# Naive Bayes Classifier

## Probability Basics -

Probability of an event A occuring = P(A) = P(A = true)

Probability of an event A not occuring = $P(\neg A) = P(A = false)$

$P(A) + P(\neg A) = 1$

An event can have different domain element.

weather can be sunny, rainy, cloudy.

So, probability weather being sunny = P(weather = sunny).

There are three kinds of probability →

1. Prior probability
2. Conditional Probability
3. Joint probability.

Let us revise these concepts using an Joint Probability Distribution table. →

Let two events A an B

| | A | $\neg$A |
|---|---|---|
| B | P(A, B) | P($\neg$A, B) |
| $\neg$B | P(A, $\neg$B) | P($\neg$A, $\neg$B) |

Here, $P(A,B)$ = Probability event A and event B occurring together or at the same time.

This is called $\boxed{\text{joint probability}}$.

We usually write it using comma. Can be written as, $P(A,B) = P(A \text{ and } B) = P(A \cap B) = P(A \wedge B)$.

From the table, we also find ⇒

| | $A$ | $\neg A$ | summation |
|---|---|---|---|
| $B$ | $P(A,B)$ | $P(\neg A,B)$ | → $P(A,B)+P(\neg A,B) = P(B)$ |
| $\neg B$ | $P(A,\neg B)$ | $P(\neg A,\neg B)$ | → $P(A,\neg B)+P(\neg A,\neg B) = P(\neg B)$ |
| summation | $P(A,B)+P(A,\neg B) = P(A)$ | $P(\neg A,B)+P(\neg A,\neg B)$ → $P(\neg A)$ | |

$\boxed{\text{Prior / Absolute probability}}$ → Prior or absolute probabity is the probability of an event before any new information is known. For example $P(A)$, $P(B)$ → These are prior/Absolute probabilities.

From the joint distribution table we can see for two events A and B we can find prior probabilities using joint probability.

$$\boxed{P(A) = P(A,B) + P(A,\neg B)} \longrightarrow (a)$$

$P(B) = P(B,A) + P(B,\neg A)$ Here $P(A,B) = P(B,A)$

so, we can write, for two events, prior probability of an even

P(one event A occuring) = P(one event A occuring and another event B occuring) + P(one event A occuring and another event B not occuring)

## Conditional Probability

conditional probability is the probability that one event happens given that another event has already occurred.

If we know an event B has occured the probability of an event A occuring will be $P(A|B)$

Given that already occured

Formula →

~~for three~~

$$\boxed{P(A|B) = \frac{P(A,B)}{P(B)}}$$

For three variable/event A, B, C, if event B, and C has occurred, probability of event A happening is →

$$P(A \mid \underbrace{B,C}_{\text{occured}}) = \frac{P(A,B,C)}{P(B,C)}$$

From the condional Probability Formula, we can derive joint Probabity →

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$\Rightarrow P(A \mid B) \times P(B) = P(A,B)$$

$$\Rightarrow \boxed{P(A,B) = P(A \mid B) \times P(B)}$$ (Relationship) —— b

## Independence in probability

1. Absolute Independecee
2. conditional Independence :

Absolute independence – Two event A, B are independent of each other if they fullfill the following equality

$$P(A,B) = P(A) \times P(B)$$

Moreover if ~~the~~ two events are independent of each other then →

$$P(A \mid B) = P(A)$$

$$P(B \mid A) = P(B)$$

**Conditional Independence** → Two event A, B are conditionally independent of each other given another event C has occurred, if the following equation stands true →

$$P(A,B|C) = P(A|C)\, P(B|C).$$

Moreover,

$$P(A|B,C) = P(A|C) \text{ and } P(B|A,C) = P(B|C)$$

→ A depends on C only

→ B depends on C only

**Chain Rule** →

Probability of 4 events occurring together →

$$P(A,B,C,D) = P(A|B,C,D)\, P(B,C,D)$$

→ $P(A,B) = P(A|B) \times P(B)$

$$= P(A|B,C,D)\, P(B|C,D)\, P(C,D)$$

$$= P(A|B,C,D)\, P(B|C,D)\, P(C|D)\, P(D)$$

Now if we assume event A, B, C ~~and D~~ are conditionally independent ~~a and depends on~~ Given that D, then we can write,

