Calculus 1 Workbook

krista king
MATH

IDEA OF THE LIMIT

1. The table below shows some values of a function $g(x)$. What does the table show for the value of $\lim_{x \to 4} g(x)$?

x	$g(x)$
3.9	1.9748
3.99	1.9975
3.999	1.9997
4.001	2.0002
4.01	2.0025
4.1	2.0248

2. How would we express, mathematically, the limit of the function $f(x) = x^2 - x + 2$ as x approaches 3?

3. How would you write the limit of $g(x)$ as x approaches ∞, using correct mathematical notation?

$$g(x) = \frac{5x^2 - 7}{3x^2 + 8}$$

4. Explain what is meant by the equation.

$$\lim_{x \to -2} (x^3 + 2) = -6$$

5. Evaluate the limit.

$$\lim_{x \to -1} \frac{-x^2 + 3x - 1}{5}$$

6. Evaluate the limit.

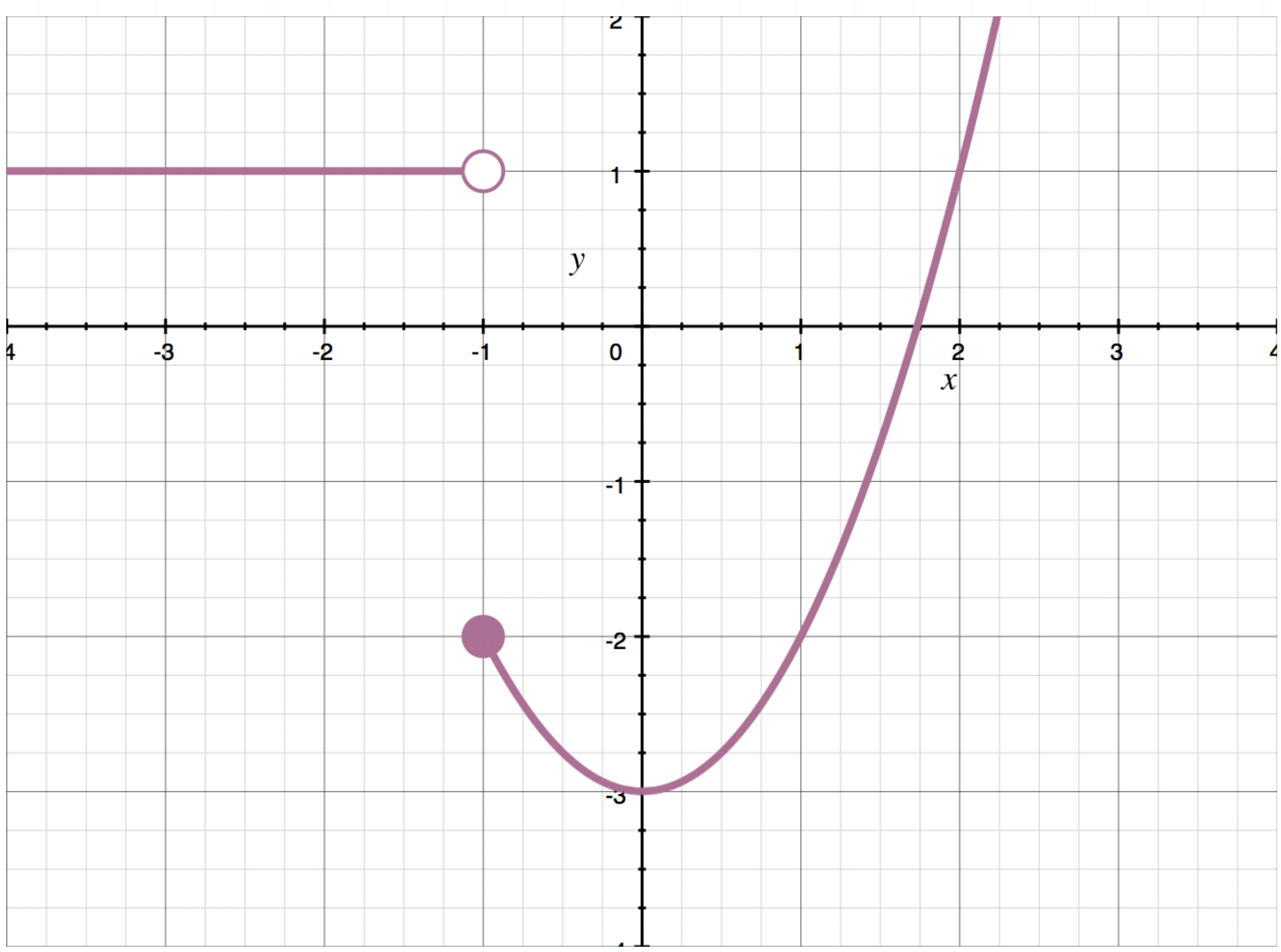
$$\lim_{x \to 0} \frac{x^2 - 5}{2}$$

ONE-SIDED LIMITS

■ 1. Find the limit.

$$\lim_{x \to -7^+} x^2\sqrt{x+7}$$

■ 2. What does the graph of $f(x)$ say about the value of $\lim_{x \to -1^+} f(x)$?



■ 3. The table shows values of $k(x)$. What is $\lim_{x \to -5^-} k(x)$?

x	-5.1	-5.01	-5.0001	-5	-4.999	-4.99	-4.9
k(x)	-392.1	-3,812	-38,012	?	37,988	3,788	368.1

4. What is $\lim_{x \to -2^-} h(x)$?

$$h(x) = \begin{cases} -2x - 1 & x < -2 \\ x & -2 \le x < 2 \\ 2x - 3 & x \ge 2 \end{cases}$$

5. What is $\lim_{x \to 6^+} g(x)$?

$$g(x) = \frac{x^2 + x - 42}{x - 6}$$

6. Find the left- and right-hand limits of the function at $x = 3$.

$$f(x) = \frac{|x - 3|}{x - 3}$$

PROVING THAT THE LIMIT DOES NOT EXIST

■ 1. Prove that the limit does not exist.

$$\lim_{x \to 0} \frac{-2|3x|}{3x}$$

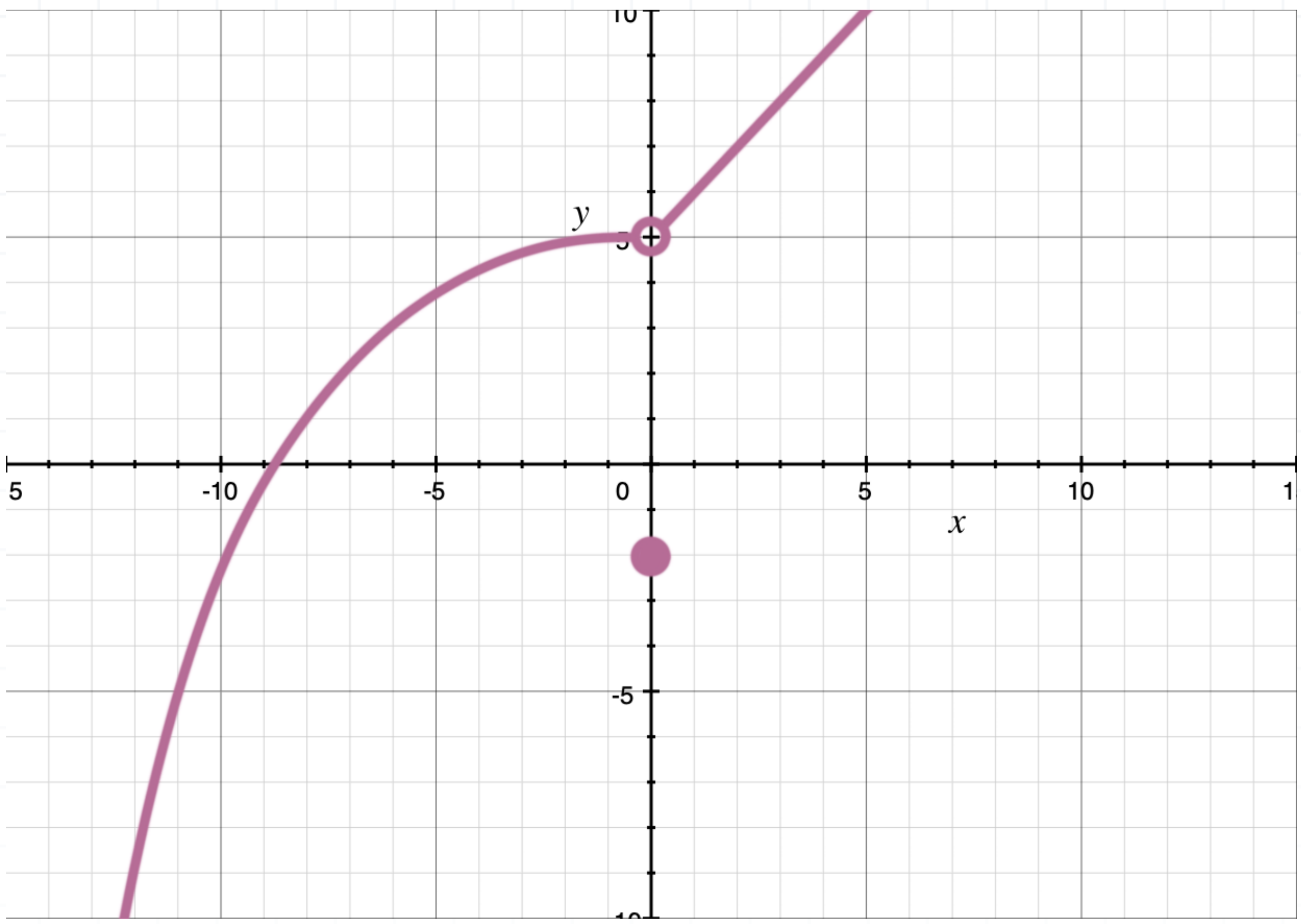
■ 2. Prove that the limit does not exist.

$$\lim_{x \to -5} \frac{x^2 + 7x + 9}{x^2 - 25}$$

■ 3. Prove that $\lim_{x \to 1} f(x)$ does not exist.

$$f(x) = \begin{cases} -3x + 2 & x < 1 \\ 3x - 2 & x \geq 1 \end{cases}$$

■ 4. Use the graph to determine whether or not the limit exists at $x = 0$.



■ 5. Suppose we know that $\lim_{x \to 5} f(x) = 12$. If possible, determine the values of the one-sided limits.

$$\lim_{x \to 5^-} f(x)$$

$$\lim_{x \to 5^+} f(x)$$

■ 6. Prove that the limit does not exist.

$$\lim_{x \to -2} \frac{x^2 - 4}{(x+2)^2}$$

PRECISE DEFINITION OF THE LIMIT

■ 1. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x\to 4}(5x-16)=4$$

■ 2. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x\to -7}(-2x+15)=29$$

■ 3. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x\to 16}\left(\frac{2}{5}x-\frac{17}{5}\right)=3$$

■ 4. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x\to 7}\frac{x^2-15x+56}{x-7}=-1$$

■ 5. Find δ when $f(x)=2x-5$, such that if $0<|x-1|<\delta$ then $|f(x)+3|<0.1$.

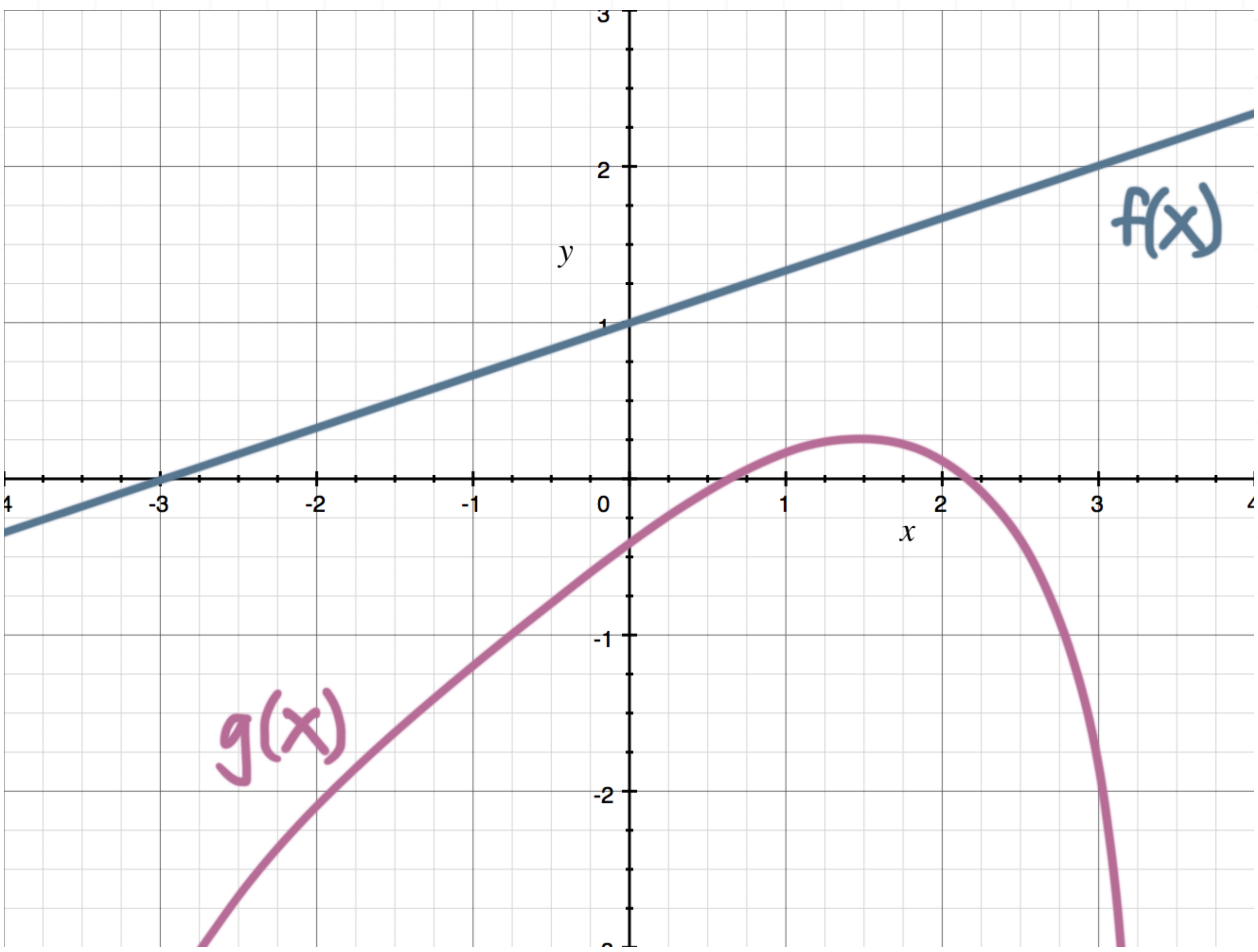
6. Find a value of δ given $\epsilon = 0.04$.

$$\lim_{x \to 2} (x-2)^2 = 0$$

LIMITS OF COMBINATIONS

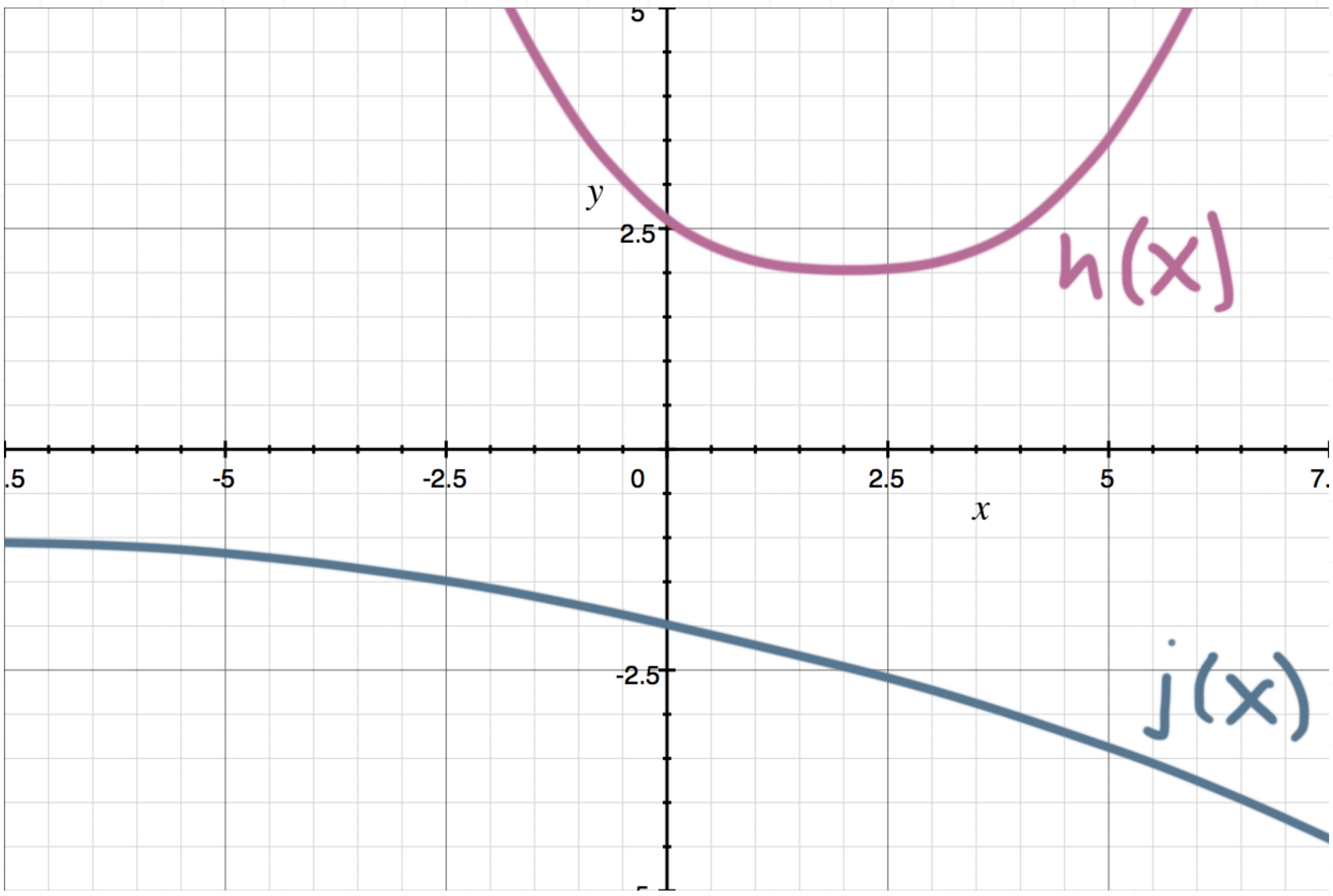
■ 1. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x\to 3}\left[4f(x)-3g(x)\right]$$



