Coin dropped from the roof

In the previous lesson, we were looking at a vertical motion pattern in which we threw an object up from the ground, or from some other height, and the object traveled upward, eventually reached a maximum height, and then fell back down to earth, eventually stopping when it hit the ground.

In this lesson, we're looking at a different vertical motion pattern. This time, we're dropping a coin, or some other object, from some height, letting it fall straight to the ground, eventually stopping when it hits the ground.

Let's work through an example of how to find different values from a position function that models this pattern of vertical motion.

Example

A watermelon is dropped from the top of a building 28 meters high. Find instantaneous velocity at $t=2$, average velocity between $t=0$ and $t=2$, and find the time when the watermelon hits the ground.

Plugging everything we know into the formula for standard projectile motion, we get

$$x(t)=-\frac{1}{2}gt^2+v_0t+y_0$$

$$x(t) = -\frac{1}{2}(9.8)t^2 + 0t + 28$$

$$x(t) = -4.9t^2 + 28$$

Find the velocity function by differentiating the position function.

$$v(t) = x'(t) = -9.8t$$

To find instantaneous velocity at $t = 2$, substitute $t = 2$ into the velocity function.

$$v(2) = -9.8(2)$$

$$v(2) = -19.6$$

This is the instantaneous velocity at $t = 2$. Find average velocity over $t = [0,2]$.

$$\Delta v(a, b) = \frac{x(b) - x(a)}{b - a}$$

$$\Delta v(0,2) = \frac{x(2) - x(0)}{2 - 0}$$

$$\Delta v(0,2) = \frac{-4.9(2)^2 + 28 - (-4.9(0)^2 + 28)}{2}$$

$$\Delta v(0,2) = \frac{-19.6 + 28 - 28}{2}$$

$$\Delta v(0,2) = \frac{-19.6}{2}$$

$$\Delta v(0,2) = -9.8$$

This is the average velocity of the watermelon between $t = 0$ and $t = 2$.

The watermelon will hit the ground when $x(t) = 0$, so we'll set the position function equal to 0 and then solve for t.

$$-4.9t^2 + 28 = 0$$

$$4.9t^2 = 28$$

$$t^2 = \frac{28}{4.9}$$

$$t \approx \sqrt{5.71}$$

$$t \approx 2.39$$

The watermelon hits the ground after $t \approx 2.39$ seconds.