(Depends on D only)

$$P(A,B,C,D) = P(A|B,C,D)\, P(B|C,D)\, P(C|D)\, P(D)$$

$$= P(A|D)\, P(B|D)\, P(C|D)\, P(D)$$

A only depends on D

B only Depends on D

C

# Bayes Theorum:

$$P(C|X) = \frac{\overset{\text{Likelihood}}{P(X|C)}\ \overset{\text{class prior probability}}{P(C)}}{\underset{\text{Predictor posterior probability}}{P(X)}}$$

$P(C|X)$ → Posterior Probability

## Math Example →

HIV global prior Probability 0.008.

$P(T|HIV) = ~~0~~ 95\%$.

$P(\neg T|\neg HIV) = 95\%$.

Perform a test and the result is positive. What is the ~~probability~~ Likelihood of having HIV?

**Solution:** Here HIV global $\boxed{\text{Prior}}$ probability,

$$P(HIV) = 0.008$$

$$\therefore P(\neg HIV) = 1 - 0.008 = 0.992$$

$P(T|HIV) = 95\% = 0.95$ $\quad\therefore P(\neg T|HIV) = 1 - 0.95 = 0.05$

$P(\neg T|\neg HIV) = 95\% = 0.95$ $\quad\therefore P(T|\neg HIV) = 1 - 0.95 = 0.05$

They have performed a test and the result is positive. This is the given situation. So T is true is given. Now, we have to find if that patient actually ~~have~~ HIV or not. In this case we can find probability of the HIV of the patient having ~~probability~~ $P(HIV|T)$ and

Has HIV  Test true

the probability of ~~of~~ patient not having HIV $(\neg HIV|T)$.

Does not have HIV  Test is true

Then we can check which is larger and can give our answer as liklihood of having HIV. [If Question asks to find likelihood

From Bayes Theorum,

$$P(HIV|T) = \frac{P(T|HIV)\,P(HIV)}{P(T)}$$

$$P(\neg HIV|T) = \frac{P(T|\neg HIV)\,P(\neg HIV)}{P(T)}$$

Both of this formula has $P(T)$ as denominator. As we are just comparing $P(HIV|T)$ and $P(\neg HIV|T)$, we do not need the denominator to decide which is larger as they both are getting divided by same $P(T)$ value.

For example → $6 > 4$ [6 is greater than 4]

Left  right

$\Rightarrow \frac{6}{2} > \frac{4}{2}$ [we divide both by 2]

$\Rightarrow 3 > 2$ [the unequality sign remains same and left is still greater than right]

So, we will skip computing $P(T)$.

Now, $P(HIV|T) \approx P(T|HIV)\,P(HIV)$ → Bayes formula without $P(T)$

$$= 0.95 \times 0.008$$

$$= 0.0076$$

and $P(\neg HIV|T) \approx \cancel{P(\neg H}\; P(T|\neg HIV)\,P(\neg HIV)$

$$= 0.05 \times 0.992$$

$$= 0.0496$$

~~Hence~~ As, $P(\neg HIV|T) > P(HIV|T)$, so we can say the patient will not have HIV.

Here $P(HIV|T) + P(\neg HIV|T) = 0.0076 + 0.0496$

$$= 0.0572 \neq 1$$

It does not give us 1 as we have skipped dividing by $P(T)$.

However, if the question we were asked to find the probability of having HIV instead of finding the likelihood. Then we have to compute $P(T)$ as well as we need the probability of having HIV if the test result is true, $P(HIV|T)$

Let us calculate that along with the $P(\neg HIV|T)$.