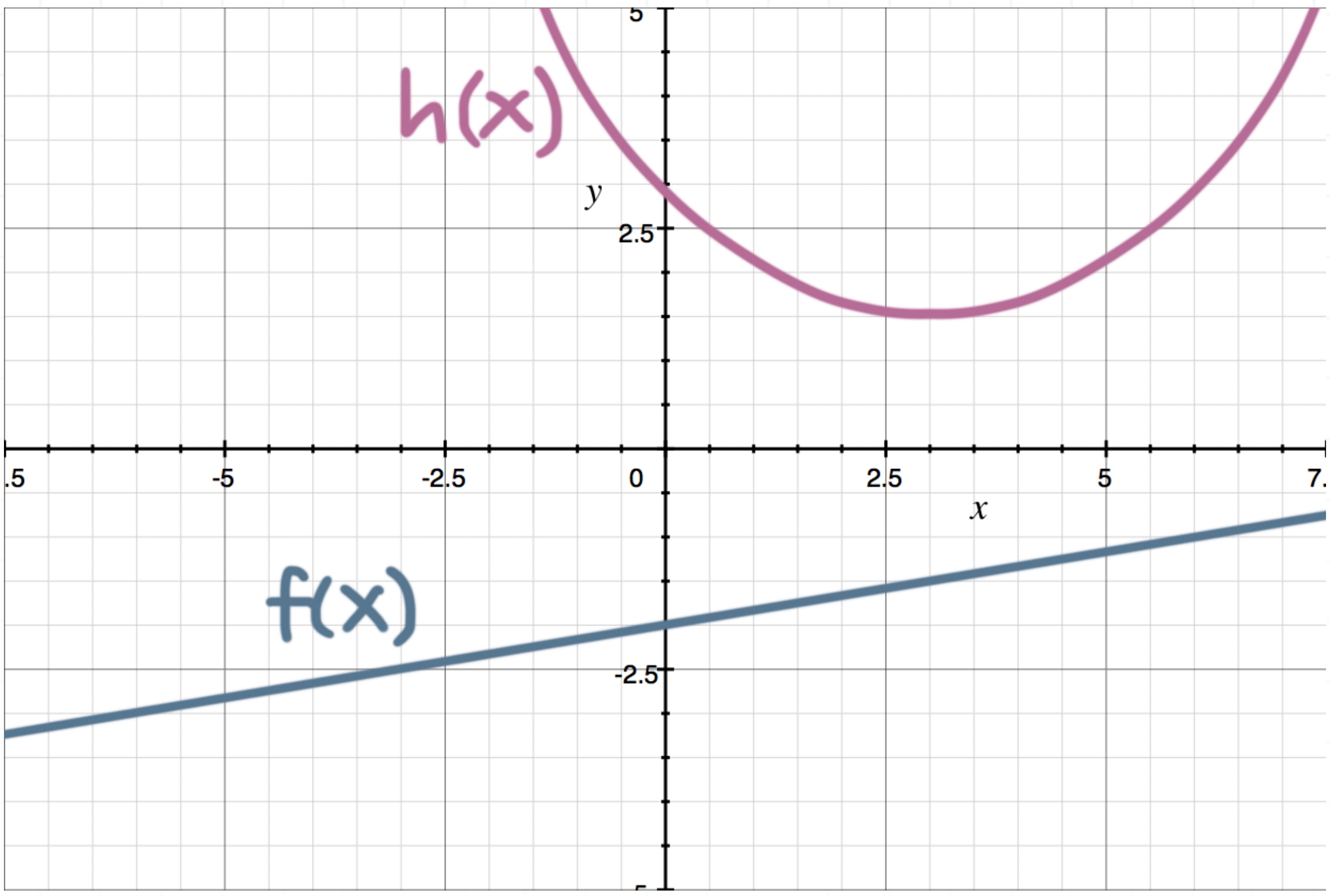
■ 2. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x\to 4}\frac{h(x)}{j(x)}$$



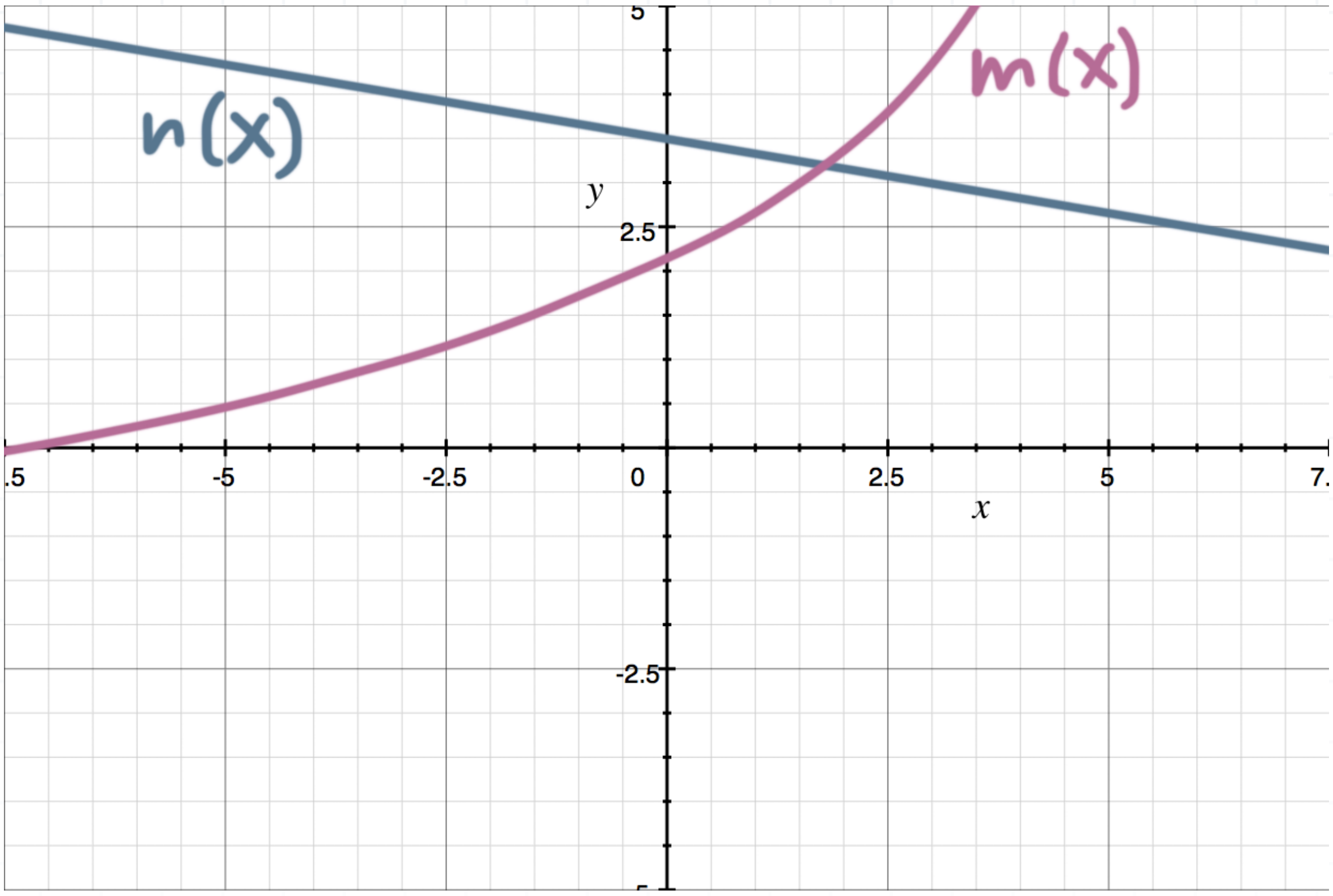
3. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x \to 0} \left[2f(x) \cdot 3h(x) \right]$$



■ 4. Use limit laws and the graph below to evaluate the limit.

$$\lim_{x\to -3}\left[\frac{5m(x)}{n(x)}-\frac{4m(x)}{n(x)}\right]$$



■ 5. Evaluate the limit.

$$\lim_{x\to 6}\left(\sqrt{x-2}+\frac{e^x}{2x+3}-x^2-12\right)$$

■ 6. If $f(x)=x^2+4$, $g(x)=x-5$, and $h(x)=-5x$, evaluate the limit.

$$\lim_{x\to 1}\sqrt{\frac{f(x)g(x)}{h(x)}}$$

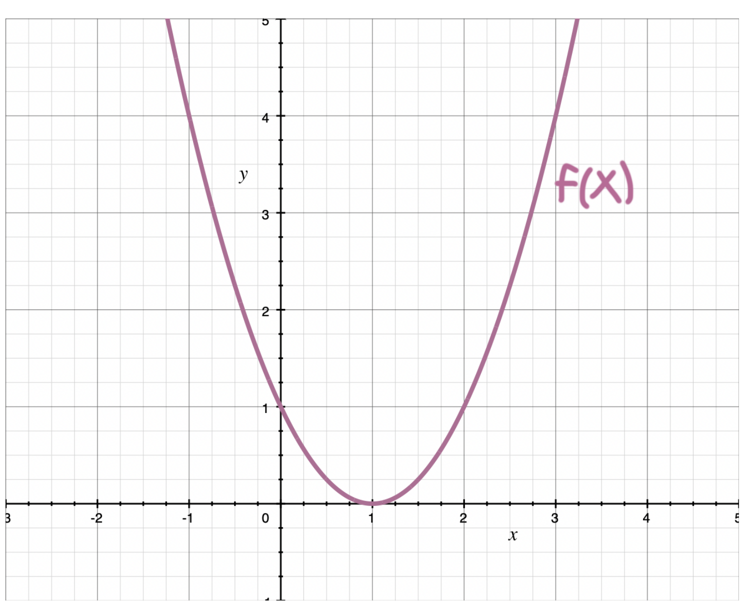
LIMITS OF COMPOSITES

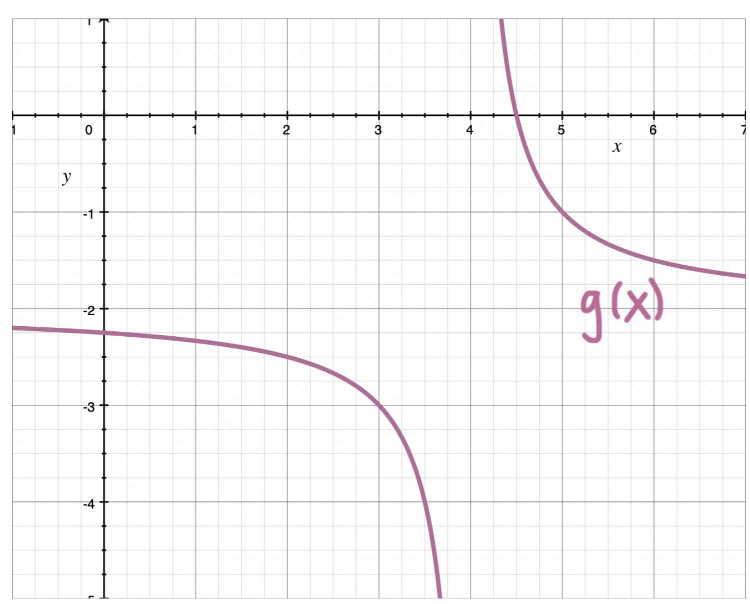
■ 1. What is $\lim_{x\to 3} f(g(x))$ if $f(x) = 4x$ and $g(x) = 6x - 9$?

■ 2. What is $\lim_{x\to -4} f(g(x))$ if $f(x) = 2x^2$ and $g(x) = 2x - 1$?

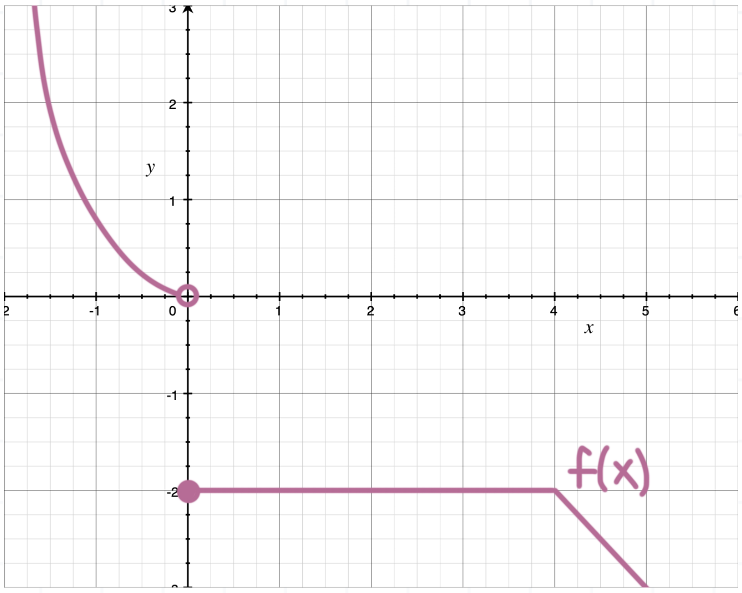
■ 3. What is $\lim_{x\to \frac{\pi}{2}} f(g(x))$ if $f(x) = \sin x$ and $g(x) = x/2$?

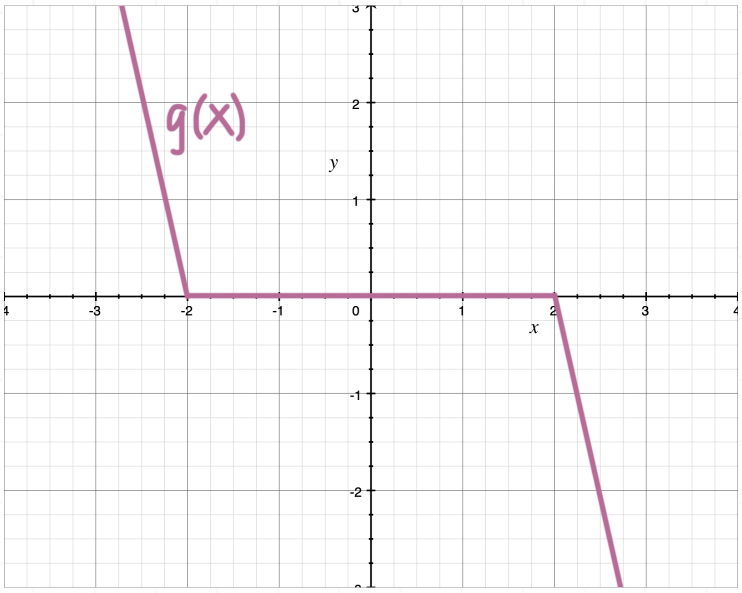
■ 4. If $f(x)$ and $g(x)$ are graphed below, find $\lim_{x\to 3} g(f(x))$.





■ 5. If $f(x)$ and $g(x)$ are graphed below, find $\lim_{x\to 2} g(f(x))$.





■ 6. If $f(x) = 2x + 1$ and $\lim_{x \to 3} h(x) = -2$, find $\lim_{x \to 3} f(h(x))$.

POINT DISCONTINUITIES

■ 1. Redefine the function as a continuous piecewise function.

$$f(x) = \frac{x^2 - 6x - 27}{x + 3}$$

■ 2. Identify the non-removable discontinuities of the function.

$$k(x) = \frac{x^3 + 3x^2 - 25x - 75}{x^2 + x - 12}$$

■ 3. What is the set of removable discontinuities of the function?

$$j(\theta) = \frac{\cos^2\theta \cdot \sin^2\theta}{\tan^2\theta}$$

■ 4. Examine whether or not the function is continuous at $x = 0$.

$$g(x) = \begin{cases} 2 - x^2 & x \le 0 \\ x - 2 & x > 0 \end{cases}$$

■ 5. Where is the removable discontinuity in the graph of the function?

