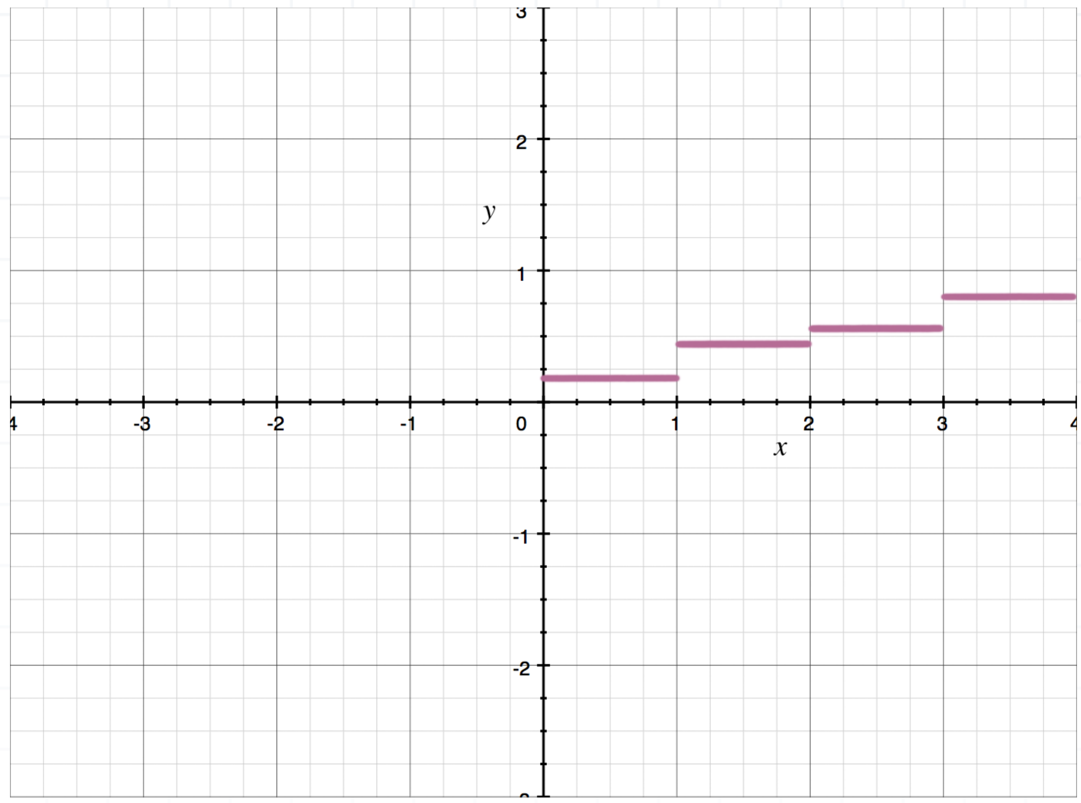
Jump discontinuities

We'll usually encounter **jump discontinuities** with piecewise-defined functions, which are functions for which different parts of the domain are defined by different expressions.

For instance, we might define the cost of postage as a function. If the cost per ounce of any package lighter than 1 pound is 20 cents per ounce, the cost of every ounce from 1 pound to anything less than 2 pounds is 40 cents per ounce, etc., then the piecewise function that defines the cost of postage might be

$$f(x) = \begin{cases} 0.2 & 0 < x < 1 \\ 0.4 & 1 \le x < 2 \\ 0.6 & 2 \le x < 3 \\ 0.8 & 3 \le x < 4 \\ 1.0 & 4 \le x \end{cases}$$

The graph of this piecewise function would be

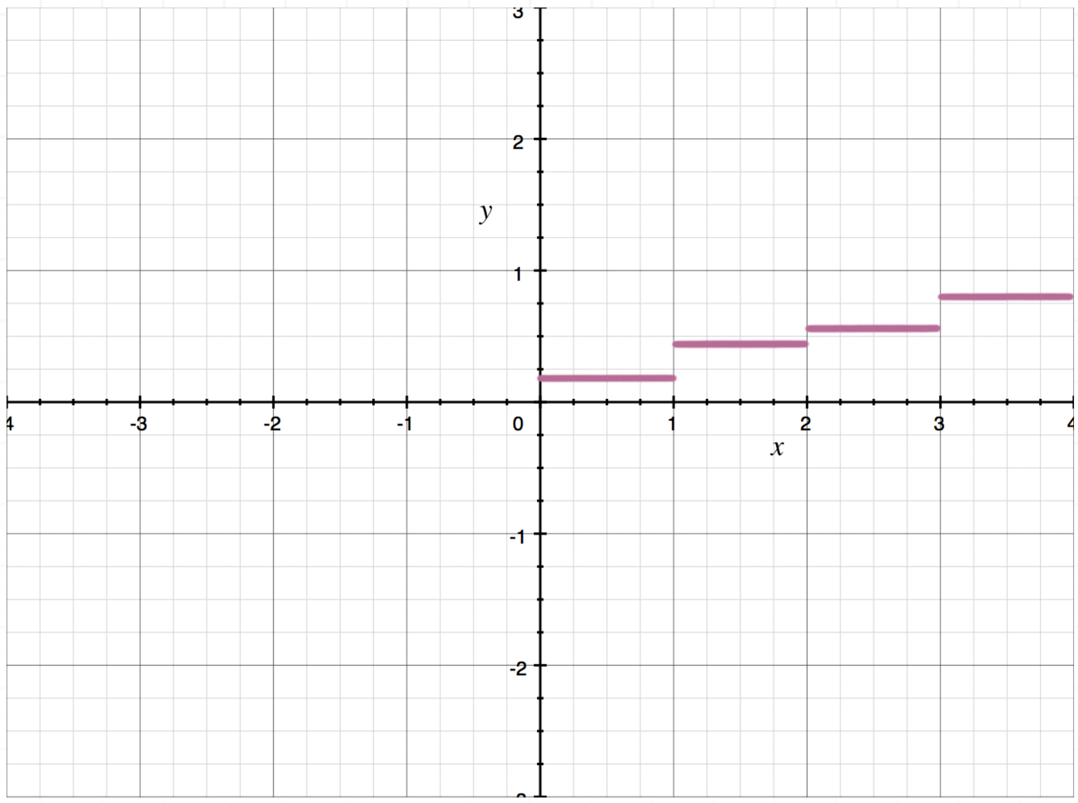


Every break in this graph is a jump discontinuity. We can remember that these are jump discontinuities if we imagine walking along on top of the first segment of the graph. In order to continue, we'd have to jump up to the second segment, then to the third, and so on.

The general limit never exists at a jump discontinuity because, while the left- and right-hand limits both exist, they're not equal to one another.

Does the limit exist at a jump discontinuity?

Keep in mind that the general limit never exists at a jump discontinuity. That's because the left-hand limit and right-hand limit both exist, but those one-sided limits aren't equal. For instance, in the graph



the left-hand limit as $x \to 1$ is 0.2, and the right-hand limit as $x \to 1$ is 0.4. Both one-sided limits exist, but they aren't equal, which means the general limit doesn't exist.