Pdf At first we need $P(T)$, the denominator.

$$P(T) = P(T, HIV) + P(T, \neg HIV)$$ [from the formula ⓐ of Joint probability]

$$= P(T|HIV)\,P(HIV) + P(T|\neg HIV)\,P(\neg HIV)$$

$= 0.95 \times 0.008 + 0.005 \times 0.992$

$= 0.076 + 0.0496$

$= 0.0572$

[as we do not have the joint probability values, From formula ⓑ of conditional probability]

$$\therefore P(HIV|T) = \frac{P(T|HIV)\,P(HIV)}{P(T)} = \frac{0.0076}{0.0572}$$

$= 0.13$

$$\therefore P(\neg HIV|T) = \frac{P(T|\neg HIV)\,P(\neg HIV)}{P(T)} = \frac{0.0496}{0.0572}$$

$= 0.87$

Now, $P(HIV|T) + P(\neg HIV|T) = 0.13 + 0.87 = 1$

as we have considered denominator $P(T)$.

# Naive Bayes Classifier -

Let us build the intution from an example →

| Day | Outlook | Temparature | Humidity | Wind | Play Tennis |
|---|---|---|---|---|---|
| Day1 | Sunny | Hot | High | Weak | NO |
| Day2 | Sunny | Hot | High | Strong | NO |
| Day3 | Overcast | Hot | High | Weak | Yes |
| Day4 | Rain | Mild | High | Weak | Yes |
| Day5 | Rain | ~~cold~~ cool | Normal | Weak | Yes |
| Day6 | Rain | cool | Normal | Strong | NO |
| Day7 | Overcast | cool | Normal | Strong | Yes |
| Day8 | Sunny | Mild | High | Weak | NO |
| Day9 | Sunny | cool | Normal | Weak | Yes |
| Day10 | Rain | Mild | Normal | Weak | Yes |
| Day11 | Sunny | Mild | Normal | Strong | Yes |
| Day12 | Overcast | Mild | High | Strong | Yes |
| Day13 | Overcast | Hot | Normal | Weak | Yes |
| Day14 | Rain | Mild | High | Strong | NO |

From this given dataset, we are told to find the Likelihood of playing Tennis if Outlook = Sunny, Temparature = ~~Hot~~ Cool, Humidity = High and Wind = Strong.

Let us use Bayes theorem like Before.
we will use short forms, outlook = O, sunny = S and so on.

So, we have to find →

$$P(\text{PlayTennis} = \text{Yes} \mid \overset{\text{outlook}}{O} = \underset{\text{sunny}}{S}, \overset{\text{Tempanatore}}{T} = \underset{\text{cool}}{\cancel{\text{cool}}}, \overset{\text{Humidity}}{Hm} = \underset{\text{High}}{H}, \overset{\text{wind}}{W} = \underset{\text{Strron}}{S})$$

and

$$P(\text{PlayTennis} : \text{NO} \mid O = S, T = c, Hm = H, W = S)$$

Then we will compare these two probabilities to find the likelihood of playing Tennis. ~~Let~~

Let us use Bayes Theorom,

$$P(\text{PlayTennis} = \text{Yes} \mid O = S, T = c, Hm = H, W = S)$$

$$= \frac{\overset{\text{Nominator}}{P(O = S, T = c, Hm = H, W = S \mid \text{PlayTennis} = \text{Yes}) P(\text{PlayTennis} = \text{Yes})}}{P(O = S, T = c, Hm = H, W = S)}$$

and

$$P(\text{PlayTennis} = \text{No} \mid O = S, T = c, Hm = H, W = S)$$

$$= \frac{\overset{\text{nominator}}{P(O = S, T = c, Hm = H, W = S \mid \text{PlayTennis} = \text{NO}) P(\text{Play} = \text{NO})}}{P(O = S, T = c, Hm = H, W = S)}$$

The denominators are same, so again we can skip it.