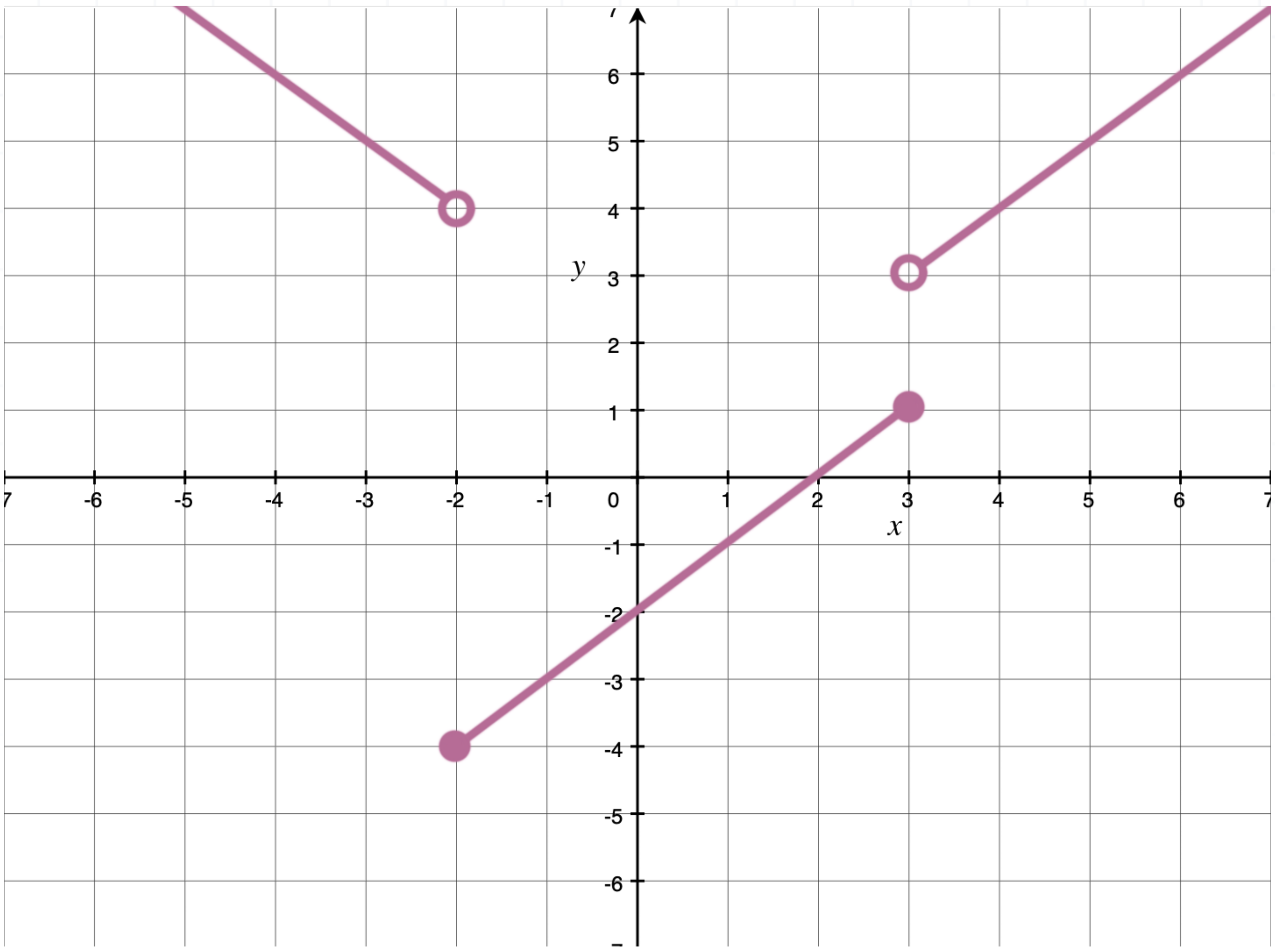
$$f(x) = \frac{x^3 + 27}{x + 3}$$

6. Identify the removable discontinuities in the function.

$$k(x) = \frac{x^4 - 2x^3 - 16x^2 + 2x + 15}{x^2 - 2x - 15}$$

JUMP DISCONTINUITIES

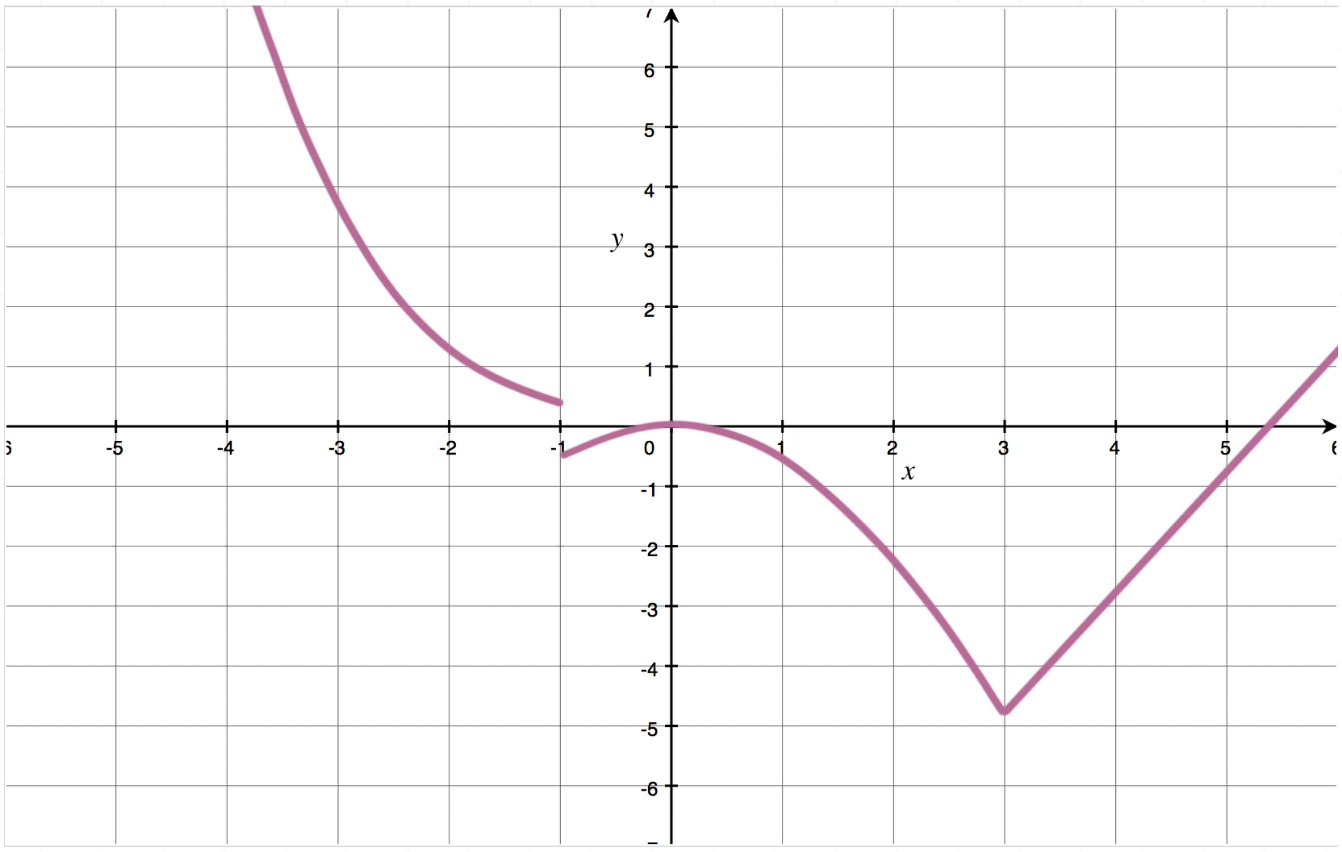
■ 1. What are the x-values where the graph of $f(x)$, shown below, has jump discontinuities?



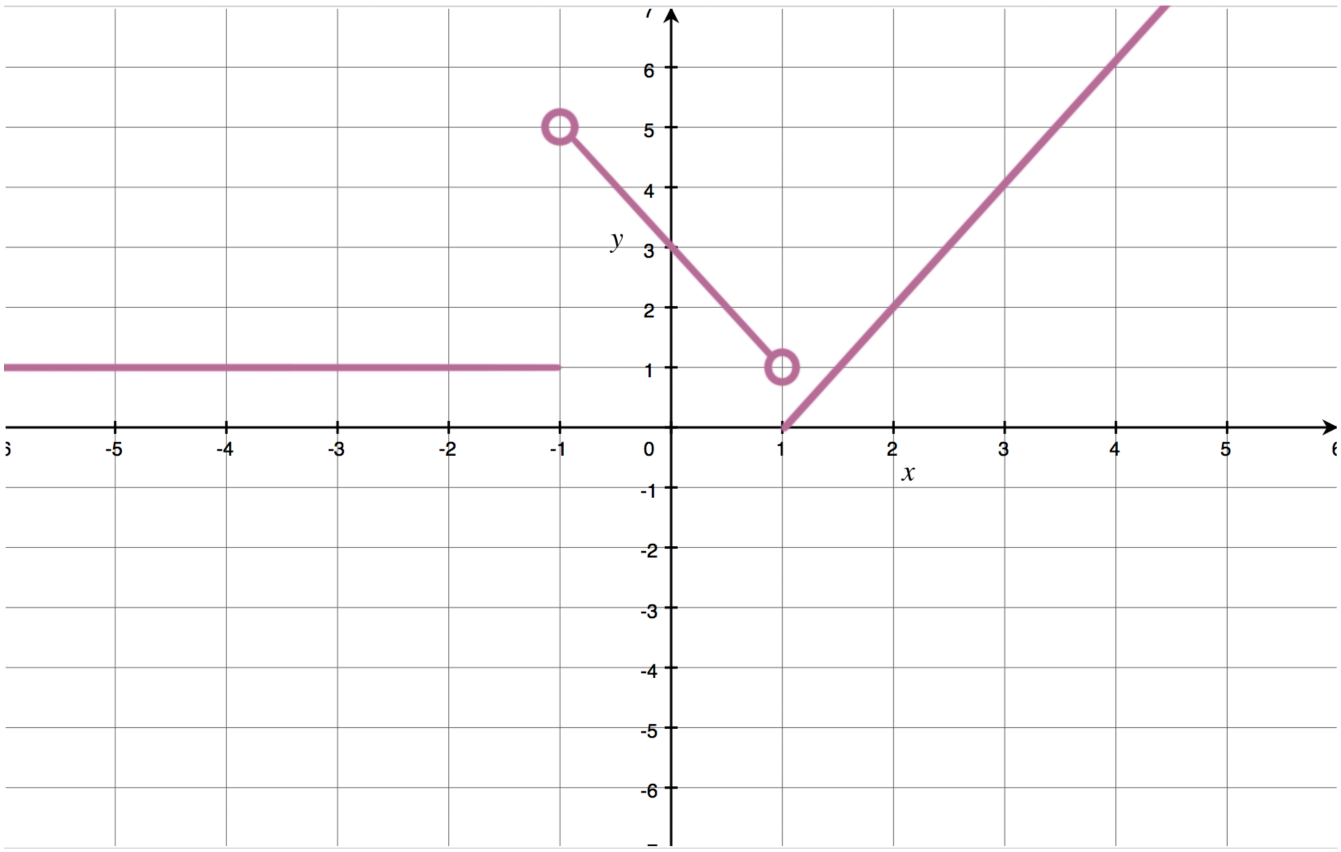
■ 2. Where are the jump discontinuities in the graph of the function?

$$h(x) = \begin{cases} -\frac{1}{3}x^2 + 2 & x < 0 \\ 3 & 0 \le x \le 1 \\ \frac{1}{3}x^2 + 4 & x > 1 \end{cases}$$

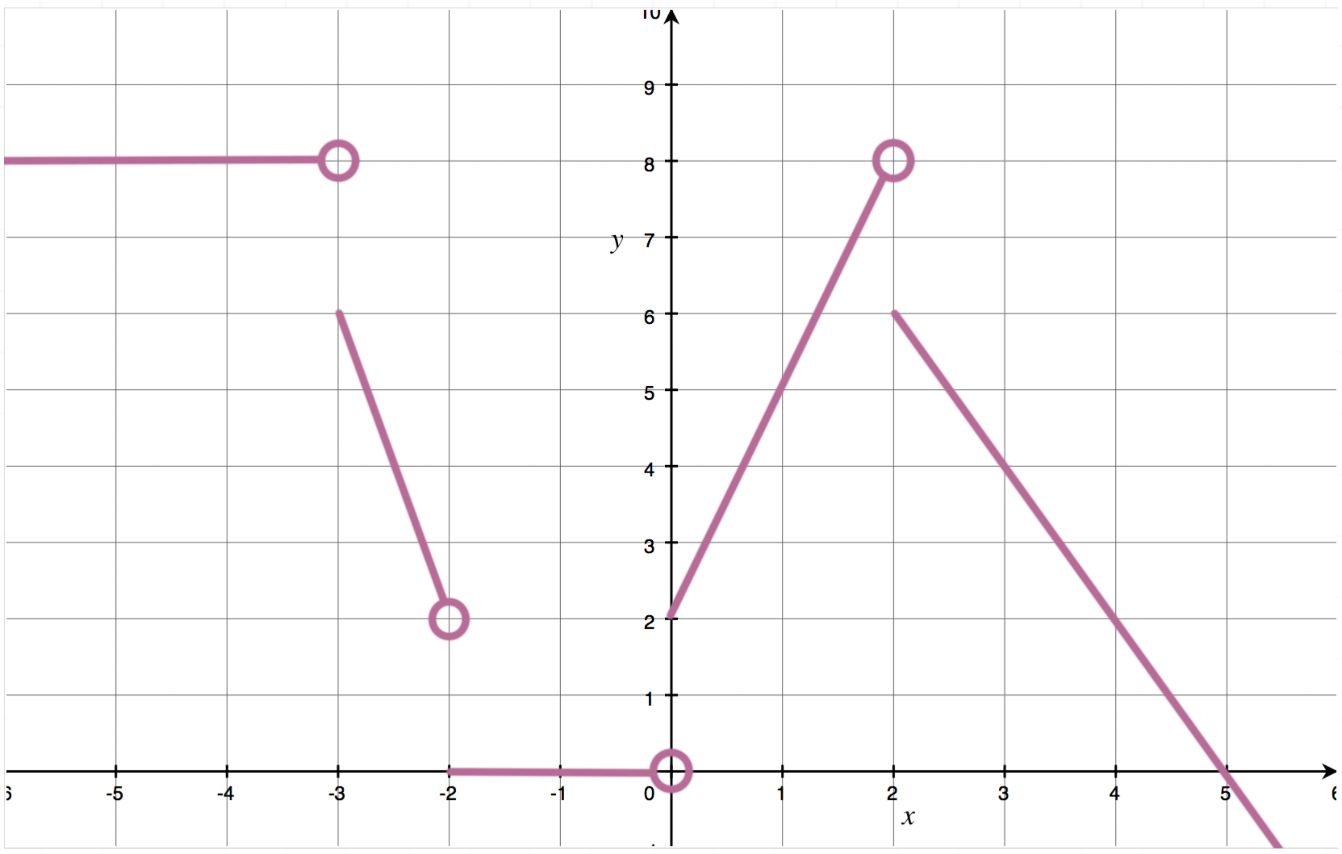
■ 3. What are the x-values where the graph of $g(x)$ has jump discontinuities?



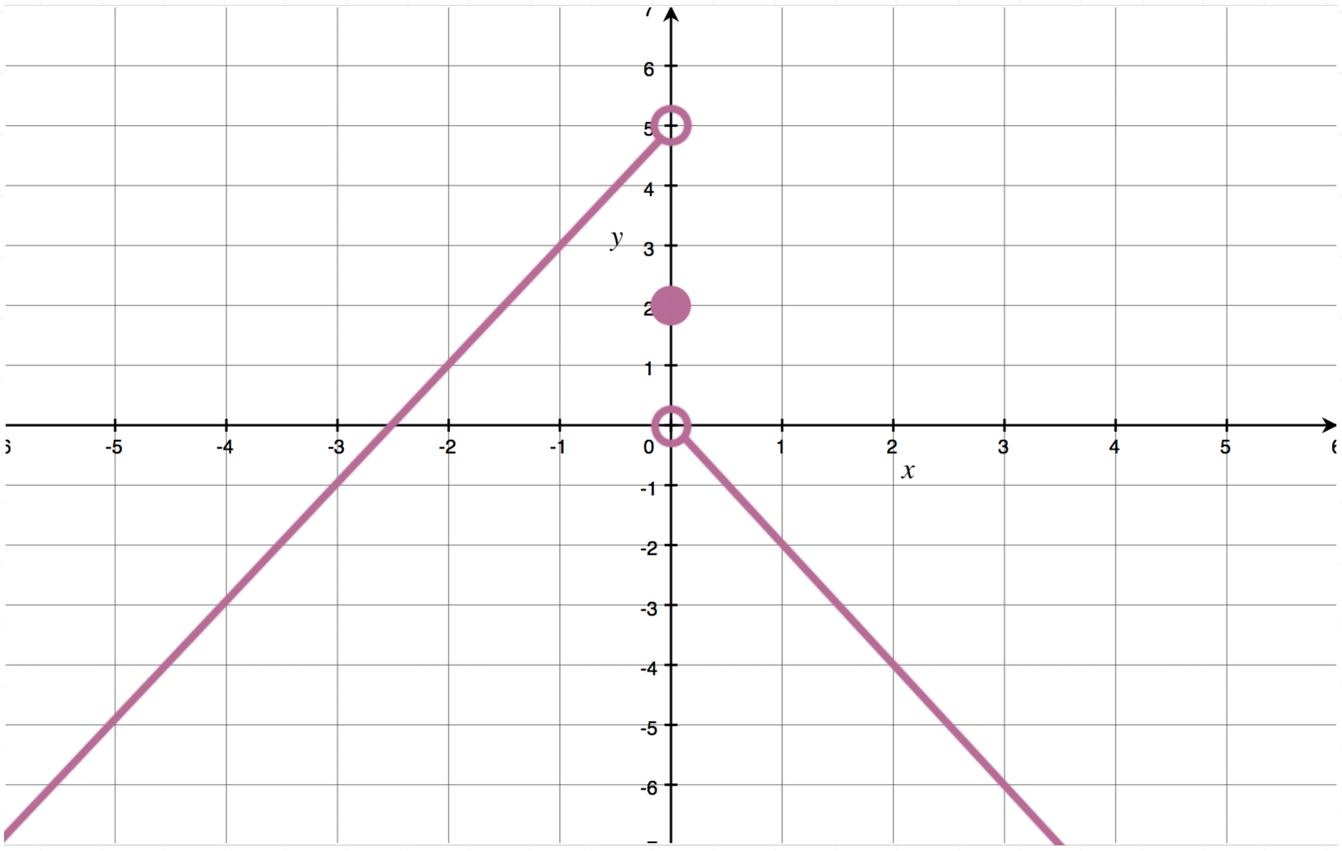
■ 4. Show that $f(x)$ has jump discontinuity at $x = -1$ and $x = 1$.



■ 5. Where are the jump discontinuities in the graph of the function shown below?



■ 6. What are the x-values where the graph of $h(x)$, shown below, has jump discontinuities?


y
x
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-5
-4
-3
-2
-1
1
2
3
4
5

INFINITE DISCONTINUITIES

■ 1. At what x-values does the function have infinite discontinuities?

$$f(x)=\frac{x^2+x-12}{x^2+x-2}$$

■ 2. Where are the infinite discontinuities of the function?

$$h(x)=\frac{x^4+3x^3-8x-24}{x^2+3x-4}$$

■ 3. At what x-values does the function have infinite discontinuities?

$$g(x)=\frac{x^2-5x+6}{x^2-1}$$

■ 4. Where are the infinite discontinuities of the function?

$$h(x)=\frac{x^2-6x+9}{x^2-4}$$

■ 5. At what x-values does the function have infinite discontinuities?

$$h(x) = \frac{x^2 - 6x + 9}{x^2 + x - 12}$$

■ 6. Classify the discontinuities of $f(x) = \cot x$ on the interval $[0,2\pi]$.

ENDPOINT DISCONTINUITIES

■ 1. What is the value of the limit on the interval $[0,3]$?

$$\lim_{x \to 3} -\sqrt{x+5}$$

■ 2. What is the value of the limit on the interval $[\pi,2\pi]$?

$$\lim_{x \to \pi} \sin x$$

■ 3. What is the value of the limit on the interval $[4,\infty)$?

$$\lim_{x \to 4} -\frac{x+7}{x^2-6x+15}$$

■ 4. What is the value of the limit on the interval $[-9/2,5/2]$?

$$\lim_{x \to \frac{5}{2}} \frac{x+3}{x^2+x+1}$$

■ 5. What is the value of the limit on the interval $(-2,2]$?

$$\lim_{x \to -2} \sqrt{2x+4}$$

■ 6. What is the value of the limit on the interval $[-\pi, \pi]$?

$$\lim_{x \to \pi} -\frac{5\cos x}{2}$$

INTERMEDIATE VALUE THEOREM WITH AN INTERVAL

■ 1. The value $c=-1$ satisfies the conditions of the Intermediate Value Theorem for the function on the interval $[-3,5]$ because $f(c)$ equals what value?

$$f(x)=\frac{1}{4}(2x+5)(x-3)^2$$

■ 2. The value $c=2$ does not satisfy the conditions of the Intermediate Value Theorem for $g(x)=2x^2-11x+4$ on the interval $[-2,4]$ because $g(c)$ equals what value?

