CSE505 HW2

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October 14, 2016

1 Interpretations and Models

For each of the solutions provided below assume that $D = N$, $zero_{\mathscr{I}} = 0$, and $s_{\mathscr{I}} :=$ the successor function over N.

1.1 Part A

1. $p_{\mathscr{I}} = \{n \in N \mid n \text{ is even}\}$.
2. $p_{\mathscr{I}} = N$.

1.2 Part B

1. $q_{\mathscr{I}} = \{n \in N \mid n \text{ is odd}\}$.
2. $q_{\mathscr{I}} = N$.

1.3 Part C

1. $p_{\mathscr{I}} = N$ and $q_{\mathscr{I}} = N$.
2. $p_{\mathscr{I}} = \{n \in N \mid n \text{ is even}\}$ and $q_{\mathscr{I}} = \{n \in N \mid n \text{ is odd}\}$.
3. $p_{\mathscr{I}} = \{n \in N \mid n \text{ is odd}\}$ and $q_{\mathscr{I}} = \{n \in N \mid n \text{ is even}\}$.
4. $p_{\mathscr{I}} = \emptyset$ and $q_{\mathscr{I}} = \emptyset$

1.4 Part D

1. $p_{\mathscr{I}} = \{n \in N \mid n \text{ is even}\}$ and $q_{\mathscr{I}} = \{n \in N \mid n \text{ is odd}\}$.
2. $p_{\mathscr{I}} = \{n \in N \mid n \text{ is even}\}$ and $q_{\mathscr{I}} = N$.
3. $p_{\mathscr{I}} = N$ and $q_{\mathscr{I}} = \{n \in N \mid n \text{ is odd}\}$.
4. $p_{\mathscr{I}} = N$ and $q_{\mathscr{I}} = N$.

1.5 Part E

Let $p_{\mathscr{I}} = \{n \in N \mid n \text{ is even}\}$ and $q_{\mathscr{I}} = N$ then $q(s(s(zero)))$ is true but $p(s(zero))$ is false so the interpretation is a model of $F_1 \wedge F_2$ but not of F_3.

1.6 Part F

Since there is some model of $F_1 \wedge F_2$ which is not a model of F_3, F_3 is not a logical consequence of $F_1 \wedge F_2$.

1.7 Part G

The only interpretations of $p/1$ and $q/1$ which model $F_1 \wedge F_3$ are:
1. $p_{\mathscr{I}} = N$ and $q_{\mathscr{I}} = N$.
2. $p_{\mathscr{I}} = \{n \in N \mid n \text{ is even}\}$ and $q_{\mathscr{I}} = \{n \in N \mid n \text{ is odd}\}$.

These interpretations for $q/1$ also model F_2, so any interpretation which models $F_1 \wedge F_3$ will also model F_2.

1.8 Part H

F_2 is a logical consequence of $F_1 \wedge F_3$ by definition of logical consequence.

2 Definite Programs

2.1 Part A

$p_{\mathscr{I}} = \{(a, a)\}$ and $q_{\mathscr{I}} = \{(a, a)\}$.

2.2 Part B

1. $p_{\mathscr{I}} = \{(a, b), (b, c), (b, d)\}$ and $q_{\mathscr{I}} = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d), (a, c), (a, d)\}$.
2. $p_{\mathscr{I}} = \{(a, b), (b, c), (b, d)\}$ and $q_{\mathscr{I}} = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d), (a, c), (a, d), (c, d)\}$.

2.3 Part C

1. $p_{\mathscr{I}} = \{(a, b), (b, c), (b, d)\}$ and $q_{\mathscr{I}} = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d), (a, c), (a, d)\}$.

2.4 Part D

Let P be a definite program, it will be shown that there exists some interpretation $\mathscr{I}$ which is a model of P. More specifically, it will be shown that there exists a Herbrand model of P. The Herbrand base of P contains all ground atomic formulas over P and as such the entire set B_P will be a Herbrand model of P.

Assume that B_P is not a Herbrand model of P. Then there exists some clause $A_0 \leftarrow A_1, ..., A_n$ in P such that all of the atoms $A_1, ..., A_n$ are true but A_0 is false under interpretation $\mathscr{I} = B_P$. Then there is some ground instance of A_0 which is not true under $\mathscr{I}$. But this implies that B_P does not contain this ground instance of A_0 which contradicts the definition of a Herbrand base.

$\therefore B_P$ is a Herband interpretation of P and P has an interpretation $\blacksquare$