However in the 1st Nominator term we have joint events of O = S, T = c, Hm = H, W = S. If we search in our Dataset There is no datapoint where outlook = sunny, Tempanatature = cool, Humidity = High and Wind = strong at the same time. So

So, $P(O=S, T=C, Hm=H, W=S \mid Play\ Tennis = Yes) = \frac{0}{9}$

Total 9 Yes target

and $P(O=S, T=C, Hm=H, W=S \mid Play\ Tennis = NO) = \frac{0}{5}$

Total 5 NO target

If we substitute these values in the naive Bayes Formula, we will get 0 probability overall. This is a $\boxed{\text{Zero Probability problem}}$. To solve this Issue we can will Assume .outlook, temparature, Humidity and wind events are conditionally independent Given ~~that~~ the event Play Tennis. So all event O, T, Hm and wind depends on play Tennis only.

According to the chain Rule →

$$P(A,B,C,D) = P(A|B,C,D)\, P(B|C,D)\, P(C|D)\, P(D)$$

After assuming, A, B, C conditionally independent given D, we got ~~in~~.

$$P(A,B,C,D) = P(A|D)\, P(B|D)\, P(C|D)\, P(D)$$

Let us divide both side by P(D) →

$$\frac{P(A,B,C,D)}{P(D)} = \frac{P(A|D)\, P(B|D)\, P(C|D)\, \cancel{P(D)}}{\cancel{P(D)}}$$

$$\Rightarrow P(A,B,C|D) = P(A|D)\, P(B|D)\, P(C|D)$$

A depend on D    B depend on D    C depends on D

$$\frac{P(A,B)}{P(B)} = P(A|B)$$

Now if we similarly enforce conditional independence rule on our ~~Ba~~ Bayes theorum to omit the zero probability problem, That means Outlook, Temparatura, Humidity and wind will be conditionally independent given Play Tennis, Then we get →

$$P(O=S, T=S, Hm=H, W=S \mid \text{Play Tennis} = \text{Yes}) =$$

$$P(O=S \mid \text{Play Tennis}=\text{Yes})\, P(T=S \mid \text{Play}=\text{Yes})\, P(Hm=H \mid \text{Ply}=\text{Yes})$$

$$P(W=S \mid \text{Play Tennis} = \text{Yes})$$

Humidity depends on play Tennis

wind depends on play tennis

As we have assumed the conditional independence in the Bayes formula, this formula is called $\boxed{\text{Naive Bayes}}$.

Now we can calculate the probabilities without zero probability issue →

$$P(Yes \mid O=S, T=C, Hm=H, W=S) = P(O=S, T=C, Hm=H, W=S \mid Yes)\ P(\text{Play Tennis} = Yes)$$

$$= P(O=S \mid Yes)\ P(T=C \mid Yes)\ P(Hm=H \mid Yes)\ P(W=S \mid Yes)\ P(\text{Play Tennis} = Yes)$$

$$= \frac{2}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{2}{9} \times \frac{9}{14}$$

→ 9 Yes Among 14 data points.

2 outlook = sunny Among 9 Yes

3 Temp = cool Among 9 Yes

3 Humidity = High Among 9 Yes

2 wind = strong Among 9 Yes

$$= 0.0053$$

Similarly, $P(NO \mid O=S, T=C, Hm=H, W=S) = P(O=S, T=C, Hm=H, W=S \mid NO) \times P(\text{Play Tennis} = NO)$

$$= P(O=S \mid NO)\ P(T=C \mid NO)\ P(Hm=H \mid NO)\ P(W=S \mid NO)\ P(\text{Play Tennis} = NO)$$

$$= \frac{3}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{5}{14}$$

$$= 0.0206$$

So, probability not playing is higher.