■ 3. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[-3,3]$ if $h(x)=3(x+1)^3$ and $h(c)=24$?

■ 4. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[-5,6]$ if $f(c)=-6$ and

$$f(x)=\begin{cases}3x-10 & \text{if } x\le 0\\ x^2+3x-10 & \text{if } 0<x<2\\ 3x-6 & \text{if } x\ge 2\end{cases}$$

■ 5. Show that the function has a zero in the interval $[2,9]$ and find the solution.

$$g(x) = \frac{x^2 - 9}{x + 3}$$

■ 6. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[3,6]$ if c is a root of $h(x)$.

$$h(x) = \frac{x^3 - 4x^2 - 11x + 30}{x^2 - 4}$$

INTERMEDIATE VALUE THEOREM WITHOUT AN INTERVAL

■ 1. Use the Intermediate Value Theorem to prove that the equation $2e^x = 3\cos x$ has at least one positive solution. In what interval is that solution?

■ 2. Use the Intermediate Value Theorem to prove that the equation $3\sin x + 7 = x^2 - 2x - 2$ has at least one positive solution. In what interval is that solution?

■ 3. Use the Intermediate Value Theorem to prove that the equation $x^6 - 9x^4 + 7 = x^5 - 8x^3 - 9$ has at least one positive solution. In what interval is that solution?

■ 4. Use the Intermediate Value Theorem to prove that the equation $4e^{x-3} = 2(x^3 - 5x + 9)$ has at least one negative solution. In what interval is that solution?

■ 5. Use the Intermediate Value Theorem to show that the equation has at least one positive solution. In what interval is that solution?

$$6e^{-x} = -\left(\frac{1}{5}x^2 - 4x + 9\right)$$

6. Use the Intermediate Value Theorem to show that the equation $2\sin(4x - 1) = \cos(2x - 3)$ has at least one negative solution. In what interval is that solution?

SOLVING WITH SUBSTITUTION

■ 1. What is the value of the limit?

$$\lim_{x\to 3}(-x^4+x^3+2x^2)$$

■ 2. What is the value of the limit?

$$\lim_{x\to 7}\frac{x^2-5}{x^2+5}$$

■ 3. What is the value of the limit.

$$\lim_{x\to -2}\frac{x^3-5x^2+4x-6}{x^2+7x+6}$$

■ 4. Evaluate the limit.

$$\lim_{y\to -2}\frac{|y-5|}{y+1}$$

■ 5. Evaluate the limit.

$$\lim_{x \to 2} \left(\sin\left(\frac{\pi x}{4}\right) + \ln\left(\frac{2e}{x}\right) \right)$$

■ 6. Evaluate the limits $\lim_{x \to -1} f(x)$ and $\lim_{x \to 2} f(x)$.

$$f(x) = \begin{cases} -3x + 5 & x < -1 \\ \frac{1}{2}x^2 - 3x + 1 & x \geq -1 \end{cases}$$

SOLVING WITH FACTORING

1. What is the value of the limit?

$$\lim_{x\to -7}\frac{6x^3+42x^2}{2x^2+26x+84}$$

2. What is the value of the limit?

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}$$

3. What is the value of the limit?

$$\lim_{x\to 0}\frac{(x+3)^2-9}{x}$$

4. What is the value of the limit?

$$\lim_{x\to 7}\frac{x^3-x^2-42x}{2x^2-20x+42}$$

5. What is the value of the limit?

$$\lim_{x \to 8} \frac{x^2 + 2x - 80}{2x^3 - 24x^2 + 64x}$$

■ 6. What is the value of the limit?

$$\lim_{x \to 0} \frac{1}{x} \left(1 - \frac{16}{(x-4)^2} \right)$$

SOLVING WITH CONJUGATE METHOD

■ 1. Use conjugate method to evaluate the limit.

$$\lim_{x\to 16} \frac{3(x-16)}{\sqrt{x}-4}$$

■ 2. What is the value of the limit?

$$\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$$

■ 3. What is the value of the limit?

$$\lim_{x\to 0} \frac{\sqrt{x^2+16}-4}{x^2}$$

■ 4. Use conjugate method to evaluate the limit.

$$\lim_{x\to 49} \frac{x-49}{3(\sqrt{x}-7)}$$

■ 5. What is the value of the limit?

$$\lim_{x \to 1} \frac{4 - \sqrt{x + 15}}{2(x - 1)}$$

■ 6. What is the value of the limit?

$$\lim_{x \to 2} \frac{\sqrt{11 - x} - 3}{\sqrt{6 - x} - 2}$$

INFINITE LIMITS AND VERTICAL ASYMPTOTES

■ 1. What is the value of the limit?

$$\lim_{x \to 2} \frac{x^2 - x - 6}{-3x^2 - 3x + 18}$$

■ 2. What is the value of the limit?

$$\lim_{x \to -1} \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

■ 3. What is the value of the limit?

$$\lim_{x \to 3} \frac{1}{|x - 3|}$$

■ 4. What is the value of the limit?

$$\lim_{x \to -4} \frac{\sqrt{x^2 - 1}}{x + 4}$$

■ 5. What is the value of the limit?

$$\lim_{x\to 3}\frac{x^2-4x}{x^2-2x-3}$$

6. What is the value of the limit?

$$\lim_{x\to -2}\frac{x^2-16}{-x^2+x+6}$$

LIMITS AT INFINITY AND HORIZONTAL ASYMPTOTES

■ 1. What is the value of the limit?

$$\lim_{x\to\infty} \frac{3x^3 - 5x + 2}{9x^3 + 7x^2 - x}$$

■ 2. What is the value of the limit?

$$\lim_{x\to\infty} \frac{\sqrt[3]{x} + 1}{\sqrt{x} - 1}$$

■ 3. What is the value of the limit?

$$\lim_{x\to\infty} \frac{x^3 + x^2 + 1}{1 + 2x}$$

■ 4. What is the value of the limit?

$$\lim_{x\to-\infty} \frac{\sqrt{x^2 + 5}}{x + 4}$$

■ 5. What is the value of the limit?

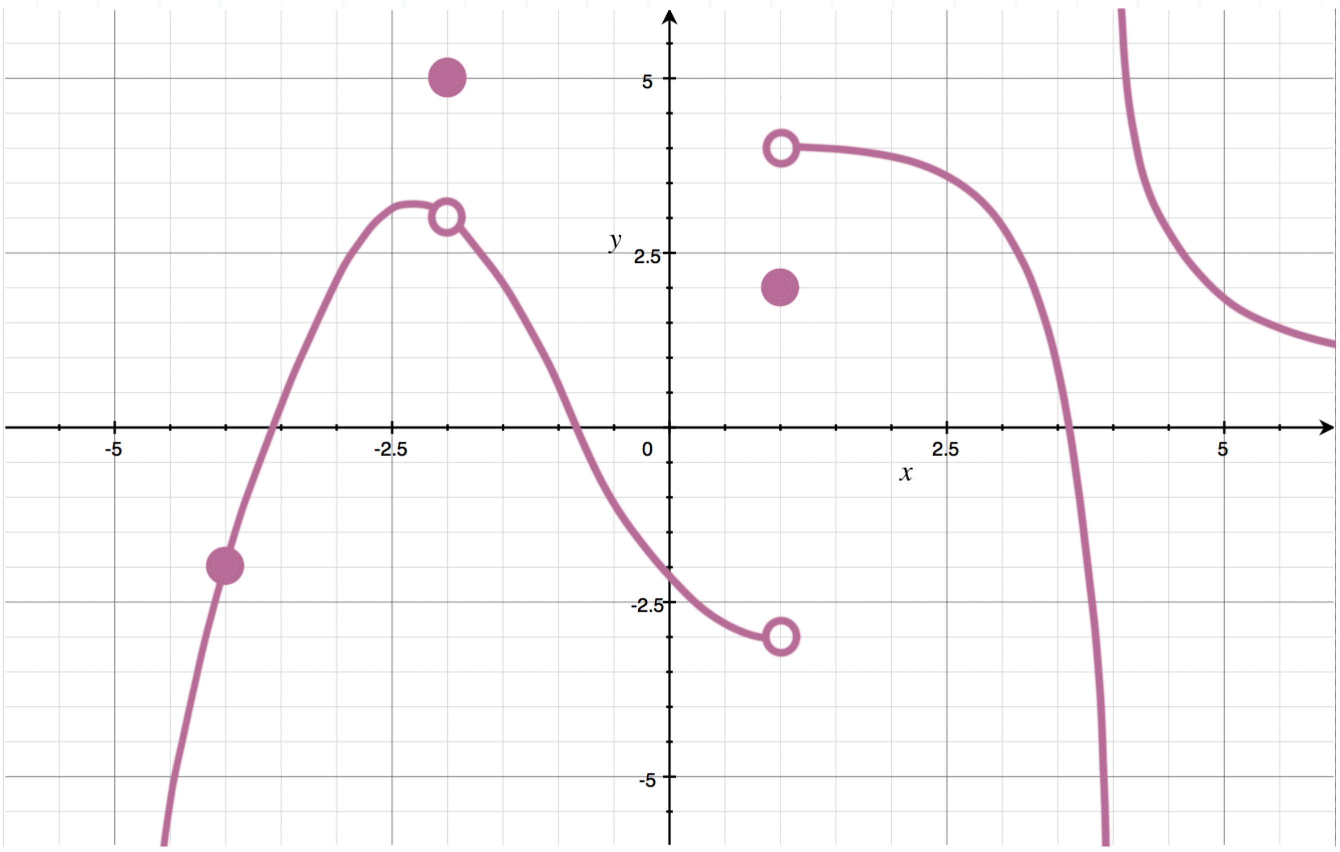
$$\lim_{x \to -\infty} \frac{19x + 21}{x^3 + 15x + 11}$$

6. What is the value of the limit?

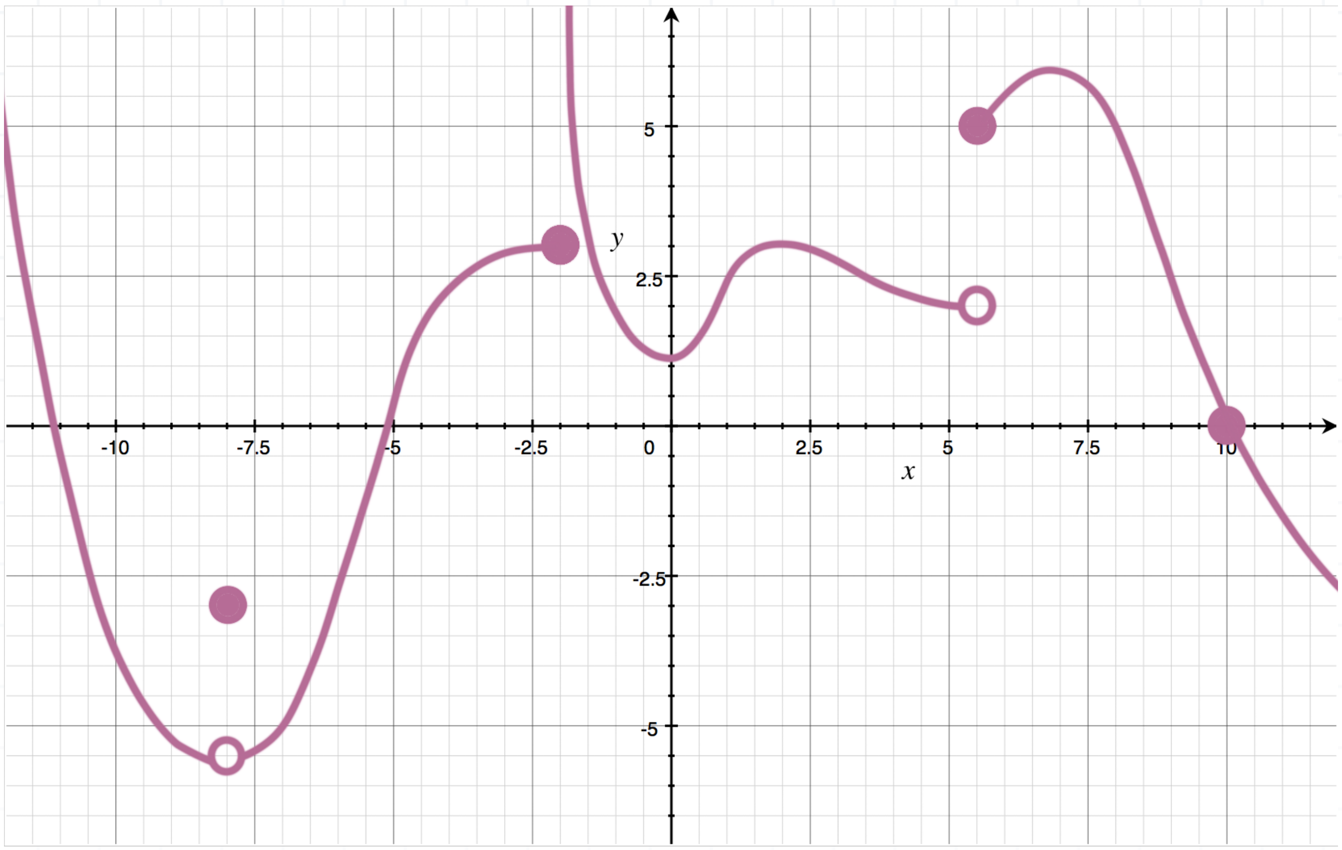
$$\lim_{x \to \infty} \left(\sqrt{x^2 + 2x} - x\right)$$

CRAZY GRAPHS

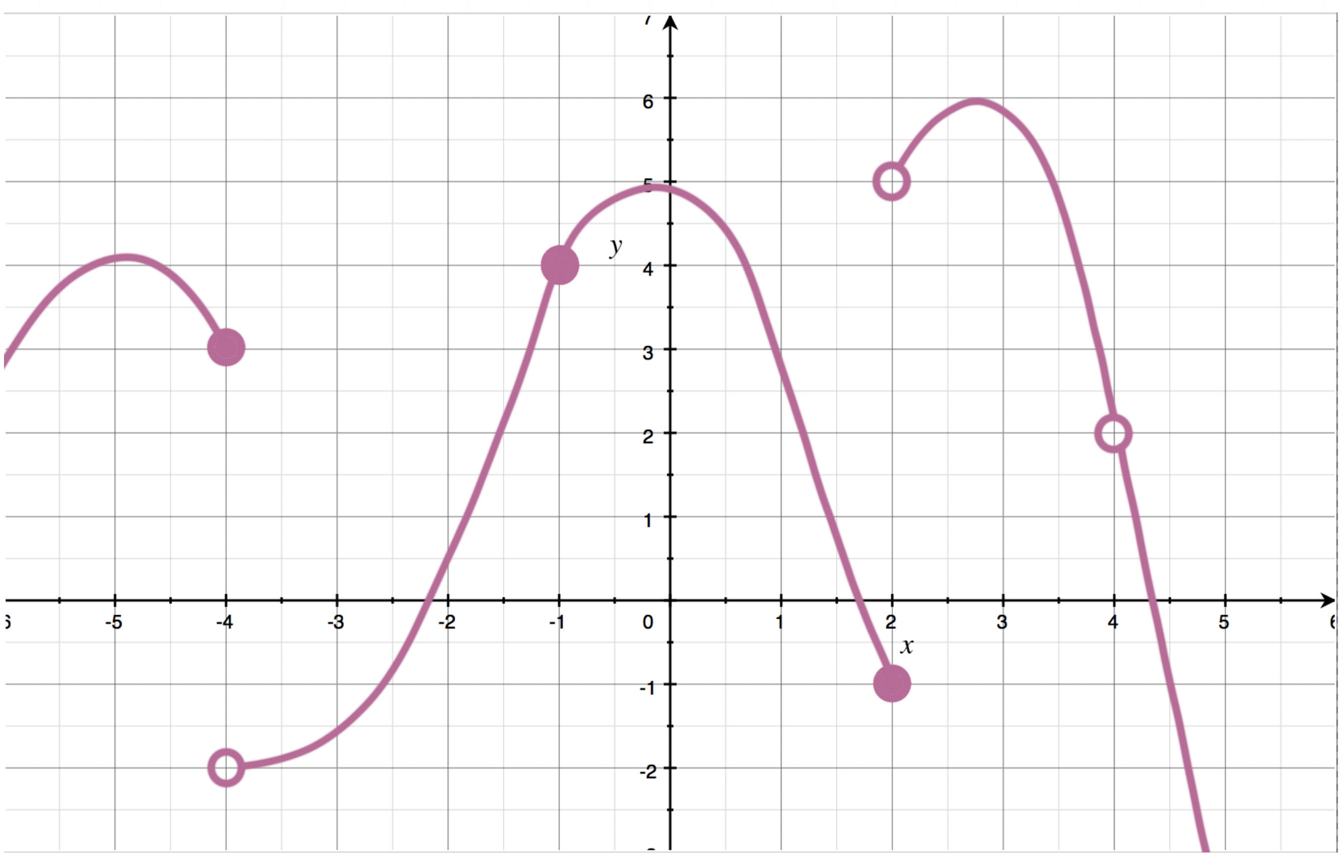
1. Use the graph to find the value of $\lim_{x \to 1} f(x)$.



