MODRIO

D5.1.9 - NMPC with inverse models

WP 5.1 Nonlinear Model Predictive Control
WP 5 Optimized system operations

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Executive Summary

In model based control inverse models of the system dynamics are a key element of successful control algorithms in applications – especially when using Modelica. The idea to be investigated in this deliverable is to combine the advantages of inverse system models and Nonlinear Model Predictive Control (NMPC).

Contents

1. Theoretical Analysis of NMPC and Inverse Models

In this Chapter we focus on the following control task: A dynamical system described by differential algebraic equations (DAEs)

$$f(\dot{x}, x, w, y, u, t) = 0 \tag{1}$$

with states x, algebraic variables w, outputs y, inputs u, time t shall be controlled by the inputs u in such a way that the outputs y follow a given reference signal $y_{ref}(t)$ for $t \in [0, t_e]$. The inputs u have to fulfill given constraints: $u_{min} \le u \le u_{max}$ because of technical limitations.

The following (global) optimal control problem

$$\min_{u_{min} \le u \le u_{max}} \int_0^{t_e} \left(y(t) - y_{ref}(t)\right)^2 dt \qquad \text{s.t.} \qquad f(\dot{x}, x, w, y, u, t) = 0 \tag{2}$$

can be formulated to solve the task by applying numerical methods on it. One possible disadvantage of this global approach is the high amount of computational power that is necessary to numerically solve the optimal control problem.

In this deliverable we will investigate a nonlinear model predictive control (NMPC) approach on the task. Generally, NMPC approaches do not cover the global time interval in the optimization problem, but the less dimensional NMPC optimization problem can be solved faster than a high dimensional optimization problem involving a long time frame of the reference signal. For NMPC the reference signal y_{ref} can be changed in each sample time of the control tasks whereas the global (i.e. for $t \in [0, t_e]$) reference signals have to be known in advance for the optimal control task (2).

To find control inputs u for system (1) the following approach is considered. We sample the inputs u such that $u(t) = u_k \in R^{n_u}$ for $t \in [t_k, t_{k+1}]$, $t_0 = 0$, $T_{N-1} = t_e$ and $k = 0,1,2,\dots,N$ is a piecewise constant signal. Using a prediction horizon $T_p > 0$ we can formulate the NMPC problem for each sample time t_k:

$$\begin{aligned}
&\min_{u_{min} \le \tilde{u} \le u_{max}} \int_{t_{k+1}}^{t_{k+1}+T_p} \left(\tilde{y}(t) - y_{ref}(t)\right)^2 dt && \text{s.t.} \\
&f\left(\dot{\tilde{x}}, \tilde{x}, \tilde{w}, \tilde{y}, \tilde{u}, t\right) = 0 && \text{for} \quad t \in \left[t_k, t_{k+1} + T_p\right], \\
&\tilde{x}(t_k) = x(t_k) && \text{and} \\
&\tilde{u}(t) := u(t) && \text{for} \quad t \in [t_k, t_{k+1}].
\end{aligned} \tag{3}$$

The solution $\tilde{u}^*$ of this problem is defined on the time interval $\left[t_k, t_{k+1} + T_p\right]$, but only the first non-fixed part is used for the global input: $u(t) := \tilde{u}^*(t)$ for $t \in [t_{k+1}, t_{k+2}]$. Afterwards, the sample index is increased $k \to k + 1$ and system (3) is solved again.

Inverse Models

In model based control inverse models of the system dynamics are a key element of successful control algorithms in applications – especially when using Modelica the user can profit by this approach [1]. The idea of the deliverable at hand is to combine the advantages of inverse system models and NMPC.

Under certain assumptions there exists an inverse model f^{-1} of system (1) such that the corresponding DAE system

$$f^{-1}(\dot{\bar{x}}, \bar{x}, \bar{w}, \bar{y}, \bar{u}, t) = 0 \tag{4}$$

has a solution that inverts the inputs and outputs of the original DAE system: $f(\dot{x}, x, w, \bar{u}, \bar{y}, t) = 0$. In simple words, the original system (1) describes the solution of the outputs y, if the inputs u are given. The inverse system (4) provides the inputs of the original system when the outputs are given.