# Continuous/numerical Feature Handling in Naive Bayes:

| Day | Outlook | Temperature | Humidity | Wind | Play Tennis |
|---|---|---|---|---|---|
| D1 | Sunny | 27.3 | High | weak | NO |
| D2 | Sunny | 30.1 | High | strong | NO |
| D3 | overcast | 25.2 | High | weak | Yes |
| D4 | Rain | 19.3 | High | weak | Yes |
| D5 | Rain | 18.5 | Normal | weak | Yes |
| D6 | Rain | 17.9 | Normal | Strong | NO |
| D7 | overcast | 21.7 | Normal | Strong | Yes |
| D8 | Sunny | 29.5 | High | weak | NO |
| D9 | Sunny | 20.3 | Normal | weak | Yes |
| D10 | Rain | 24.3 | Normal | weak | Yes |
| D11 | Sunny | 22.8 | Normal | Strong | Yes |
| D12 | overcast | 23.1 | High | Strong | Yes |
| D13 | Overcast | 19.8 | Normal | Weak | Yes |
| D14 | Rain | 15.1 | High | Strong | NO |

↓ [numerical feature]

Let's say, we have ~~check~~ Predict the likelihood of playing given Outlook = Sunny, Temperature = 25, Humidity = High, and wind = Strong)

So, using naive Bayes,

$P(Yes \mid O=S, T=25, Hm=H, W=S)$

$= P(O=S \mid Yes)\, \underbrace{P(T=25 \mid Yes)}\, P(Hm=H \mid Yes)\, P(W=S \mid Yes)\, P(Yes)$

we have to find this value as we cannot find it using probability from the Dataset.

To handle Numerical feature we will use $\boxed{\text{Gaussian Naive Bayes}}$ that uses Gaussian Probability Density function.

First we need to calculate the mean ($\mu$) and standard deviation ($\sigma$) of the numerical feature for each class.

Calculating mean:

~~The tempature values in~~

The temparature values in 'Yes' class are:

25.2, 19.3, 18.5, 21.7, 20.1, 24.3, 22.8, 23.1 and 19.8.

There are total 9 data points.

$$\therefore \mu_{yes} = \frac{25.2 + 19.3 + 18.5 + 21.7 + 20.1 + 24.3 + 22.8 + 23.1 + 19.8}{9}$$

$$= 21.64$$

Similarly the mean of temparature in "NO" ~~cat~~ class

$$\therefore \mu_{NO} = \frac{27.3 + 30.1 + 17.4 + 29.5 + 15.1}{5}$$

$$= 23.88$$

Now we have to calculate standard deviation for both 'Yes' and 'NO' class.

| Temparature ($x$) | $x-\mu$ | $(x-\mu)^2$ |
|---|---|---|
| 27.3 | 27.3 − 23.88 = 3.42 | 11.6964 |
| 30.1 | 30.1 − 23.88 = 6.22 | 38.6884 |
| 25.2 | 25.2 − 21.64 = 3.56 | 12.6736 |
| 19.3 | 19.3 − 21.64 = −2.34 | 5.4756 |
| 18.5 | 18.5 − 21.64 = −3.14 | 9.8596 |
| 17.4 | 17.4 − 23.88 = −6.48 | 41.9904 |
| 21.7 | 21.7 − 21.64 = 0.06 | 0.0036 |
| 29.5 | 29.5 − 23.88 = 5.62 | 31.5844 |
| 20.1 | 20.1 − 21.64 = −1.54 | ~~0.2916~~ 2.3716 |
| 24.3 | 24.3 − 21.64 = 2.66 | 7.0756 |
| 22.8 | 22.8 − 21.64 = 1.16 | 1.3456 |
| 23.1 | 23.1 − 21.64 = 1.46 | 2.1316 |
| 19.8 | 19.8 − 21.64 = −1.84 | 3.3856 |
| 15.1 | 15.1 − 23.88 = −8.78 | 77.0884 |