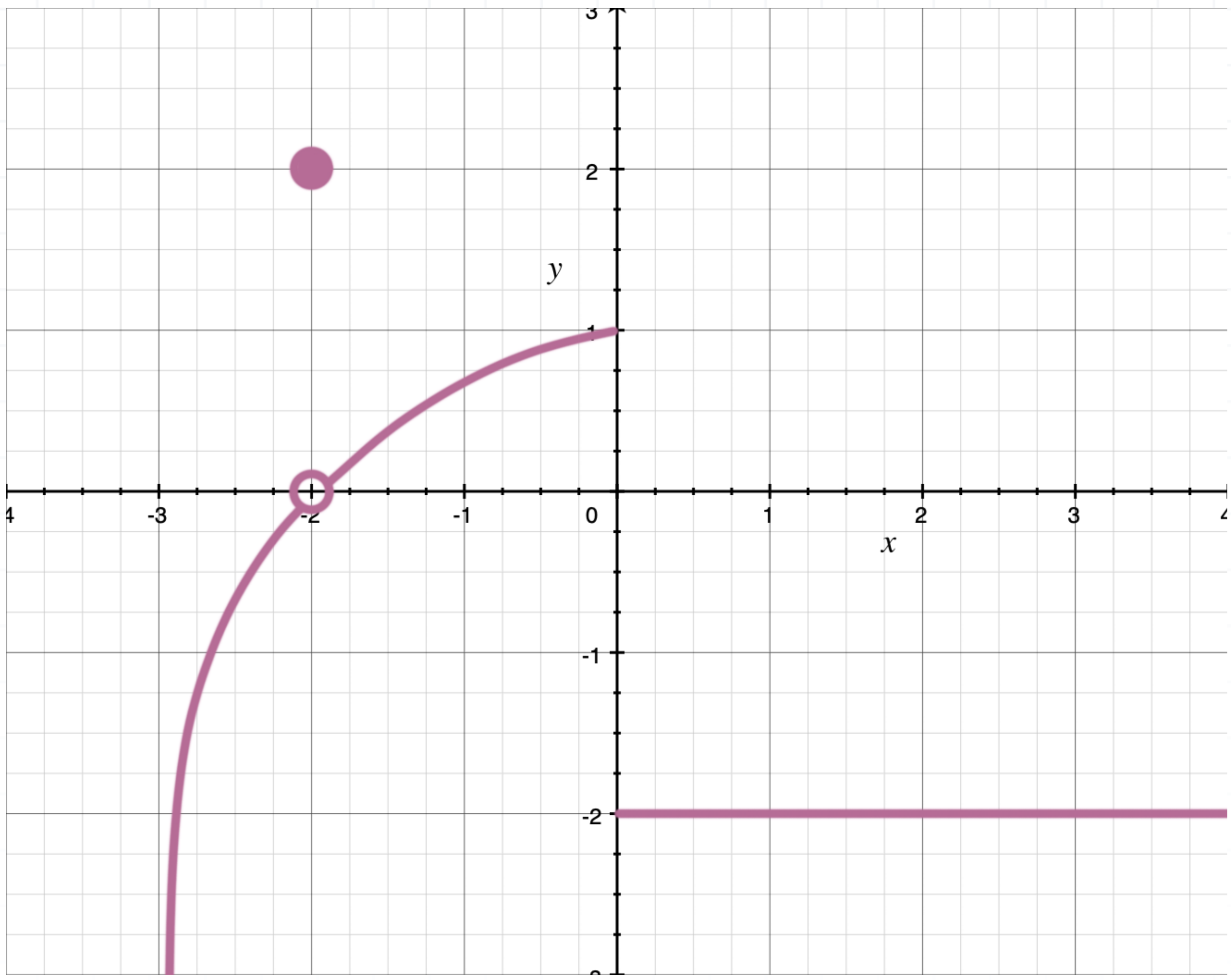
2. Use the graph to find the value of $\lim_{x \to 5.5} g(x)$.



■ 3. Use the graph to find the value of $\lim_{x \to 4} h(x)$.



4. Use the graph to determine whether or not the limit exists at $x = 0$.



5. Sketch the graph of a function that satisfies each of the following conditions.

$$\lim_{x \to -1^-} f(x) = 2 \qquad \lim_{x \to -1^+} f(x) = -1 \qquad f(-1) = 0$$

6. Sketch the graph of a function that satisfies each of the following conditions.

$\lim_{x \to 0} f(x) = -5$ $\qquad$ $f(0)$ does not exist

$\lim_{x \to 3^-} f(x) = -2$ $\lim_{x \to 3^+} f(x) = -4$ $f(3) = 0$

TRIGONOMETRIC LIMITS

1. Find $\lim_{x\to\pi} f(x)$ if $f(x) = 3\cos x - 2$.

2. Evaluate the limit.

$$\lim_{x\to 0} \frac{\sin(8x)}{x}$$

3. Evaluate the limit.

$$\lim_{x\to \frac{\pi}{4}} \frac{1-\cot x}{\cos x - \sin x}$$

4. Evaluate the limit.

$$\lim_{x\to 0} \frac{\tan(4x)}{\sin(2x)}$$

5. Evaluate the limit.

$$\lim_{x\to 0} \frac{x}{\sin \frac{x}{3}}$$

6. Evaluate the limit.

$$\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos x}$$

MAKING THE FUNCTION CONTINUOUS

■ 1. What value of c makes the function's pieces meet each other at $x=4$?

$$h(x)=\begin{cases} x^2 & x\le 4 \\ 3x+c & x>4 \end{cases}$$

■ 2. What value of k makes the function's pieces meet each other at $x=3$?

$$f(x)=\begin{cases} kx^2-2x+1 & x\le 3 \\ kx+1 & x>3 \end{cases}$$

■ 3. What values of a and b make the function's pieces meet each other at $x=-2$ and $x=2$?

$$g(x)=\begin{cases} 3 & x\le -2 \\ ax-b & -2<x<2 \\ -2 & x\ge 2 \end{cases}$$

■ 4. What value of c makes the function's pieces meet each other at $x=1$?

$$f(x)=\begin{cases} 2x^3-6x^2+8x+3 & x\le 1 \\ cx+9 & x>1 \end{cases}$$

5. What value of c makes the function $g(x)$ continuous?

$$g(x) = \begin{cases} \sqrt{x} + 18 & x \leq 16 \\ x - 2c & x > 16 \end{cases}$$

6. What values of a and b make the function $h(x)$ continuous?

$$h(x) = \begin{cases} ax^2 & x \leq -1 \\ ax + b & -1 < x < 3 \\ bx + 2 & x \geq 3 \end{cases}$$

SQUEEZE THEOREM

1. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right) - 2 \right)$$

2. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x \to \infty} \frac{3 \sin x}{4x}$$

3. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x \to 0} \left(x^2 \cos\left(\frac{1}{x^2}\right) + 1 \right)$$

4. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x \to \infty} \frac{e^{-x}}{x}$$

5. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x \to \infty} \frac{x^2 + x \sin \sqrt{x}}{4x^2 + 7}$$

■ 6. Use the Squeeze Theorem to evaluate the limit.

$$\lim_{x \to \infty} \frac{\sin x}{x}$$

DEFINITION OF THE DERIVATIVE

■ 1. Use the definition of the derivative to find the derivative of $f(x) = 2x^2 + 2x - 12$ at $(4,28)$.

■ 2. Use the definition of the derivative to find the derivative of $g(x) = 3x^3 - 4x + 7$ at $(-2, -9)$.

■ 3. Use the definition of the derivative to find the derivative at $(-1, -1)$.

$$f(x) = \frac{x}{x+2}$$

■ 4. Use the definition of the derivative to find the derivative of $f(x) = \sqrt{5x - 4}$ at $x = 4$.

■ 5. Use the definition of the derivative to find the derivative of $g(x) = \cos(x - 1)$ at $x = \pi/2$.

■ 6. Use the definition of the derivative to find the derivative of $g(x) = |x|$ at $x = 0$.

POWER RULE

■ 1. Find the derivative of $f(x) = 7x^3 - 17x^2 + 51x - 25$ using the power rule.

■ 2. Find the derivative of $g(x) = 2x^4 + 8x^3 + 6x^2 - 32x + 16$ using the power rule.

■ 3. Find the derivative of $h(x) = 22x^3 - 19x^2 + 13x - 17$ using the power rule.

■ 4. Find the derivative of $h(s) = s^4 - s^3 + 3s - 7$ using the power rule.

■ 5. Find the derivative using the power rule.

$$g(t) = \frac{2}{3}t^3 - \frac{5}{2}t^6$$

■ 6. Find the derivative of $f(x) = 20x^{100} + 5x^{21} - 3x - 1$ using the power rule.

POWER RULE FOR NEGATIVE POWERS

1. Find the derivative of the function using the power rule.

$$f(x) = \frac{7}{x^2} - \frac{5}{x^4} + \frac{2}{x}$$

2. Find the derivative of the function using the power rule.

$$g(x) = \frac{1}{9x^4} + \frac{2}{3x^5} - \frac{1}{x}$$

3. Find the derivative of the function using the power rule.

$$h(x) = -\frac{7}{6x^6} - \frac{1}{4x^4} + \frac{9}{2x^2}$$

4. Find the derivative of the function using the power rule.

$$g(x) = \frac{3}{x^2} + \frac{3}{2x^4} + \frac{1}{2}$$

5. Find the derivative of the function using the power rule.

$$f(x) = -2x^{-4} + \frac{1}{x^2} + 7x$$

6. Find the derivative of the function using the power rule, if a, b, and c are constants.

$$f(x) = 2ax^{-3a} + \frac{b}{cx^{2c}} - 2a$$

POWER RULE FOR FRACTIONAL POWERS

■ 1. Find the derivative of the function using the power rule.

$$f(x) = 4x^{\frac{3}{2}} - 6x^{\frac{5}{3}}$$

■ 2. Find the derivative of the function using the power rule.

$$g(x) = 6x^{\sqrt{3}} - 4x^{\sqrt{5}}$$

■ 3. Find the derivative of the function using the power rule.

$$h(x) = \frac{1}{3}x^{\frac{6}{5}} + \frac{1}{4}x^{\frac{8}{3}} - \frac{1}{5}x^{\frac{5}{2}}$$

■ 4. Find the derivative of the function using the power rule.

$$h(x) = \sqrt{x} + 2\sqrt[3]{x} - 3\sqrt[5]{x^2}$$

■ 5. Find the derivative of the function using the power rule.

$$f(z) = \frac{3}{\sqrt{z^5}} + \frac{5}{4z^4} - 2z^{-2}$$

6. Find the derivative of the function using the power rule.

$$h(t) = \frac{2}{3t^6} + \frac{t^4}{4} - 9t^3 + \sqrt{t^3} + \frac{1}{2\sqrt[3]{t^2}}$$

PRODUCT RULE WITH TWO FUNCTIONS

■ 1. Use the product rule to find the derivative of the function.

$$h(x) = (3x + 5)(2x^2 - 3x + 1)$$

■ 2. Use the product rule to find the derivative of the function.

$$h(x) = 8x^3\sqrt[3]{x^2}$$

■ 3. Use the product rule to find the derivative of the function.

$$h(x) = (5x^2 - x)\left(\frac{1}{x^4} - 6\right)$$

■ 4. Use the product rule to find the derivative of the function.

$$h(x) = (1 + \sqrt{x^3})(x^{-2} - 3\sqrt[3]{x})$$

■ 5. If $f(3) = -4$, $f'(3) = 2$, $g(3) = -1$, and $g'(3) = 3$, determine the value of $(fg)'(3)$.

6. If $h(x)=2x^3g(x)$, $g(-4)=-5$, and $g'(-4)=1$, determine the value of $h'(-4)$.

PRODUCT RULE WITH THREE OR MORE FUNCTIONS

■ 1. Use the product rule to find the derivative of the function.

$$y = 5x^4(2x - x^2)\left(\frac{1}{x^2} - 5\right)$$

■ 2. Use the product rule to find the derivative of the function.

$$y = 30\left(\frac{1}{x^3} + x^2\right)(2x^4 - x^2 - x)$$

■ 3. Use the product rule to find the derivative of the function.

$$y = (x^2 - 3x + 5)(7 + 2x - 5x^2)(2 - 2\sqrt{x})$$

■ 4. Use the product rule to find the derivative of the function.

$$y = \left(x - \frac{3}{x}\right)(x^2 + 4x)(7x^4)\left(-5x^2 - \frac{1}{2}\right)$$

■ 5. Use $f(-2) = 5$, $f'(-2) = -7$, $g(-2) = -8$, $g'(-2) = -3$, $h(-2) = 1$ and $h'(-2) = 0$ to determine the value of $(fgh)'(-2)$.

6. Use $f(5) = 4$, $f'(5) = 2$, $g(5) = -2$, $g'(5) = 3$, $h(5) = -3$, and $h'(5) = -8$ if $y = [x^2 - f(x)]g(x)h(x)$, to determine the value of $y'(5)$.

QUOTIENT RULE

■ 1. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{2x + 6}{7x + 5}$$

■ 2. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{\sqrt[3]{x}}{1 + 2x^2}$$

■ 3. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{-8x}{5x + 2}$$

■ 4. Use the quotient rule to find the derivative of the function.

$$h(x) = \frac{2 - 4x + 5x^2}{5x + x^3}$$

■ 5. Use the quotient rule to find the derivative of the function.

$$k(x) = \frac{(2-3x)(1+x)}{2+3x^2}$$

■ 6. Use $f(5)=4$, $f'(5)=2$, $g(5)=-2$, $g'(5)=3$, $h(5)=-3$, and $h'(5)=-8$ to determine the value of $k'(5)$.

$$k'(5) = \left(\frac{fg}{h}\right)'(5)$$

TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(x)$ if $f(x) = 3x^{-4} + x^2 \cot x$.

■ 2. Find $h'(x)$.

$$h(x) = \frac{\sin x}{5 - 2\cos x}$$

■ 3. Find $h'(x)$ if $h(x) = 3\sin x \cos x + 5\sec x$.

■ 4. Find the derivative of the trigonometric function.

$$y = 3 - 2\sqrt{x}\csc x$$

■ 5. Find the derivative of the trigonometric function.

$$y = \frac{2}{4\cos x - 5\sin x}$$

■ 6. Find the derivative of y.

$$y = 2x^4 + \frac{x \tan x}{x^2 + 1}$$

EXPONENTIAL DERIVATIVES

■ 1. Find $f'(x)$ if $f(x) = (x^3 - x)e^x$.

■ 2. Find $g'(x)$ if $g(x) = 5^x(x^2 - 7x + 1)$.

■ 3. Find $h'(x)$ if $h(x) = \sin xe^x - x^2 \cos x$.

■ 4. Find $f'(x)$.

$$f(x) = \frac{4e^x}{3e^x - 1}$$

■ 5. Find $g'(x)$ if $g(x) = 8^x + 3e^x \cot x$.

■ 6. Find $h'(x)$ if $h(x) = \dfrac{x^3e^x}{x + 3^x}$.

LOGARITHMIC DERIVATIVES

■ 1. Find $f'(x)$.

$$f(x) = 2\log_5 x - 11\log_{13} x$$

■ 2. Find $g'(x)$.

$$g(x) = \log_4 x - x^6 \ln x$$

■ 3. Find $h'(x)$.

$$h(x) = \log_7 x \ln x$$

■ 4. Find $y'(x)$.

$$y = \frac{1 + 7\ln x}{6x^4}$$

■ 5. Find $y'(x)$.

$$y = \frac{x^3 + \log_5 x}{5^x}$$

■ 6. Find $y'(x)$.

$$y = \frac{x^7 e^x}{\ln x}$$

CHAIN RULE WITH POWER RULE

■ 1. Find $h'(x)$ if $h(x) = (3x^2 - 7)^4$.

■ 2. Find $h'(x)$ if $h(x) = \sqrt{2 - 4x^2}$.

■ 3. Find $h'(x)$ if $h(x) = (2x^2 - 6x + 5)^7$.

■ 4. Find $h'(x)$ if $h(x) = 2(x^3 + 4x^2 - 2x)^{-5}$.

■ 5. Find $f'(x)$ if $f(x) = 3(5x^2 + \sin x)^4$.

■ 6. Find $g'(y)$ if $g(y) = \sqrt{3y + (2y + y^2)^2}$.

CHAIN RULE WITH TRIG, LOG, AND EXPONENTIAL FUNCTIONS

■ 1. Find $f'(x)$.

$$f(x) = \ln(x^2 + 6x + 9)$$

■ 2. Find $g'(x)$ if $g(x) = 3\sin(4x^3) - 4\cos(6x) + 3\sec(2x^4)$.

■ 3. Find $h'(x)$ if $h(x) = \cos(\sin x + 3x^3)$.

■ 4. Find $f'(y)$ if $f(y) = e^{y+\ln y} + 8^{\cos y}$.

■ 5. Find $f'(x)$ if $f(x) = \tan^5 x + \tan x^5$.

■ 6. Find $g'(x)$ if $g(x) = \ln(e^{\sin x} - \sin^2 x)$.

CHAIN RULE WITH PRODUCT RULE

■ 1. Find $y'(x)$ if $y(x) = (3x - 2)(5x^3)^5$.

■ 2. Find $h'(x)$ if $h(x) = (x^2 - 5x)^2(2x^3 - 3x^2)^5$.

■ 3. Find the derivative of the function.

$$y = (\sin(7x))(7e^{4x})(2x^6 + 1)$$

■ 4. Find $h'(x)$ if $h(x) = \sin(4x)e^{3x^2+4}$.

■ 5. Find the derivative of the function.

$$y = \sin(x^2 e^{x^2})$$

■ 6. Find $h'(x)$ if $h(x) = \ln(x^3\sqrt{3x^4 - 2x^2 + 3})$.

CHAIN RULE WITH QUOTIENT RULE

■ 1. Find $h'(x)$.

$$h(x) = \frac{(2x+1)^3}{(3x-2)^2}$$

■ 2. Find $h'(x)$.

$$h(x) = \frac{(4x+5)^5}{(x+3)^2}$$

■ 3. Find $h'(x)$.

$$h(x) = \ln\left(\frac{x^3}{x^2+3}\right)$$

■ 4. Find $h'(x)$.

$$h(x) = \frac{\sec(2-x)}{2x+e^{-x}}$$

■ 5. Find $h'(x)$.

$$h(x) = \frac{2 + \ln(3x)}{x + \cot(2x)}$$

■ 6. Find $h'(x)$.

$$h(x) = x^2 \sin\left(\frac{x^3 + 4x}{\sqrt{x^4 - 2}}\right)$$

INVERSE TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(t)$.

$$f(t) = 4\sin^{-1}\left(\frac{t}{4}\right)$$

■ 2. Find $g'(t)$.

$$g(t) = -6\cos^{-1}(2t+3)$$

■ 3. Find $h'(t)$.

$$h(t) = 2\sec^{-1}(6t^2+3) - 8\cot^{-1}\left(\frac{t^3}{3}\right)$$

■ 4. Find the derivative.

$$y = (x^4 + x^2)\csc^{-1}x + \sin(5x^3)$$

■ 5. Find the derivative.

$$y = \frac{\sin^{-1}\left(x + \frac{x^2}{2}\right)}{1 + x}$$

6. Find the derivative.

$$y = \frac{1 - \sin^{-1}(2x)}{1 + \cos^{-1}(2x)}$$

HYPERBOLIC DERIVATIVES

■ 1. Find $f'(\theta)$ if $f(\theta) = 3\sinh(2\theta^2 - 5\theta + 2)$.

■ 2. Find $g'(\theta)$ if $g(\theta) = 2\cosh(5\theta^{\frac{3}{2}} + 6\theta)$.

