Power rule for negative powers

We also use power rule to differentiate power functions with negative powers. In order to use it, we need to remember a key exponent rule.

$$x^{-a} = \frac{1}{x^a}$$

This rule tells us that, when we have a power function with a negative exponent, x^{-a}, we can turn that exponent from negative to positive by moving the power function to the denominator.

Similarly, if we find ourselves with a negative exponent already in the denominator,

$$\frac{1}{x^{-a}} = x^a$$

we can turn that exponent from negative to positive by moving the power function to the numerator.

So regardless of where the negative exponent exists, we can change it to a positive exponent by moving it to the opposite location (from the numerator to the denominator, or from the denominator to the numerator).

And regardless of whether the exponent is positive or negative, we can apply power rule in the same way we've used it up to this point.

Let's do an example where we use power rule to differentiate a power function with negative exponents.

Example

Use power rule to take the derivative.

$$y = x^{-3} + 2x^{-6}$$

We'll apply power rule, one term at a time, in the same way we applied it previously. In other words, we'll bring the exponent down to multiply it by the coefficient, and we'll subtract 1 from the exponent.

$$y' = -3x^{-3-1} + 2(-6)x^{-6-1}$$

$$y' = -3x^{-4} - 12x^{-7}$$

We've found the derivative, so we can stop here. Or, if we want to make the exponents positive, we can move those terms into their denominators.

$$y' = -\frac{3}{x^4} - \frac{12}{x^7}$$

Now let's work through an example in which we need to move a power function from the denominator into the numerator, before we can apply the power rule.

Example

Find the derivative of the function.

$$y = \frac{1}{x^2} - \frac{3}{x^{-3}}$$

In order to be able to apply power rule, we want to move the power functions into the numerator. When we do, the signs of the exponents will flip.

$$y = 1(x^{-2}) - 3(x^3)$$

$$y = x^{-2} - 3x^3$$

Now that we've moved the power functions into the numerator, we can apply power rule in order to differentiate y.

$$y' = -2x^{-2-1} - 3(3)x^{3-1}$$

$$y' = -2x^{-3} - 9x^2$$

We can leave the derivative this way, or we can rewrite it so that all the exponents are positive.

$$y' = -\frac{2}{x^3} - 9x^2$$