~~Bat~~ Black = NO
Red = Yes

$$\therefore \sigma^2_{Yes} = \frac{1}{N}\sum_{n=1}^{n=9}(x_n - \mu_{Yes})^2 = \frac{1}{9}\big(12.6736 + 5.4756 + 9.8596 + 0.0036 + \cancel{0.2916}\ 2.3716 + 7.0756 + 1.3456 + 2.1316 + 3.3856\big)$$

$$= \frac{44.3204}{9} = 4.92449$$

$$\therefore \sigma_{yes} = \sqrt{4.92449} = 2.219$$

Similarly,

$$\sigma_{NO}^2 = \frac{1}{N}\sum_{n=1}^{n=5}(x_n - \mu_{NO})^2 = \frac{1}{5}(11.6964 + 38.6884 + 41.9904 + 31.5844 + 77.0884)$$

$$= \frac{201.048}{5} = 40.2096$$

$$\therefore \sigma_{NO} = \sqrt{40.2096} = 6.341$$

The Gaussian Probability Density Function →

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

Using this function we can find,

$$P(T=25|Yes) = \frac{1}{\sigma_{Yes}\sqrt{2\pi}} e^{-\left[\frac{(25-\mu_{Yes})^2}{2\sigma_{Yes}^2}\right]}$$

$$= \frac{1}{2.219 \times \sqrt{2\pi}} e^{-\left[\frac{(25-21.64)^2}{2\times(2.219)^2}\right]}$$

$$= \boxed{0.057}$$

and

$$P(T=25|NO) = \frac{1}{\sigma_{NO}\sqrt{2\pi}} e^{-\left[\frac{(25-\mu_{NO})^2}{2\sigma_{NO}^2}\right]}$$

$$= \frac{1}{6.341 \times \sqrt{2\pi}} e^{-\left[\frac{(25-23.88)^2}{2\times(6.341)^2}\right]}$$

$$= \boxed{0.0619}$$

Now we can use these probability values to find the likelihood in given situation →

$$P(Yes \mid O=S, T=25, Hm=H, W=S)$$

$$= P(O=S \mid Yes)\,\boxed{P(T=25 \mid Yes)}\,P(Hm=H \mid Yes)\,P(W=S \mid Yes) \times P(Yes)$$

$$= \underbrace{\frac{2}{9}}_{\text{From dataset}} \times \underbrace{\boxed{0.057}}_{\text{From Gaussian PDF}} \times \underbrace{\frac{3}{9} \times \frac{2}{9} \times \frac{9}{14}}_{\text{From Dataset}}$$

$$= 0.0006$$

and $P(NO \mid O=S, T=25, Hm=H, W=S)$

$$= P(O=S \mid NO)\,P(T=25 \mid NO)\,P(Hm=H \mid NO)\,P(W=S \mid NO)\,P(NO)$$

$$= \frac{3}{5} \times 0.0619 \times \frac{4}{5} \times \frac{2}{5} \times \frac{5}{14}$$

$$= 0.004$$

So, the prediction is NO. The likelihood of not playing is more.

# Relevant Issues with Naive Bayes:

## (1) Violation of independent assumption:

Naive Bayes assumes all features $x_1, x_2 \ldots x_1$ are conditionally independent Given the class $c$. Mathematically,

$$P(x_1, x_2 \ldots x_n | c) = P(x_1|c)\, P(x_2|c) \ldots P(x_n|c)$$

This makes calculation simpler and helps us to omit the zero probability problem somehow.

However, in most real-world problems, this assumption is not true as features often depends on each other. Despite this unrealistic Assumption. Naive Bayes works surprisingly well.

## (2) Zero Conditional Probability problem:

Let's assume we have the dataset given below now. Here Again Like before we have 9 Yes and 5 No.

