Limits of combinations

Up to this point, we've taken the limit of just one function at a time. But we can find the limit of combinations of different functions.

For instance, assume that $f(x) = x + 1$ and that $g(x) = x^2 - 4$. We could find the limit for the combination like

$$\lim_{x \to 1}[f(x) + g(x)]$$

Here, we're finding the limit of the sum of the functions $f(x)$ and $g(x)$. The sum $f(x) + g(x)$ is the combination, so we're finding the limit of the combination.

Properties of limits

But the sum isn't the only kind of combination we can take. If the limits of two functions at the same value both exist,

$$\lim_{x \to a} f(x) \text{ and } \lim_{x \to a} g(x)$$

then we can define five kinds of combinations. These are the **properties of limits**, and they apply to any number of functions.

$$\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x) \qquad \text{for any real number } c$$

$$\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \qquad \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n \qquad \text{for any real number } n$$

Two ways to evaluate

There are two ways to find the limit of the combination.

1. Find the combination of the functions, then take the limit of the combination.
2. Take the limit of each function, then find the combination of the limits.

We'll get to the same result, regardless of which method we use. Let's work through the example given above, and use both methods to find the limit of the combination.

Example

Evaluate the limit, given $f(x) = x + 1$ and $g(x) = x^2 - 4$.

$$\lim_{x\to 1}[f(x) + g(x)]$$

Using the first method, we'll find the combination of the functions.

$$f(x) + g(x)$$

$$x + 1 + x^2 - 4$$

$$x^2 + x - 3$$

Now we'll take the limit of the combination.

$$\lim_{x \to 1}[f(x) + g(x)]$$

$$\lim_{x \to 1}(x^2 + x - 3)$$

$$1^2 + 1 - 3$$

$$-1$$

Let's use the second method to double-check the result. We'll take the limit of each function individually.

$$\lim_{x \to 1} f(x) = \lim_{x \to 1}(x + 1) = 1 + 1 = 2$$

$$\lim_{x \to 1} g(x) = \lim_{x \to 1}(x^2 - 4) = 1^2 - 4 = -3$$

Now we'll take the combination of the limits. Because we're using the combination $f(x) + g(x)$, we'll sum the limits.

$$\lim_{x \to 1} f(x) + \lim_{x \to 1} g(x)$$

$$2 + (-3)$$

$$2 - 3$$

$$-1$$

Using both methods, we got a result of -1.
