$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}(m_1+m_2)l_1^2+m_2l_2^2+2m_2l_1l_2\cos\theta_2 & m_2l_2^2+m_2l_1l_2\cos\theta_2\\ m_2l_2^2+m_2l_1l_2\cos\theta_2 & m_2l_2^2\end{bmatrix}\begin{bmatrix}\ddot\theta_1\\ \ddot\theta_2\end{bmatrix}$$

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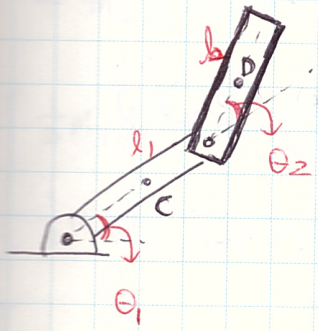
$$+\begin{bmatrix}0 & -m_2l_1l_2\sin\theta_2\\ m_2l_1l_2\sin\theta_2 & 0\end{bmatrix}\begin{bmatrix}\dot\theta_1^2\\ \dot\theta_2^2\end{bmatrix}+\begin{bmatrix}-m_2l_1l_2\sin\theta_2 & -m_2l_1l_2\sin\theta_2\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot\theta_1\dot\theta_2\\ \dot\theta_2\dot\theta_1\end{bmatrix}$$

$$+\begin{bmatrix}(m_1+m_2)g\,l_1\cos\theta_1+m_2gl_2\cos(\theta_1+\theta_2)\\ m_2gl_2\cos(\theta_1+\theta_2)\end{bmatrix}$$



$$x_D=l_1\cos\theta_1+0.5l_2\cos(\theta_1+\theta_2)$$

$$y_D=l_1\sin\theta_1+0.5l_2\sin(\theta_1+\theta_2)$$

$$\dot x_D=-l_1\dot\theta_1\sin\theta_1-0.5l_2(\dot\theta_1+\dot\theta_2)\sin(\theta_1+\theta_2)$$

$$\dot y_D=l_1\dot\theta_1\cos\theta_1+0.5l_2(\dot\theta_1+\dot\theta_2)\cos(\theta_1+\theta_2)$$

링크 2의 질량중심의 총 속도

$$V_D^2=\dot x_D^2+\dot y_D^2$$

$$=l_1^2\dot\theta_1^2+0.25l_2^2(\dot\theta_1+\dot\theta_2)^2$$
$$+l_1l_2\dot\theta_1(\dot\theta_1+\dot\theta_2)\sin\theta_1\sin(\theta_1+\theta_2)$$
$$+l_1l_2\dot\theta_1(\dot\theta_1+\dot\theta_2)\cos\theta_1\cos(\theta_1+\theta_2)$$

$$s_1(s_1c_2+c_1s_2)+c_1(c_1c_2-s_1s_2)$$

$$V_D^2=l_1^2\dot\theta_1^2+0.25l_2^2(\dot\theta_1+\dot\theta_2)^2$$
$$+l_1l_2\dot\theta_1(\dot\theta_1+\dot\theta_2)\cos\theta_2$$
$$=(l_1^2+0.25l_2^2+l_1l_2\cos\theta_2)\dot\theta_1^2$$
$$+0.25l_2^2\dot\theta_2^2+0.5l_2^2\dot\theta_1\dot\theta_2$$
$$+l_1l_2\dot\theta_1\dot\theta_2\cos\theta_2$$

$$K=K_1+K_2$$
$$=\left[\tfrac12J_A\dot\theta_1^2\right]+\left[\tfrac12J_D(\dot\theta_1+\dot\theta_2)^2+\tfrac12m_2V_D^2\right]$$
$$=\left[\tfrac12\left(\tfrac13m_1l_1^2\right)\dot\theta_1^2\right]+\left[\tfrac12\left(\tfrac{1}{12}m_2l_2^2\right)(\dot\theta_1+\dot\theta_2)^2+\tfrac12m_2V_D^2\right]$$
$$K=\left[\tfrac16m_1l_1^2+\tfrac{1}{24}m_2l_2^2+\tfrac12m_2l_1^2+\tfrac18m_2l_2^2\right.$$
$$\left.+\tfrac12m_2l_1l_2\cos\theta_2\right]\dot\theta_1^2$$
$$+\left[\tfrac{1}{24}m_2l_2^2+\tfrac18m_2l_2^2\right]\dot\theta_2^2$$
$$+\tfrac{1}{12}m_2l_2^2\dot\theta_1\dot\theta_2+\tfrac14m_2l_2^2\dot\theta_1\dot\theta_2$$
$$+\tfrac12m_2l_1l_2\dot\theta_1\dot\theta_2\cos\theta_2$$

