Big Θ notation

$$\Theta(g) := \{f \mid f \in O(g) \text{ and } g \in O(f)\}$$

Example 2.9.

$$f \in \Theta(g)$$

Notation

$\longrightarrow$ "from ... to" $\mathbb{N} \to \mathbb{N}$

$\longmapsto$ "maps to" $f : n \mapsto f(n)$

colon $:$

$$f : \mathbb{N} \to \mathbb{N} : n \mapsto f(n)$$

$$f : A \to B : n \mapsto f(n)$$

colon   from ... to   colon   maps to

Polynomial

with integer coefficients of degree $r \in \mathbb{N}$

is a function of the form

$$f : \mathbb{N} \to \mathbb{Z} : n \mapsto n^r + c_{r-1} \cdot n^{r-1} \cdots$$

$$c_1 \cdot n_1 + c_0$$

with $r > 0 \in \mathbb{N}$

coefficients $c_i \in \mathbb{Z}$ $(i = 1, \ldots, r)$, $c_r \neq 0$

$(c_r > 0)\, n^3 \underline{\qquad\qquad}$ $c_{r-1} \cdots c_0$ are 0

$f, g : \mathbb{N} \to \mathbb{Z}$, we can say that

$$f \in O(g) \quad \text{if}$$

$$|f| \in O(|g|)$$

$$|f| : \mathbb{N} \to \mathbb{N} : n \to |f(n)|$$

Absolute Value

$$|n| = \begin{cases} -n & \text{if } n < 0 \\ 0 & \text{if } n = 0 \\ n & \text{if } n > 0 \end{cases}$$

$f(n) = n^2 + 2n + 5$ Ex. 2.9

$g(n) = -n^2 = -(n^2)$

Show that $g \in O(f)$

$|g| : n \mapsto n^2 \qquad |f| = f$

$|g| \in O(|f|)$

$f(n) = n^2 + 2n + 5$

$g(n) = -(n^2)$

Show that $f \in O(g)$

$|f| \equiv f$

$|g| : n \mapsto n^2$

ex,

follow 2.9

$f, g : \mathbb{N} \to \mathbb{R}$

$f \in O(g)$

if

$\lfloor f \rfloor \in O(\lfloor g \rfloor)$

floor function

$\lfloor 1.2 \rfloor = 1 \in \mathbb{Z}$

$\lfloor .2 \rfloor = 0 \in \mathbb{Z}$

$\log_a(n) \in O(\log_b(n))$ for all $1 < a, b \in \mathbb{N}$

$$\log_a(n) = \frac{\log_b(n)}{\log_b(a)}$$

$$\log_b a = \frac{1}{\log_a(b)}$$

$\log_a(n) = c \cdot \log_b n$

$$\log_a(n) = \log_b(n)\overbrace{(\log_a b)}^{c}$$

constant