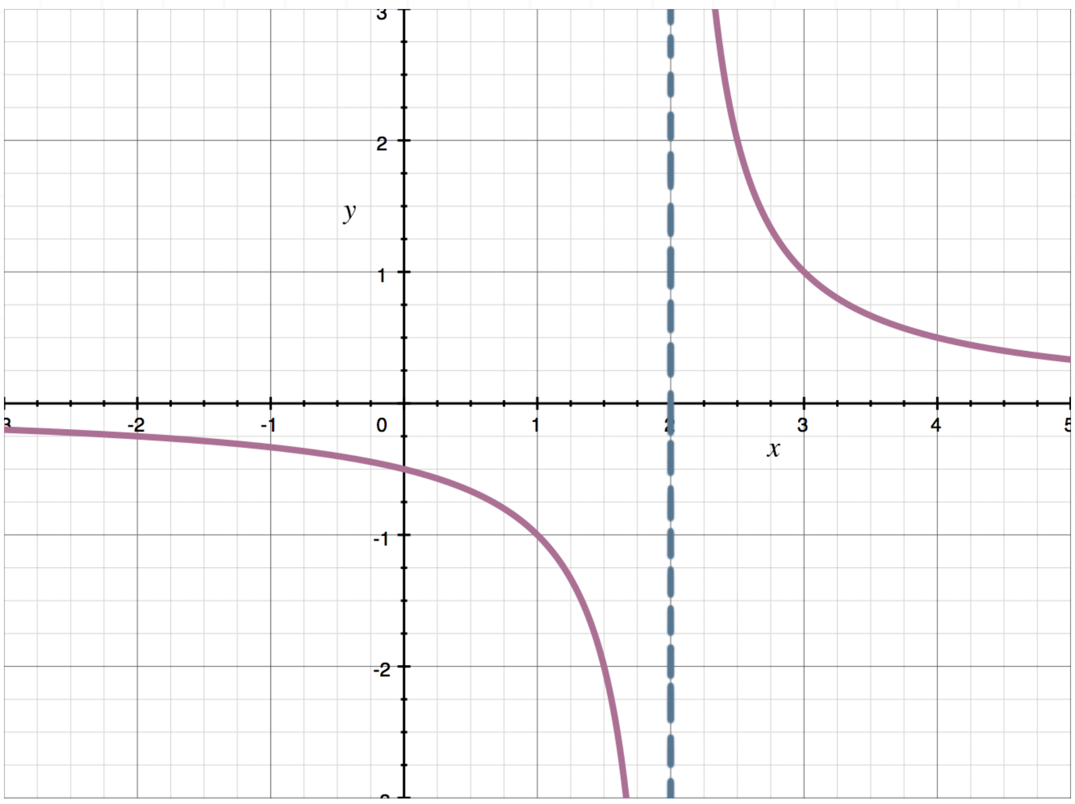
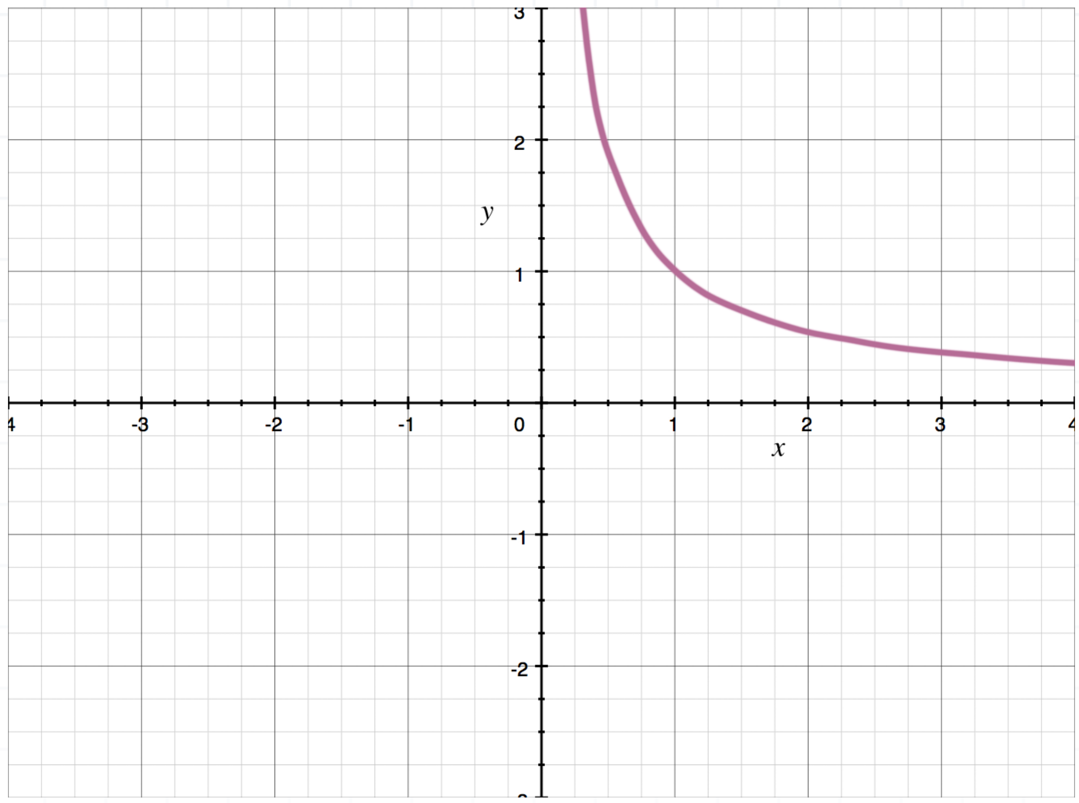
Infinite discontinuities

An **infinite discontinuity**, or essential discontinuity, is the kind of discontinuity that occurs at an asymptote. When the graph exists on both sides of a vertical asymptote, the graph has an infinite discontinuity at the asymptote.



The vertical asymptote in the graph below at $x = 0$ is not a discontinuity, because the graph doesn't exist on both sides of the asymptote, which means the asymptote doesn't break up any part of the graph.



We often find both removable (point) and nonremovable (infinite) discontinuities within rational functions. Going back to the example we used previously for point discontinuities,

$$f(x) = \frac{x-2}{x^2+x-6}$$

we factored the denominator to get

$$f(x) = \frac{x-2}{(x-2)(x+3)}$$

In this form, we can see that the denominator is 0 at both $x = 2$ and $x = -3$. Because the factor of $x - 2$ can be canceled,

$$f(x) = \frac{1}{x+3}$$

there's a point (removable) discontinuity at $x = 2$. Since the factor of $x + 3$ can't be canceled, and therefore $x = -3$ will always make the denominator 0, there's a vertical asymptote and an infinite discontinuity at $x = -3$.

Does the limit exist at an infinite discontinuity?

The general limit may or may not exist at an infinite discontinuity. If the function approaches $-\infty$ on one side and ∞ on the other side, then the general limit doesn't exist, because the left- and right-hand limits aren't equal.

The story gets a little more complicated when the left- and right-hand limits both approach $-\infty$ or both approach ∞. Technically, the function must approach a finite value for the limit to be defined, and neither $-\infty$ nor ∞ are finite values, so the general limit would never exist at a vertical asymptote.

But oftentimes, for the sake of providing more information about how the function behaves, we'll say that the value of the general limit is ∞ when both the left- and right-hand limits are ∞, or that the general limit is $-\infty$ when both the left- and right-hand limits are $-\infty$.