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$$K = \dot{\theta}_1^2 \left[\frac{1}{6} m_1 l_1^2 + \frac{1}{6} m_2 l_2^2 + \frac{1}{2} m_2 l_1^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right]$$

$$+ \dot{\theta}_2^2 \left[\frac{1}{6} m_2 l_2^2 \right] + \dot{\theta}_1 \dot{\theta}_2 \left(\frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right)$$

$$P = P_1 + P_2 = m_1 g \frac{l_1}{2} \sin\theta_1 + m_2 g \left(l_1 \sin\theta_1 + \frac{l_2}{2} \sin(\theta_1 + \theta_2) \right)$$

$$L = K - P = \dot{\theta}_1^2 \left(\frac{1}{6} m_1 l_1^2 + \frac{1}{6} m_2 l_2^2 + \frac{1}{2} m_2 l_1^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right)$$

$$+ \dot{\theta}_2^2 \left(\frac{1}{6} m_2 l_2^2 \right) + \dot{\theta}_1 \dot{\theta}_2 \left(\frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right)$$

$$- m_1 g \frac{l_1}{2} \sin\theta_1 - m_2 g \left(l_1 \sin\theta_1 + \frac{l_2}{2} \sin(\theta_1 + \theta_2) \right)$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = 2\dot{\theta}_1 \left(\frac{1}{6} m_1 l_1^2 + \frac{1}{6} m_2 l_2^2 + \frac{1}{2} m_2 l_1^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right)$$
$$+ \dot{\theta}_2 \left(\frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1} \right) = 2\ddot{\theta}_1 \left(\frac{1}{6} m_1 l_1^2 + \frac{1}{6} m_2 l_2^2 + \frac{1}{2} m_2 l_1^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right)$$

$$+ 2\dot{\theta}_1 \left(-\frac{1}{2} m_2 l_1 l_2 \dot{\theta}_2 \sin\theta_2 \right) - \dot{\theta}_2^2 \left(\frac{1}{2} m_2 l_1 l_2 \sin\theta_2 \right) + \ddot{\theta}_2 \left(\frac{1}{3} m_2 l_2^2 \right.$$
$$\left. + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right)$$

$$= \ddot{\theta}_1 \left(\frac{1}{3} m_1 l_1^2 + \frac{1}{3} m_2 l_2^2 + m_2 l_1^2 + m_2 l_1 l_2 \cos\theta_2 \right)$$

$$- \dot{\theta}_1 \dot{\theta}_2 \left(m_2 l_1 l_2 \sin\theta_2 \right) + \ddot{\theta}_2 \left(\frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right) - \dot{\theta}_2^2 \left(\frac{1}{2} m_2 l_1 l_2 \right.$$
$$\left. \sin\theta_2 \right)$$

$$\frac{\partial L}{\partial \theta_1} = - m_1 g \frac{l_1}{2} \cos\theta_1 - m_2 g l_1 \cos\theta_1 - m_2 g \frac{l_2}{2} \cos(\theta_1 + \theta_2)$$

$$T_1 = \left(\frac{1}{3} m_1 l_1^2 + m_2 l_1^2 + \frac{1}{3} m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2 \right) \ddot{\theta}_1$$

$$+ \left(\frac{1}{3} m_2 l_2^2 + \frac{1}{2} m_2 l_1 l_2 \cos\theta_2 \right) \ddot{\theta}_2 - \left(m_2 l_1 l_2 \sin\theta_2 \right) \dot{\theta}_1 \dot{\theta}_2 - \left(\frac{1}{2} m_2 l_1 l_2 \sin\theta_2 \right)$$

$$+ \left(\frac{1}{2} m_1 + m_2 \right) g l_1 c_1 + \frac{1}{2} m_2 g l_2 \cos(\theta_1 + \theta_2)$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = 2\dot{\theta}_2\left(\frac{1}{6}m_2 l_2^2\right) + \dot{\theta}_1\left(\frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos\theta_2\right)$$

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$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = \frac{1}{3}m_2 l_2^2 \ddot{\theta}_2 + \ddot{\theta}_1\left(\frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos\theta_2\right) - \dot{\theta}_1\dot{\theta}_2 \frac{1}{2}m_2 l_1 l_2 \sin\theta_2$$

$$\frac{\partial L}{\partial \theta_2} = -\dot{\theta}_1^2 \frac{1}{2}m_2 l_1 l_2 \sin\theta_2 - \dot{\theta}_1\dot{\theta}_2 \frac{1}{2}m_2 l_1 l_2 \sin\theta_2$$

$$\tau_2 = \ddot{\theta}_1\left(\frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos\theta_2\right) + \ddot{\theta}_2\left(\frac{1}{3}m_2 l_2^2\right) + \dot{\theta}_1^2\left(\frac{1}{2}m_2 l_1 l_2 \sin\theta_2\right) + m_2 g \frac{l_2}{2}\cos(\theta_1+\theta_2)$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3}m_1 l_1^2 + m_2 l_1^2 + \frac{1}{3}m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2 & \frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos\theta_2 \\ \frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos\theta_2 & \frac{1}{3}m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -\frac{1}{2}m_2 l_1 l_2 \sin\theta_2 \\ \frac{1}{2}m_2 l_1 l_2 \sin\theta_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{1}{2}m_2 l_1 l_2 \sin\theta_2 & -\frac{1}{2}m_2 l_1 l_2 \sin\theta_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_2\dot{\theta}_1 \end{bmatrix}$$

$$+ \begin{bmatrix} \left(\frac{1}{2}m_1 + m_2\right) g l_1 \cos\theta_1 + \frac{1}{2}m_2 g l_2 \cos(\theta_1+\theta_2) \\ m_2 g \frac{l_2}{2}\cos(\theta_1+\theta_2) \end{bmatrix}$$

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