■ 3. Find $h'(\theta)$ if $h(\theta) = 9\tanh(3\theta^2 - \theta^{\sqrt{3}})$.

■ 4. Find the derivative of the hyperbolic function.

$$y = \coth(x^2 + 3x) - x^4\text{csch}(x^2)$$

■ 5. Find the derivative of the hyperbolic function.

$$y = \frac{2x + 3e^x}{\cosh(x^{-5})}$$

■ 6. Find the derivative of the hyperbolic function.

$$y = \tanh(x^2)\tan(x^2)$$

INVERSE HYPERBOLIC DERIVATIVES

■ 1. Find $f'(t)$ if $f(t)=7\sinh^{-1}(5t^4)$.

■ 2. Find $g'(t)$ if $g(t)=4\cosh^{-1}(2t-3)$.

■ 3. Find $h'(t)$ if $h(t)=9\tanh^{-1}(-7t+2)$.

■ 4. Find the derivative of the inverse hyperbolic function.

$$y=\cosh^{-1}(3x^3+4x^2)-x^2\sinh^{-1}(e^x)$$

■ 5. Find the derivative of the inverse hyperbolic function.

$$y=\left(\operatorname{csch}^{-1}\left(\frac{x^2}{3x^4+1}\right)\right)^5$$

■ 6. Find the derivative of the inverse hyperbolic function.

$$y=-\frac{\coth^{-1}x}{\tanh^{-1}(2x^4)}$$

LOGARITHMIC DIFFERENTIATION

■ 1. Use logarithmic differentiation to find dy/dx.

$$y = (\ln x)^{\ln(x^2)}$$

■ 2. Use logarithmic differentiation to find dy/dx.

$$y = 5x^4e^{3x}\sqrt[4]{x}$$

■ 3. Use logarithmic differentiation to find dy/dx.

$$y = (7 - 4x^3)^{x^2+9}\sqrt[3]{1 - \cos(3x)}$$

■ 4. Use logarithmic differentiation to find dy/dx.

$$y = \frac{(2e)^{\cos x}}{(3e)^{\sin x}}$$

■ 5. Use logarithmic differentiation to find dy/dx.

$$y = e^x(2e)^{\sin x}(3e)^{\cos x}$$

6. Use logarithmic differentiation to find dy/dx.

$$y = \frac{(1-2x)^{\sin x}}{(x^3-2x)^{5x+7}}$$

TANGENT LINES

■ 1. Find the equation of the tangent line to the graph of the equation at $(1/2,\pi)$.

$$f(x) = 4\arctan 2x$$

■ 2. Find the equation of the tangent line to the graph of the equation at $(-1,-9)$.

$$g(x) = x^3 - 2x^2 + x - 5$$

■ 3. Find the equation of the tangent line to the graph of the equation at $(0,-4)$.

$$h(x) = -4e^{-x} + 3x$$

■ 4. Find the equation of the tangent line to the graph of the equation at $(1,1)$.

$$f(x) = -6x^4 + 4x^3 - 3x^2 + 5x + 1$$

■ 5. At what point(s) is the tangent line of $f(x) = 2x(3-x)^2$ horizontal?

■ 6. Find the constants a, b, and c such that the function $f(x) = ax^2 + bx + c$ intersects the point $(-2,5)$ and has a horizontal tangent line at $(0, -3)$.

VALUE THAT MAKES TWO TANGENT LINES PARALLEL

■ 1. Find the value of a such that the tangent lines to $f(x)=2x^3+2$ at $x=a$ and $x=a+1$ are parallel.

■ 2. Find the value of a such that the tangent lines to $g(x)=x^3+x^2+7$ at $x=a$ and $x=a+1$ are parallel.

■ 3. Find the value of a such that the tangent lines to $h(x)=\tan^{-1}x$ at $x=a$ and $x=a+1$ are parallel.

■ 4. Find parallel tangent lines to $f(x)=4x^3-6x+7$ at $x=a$ and $x=a+1$.

■ 5. Find the value of a such that the tangent lines to $g(x)=(x-2)^3+x^2+3$ at $x=a$ and $x=a+1$ are parallel.

■ 6. Find the approximate value of a, rounded to the nearest hundredth, such that the tangent lines to $h(x)=e^x-3x^2$ at $x=a$ and $x=a+1$ are parallel.

VALUES THAT MAKE THE FUNCTION DIFFERENTIABLE

■ 1. What value of a and b will make the function differentiable?

$$f(x) = \begin{cases} x^2 & x \le 3 \\ ax - b & x > 3 \end{cases}$$

■ 2. What value of a and b will make the function differentiable?

$$g(x) = \begin{cases} ax + b & x \le -1 \\ bx^2 - 1 & x > -1 \end{cases}$$

■ 3. What value of a and b will make the function differentiable?

$$h(x) = \begin{cases} ax^3 & x \le 2 \\ x^2 - b & x > 2 \end{cases}$$

■ 4. What value of a and b will make the function differentiable?

$$f(x) = \begin{cases} 3 - x & x \le 1 \\ ax^2 - bx & x > 1 \end{cases}$$

■ 5. What value of a and b will make the function differentiable?

$$g(x) = \begin{cases} x^3 & x \le 1 \\ a(x-2)^2 - b & x > 1 \end{cases}$$

■ 6. What value of a and b will make the function differentiable?

$$h(x) = \begin{cases} ax^2 + b & x \le 3 \\ bx + 4 & x > 3 \end{cases}$$

NORMAL LINES

■ 1. Find the equation of the normal line to the graph of $f(x) = 5x^4 + 3e^x$ at $(0,3)$.

■ 2. Find the equation of the normal line to the graph of $g(x) = \ln e^{4x} + 2x^3$ at $(2,24)$.

■ 3. Find the equation of the normal line to the graph of $h(x) = 5\cos x + 5\sin x$ at $(\pi/2,5)$.

■ 4. Find the equation of the normal line to the graph of $f(x) = 7x^3 + 2x^2 - 5x + 9$ at $(2,63)$.

■ 5. Find the equation of the normal line to the graph of $g(x) = 5\sqrt{x^2 - 14x + 49}$ at $(2,25)$.

■ 6. Find the equations of the tangent and normal lines of $g(x) = (2x^2 - 5x + 3)^2$ at $(0,9)$.

AVERAGE RATE OF CHANGE

■ 1. Find the average rate of change of the function over the interval $[4,9]$.

$$f(x) = \frac{5\sqrt{x} - 2}{3}$$

■ 2. Find the average rate of change of the function over the interval $[16,25]$.

$$g(x) = \frac{2x - 8}{\sqrt{x} - 2}$$

■ 3. Find the average rate of change of the function over the interval $[0,4]$.

$$h(x) = \frac{x^3 - 8}{x^2 - 4x - 5}$$

■ 4. Find the average rate of change of the function over the interval $[-2, -3/2]$.

$$f(x) = -\frac{1}{4 - x}$$

■ 5. On Thursday, the price of a gallon of gas was $\$3.24$. What was the price of a gallon of gas on Sunday, if the average rate of change of the price of a gallon of gas from Thursday to Sunday is $\$0.09$ per day?

■ 6. Find an expression in terms of a that models the average rate of change of the function $f(x) = 2x^2 + 5x - 4$ over the interval $[0,2a]$.

IMPLICIT DIFFERENTIATION

■ 1. Use implicit differentiation to find dy/dx at $(3,4)$.

$$4x^3 - 3xy^2 + y^3 = 28$$

■ 2. Use implicit differentiation to find dy/dx.

$$5x^3 + xy^2 = 4x^3y^3$$

■ 3. Use implicit differentiation to find dy/dx.

$$3x^2 = (3xy - 1)^2$$

■ 4. Use implicit differentiation to find dy/dx.

$$\sin(2x + 5y) = \cos^2 x + \cos^2 y$$

■ 5. Use implicit differentiation to find dy/dx.

$$e^{2xy} = 3x^3 - \ln(xy^2)$$

■ 6. Use implicit differentiation to find dy/dx at $(0,2)$.

$$\frac{2x - y^3}{y + x^2} = 5x - 4$$

EQUATION OF THE TANGENT LINE WITH IMPLICIT DIFFERENTIATION

■ 1. Use implicit differentiation to find the equation of the tangent line to $5y^2 = 2x^3 - 5y + 6$ at $(3,3)$.

■ 2. Use implicit differentiation to find the equation of the tangent line to $5x^3 = -3xy + 4$ at $(2,-6)$.

■ 3. Use implicit differentiation to find the equation of the tangent line to $4y^2 + 8 = 3x^2$ at $(6,-5)$.

■ 4. Use implicit differentiation to find the equation of the tangent line to $2x + 3y - 5 = \ln(x^5 + y^5)$ at $(1,0)$.

■ 5. Use implicit differentiation to find the equations of the tangent and normal line to $\cos x = \sin(2y) + 9$ at $(\pi/2, \pi)$.

■ 6. Use implicit differentiation to find the equation of the tangent line to $4x^2 - xy + y^2 = 6$ at the points in the second and third quadrant when $x = -1$.

HIGHER-ORDER DERIVATIVES

■ 1. Find the second and third derivatives of the function at $x = -1$.

$$y = 2x^5 - 3x^4 + x^3 + x^2 - 7$$

■ 2. Find the second derivative of the function $y = -3x^{\frac{2}{3}} + x^{-\frac{1}{2}}$.

■ 3. Find the second derivative of the function.

$$y = -3x^7 \sin x$$

■ 4. Find the second and the third derivatives of the function.

$$y = \ln(x^5\sqrt{x})$$

■ 5. Find the second derivative of the function.

$$y = \frac{2x}{\sin(x^2)}$$

■ 6. Find the second derivative of the function at $x = 0$.

$$y = \frac{e^x}{4x - 9}$$

SECOND DERIVATIVES WITH IMPLICIT DIFFERENTIATION

■ 1. Use implicit differentiation to find d^2y/dx^2.

$$2x^3 = 2y^2 + 4$$

■ 2. Use implicit differentiation to find d^2y/dx^2.

$$4x^2 = 2y^3 + 4y - 2$$

■ 3. Use implicit differentiation to find d^2y/dx^2 at $(0,3)$.

$$3x^2 + 3y^2 = 27$$

■ 4. Use implicit differentiation to find d^2y/dx^2 at $(2,1)$.

$$e^{x-2y} = 2x - y$$

■ 5. Use implicit differentiation to find y''.

$$y \sin x = 7 - 2y^2$$

■ 6. Use implicit differentiation to find y'' at $(0,3)$.

$$e^{2y} - 2x = y^4 - 2$$

CRITICAL POINTS AND THE FIRST DERIVATIVE TEST

■ 1. Identify the critical point(s) of the function on the interval $[-3,2]$.

$$f(x) = x^{\frac{2}{3}}(x+2)$$

■ 2. Identify the critical point(s) of the function on the interval $[-2,2]$.

$$g(x) = x\sqrt{4-x^2}$$

■ 3. Determine the intervals in which the function is increasing and decreasing.

$$f(x) = \frac{5}{4}x^4 - 10x^2$$

■ 4. Determine the intervals in which the function is increasing and decreasing.

$$f(x) = (4-3x)e^x$$

■ 5. Identify the critical point(s) of the function.

$$f(x) = x + 3\ln(2x+3)$$

■ 6. Find the values a and b such that $f(x) = x^3 + ax^2 + b$ will have a critical point at $(-1,5)$.

INFLECTION POINTS AND THE SECOND DERIVATIVE TEST

■ 1. Find the inflection points of the function.

$$f(x) = \frac{1}{3}x^3 + x^2$$

■ 2. For $g(x) = -x^3 + 2x^2 + 3$, find inflection points and identify where the function is concave up and concave down.

■ 3. For $h(x) = x^4 + x^3 - 3x^2 + 2$, find inflection points and identify where the function is concave up and concave down.

■ 4. Use the second derivative test to identify the extrema of $f(x) = x^3 - 12x - 2$ as maximum values or minimum values.

■ 5. Use the second derivative test to identify the extrema of $g(x) = -4xe^{-\frac{x}{2}}$ as maxima or minima.

■ 6. Use the second derivative test to identify the extrema of $h(x) = 2x^4 - 4x^2 + 1$ as maximum values or minimum values.

INTERCEPTS AND VERTICAL ASYMPTOTES

■ 1. Find the x-intercepts and any vertical asymptote(s) of the function.

$$f(x)=\frac{-x^2+16x-63}{x^2-2x-35}$$

■ 2. Find any vertical asymptote(s) of the function.

$$g(x)=\frac{x^2-3x-10}{x^2+x-2}$$

■ 3. Find any vertical asymptote(s) of the function.

$$h(x)=\frac{8+x-8x^2-x^3}{9x^2+63x-72}$$

■ 4. Find the y-intercepts and any vertical asymptote(s) of the function.

$$f(x)=\frac{x^2+-2x-8}{x^2-9x+20}$$

■ 5. Find any vertical asymptote(s) of the function.

$$g(x)=\ln(x^2+5x)$$

■ 6. Find any vertical asymptote(s) of the function.

$$h(x) = \sec\left(x + \frac{\pi}{2}\right)$$

HORIZONTAL AND SLANT ASYMPTOTES

■ 1. Find the horizontal asymptote(s) of the function.

$$f(x) = \frac{8x^4 - x^2 + 1}{4x^4 - 1}$$

■ 2. Find the horizontal asymptote(s) of the function.

$$g(x) = \frac{2x^2 - 5x + 12}{3x^2 - 11x - 4}$$

■ 3. Find the horizontal asymptote(s) of the function.

$$h(x) = \frac{x^3 - x^2 + 6x - 1}{7x^4 - 1}$$

■ 4. Find the slant asymptote of the function.

$$f(x) = \frac{3x^4 - x^3 + x^2 - 4}{x^3 - x^2 + 1}$$

■ 5. Find the slant asymptote of the function.

$$g(x) = \frac{8x^2 + 14x - 7}{4x - 1}$$

6. Determine whether the function has a horizontal asymptote, slant asymptote, or neither.

$$h(x) = \frac{x^4 - x^3 - 8}{x^2 - 5x + 6}$$

SKETCHING GRAPHS

1. Sketch the graph of the function.

$$f(x) = x^3 - 4x^2 + 8$$

2. Sketch the graph of the function.

$$g(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + 1$$

3. Sketch the graph of the function.

$$h(x) = \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

4. Sketch the graph of the function.

$$f(x) = \frac{4}{1 + x^2}$$

5. Sketch the graph of the function.

$$f(x) = 2x \ln x$$

6. Sketch the graph of the function.

$$f(x) = x^2\sqrt{x+4}$$

EXTREMA ON A CLOSED INTERVAL

■ 1. Find the extrema of $f(x)=x^3-3x^2+5$ over the closed interval $[-3,4]$.

■ 2. Find the extrema of $g(x)=\sqrt[3]{2x^2+3}$ over the closed interval $[-1,5]$.

■ 3. Find the extrema of $h(x)=-4x^3+6x^2-3x-2$ over the closed interval $[-4,6]$.

■ 4. Find the extrema of the function over the closed interval $[-1,3]$.

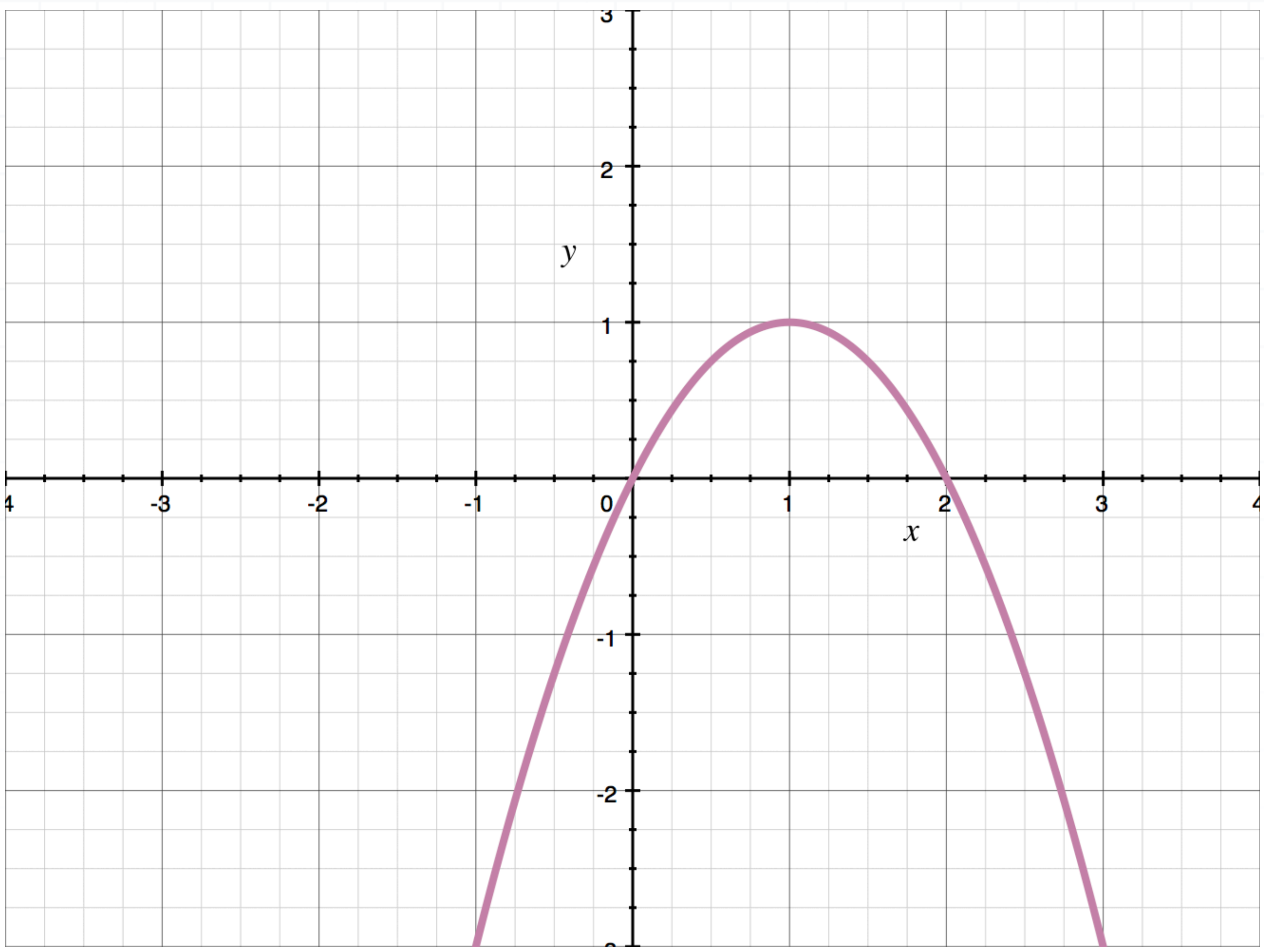
$$f(x)=\frac{x^2}{x^2+7}$$

■ 5. Find the extrema of $g(x)=e^{2x^3+4x^2-8x+3}$ over the closed interval $[-4,0]$.

