 3 shows the top-level structure of the detailed engine model. This model contains the hierarchical engine model, the flywheel inertia, a rotational connector for connecting the crankshaft to downstream components, and a signal bus connector used to provide relevant control signals to the engine model.

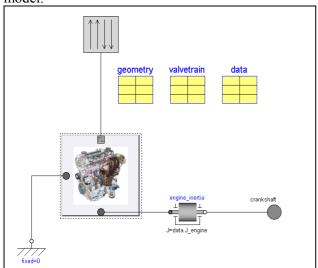


Figure 3 Engine model

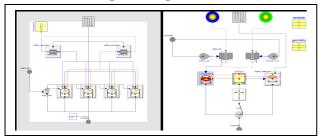


Figure 4 Hierarchical engine model structure.

Figure 4 shows the engine model being hierarchically composed of cylinders, with each cylinder consisting of models for the intake, exhaust and cylinder components. The model has another mechanical connection to the powertrain mounts which accounts for the reaction torque transmitted to the powertrain mount system.

## 3.1.2 Parameterization

The engine model parameters include engine-specific design data, initial and boundary conditions, and advanced parameters to customize the heat transfer and combustion characteristics of the engine. The parameters are summarized as follows:

- Parameters for initial conditions
- Engine geometry for overall engine specification
- Valve train for specification of the valve geometry and cam timing
- Engine friction look-up table for modeling friction torque (mechanical/rubbing) as a function of engine speed
- Manifold conditions that specify intake manifold temperature and exhaust manifold pressure and temperature
- Heat transfer parameters related to in-cylinder heat transfer
- Combustion parameters for specifying burn rate profile

## 3.1.3 Engine controller

Dynamic operating conditions are provided to the engine models via an engine controller. The engine controller takes a normalized torque input (0 to 1) and determines the dynamic operating conditions to match the engine torque profile based on a mapping process using the engine model. The engine controller component model takes the normalized torque as input and outputs the dynamic operating conditions to the control bus which will be provided to the engine model. The dynamic operating conditions specified by the controller are as follows:

- Intake manifold pressure (throttling/boost)
- Air-fuel ratio
- Firing flag (true or false)
- Start of combustion (for diesel)
- Spark advance and burn duration (for SI)

#### 3.2 Isolator Model

The isolator model captures the vibration isolation characteristics of the clutch assembly. It is a non-linear spring-damper, modeled in three stages. Each stage is parameterized to incorporate spring stiffness and hysteresis. It is modeled in such a way that the effect of first stage spring is always felt at the output of the isolator, whereas the springs in the second and third stage will be in effect (in parallel to the first stage), based on whether or not the respective backlash is taken up. These stages engage when

the relative angle between the input and output shaft of isolator exceeds the specified values of backlash. The model also includes the inertia of various components of clutch like the clutch disc, clutch facing, hub, and flange. This generic model allows a particular stage effect to be present only on one direction (positive/negative torque) by specifying asymmetric values to the specially made backlash element for the positive (clockwise) and negative (anti-clockwise) sides. This helps in parameterizing the model accurately for any isolator characteristic data available. The model for a three-staged isolator is shown in Figure 5.

inertia discfa...

J=solator\_dat...

inertia flange thirdstageelas...

fourthstageel...

Figure 5 Multi-stage clutch (Isolator) model

#### 3.3 Transmission Model

The transmission subsystem represents the gearing involved in delivering power from the engine to the wheels. One side of the transmission is connected to the engine while the other side is connected to the driveline. Like the engine, the transmission is also connected to the powertrain mounts. The consideration of the mounts is an important aspect that differentiates this architecture from many vehicle level models because it accounts for the influence of reaction torques in the power plant, transmission and driveline on the motion of the powertrain. This part of the physics is particularly important for the transmission because it can be the source of large amplitude, low frequency disturbances not effectively isolated by the mounting system [6, 7].

The transmission model is built from basic components of Modelica Standard Library (MSL) that includes a gear pair with a synchronizer (clutch) in between. Both of the gears in this component are fixed either to the input or output shaft. Any transmission architecture can be modeled by using combinations of this gear pair component.

