Van Emde Boas Trees

Goal: maintain n elements among $0, 1, ..., u-1$ subject to Insert, Delete, Successor in $O(\log \log u$ time.

In applications, such as in network routers, u is typically 32 or 64 bits.

Intuition

Let's binary search on the height of a tree, which is itself $O(\log u)$.

$$\begin{aligned} T(k) &= T(k/2) + O(1) \\ &= O(\log k) \\ T(\log u) &= T(\log u/2) + 1 \\ T'(u) &= T'(\sqrt{u}) + O(1) \\ &= O(\log \log u) \end{aligned}$$

Through the substitution method, one can show that the recurrence with $\sqrt{u}$ is indeed $O(\log \log u)$.

Let's try an approach with a **bit vector**, an array of size u where 0 denotes absence and 1 denotes presence.