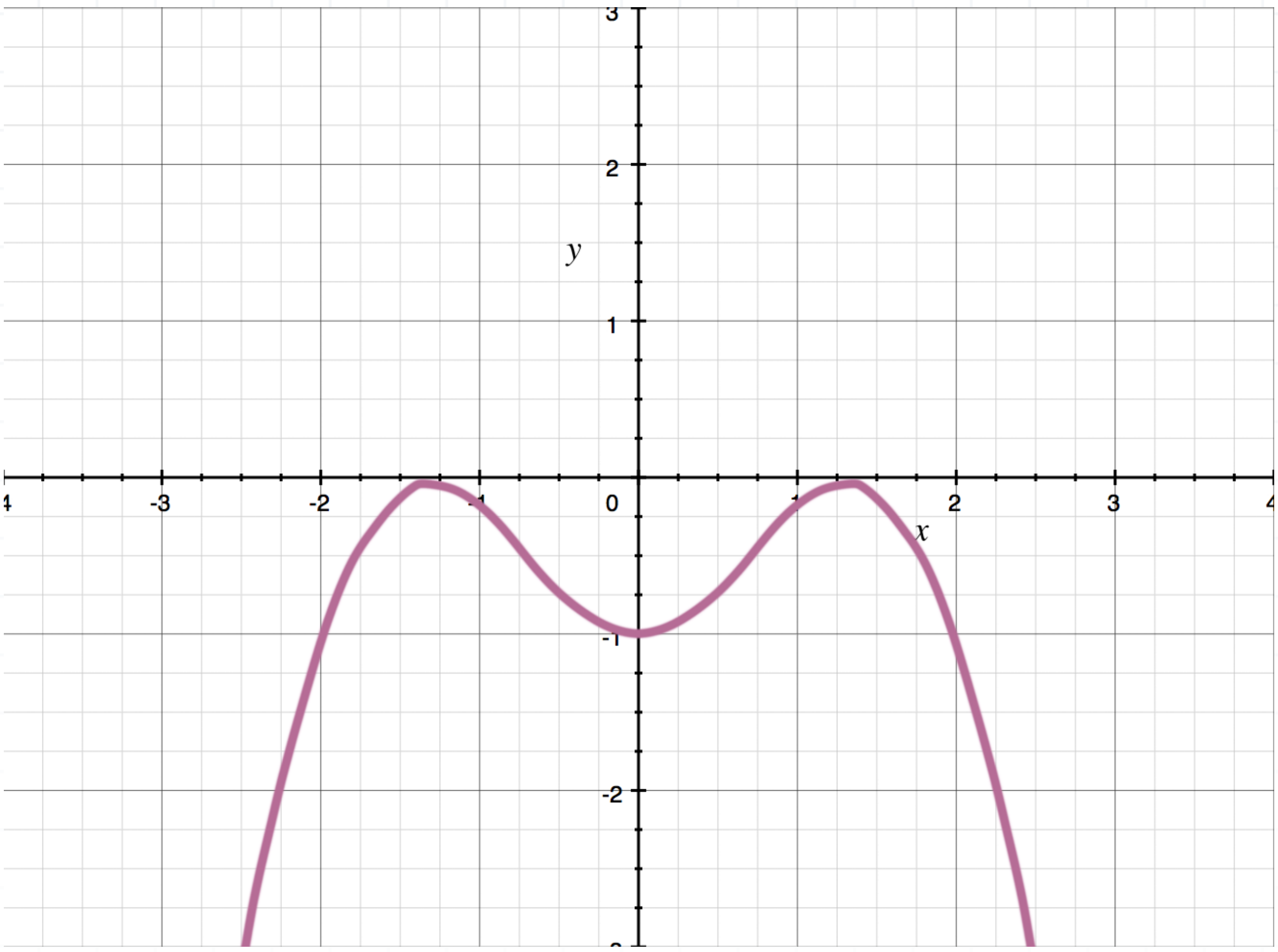
■ 6. Find the extrema of $h(x)=x-\cos x$ over the closed interval $[0,\pi]$.

SKETCHING F(X) FROM F'(X)

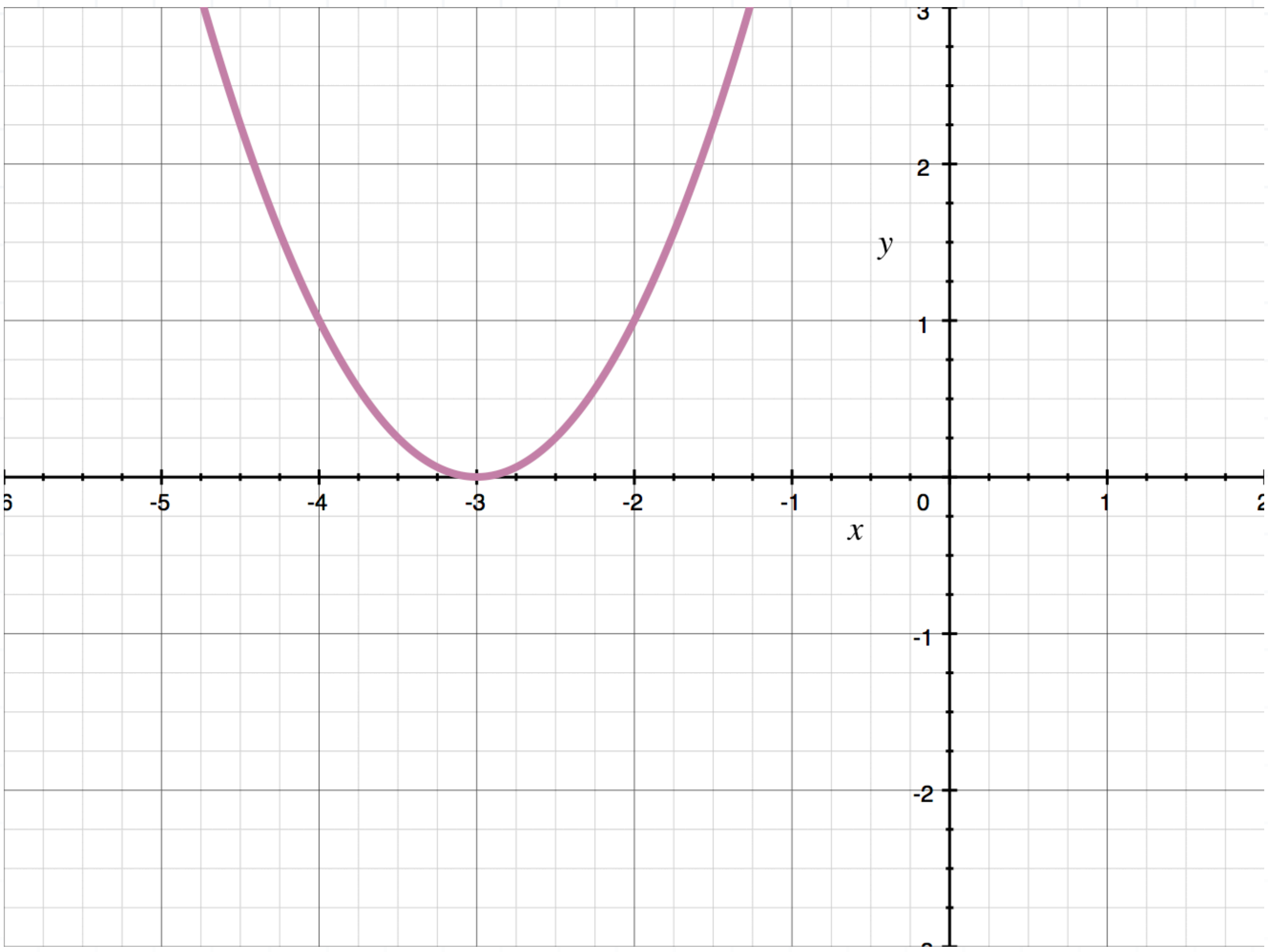
■ 1. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.



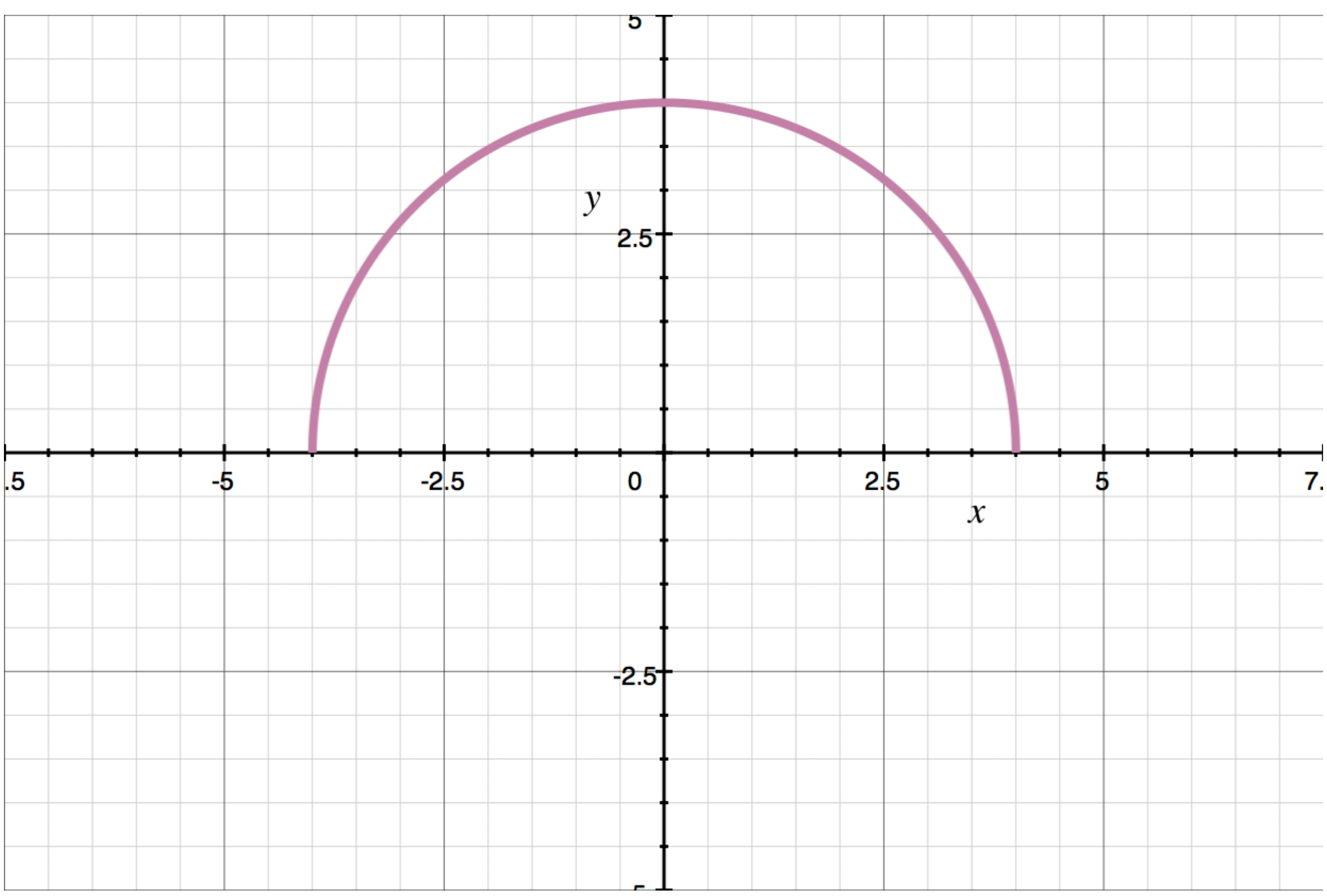
■ 2. Sketch a possible graph of $g'(x)$ given the graph below of $g(x)$.



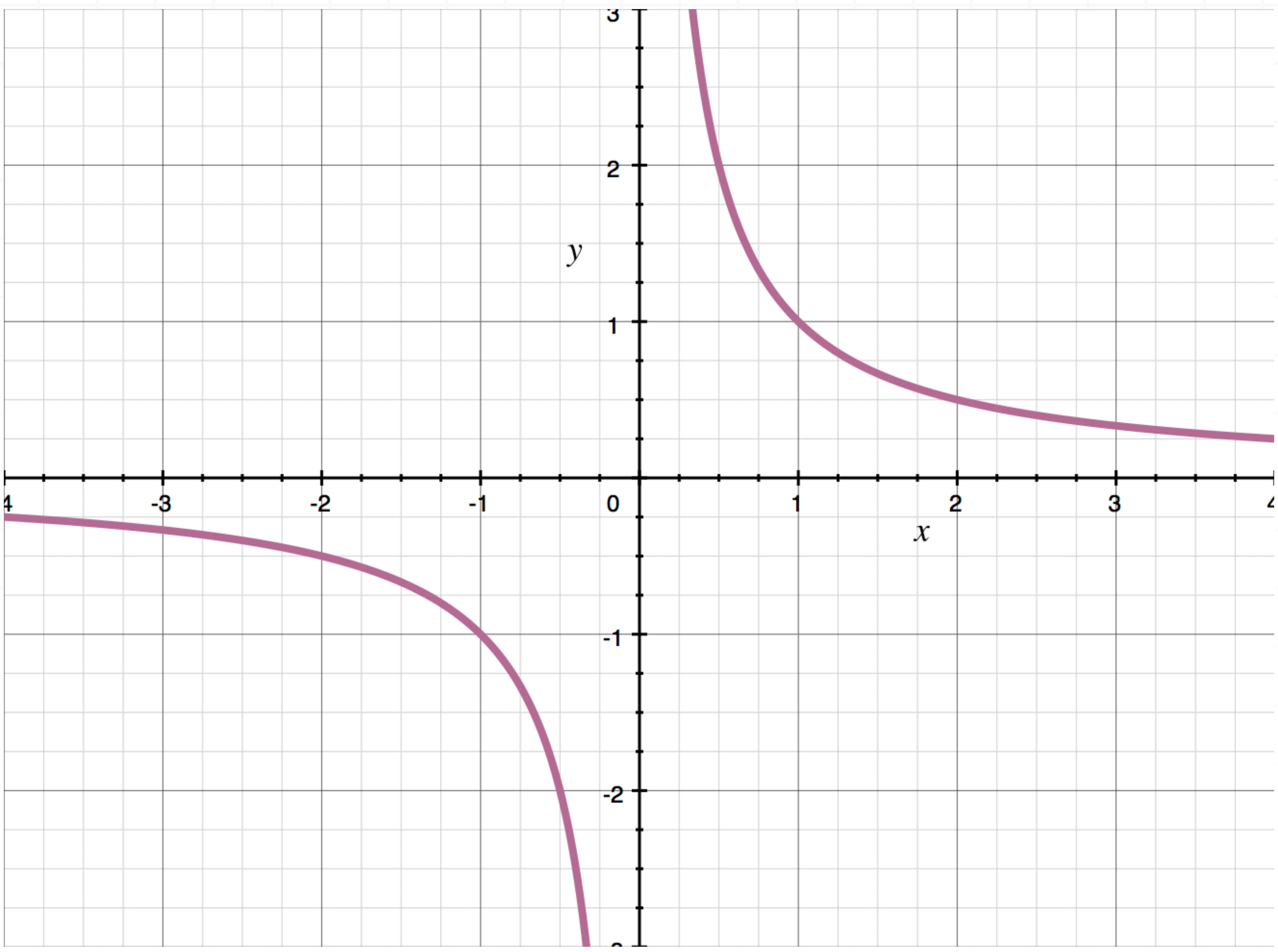
■ 3. Sketch a possible graph of $h(x)$ given the graph below of $h'(x)$.



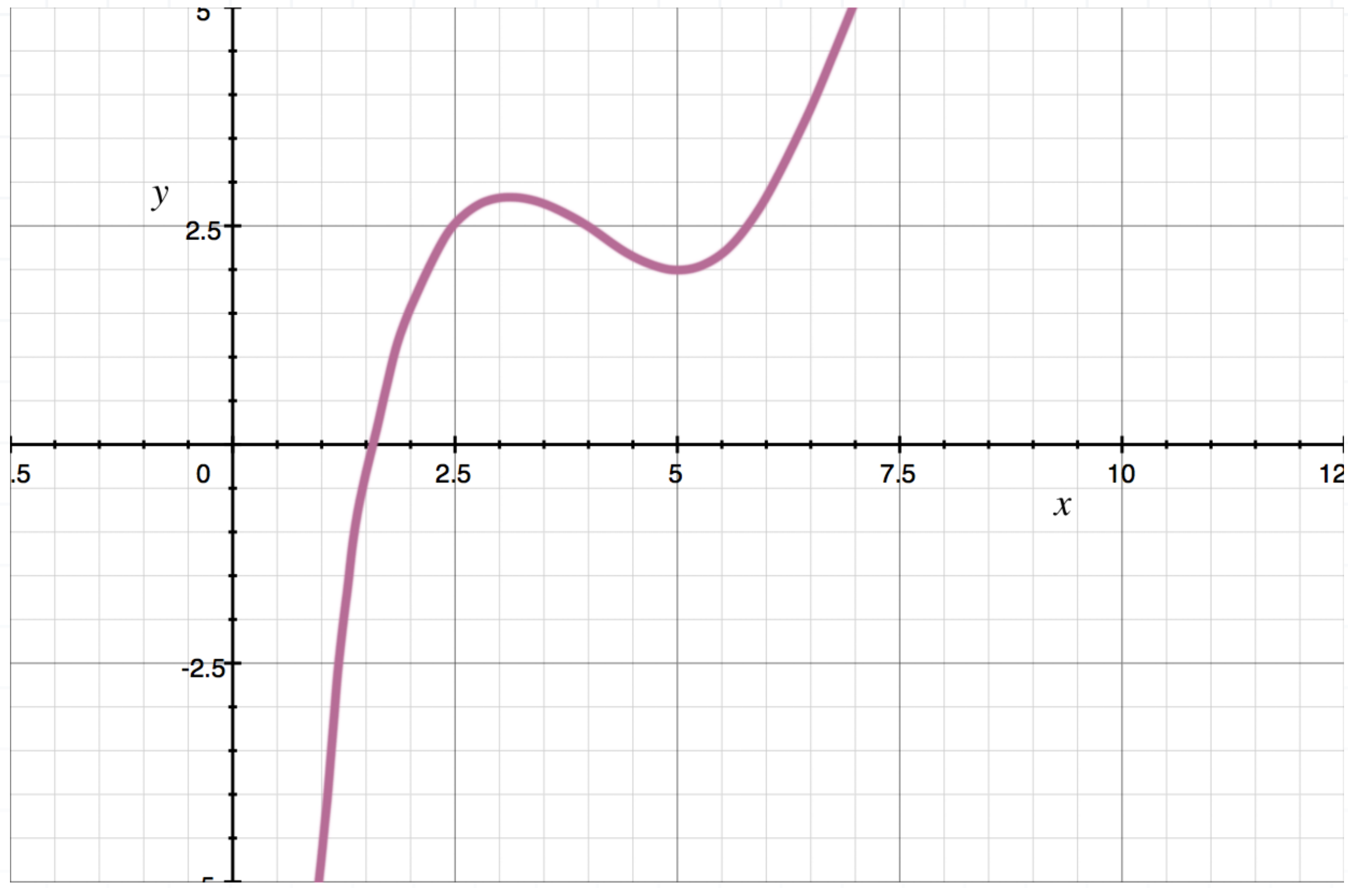
■ 4. Sketch a possible graph of $f'(x)$ given the graph below of $f(x)$.