Let us again find Likelihood of playing Given outlook = sunny, Temparature = cool, Humidity = High and Wind = strong.

| Day | Outlook | Temperature | Humidity | Wind | Play Tennis |
|---|---|---|---|---|---|
| D1 | ~~Sunny~~ Overcast | ~~27.7~~ 27.3 | High | Weak | No |
| D2 | ~~Sunny~~ Rain | 30.1 | High | Strong | No |
| D3 | Overcast | 25.2 | High | Weak | Yes |
| D4 | Rain | 19.3 | High | Weak | Yes |
| D5 | Rain | 18.5 | Normal | Weak | Yes |
| D6 | Rain | 17.4 | Normal | Strong | No |
| D7 | Overcast | 21.7 | Normal | Strong | Yes |
| D8 | ~~Sunny~~ Rain | 29.5 | High | Weak | No |
| D9 | ~~Sunny~~ Overcast | 20.1 | Normal | Weak | Yes |
| D10 | Rain | 24.3 | Normal | Weak | Yes |
| D11 | ~~Sunny~~ Rain | 22.8 | Normal | Strong | Yes |
| D12 | Overcast | 23.1 | High | Strong | Yes |
| D13 | Overcast | 19.8 | Normal | Weak | Yes |
| D14 | Rain | 15.1 | High | Strong | No |

We have Removed all $\boxed{\text{Sunny}}$ examples for the process.

No, Let us calculate

$$P(Yes \mid O=S, T=25, Hm=H, W=S)$$
$$= P(O=S \mid Yes)\, P(T=25 \mid Yes)\, P(Hm=H \mid Yes)\, P(W=S \mid Yes)\, P(Yes)$$

$$= \frac{0}{9} \times \frac{0.057}{\cancel{0.061}} \times \frac{3}{9} \times \frac{2}{9} \times \frac{9}{14} = 0$$

There is no example with Sunny in the dataset.

we are facing the zero conditional Probability Problem here. To solve this issue, we will Apply $\boxed{\text{Laplace correction}}$ on $\boxed{\text{Laplace Smoothing}}$ Tecnique here.

Let us find,

$$P(O=\text{sunny} \mid \text{Yes}) = \frac{0}{9}$$

$$P(O=\text{overcast} \mid \text{Yes}) = \frac{5}{9}$$

$$P(O=\text{Rain} \mid \text{Yes}) = \frac{4}{9}$$

~~does~~ Now we will Add an extra sample for each category of ~~overcast~~ outlook feature

$$P(O=\text{sunny} \mid \text{Yes}) = \frac{0+1}{9+\boxed{3}}$$

$$P(O=\text{overcast} \mid \text{Yes}) = \frac{5+1}{9+\boxed{3}}$$

$$P(O=\text{Rain} \mid \text{Yes}) = \frac{4+1}{9+\boxed{3}}$$

As we have added 3 extra sample for each Category of Outlook feature, we have added ~~to~~ 3 with the denominator.

$$\therefore P(O=\text{sunny} \mid \text{Yes}) = \frac{1}{12}$$

Similarly,

$$P(O = sunny \mid No) = \frac{0}{5}$$

After laplace correction,

$$P(O = sunny \mid No) = \frac{0+1}{5+3} = \frac{1}{8}$$

$\therefore P(Yes \mid O=S, T=25, Hm=H, W=S)$

$$= P(O=S \mid Yes)\, P(T=25 \mid Yes)\, P(Hm=H \mid Yes)\, P(W=S \mid Yes)\, P(Yes)$$

$$= \frac{1}{12} \times \underbrace{\overset{0.057}{\cancel{0.061}} \times \frac{3}{9} \times \frac{2}{9} \times \frac{9}{14}}_{\text{Other probabilities remains same as befor}}$$

Laplace correction

$$= 0.000226$$

$\therefore P(No \mid O=S, T=25, Hm=H, W=S)$

$$= P(O=S \mid No)\, P(T=25 \mid No)\, P(Hm=H)\, P(W=S \mid No)\, P(No)$$

$$= \frac{1}{8} \times \overset{0.0619}{\cancel{0.055}} \times \frac{4}{5} \times \frac{2}{5} \times \frac{5}{14}$$

$$= 0.00088$$

$\therefore$ Prediction is NO here.