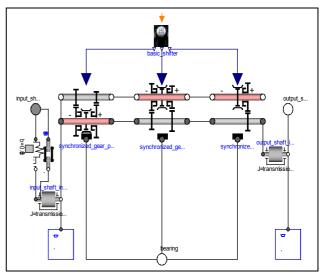


Figure 6 Transmission model

As seen in Figure 6, the transmission model includes the input and output shaft inertias. The shifter component sends the gear selection command to the appropriate gear pair in which the gear needs to be engaged. The commanded gear is then engaged by the synchronizer onto the shaft on which it is floating in neutral condition.

The model includes the coulombic drag torque associated with the input and output shafts. In physical terms, this drag comes from the stir resistance of oil around the transmission input and output shafts. There is a provision in the model to include the backlash between the isolator and the transmission input shaft. If not required, this effect can be eliminated by activating the rigid-bypass model connected in parallel to the backlash model. Rigid-bypass model is capable of locking-up the flanges that are connected to its two sides. This component does not reflect a physical component in the system, it is merely a way to control what details are included in the model.

The gear pair component has the flexibility to include the effect of backlash between the mating gears. The details regarding this backlash model are provided as a code fragment below. The backlash model is instrumented with the following metrics.

- Average\_omega: Average relative angular velocity between the mating gears
- Average\_alpha: Average relative angular acceleration between the mating gears
- Average\_power\_contact\_damper: Average power associated with contact damper between the mating gears

These metrics are calculated right after each impact between the gear pair. They are averaged over the number of impact events within a chosen time window.

```
model InstrumentedBacklash
  "Backlash w/rattle instrumentation"
  import Modelica.Mechanics.Rotational;
  import Modelica.SIunits.*;
  extends Rotational.ElastoBacklash;
protected
  Real event rate "Collisions/sec";
  Integer events "Collisions/interval";
  AngularVelocity omega rel =
                  der(phi rel);
  AngularAcceleration alpha rel =
                  der(omega rel);
  Angular Velocity avg omega
    "Average rel omega per collision";
  AngularVelocity omega sum;
  AngularAcceleration avg alpha
    "Average rel alpha per collision";
  AngularAcceleration alpha sum;
  Power power sum;
  Power avg power
    "Average power per collision";
algorithm
  when sample (0,0.02) then
    event rate := events/0.02;
    if (events == 0.0) then
      avg omega := 0.0;
      avg alpha := 0.0;
      avg power:= 0.0;
    else
      avg omega := omega sum/events;
      avg alpha := alpha sum/events;
      avg power:= power sum/events;
    end if;
    events := 0;
    omega sum := 0;
    alpha sum := 0;
    power sum:= 0;
  end when;
  when phi rel>=b2 or phi rel<=-b2 then</pre>
    events := pre(events) + 1;
    omega sum := pre(omega sum) +
                 omega rel;
    alpha sum := pre(alpha sum) +
                 alpha rel;
    power_sum := pre(power sum) +
                 omega rel*tau;
 end when;
end InstrumentedBacklash;
```

### 3.4 Driveline Model

The driveline subsystem models the distribution of transmission output torque to each of the wheels. For many vehicles, this distribution is determined by simple mechanical connections (e.g. differentials in strictly front-wheel or rear-wheel drive vehicles). In other cases, this distribution is actively controlled (e.g. on-demand four wheel drive systems). In this study, a non-active front-wheel drive system is con-

sidered. The transmission output shaft connects to a final drive gear followed by a differential that splits the torque to the front two wheels as shown in Figure 7. The final drive gear as well as the differential gear models are taken from the Modelica Standard Library. The driveline subsystem is connected to the mounting system as well. The right and left half shafts are modeled with inertia and compliance.

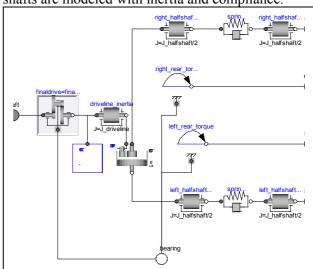


Figure 7 Driveline model

#### 3.5 Chassis Model

The chassis models the tire inertia and neglects the stiffness of the tire and the compliance of the shock absorber. The aerodynamic drag and rolling resistance are modeled using representative equations. The vehicle mass is represented as a translational mass.