■ 5. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.



■ 6. Sketch a possible graph of $g'(x)$ and $g''(x)$ given the graph below of $g(x)$.



y
x
5
2.5
-2.5
.5
0
2.5
5
7.5
10
12

LINEAR APPROXIMATION

■ 1. Find the linear approximation of $f(x) = x^3 - 4x^2 + 2x - 3$ at $x = 3$ and use it to approximate $f(3.02)$.

■ 2. Find the linear approximation of $g(x) = \sqrt{8x - 15}$ at $x = 8$ and use it to approximate $f(8.05)$.

■ 3. Find the linear approximation of $h(x) = 2e^{x-4} + 6$ at $x = 5$ and use it to approximate $h(5.1)$.

■ 4. Find the linear approximation of $f(x) = \ln(2x - 7)$ at $x = 4$ and use it to approximate $f(3.8)$.

■ 5. Use linear approximation to estimate $f(3.1)$.

$$f(x) = \sin(3x)$$

■ 6. Use linear approximation to estimate $f(6.1)$.

$$f(x) = e^{\cos x}$$

ESTIMATING A ROOT

■ 1. Use linear approximation to estimate $\sqrt[5]{34}$.

■ 2. Use linear approximation to estimate $\sqrt[8]{260}$.

■ 3. Use linear approximation to estimate $\sqrt[4]{85}$.

■ 4. Use linear approximation to estimate $\sqrt[4]{615}$.

■ 5. Use linear approximation to estimate $\sqrt{95}$.

■ 6. Use linear approximation to estimate $\sqrt[3]{700}$.

ABSOLUTE, RELATIVE, AND PERCENTAGE ERROR

■ 1. Use a linear approximation to estimate the value of $e^{0.002}$, then find the absolute error of the estimate.

■ 2. Use linear approximation to estimate $f(2.15)$, then find the relative error of the estimate.

$$f(x) = 4xe^{3x-6}$$

■ 3. Use linear approximation to estimate $f(1.2)$, then find the percentage error of the estimate.

$$f(x) = \sqrt[3]{x+1}$$

■ 4. Use a linear approximation to estimate the value of $\sqrt[3]{30}$, then find the relative error of the estimate.

■ 5. Find the absolute, relative, and percentage error of the approximation 2.7 to the value of e.

6. Use a linear approximation to estimate the value of $\sin(93^\circ)$, then find the absolute error of the estimate.

RELATED RATES

1. A boy is standing 15 feet from the base of a 100 feet cliff. As a boulder falls from the top of the cliff, the boy begins running away at 8 ft/s. At what rate is the distance between the boy and the boulder changing after 2 seconds?

The height of the falling boulder is modeled by the position function $s = -16t^2 + v_0t + s_0$, where s_0 is the initial height and v_0 is the initial velocity of the boulder.

2. Water is flowing out of a cone-shaped tank at a rate of 6 cubic inches per second. If the cone has a height of 5 inches and a base radius of 4 inches, how fast is the water level falling when the water is 3 inches deep?

3. A ladder 25 feet long leans against a vertical wall of a building. If the bottom of the ladder is pulled away horizontally from the building at 3 feet per second, how fast is the angle formed by the ladder and the horizontal ground decreasing when the bottom of the ladder is 7 feet from the base of the wall?

■ 4. The radius of a spherical balloon is increasing at a rate of 4.5 ft/hr. At what rate are the sphere's surface area and volume increasing when the surface area is 36π ft^2?

■ 5. A price p and demand q for a product are related by $q^2 - 2qp + 30p^2 = 10{,}125$. If the price is increasing at a rate of 2.5 dollars per month when the price is 15 dollars, find the rate of change of the demand.

■ 6. A trough of water 15 meters long, 8 meters wide, and 10 meters high has ends shaped like isosceles triangles. If water is being pumped in at a constant rate of 6 m^3/s, how fast are the height and width of the water changing when the water has a height of 250 cm?

APPLIED OPTIMIZATION

■ 1. A boater finds herself 2 miles from the nearest point to a straight shoreline, which is 10 miles down the shore from where she parked her car. She plans to row to shore and then walk to her car. If she can walk 4 miles per hour but only row 3 miles per hour, toward what point on the shore should she row in order to reach her car in in the least amount of time?

■ 2. Mr. Quizna wants to build in a completely fenced-in rectangular garden. The fence will be built so that one side is adjacent to his neighbor's property. The neighbor agrees to pay for half of that part of the fence because it borders his property. The garden will contain 432 square meters. What dimensions should Mr. Quizna select for his garden in order to minimize his cost?

■ 3. A company is designing shipping crates and wants the volume of each crate to be 6 cubic feet, and each crate's base to be a square between 1.5 feet and 2.0 feet per side. The material for the bottom of the crate costs $\$5$ per square foot, the sides $\$3$ per square foot, and the top $\$1$ per square foot. What dimensions will minimize the cost of the shipping crates?

■ 4. We want to construct a cylindrical can with a bottom and no top, that has a volume of 50 cm^3. Find the dimensions of the can that minimize its surface area.

■ 5. We're building a rectangular window with a a semicircular top. If we have 16 meters of framing material, what dimensions should we use in order to maximize the size of the window in order to let in the most light?

■ 6. Determine the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 3 cm.

MEAN VALUE THEOREM

■ 1. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[1,5]$.

$$f(x) = x^3 - 9x^2 + 24x - 18$$

■ 2. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[1,4]$.

$$g(x) = \frac{x^2 - 9}{3x}$$

■ 3. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[0,5]$.

$$h(x) = -\sqrt{25 - 5x}$$

■ 4. If we know that $g(x)$ is continuous and differentiable on $[2,7]$, $g(2) = -5$ and $g'(x) \le 15$, find the largest possible value for $g(7)$.

■ 5. If we know that $f(x)$ is continuous and differentiable on $[-4,3]$, $f(3) = 12$ and $f'(x) \le 4$, find the smallest possible value for $f(-4)$.

6. When a cake is removed from an oven and placed in an environment with an ambient temperature of 20° C, its core temperature is 180° C. Two hours later, the core temperature has fallen to 30° C. Explain why there must exist a time in the interval when the temperature is decreasing at a rate of 75° C per hour.

ROLLE'S THEOREM

■ 1. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-1,2]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$f(x) = x^3 - 2x^2 - x - 3$$

■ 2. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-3,5]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$g(x) = \frac{x^2 - 2x - 15}{6 - x}$$

■ 3. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-\pi/2,\pi/2]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$h(x) = \sin(2x)$$

■ 4. Determine whether Rolle's Theorem can be applied to $f(x) = \sqrt{4 - x^2}$ on the interval $[-2,2]$. If Rolle's Theorem applies, find the value(s) of c in the interval such that $f'(c) = 0$.

5. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[3,5]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$f(x) = |x - 2|$$

6. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-1,1]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$f(x) = \ln(9 - x^2)$$

NEWTON'S METHOD

■ 1. Use four iterations of Newton's Method to approximate the root of $g(x) = x^3 - 12$ in the interval $[1,3]$ to the nearest three decimal places.

■ 2. Use four iterations of Newton's Method to approximate the root of $f(x) = x^4 - 14$ in the interval $[-2, -1]$ to the nearest four decimal places.

■ 3. Use four iterations of Newton's Method to approximate the root of $h(x) = 3e^{x-3} - 4 + \sin x$ in the interval $[2,4]$ to the nearest four decimal places.

■ 4. Use four iterations of Newton's Method to approximate $\sqrt[65]{100}$ to four decimal places.

■ 5. Use Newton's Method to approximate to three decimal places the root of the function in the interval $[3,7]$.

$$5x^2 + 3 = e^x$$

■ 6. Use Newton's Method to find an approximation of the root of the function to four decimal places.

$$2 \ln x = \cos x$$

L'HOSPITAL'S RULE

1. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to 0}\frac{2\sqrt{x+4}-4-\frac{1}{2}x}{x^2}$$

2. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to \frac{\pi}{2}}\frac{\sec x}{3+\tan x}$$

3. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to \infty}\frac{\ln x}{4\sqrt{x}}$$

4. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to \infty}\frac{e^x}{x^2}$$

5. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to 0^+} (\cos x)^{\cot x}$$

6. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to \infty} (e^x + 4x)^{\frac{4}{x}}$$

POSITION, VELOCITY, AND ACCELERATION

■ 1. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 2$ of the position function.

$$s(t) = -\frac{t^3}{3} + t^2 + 3t - 1$$

■ 2.The position of a particle which moves along the x-axis is given by $s(t) = \cos t + \sin t$. What is the acceleration of the particle at the point where the velocity is first equal to zero?

■ 3. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 4$ of the position function.

$$s(t) = \frac{t^2}{2t + 4}$$

■ 4. Let $s(t) = 2t^3 - 12t^2 + 18t + 2$ be the position of a particle. What is the velocity when acceleration is zero? What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

■ 5. The position of a particle moving along a line is given. For what values of t is the speed of the particle decreasing?

$$s(t) = \frac{4}{3}t^3 - 12t^2 + 32t - 12 \text{ for } t \geq 0$$

■ 6. A particle moves along the x-axis with its position at time t given by $s(t) = a(t + a)(t - b)$, where a and b are constants and $a \neq b$. Find the values of t when the particle is at rest.

BALL THROWN UP FROM THE GROUND

■ 1. A ball is thrown straight upward from the ground with an initial velocity of $v_0 = 86$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

■ 2. A ball is thrown straight upward from the top of a building, which is 56 feet above the ground, with an initial velocity of $v_0 = 48$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

■ 3. A ball is thrown straight upward from a bridge, which is 24 meters above the water, with an initial velocity of $v_0 = 20$ m/sec. Assuming constant gravity, find the maximum height, in meters, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in m/sec, when it hits the water below.

■ 4. A boy needs to jump 2.8 ft in the air in order to dunk a basketball. The height that the boy's feet are above the ground is given by the function $h(t) = -16t^2 + 10t$. What is the maximum height the boy's feet will ever be above the ground, and will he be able to dunk the basketball?

5. A diver jumps up from a platform and then falls down into a pool. His height as a function of time can be modeled by $h(t) = -16t^2 + 12t + 60$, where t is the time in seconds and h is the height in feet. How long did it take for the diver to reach his maximum height? What was the highest point that he reached? In how many seconds does he hit the water?

6. An amateur rocketry club is holding a competition. There is cloud cover at 890 ft. If they launch a rocket with an initial velocity of 365 ft/s, determine the amount of time that the rocket is out of site in the cloud cover.

COIN DROPPED FROM THE ROOF

■ 1. A rock is dropped from the top of an 800 foot tall cliff, with an initial velocity of $v_0 = 0$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

■ 2. A rock is tossed from the top of a 300 foot tall cliff, with an initial velocity of $v_0 = 15$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

■ 3. A coin is tossed from the top of a 36 meter tall building, with an initial velocity of $v_0 = 6$ m/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

■ 4. A raindrop falls from the sky and takes 25 seconds to reach the ground. Assuming constant gravity, what is the raindrop's velocity at impact? How far did it fall? What is its acceleration when $t = 5$ seconds?

■ 5. You drop a rock into the Grand Canyon and it takes 7.55 seconds to hit the ground. Calculate the velocity of the rock at impact in m/s and then find the distance the rock fell in meters.

■ 6. A coin is dropped into a very deep wishing well. It hits the water 4.5 s later. How far is it from the top of the well to the water at the bottom? At what velocity does the coin hit the water? How far had the coin fallen when it reached -20m/s?

MARGINAL COST, REVENUE, AND PROFIT

■ 1. A company manufactures and sells basketballs for $\$9.50$ each. The company has a fixed cost of $\$395$ per week and a variable cost of $\$2.75$ per basketball. The company can make up to 300 basketballs per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 150 basketballs.

■ 2. A company manufactures and sells high end folding tables for $\$250$ each. The company has a fixed cost of $\$3{,}000$ per week and variable costs of $85x + 150\sqrt{x}$, where x is the number of tables manufactured. The company can make up to 200 tables per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 64 tables.

■ 3. A company manufactures and sells electric food mixers for $\$150$ each. The company has a fixed cost of $\$7{,}800$ per week and variable costs of $24x + 0.04x^2$, where x is the number of mixers manufactured. The company can make up to 200 mixers per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 75 mixers.

■ 4. A coffee machine manufacturer determines that the demand function for their coffee machines is given by p, while the cost of producing x coffee machines is given by $C(x) = 25x + 10\sqrt{x^3} + 1{,}250$. What is the marginal cost, marginal revenue, and marginal profit at $x = 25$?

$$p = \frac{750}{\sqrt{x^3}}$$

■ 5. For the given cost and demand functions, find the number of units the company needs to produce in order to maximize profit.

$$C(x) = 15x + 300$$

$$p = 250 - 2x$$

■ 6. A company manufactures and sells kids' toys. The total cost of producing x toys is $C(x) = -0.3x^2 + 25x + 975$, and demand is given by $p(x) = 12 + 3x$. Calculate the marginal profit from selling the 10th toy.

HALF LIFE

■ 1. Find the half-life of Tritium if its decay constant is 0.0562.

■ 2. Find the half-life of Cobalt-60 if its decay constant is 0.1315.

■ 3. Find the half-life of Berkelium-97 if its decay constant is 0.000503.

■ 4. Radium-224 has a half life of 3.66 days. If 3.25 g of Radium-224 remains after 9 days, what was the original mass of Radium-224?

■ 5. The half-life of Potassium-40 is 1.25 billion years. A scientist analyzes a rock that contains only 9.5% of the Potassium-40 it contained originally when the rock was formed. How old is the rock?

■ 6. 25 grams of a substance decayed to 13.25 grams in 13 seconds. Determine the half-life of a substance.

NEWTON'S LAW OF COOLING

■ 1. A cup of coffee is 195° F when it's brewed. Room temperature is 74° F. If the coffee is 180° F after 5 minutes, to the nearest degree, how hot is the coffee after 25 minutes?

■ 2. A boiled egg that's 99° C is placed in a pan of water that's 24° C. If the egg is 62° C after 5 minutes, how much longer, to the nearest minute, will it take the egg to reach 32° C.

■ 3. Suppose a cup of soup cooled from 200° F to 161° F in 10 minutes in a room whose temperature is 68° F. How much longer will it take for the soup to cool to 105° F?

■ 4. A thermometer is measuring 18° C indoors. The thermometer is moved outdoors where the temperature is -5° C, and after 2 minutes the thermometer reads 11° C. How many more minutes will it take for the thermometer to read 0° C?

■ 5. A cake baking inside an oven currently has a temperature of 220° C. Find the decay constant if the cake's temperature is 168° C 5 minutes after it's removed from the oven, given that the room temperature is 23° C.

■ 6. Using the decay constant we calculated in the previous problem, determine the number of minutes that will pass before the cake's temperature will be 50° C.

SALES DECLINE

■ 1. Suppose a pizza company stops a special sale for their three-topping pizza. They will resume the sale if sales drop to 70% of the current sales level. If sales decline to 90% during the first week, when should the company expect to start the special sale again?

■ 2. Suppose a donut store experiments with raising the price of a dozen donuts to see if sales are affected. They'll resume the sale if sales drop to 80% of the current sales level. If sales decline to 90% after two weeks, when should the store change back to the original price?

■ 3. Suppose a flower shop decides to stop ordering roses in the winter time to see if sales are affected. They will resume the sale if sales drop to 90% of the current sales level. If sales decline to 96% after three weeks, when should the shop begin ordering roses again?

■ 4. Mark has been selling lemonade for the last 5 years. Five years ago, he sold $3{,}850$ glasses of lemonade, but this year he's only sold $2{,}985$. Assuming that sales have declined exponentially, what's been the annual rate of decline?

■ 5. A bakery sold $5{,}465$ croissants 3 years ago. If the sales declined at a rate of $1.5\,\%$ per month, how many croissants were sold last year?

■ 6. Suppose a convenience store decides to stop their sale on ice cream in the summer time to see if sales are affected. They will resume the sale if sales drop to $87\,\%$ of their current level. If sales fall to $87\,\%$ of their current level after two weeks, what was the monthly rate of decline?

COMPOUNDING INTEREST

■ 1. Suppose you borrow $\$15{,}000$ with a single payment loan, payable in 2 years, with interest growing exponentially at $1.82\,\%$ per month, compounded continuously. How much will it cost to pay off the loan after 2 years?

■ 2. Your parents deposit $\$5{,}000$ into a college savings account, with interest growing exponentially at $0.875\,\%$ per quarter, compounded continuously. How much will be in the account after 18 years?

■ 3. Suppose you win $\$50{,}000$ in a contest and you decide to save it for your retirement. You deposit it into an annuity account that pays $2.4\,\%$ semi-annually, compounded continuously. How much will the account contain after 25 years, when you plan to retire?

■ 4. At a $7.5\,\%$ yearly interest rate compounded semi-annually, how much money would we have to deposit now to have $\$15{,}500$ after 10 years?

■ 5. How many years would it take for $\$25{,}000$ to turn into $\$50{,}000$, at a yearly interest rate of $4.75\,\%$, compounded annually?

6. At a yearly interest rate of 6.5% compounded quarterly, how long would it take to triple an initial investment of $\$10{,}000$?

W W W . K R I S T A K I N G M A T H . C O M