Normal lines

We know how to find the equation of the tangent line, but in this lesson we want to turn toward the equation of the normal line.

Equation of the normal line

For every tangent line, we can find a corresponding normal line, because the **normal line** to a function at a particular point is the line that's perpendicular to the tangent line to the function at that same point.

So if the slope of the tangent line is m, then the slope of the normal line is the negative reciprocal of m, or $-1/m$.

We can find the equation of the normal line by following these steps:

1. Take the derivative of the original function, and evaluate it at the given point. This is the slope of the tangent line, which we'll call m.

2. Find the negative reciprocal of m, which will be $-1/m$. This is the slope of the normal line, which we'll call n. So $n = -1/m$.

3. Plug n and the given point into the point-slope formula for the equation of the line, $(y - y_1) = n(x - x_1)$.

4. Simplify the normal line equation by solving for y.

Let's do an example where we walk through these steps in order to find the equation of the normal line.

Example

Find the equation of the normal line to the function $f(x)$ at $(1,9)$.

$$f(x) = 6x^2 + 3$$

Let's follow the steps we just outlined. First, we'll take the derivative of the function, and then evaluate it at $(1,9)$.

$$f'(x) = 12x$$

$$f'(1) = 12(1)$$

$$f'(1) = 12$$

This is the slope of the tangent line at $(1,9)$. Since $m = 12$, we'll take the negative reciprocal to find n, the slope of the normal line.

$$n = -\frac{1}{12}$$

We'll plug $n = -1/12$ and the point $(1,9)$ into the point-slope formula for the equation of the line. Once we simplify, we'll have the equation of the normal line to the function at $(1,9)$.

$$y - y_1 = n(x - x_1)$$

$$y - 9 = -\frac{1}{12}(x - 1)$$

$$12y - 108 = -(x - 1)$$

$$12y - 108 = -x + 1$$

$$12y = -x + 109$$

$$y = -\frac{1}{12}x + \frac{109}{12}$$