# 4 Component Validation

To study driveline torsional vibration, it is imperative to have an accurate prediction of the engine torque fluctuation, as well as an accurate prediction of the behavior of the transferring system namely the isolator. In this section, validation of the engine and isolator components is presented.

### 4.1 Engine Model Validation

The vehicle system model has to be employed to study the powertrain torsion characteristics for three different conditions namely, idling, driving and coasting. For this purpose, unit tests were carried out to validate the engine model against engine test data at each of these three conditions.

## 4.1.1 Engine Vibration at Idling

Here the engine is at steady-state idling condition with engine friction, A/C and alternator loads being applied. Figure 8 shows the acceleration fluctuation levels at the specified idling speed for 2 different engine load levels. The model results show good agreement with the test results.

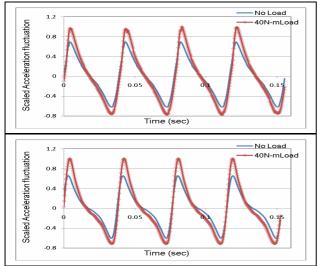


Figure 8 Experimental (Top) vs. Model (Bottom) results for engine vibration at idle condition.

## 4.1.2 Engine Vibration at Driving

For the engine in driving condition, both acceleration and deceleration validation tests were performed. The acceleration test is done on the vehicle model, with the engine and vehicle accelerating and the transmission in third gear. By sweeping the engine through the operating range (via a tip-in throttle command), the speed fluctuation values at various engine speeds were captured. For the deceleration test, the tip-out maneuver is executed from a higher engine speed.

To facilitate this analysis, a Fourier Transform computation block in Modelica was developed inhouse. This model is connected to the engine crankshaft and performs FFT (Fast Fourier Transform) on the engine speed signature on-the-fly. The Fourier transform is computed for a user-specified fundamental frequency and specified number of harmonics of this fundamental frequency. The model computes the continuous Fourier integrals which are used to compute Fourier coefficients, which in turn are used to compute the magnitude and phase for the specified frequencies.

Figure 9 shows the speed fluctuation of the dominant second order harmonic against engine speed. It is noted that the engine speed fluctuation first decreases and then increases as a function of nominal speed. The point at which this trend reversal in speed fluctuation happens is dependent on the relative

magnitudes of the inertia forces due to the piston mass and the combustion force.

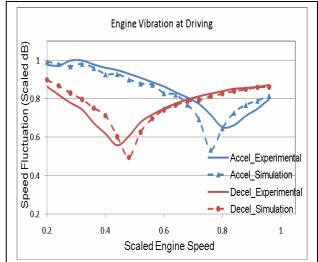


Figure 9 Experimental vs. Model results for engine driving vibration

As seen in Figure 9, a good correlation between the simulation and experimental results for engine driving vibration during both acceleration and deceleration is observed.

## 4.1.3 Engine Vibration at Coasting

To illustrate the applicability of the vehicle system model to study vibration phenomenon under various conditions, a generic maneuver different from idling or driving, namely coasting was considered. In this maneuver, the transmission is engaged in third gear and the engine is initially at a speed higher than idle speed. The driver lets off the throttle with the transmission still engaged, resulting in the engine speed decreasing due to engine braking.

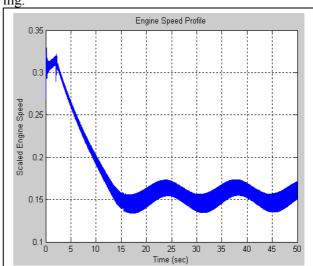


Figure 10 Engine speed prediction during coasting

When the engine speed comes down close to idle, Idle Speed Control (ISC) takes over. To be able to

study rattle behavior during such a maneuver, it is first essential to ensure that the engine model is able to simulate the engine speed fluctuation accurately at different stages in the maneuver. For this purpose, an appropriate engine model control to capture the exact behavior during coasting condition is developed. Figure 10 shows the predicted engine speed during coasting condition that includes engine braking and ISC regions. It is noted that the model is able to adequately represent both the mean value as well as the variation in the engine speeds as a qualitative validation of the model result with experimental data was performed.

