Price of the product

When we looked at the inflating/deflating balloon problem, we weren't given an equation as part of the question. But we knew the formula for the volume of a sphere, so we were able to use that equation to relate the variables for volume and radius.

When we have a "price of the product" problem, we're usually given the equation that relates quantity and price as part of the problem. For instance, we might be told that the quantity and price of an item are related by $q = 20e^{-2p}$.

Along with the equation, we'll be given information about the sale price, the rate of change of the sale price, the quantity (supply) of the item, or the rate of change of the supply.

If we're given the equation that relates price to quantity, then we can differentiate it implicitly, substitute for the values we know, and then solve for the value we're trying to find.

Let's do an example.

Example

The manufacturer is increasing production of an item by 14 units per week. The item currently sells for $\$2$. How fast is the price of the item changing as the manufacturer produces more units?

$$q = 1000e^{-0.5p}$$

The equation for q in terms of p gives us the relationship between the quantity (number of items being manufactured) and price (the retail price at which the item is being sold).

We need to start by differentiating the equation, taking the derivative of both q and p as functions of time t, and therefore multiplying by q' when we differentiate q, and by p' when we differentiate p.

$$1(q') = 1000e^{-0.5p}(-0.5)(1)(p')$$

$$q' = -500e^{-0.5p}p'$$

We know that production (quantity) is increasing by 14 units per week, so $q' = 14$.

$$14 = -500e^{-0.5p}p'$$

And we know that the current price of the item is $\$2$, so $p = \$2$.

$$14 = -500e^{-0.5(2)}p'$$

We were asked to solve for the rate of change of the price, which means we need to solve for p'.

$$-\frac{14}{500} = e^{-1}p'$$

$$-\frac{7}{250} = e^{-1}p'$$

$$p' = -\frac{7e}{250}$$

$$p' \approx -0.08$$

Since we were told in the problem that the manufacturer was increasing production by 14 units per *week*, this result tells us that the retail price of the item is changing by $-\$0.08$ per week (decreasing by 8 cents per week). In other words, the price is going down by 8 cents per week.
