Marginal cost, revenue, and profit

We've been looking at physical applications of derivatives, but there are also economics applications.

In this lesson, we'll look at marginal cost, revenue, and profit. But before we jump into these marginal values, let's look at cost, revenue, and profit in general.

Cost, revenue, and profit

If a business wants to calculate the revenue generated, the cost incurred, and the profit gained by producing x units of a product, it can use the specific formulas.

Revenue	$R(x) = xp$
Cost	$C(x) = F + V(x)$
Profit	$P(x) = R(x) - C(x)$

In these formulas, p is the selling price of an individual unit, so **revenue** is given by the product of selling price and the number of units sold. F is fixed cost and $V(x)$ is variable cost, so **cost** is the sum of the fixed and variable costs. The **profit** is then the difference between the revenue and the cost.

In other words, if a company is making 100 units of their product, the revenue function will tell them how much revenue will be generated by the

100 units, the cost function will tell them how much it'll cost to produce the 100 units, and the profit function will find the total profit gained from producing and then selling the 100 units.

The marginal functions

Of course, every company wants to maximize its profits, but increasing the number of units they produce doesn't always translate to higher profits.

For instance, if an airplane company is making as many planes each month as their current manufacturing space allows, they might need to build a second factory in order to make even one more plane per month. But building a second factory, to make only one more plane, won't necessarily be profitable. On the other hand, if they build a second factory in order to produce 100 more planes each month, that might be a profitable decision.

The marginal revenue, cost, and profit functions are what the company can use to determine whether or not they should increase production. These marginal functions are the derivatives of their associated functions. So the marginal revenue function is the derivative of the revenue function; the marginal cost function is the derivative of the cost function; and the marginal profit function is the derivative of the profit function.

The **marginal revenue** function models the revenue generated by selling one more unit, the **marginal cost** function models the cost of making one more unit, and the **marginal profit** function models the profit made by selling one more unit.

This understanding of what the marginal functions model should make sense to us. Because these marginal functions are derivative functions, they model the slope of the original function, or the change per unit. So if we, for instance, find a marginal cost function as the derivative of the cost function, the marginal cost function should be modeling the change, or slope, of the cost function. And that slope is really just how much the original cost function is increasing or decreasing, per unit.

Let's work through an example where we find all three marginal functions.

Example

Calculate a smartphone manufacturer's marginal cost, marginal revenue, and marginal profit when they're producing 75 smartphones, if the selling price is $p = 6x$ and the cost of making the smartphones is modeled by $C(x)$.

$$C(x) = 6x^2 + 34x + 2{,}500$$

To calculate marginal cost at 75 units, we take the derivative of the cost function and then evaluate the derivative at $x = 75$.

$$C'(x) = 12x + 34$$

$$C'(x) = 12(75) + 34$$

$$C'(x) = 934$$

The marginal cost at $x = 75$ is $\$934$, which means the additional cost associated with producing the 76th unit is $\$934$.

To calculate marginal revenue at 75 units, we need to find a revenue function, take its derivative, and then evaluate the derivative at $x = 75$.

The revenue equation is $R(x) = xp$ where p is the selling price, $p = 6x$.

$$R(x) = x(6x)$$

$$R(x) = 6x^2$$

Taking the derivative of revenue to get marginal revenue, and then evaluating at $x = 75$, we get

$$R'(x) = 12x$$

$$R'(x) = 12(75)$$

$$R'(x) = 900$$

The marginal revenue at $x = 75$ is $\$900$, which means the additional revenue associated with selling the 76th unit is $\$900$.

Finally, to solve for marginal profit we need to find a profit function, take its derivative, and then evaluate the derivative at $x = 75$.

The profit equation is $P(x) = R(x) - C(x)$, where R is the revenue function we found earlier and C is the cost function we were given.

$$P(x) = R(x) - C(x)$$

$$P(x) = (6x^2) - (6x^2 + 34x + 2{,}500)$$

$$P(x) = -34x - 2{,}500$$

Taking the derivative of profit to get marginal profit, and then evaluating at $x = 75$ gives

$$P'(x) = -34$$

$$P'(75) = -34$$

The marginal profit at $x = 75$ is $-\$34$, which means that the smartphone company's profit declines by $\$34$ when they produce and sell the 76th smartphone.

We can also find marginal profit just by subtracting marginal cost from marginal revenue.

$$P'(x) = R'(x) - C'(x)$$

$$P'(75) = 900 - 934$$

$$P'(75) = -34$$

Either way, we find $P'(75) = -34$ and we can say that, if the manufacturer's goal is to maximize profit, they should not increase production.