#### 4.2 Isolator Model Validation

The unit test for the isolator component uses a low frequency sinusoidal torque as input to the isolator model. Figure 11 shows isolator torque plotted against isolator relative angle. The experimental curve was used to determine the parameters for isolator model such as backlash (which governs the onset for each stage) and the stiffness and hysteresis values for each stage.

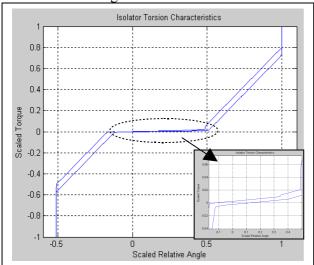


Figure 11 Isolator torsion curve

It is observed that this particular isolator design has 3 stages on the positive side but only the first and third stage on the negative side (see the inset picture of Figure 11).

# 5 Vehicle System Validation

In the previous sections, component model development and validation for the vibration source component (engine), the vibration transfer component (isolator) and the noise source component (transmission and driveline) was discussed. After the component validation is done for different vehicle condi-

tions, the vehicle system vibration validation and analysis at these conditions needs to be performed. This section presents the validation results for the vehicle system at idle, driving and coating conditions.

#### 5.1 Idle Condition Validation

This test is done on the vehicle model with the engine idling and the transmission in neutral, and hence is intended to show the torsional vibration isolation provided by the isolator first stage.

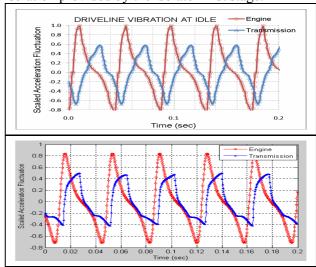


Figure 12 Experimental (top) vs. Model (bottom) results for driveline vibration during idling

Figure 12 shows the torsional vibrations being dampened by the isolator from the engine to the transmission input shaft. The amplitudes at the engine and transmission shaft show good agreement between simulation and experimental data.

#### 5.2 Driving Condition Validation

This test case is very similar to the engine acceleration test, where the engine speed is swept and the corresponding speed fluctuation of engine is captured. Here the speed fluctuation for the transmission is also captured. In this test, the transmission resonance where the transmission speed fluctuation overshoots the engine speed fluctuation near the transmission resonant speed has to be observed. The effect of system parameters such as isolator hysteresis on the system resonance is of great interest.

A frequency domain analysis of the vehicle system model can determine the resonant speed at which the engine speed fluctuation excites the transmission. A frequency analysis of this system will provide us the different resonant speeds of the system.

To perform this analysis, the model linearization functionality of Dymola was used to generate a linear time invariant system from the developed models.

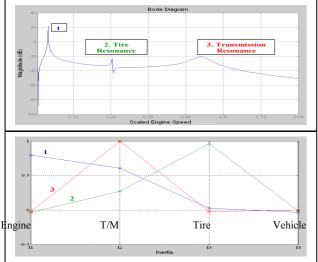
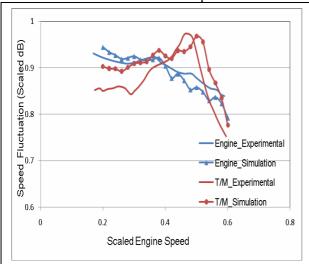


Figure 13 Bode & mode shape plots obtained from linearized model

Figure 13 shows the Bode plot and mode shape plot obtained after vehicle model linearization. The Bode plot represents the transfer function or frequency response of a linear, time invariant system and identifies the system resonant frequencies. The mode shape plot helps identification of the particular system components that are involved in each of the resonant frequencies. For example, the resonance frequencies arising from the transmission and the tire inertias are identified in the Bode plot.



