Idea of the limit

The **limit** of a function is the value the function approaches at a given value of x, regardless of whether the function actually reaches that value.

For an easy example, consider the function

$$f(x) = x + 1$$

To find the value of the function $f(x)$ when $x = 5$, we plug $x = 5$ into the function, and we get

$$f(5) = 5 + 1$$

$$f(5) = 6$$

So 6 is the limit of the function at $x = 5$, because 6 is the value that the function approaches as the value of x gets closer and closer to 5.

It's strange to talk about the value that a function "approaches," but if we look at some of the other values around $x = 5$, we start to get a better idea of what we mean. For instance,

if we plug $x = 4.9999$ into $f(x)$, then $f(x) = 5.9999$, or

if we plug $x = 5.0001$ into $f(x)$, then $f(x) = 6.0001$.

We start to see that, as we get closer to $x = 5$, whether we're approaching it from the 4.9999 side or the 5.0001 side, the value of $f(x)$ gets closer and closer to 6.

x	4.9998	4.9999	5	5.0001	5.0002
f(x)	5.9998	5.9999	6	6.0001	6.0002

In this simple example, the limit of the function is 6, because that's the actual value of the function at that point; the point is defined. In limit notation, here's how that looks:

$$\lim_{x \to 5}(x+1) = 6$$

This notation tells us that "the limit of the function $x+1$, as x approaches 5, is 6." If we generalize this, we say that the limit of the function $f(x)$ as x approaches a is L.

$$\lim_{x \to a} f(x) = L$$

Let's work through another example of how to find L.

Example

Find the limit.

$$\lim_{x \to 16}(\sqrt{x}+2)$$

To find the limit, substitute the value that x approaches, $x = 16$, into the function.

$$\sqrt{16}+2$$

$$4+2$$

$$6$$

So the value of the limit is 6.

$$\lim_{x \to 16} (\sqrt{x} + 2) = 6$$