# Sales decline

If sales of a product are consistently declining at an exponential rate, we can model that decline with the formula

$$F = Pe^{-rt}$$

where $P$ is the number of items being sold, $r$ is the rate of decline, and $F$ is the number of items being sold after sales have continued to decline for some specified amount of time $t$.

It's important to remember that the rate $r$ and the time $t$ need to have complementary units. For instance, if $r$ is given as some rate per *month*, then time $t$ needs to be measured in *months*. If $r$ is in years, then $t$ should also be in years.

When the units of rate and time don't match, we'll need to convert one to match the other. For example, if $r$ was given as a rate per year, and $t$ was given in months, we could either divide $t$ by $12$ to convert it into years to match the rate, or we could divide $r$ by $12$ to convert it into months to match the time.

Let's work through an example where we solve for the rate of decline.

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**Example**

Sales of Isaac Newton bobbleheads have decreased over the last $4$ years. Four years ago, $285{,}674$ bobbleheads were sold, but over the last year, sales were only $97{,}546$ bobbleheads. Assuming that sales have declined at a steady exponential rate, what is the rate of decline?

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We know that fourth year sales are $F = 97{,}546$, and that first year sales were $P = 285{,}674$. Time is $t = 4$ and, we need to calculate $r$, so we'll substitute what we know into the sales decline formula.

$$F = Pe^{-rt}$$

$$97{,}546 = 285{,}674e^{-r(4)}$$

Solve for $r$.

$$\frac{97{,}546}{285{,}674} = e^{-4r}$$

$$0.3415 = e^{-4r}$$

$$\ln 0.3415 = \ln(e^{-4r})$$

$$-1.075 = -4r$$

$$r = 0.269$$

This result tells us that sales of Isaac Newton bobbleheads have declined at an annual rate of about $r = 0.269$, or $r = 26.9\,\%$ per year.

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We'll do one more example, and in this one we'll use the exponential model we develop to predict the future sales decline of a product.

**Example**

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One year ago, a company sold $5{,}698$ disposable sandwich bags in a month. But over the last $12$ months, sales have decreased at a steady exponential rate of $15\,\%$ per month. How many disposable sandwich bags did the company sell last month, and how many are predicted to sell in $6$ months from now?

The rate of decline is $r = 0.15$, the original amount from one year ago ($12$ months ago) was $P = 5{,}698$. Therefore, we could say that last month is at $t_{\text{last month}} = 12$, and $6$ months from now is $t_{\text{in 6 months}} = 18$.

We'll substitute into the sales decline formula to find sales for last month.

$$F = Pe^{-rt}$$

$$F_{\text{last month}} = 5{,}698e^{-0.15(12)}$$

$$F_{\text{last month}} \approx 941.87$$

In the past month, about $942$ units of disposable sandwich bags were sold. Now we'll calculate expected sales for $6$ months from now.

$$F_{\text{in 6 months}} = 5{,}698e^{-0.15(18)}$$

$$F_{\text{in 6 months}} \approx 382.94$$

Assuming the rate of decline remains the same, in $6$ months from now, the company will sell about $383$ disposable sandwich bags.

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