**Figure 14 Transmission Frequency Response** 

After finding the resonant speed, the nonlinear system model is used to find out the magnitude of speed fluctuation for both the engine and transmission. It is observed from Figure 14 that at around the predicted resonant speed, the speed fluctuation of the transmission increases well over that of the engine.

The predicted frequency response agrees well with the experimental data as well.

# 6 Model Based Design

One advantage of a model-based design approach is the ability to analyze the effect of a design change via simulation rather than requiring fabrication and testing of actual hardware. The original isolator design has the characteristics shown in Figure 15. This isolator was observed to have a rattle noise peak during the experiment in test bed. The modified isolator with improved characteristics as in Figure 15 was shown to eliminate the rattle noise peak by the results obtained from the simulation of developed models.

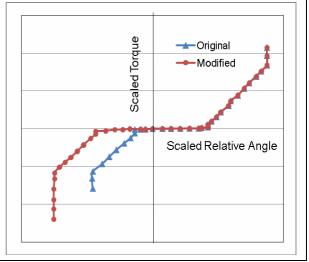


Figure 15 Comparison of Torsion Characteristic of the Original and Modified Isolator

The original isolator design did not include a second-stage spring on the negative side and this resulted in a sudden transition from the negative maindamper (third stage) to negative pre-damper (first stage) which led to rattle. The isolator design can be improved with the addition of a second stage spring in the negative direction as shown in Figure 15 in red color. This design change is implemented in the models and simulated to observe the new rattle metrics.

Figure 16 shows the comparison of the rattle metrics for the original and modified isolators for the driving gear. These rattle metrics were captured in the gear models of the transmission sub-system as explained in section 3.3. Clearly, the magnitudes of the rattle metrics are reduced in the case of the modified isolator, a result which again validates the design proposal for eliminating rattle.

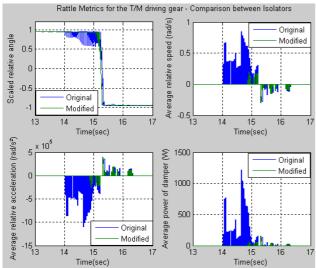


Figure 16 Comparison of Rattle Metrics with the Original and Modified isolator for the T/M driving gear

The conclusions drawn from the model results (Figure 16) were found to agree with the experimental results obtained from the test bed as shown in Figure 17.

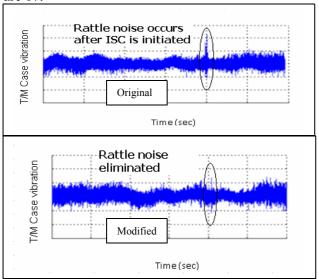


Figure 17 Comparison of vibration between original and modified isolator in test bed

### 7 Conclusion

In this work, a systematic approach for development and application of an overall vehicle system simulation model to study the powertrain torsional vibration characteristics using Modelica is presented. The resulting comprehensive vehicle system model is capable of analyzing a wide range of vehicle operating conditions such as idling, driving and coasting. Comprehensive tests were done both at the component level and overall system level to demonstrate quantitative model validation.

To illustrate the applicability of the developed system model, a vehicle coasting maneuver was studied in detail and it was found to introduce rattle noise in the system. Having an overall system model was important since it captures the intimate interactions between subsystems. In this particular case, the system model clearly brought out the influence of isolator design on backlash resonance in the transmission. The observation of rattle metrics demonstrated the effectiveness of the model in the identification of a design change in the isolator to eliminate rattle during the maneuver. The effectiveness of the design change was confirmed by vehicle testing. The use of Modelica in modeling a complex system and its use in analyzing complex phenomenon is demonstrated in this work.

In terms of future applications, the simulation model instrumented with the identified rattle metrics could be used to assess likelihood of rattle noise in other vehicle maneuvers without the need to run costly vehicle tests. The system model has been developed in such a way that the component models are readily replaceable. It would be fairly straightforward to integrate alternative components such as a dual mass flywheel (DMF) in the architecture. In this paper, the effectiveness of a design change in the clutch disk isolator towards reducing rattle noise is shown. Similarly design changes in other isolation components such as engine and transmission mounts can be studied in future.

# 8 Acknowledgements

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