By using the inverse model the necessary inputs u could be determined to achieve the outputs y_{ref}. If this is possible, the optimal control problem (2) is solved without optimization algorithm, because the solution is trivial under the assumption that the determined inputs u fulfill the constraints $u_{min} \le u \le u_{max}$. In applications this assumption is often not true and therefore the solution is not as trivial as it seemed.

Therefore, we combine the inverse model approach and the NMPC system (3) in the following way:

$$\begin{aligned}
&\min_{\tilde{y}} \int_{t_{k+1}}^{t_{k+1}+T_p} \left(\tilde{y}(t) - y_{ref}(t)\right)^2 dt \qquad \text{s.t.} \\
&u_{min} \le \tilde{u}(t) \le u_{max} \quad \text{for} \quad t \in \left[t_k, t_{k+1} + T_p\right], \\
&f^{-1}\left(\dot{\tilde{x}}, \tilde{x}, \tilde{w}, \tilde{u}, \tilde{y}, t\right) = 0 \quad \text{for} \quad t \in \left[t_k, t_{k+1} + T_p\right], \\
&\tilde{x}(t_k) = x(t_k) \qquad \text{and} \\
&\tilde{y}(t) := y(t) \qquad \text{for} \quad t \in [t_k, t_{k+1}].
\end{aligned} \tag{5}$$

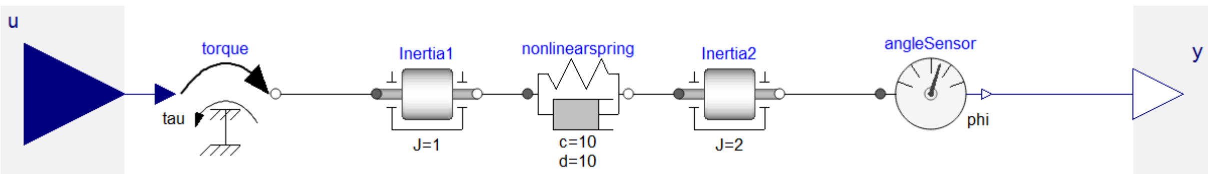
The solution $\tilde{y}^*$, resp. $\tilde{u}^*$ of this problem is defined on the time interval $\left[t_k, t_{k+1} + T_p\right]$, but only the first non-fixed part is used for the global input: $u(t) := \tilde{u}^*(t)$ for $t \in [t_{k+1}, t_{k+2}]$. Afterwards, the sample index is increased $k \to k+1$ and system (5) is solved again.

The advantage of this approach should be the simpler optimization problem compared to problem (3). In problem (5) the solution of the optimization problem is trivial (take $\tilde{y}(t) = y_{ref}(t)$) as long as the inverse model generates a signal $\tilde{u}$ that fulfills the constraints. Only in the phases when the constraints for $\tilde{u}$ are active, the optimization problem has to be solved numerically.

2. Numerical Tests by an Example

In the Modrio deliverable D5.2.3 [2] we introduce a prototype implementation of NMPC using Dymola's Optimization library. We apply the tool to a nonlinear plant model and test the approach using inverse dynamical models combined by NMPC as suggested in Section 1.

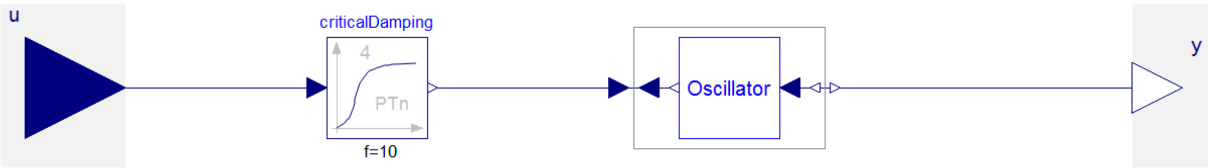
The example model is an oscillator composed of two inertias that are coupled by a nonlinear spring-damper system. The diagram of the corresponding Modelica models is shown in the following:



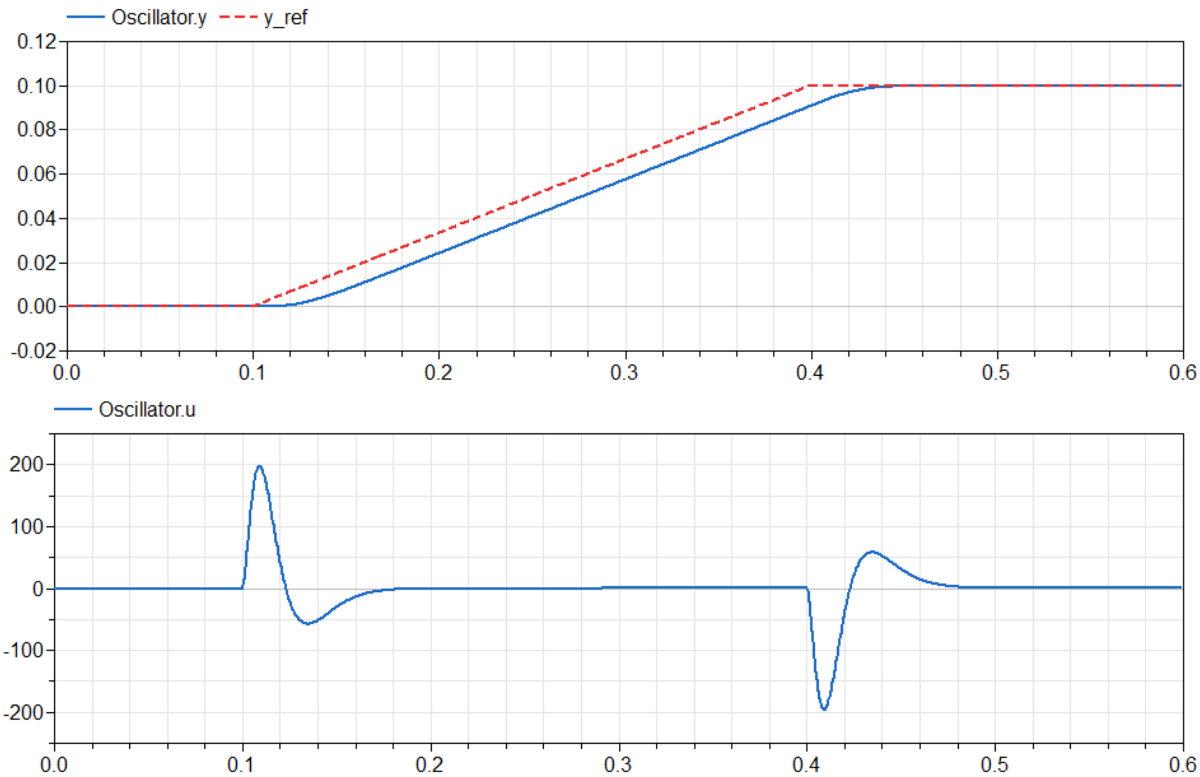
The control input of the system is the torque acting on one side of the oscillator (motor side), the output of the system to be controlled is the position of the inertia on the other side (output side) of the oscillator.

2.1. Inverse Model

In a first test only the inverse model is applied to a reference signal of the position of `Inertia2`. A typical property of inverse models is the necessity to differentiate the input signal. Most commonly the differentiation is provided by filtering the input signal and using the derivatives of the filtered signal as approximation for the derivative of the input signal. In case of the oscillator model the object diagram of the inverse model looks like the following:



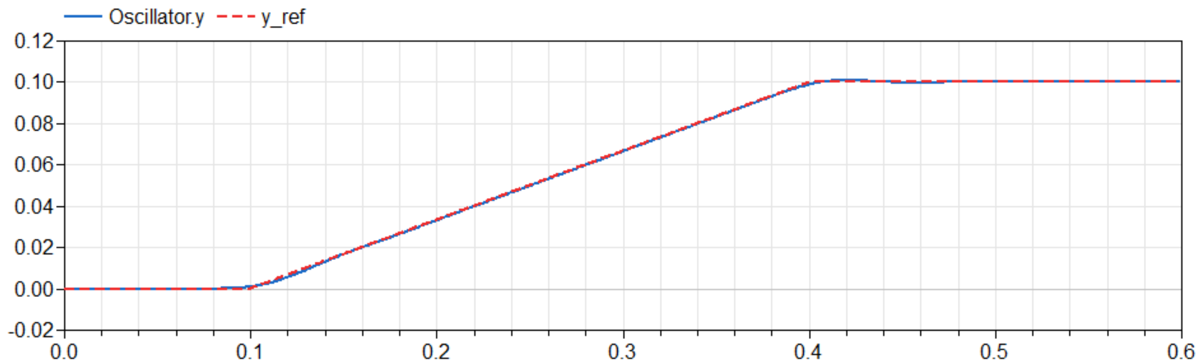
The input of the inverse model is the position of `Inertia2` and the output of the inverse model is the acting torque at the motor side of `Inertia1`. A ramp signal is chosen as reference signal for the position of `Inertia2`, see the red line in the following time plot :

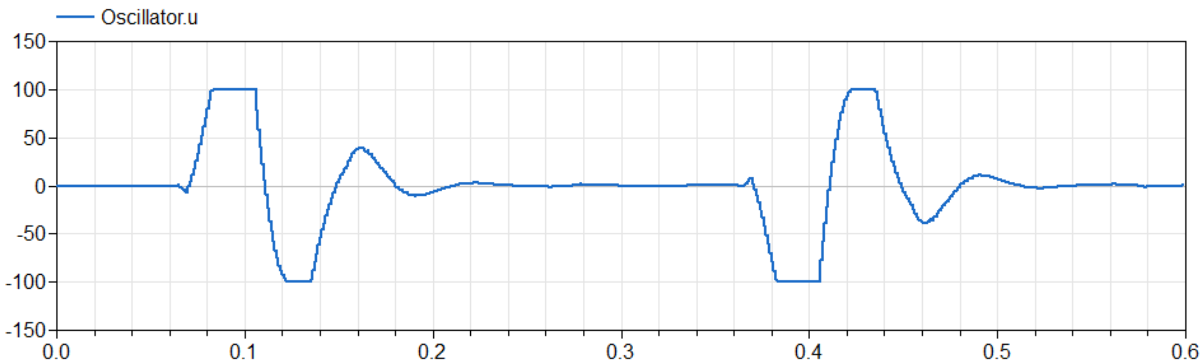


The numerical integration of the inverse model provides a signal for the output of the inverse model, i.e. the input torque `Oscillator.u` of the oscillator, see in the second part of the plot above. The corresponding position `Oscillator.y` of `Inertia2` is plotted in comparison to the reference signal in the plot above. The deviations are only because of the filter (`criticalDamping`) in the inverse model, but this is a typical effect when applying inverse models.

2.2. Classical NMPC

In the second test we apply the oscillator model to the prototype NMPC implementation in Dymola's Optimization library using a sample time of 1 ms and prediction horizon of 40 ms. The input torque `Oscillator.u` to the oscillator is restricted by bounds of ±100 Nm. The generated nonlinear model predictive controller is working well as can be seen in the following plots:

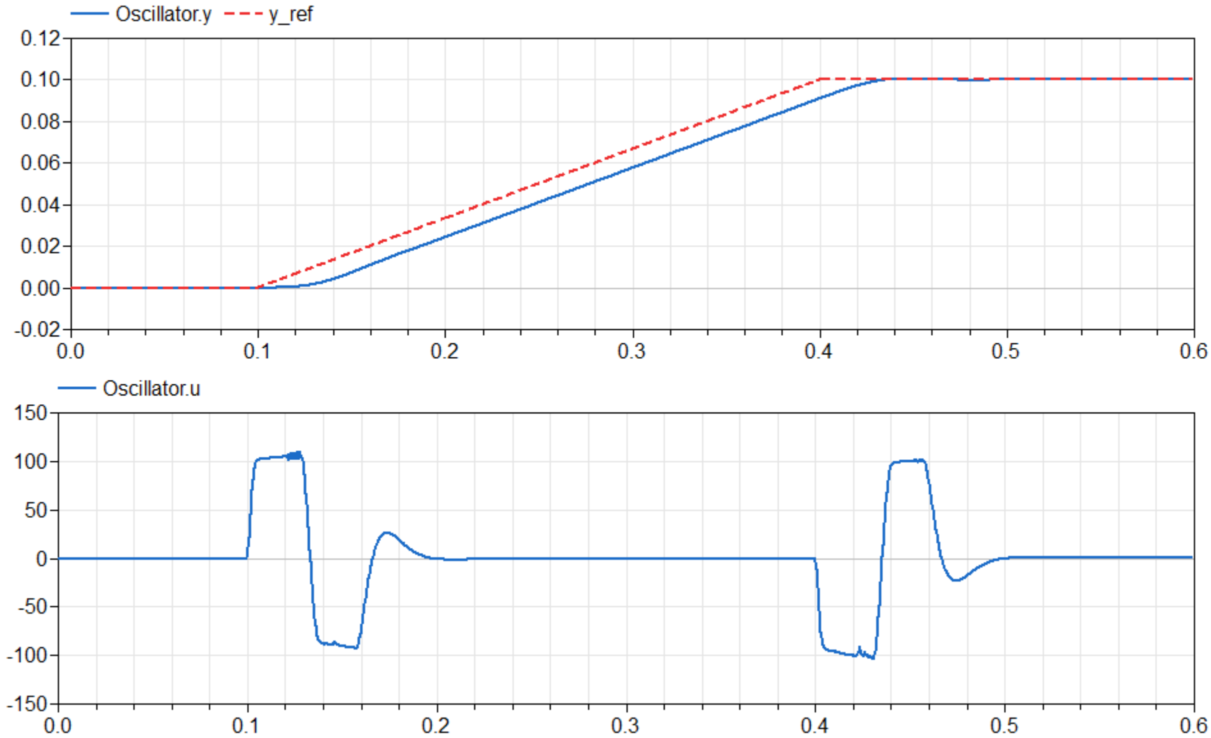




The constraints of the input torque are fulfilled and the inertia position of the output side is coinciding very well with the reference signal `y_ref`. Due to the prediction property of the controller the torque input is acting several ms before the ramp begins to rise or to settle.

2.3. NMPC and Inverse Model

In the last test the proposed NMPC variant using the inverse oscillator model is generated. The sample time has to be selected to 0.4 ms and the prediction horizon to 4 ms. The resulting behavior of the output position is comparable with the behavior of the inverse model on page 6. It can also be seen that the input torque `Oscillator.u` does not meet the constraints of ±100 Nm.



The execution times of the presented classical NMPC test and the NMPC using the inverse model are comparable. Unfortunately, greater values for the prediction horizon do not result in a stable behavior of the controller. The reason is, that the corresponding optimization problems cannot be accurately solved by the optimization algorithm. It seems, that solving the optimization problem (5) is much more difficult than solving problem (3) from a numerical point of view – at least for the oscillator model. It means, that the theoretical advantages discussed in Section 1 could not be confirmed by the numerical tests. Further studies on other examples would be necessary to investigate the reasons and possible solutions in detail.

3. References

[1] Michael Thümmel, Gertjan Looye, Matthias Kurze, Martin Otter, Johann Bals: *Nonlinear Inverse Models for Control.* Proc. of 4th International Modelica Conference, pp. 267-279, Hamburg, Germany, March 2005.

[2] Deliverable D5.2.3: *Prototype for optimization toolchain in Dymola-Optimization*, ITEA2 project MODRIO (11004), Version 1.0, 2016.