Column Generation Techniques for GAP

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1 Literature Review

- Branch-and-Price : Column Generation for Solving Huge Integer Programs (Barnhart, et. al., 1998)
- A Branch-and-Price Algorithm for the Generalized Assignment Problem (Savelsbergh and Martin, 1997)
  - Dantzig-Wolfe decomposition for GAP (construct master and sub problems)
  - Column generation : additional columns of the restricted master problem are generated by solving the pricing problem.
  - Branching strategies : variable dichotomy(sing variable) and GUB dichotomy(set of variables)
- Chebyshev center based column generation (Lee and Park, 2011)
  - The column generation procedure based on the simplex algorithm often shows desperately slow convergence. (zig-zag movement)
  - Chebyshev center based column generation techniques
    - Chebyshev center
    - Proximity adjusted Chebyshev center
    - Chebyshev center + Stabilization
    - Proximity adjusted Chebyshev center + Stabilization
  - Computational experiments on the binpacking, VRP, GAP
  - The proposed algorithm could accelerate the column generation procedure.
- Comparison of bundle and classical column generation (O.Briant, et. al., 2006)
  - Bundle method : the dual solution is often constrained to a given interval, and any deviation from the interval is penalized by a penalty function.
  - The penalty function for stabilized column generation : a simple V-shaped function (stabilizing center, ϵ)
- Stabilized Column Generation (O. Du Merle, et. al., 1997)

2 Problems

2.1 A case study : Generalized Assignment Problem

Dantzig-Wolfe Decomposition [1]

$$
\begin{aligned}
(\mathrm{P})\ \ \min\ & \sum_{i\in I}\sum_{k\in K} c_k^i x_k^i, \\
\text{s.t.}\ & \sum_{i\in I}\sum_{k\in K_i} \delta_k^j x_k^i \geq 1, \quad j \in J, \\
& -\sum_{k\in K_i} x_k^i \geq -1, \quad \forall i \in I, \\
& x_k^i \geq 0, \quad \forall k \in K_i, i \in I.
\end{aligned}
$$

$$
\begin{aligned}
(\mathrm{D})\ \ \max\ & \sum_{j\in J}\pi_j - \sum_{i\in I}\phi_i, \\
\text{s.t.}\ & \sum_{j\in J}\delta_k^j \pi_j - \phi_i \leq c_k^i, \quad \forall k \in K_i, i \in I, \\
& \pi_j \geq 0, \quad \forall j \in J, \\
& \phi_i \geq 0, \quad \forall i \in I.
\end{aligned}
$$

The GAP oracle finds an assignment pattern while satisfying the knapsack constraints :

$$
\max \sum_{j\in J}(\pi_j - c_{ij})\,\delta_j, \qquad \text{s.t.} \sum_{j\in J} a_{ij}\delta_j \leq b_i, \delta_j \in \{0,1\}, \quad \forall j \in J
$$

Stabilization

$$
\begin{aligned}
(\tilde{P})\ \ \min\ & \sum_{i\in I}\sum_{k\in K} c_k^i x_k^i + \sum_{j\in J}\delta_j(\gamma_j^+ - \gamma_j^-) + \sum_{i\in I}\phi_i(y_i^+ - y_i^-), \\
\text{s.t.}\ & \sum_{i\in I}\sum_{k\in K_i}\delta_k^j x_k^i + \gamma_j^+ - \gamma_j^- \geq 1, \quad \forall j \in J, \\
& -\sum_{k\in K_i} x_k^i + y_i^+ - y_i^- \geq -1, \quad \forall i \in I, \\
& \gamma_j^+ \leq \epsilon,\ \gamma_j^- \leq \epsilon, \quad \forall j \in J, \\
& y_i^+ \leq \epsilon,\ y_i^- \leq \epsilon, \quad \forall i \in I, \\
& x_k^i \geq 0, \quad \forall k \in K_i, i \in I, \\
& y_i^+ \geq 0, \quad \forall k \in K_i, i \in I.
\end{aligned}
$$

3 Preliminary Tests

Github page : `https://github.com/mody3062/CG`

[1] The written mathematical formulation are from (Lee and Park, 2011)

Testing algorithms

- Classical column generation (Kelly's cutting plane)
- Stabilized column generation (O. Du Merle, et. al., 1997)

All codes for the both of algorithms are based on the pseudo code described in Figure 1 of (O. Du Merle, et. al., 1997).

Algorithmic parameters RMP was constructed with a single decision variable which is dummy. The coefficient of the dummy variable on the objective function was set to a sufficiently large value, which is the sum of listed values such that `np.sum(c,axis=1)`. For stabilized column generation algorithm, I fixed the parameter ϵ to 0.0001. ($\because$ I don't understand the criteria for changing the parameter value(ϵ).)

4 New Column Generation Approach

Consider $J = J_1 \cup J_2$ and the dual solutions π_j for all $j \in J_2$ are fixed to $\Pi' = \bigcup_{j \in J_2} \pi'_j$ (π' is a feasible solution). Then, the reformulation of the dual problem and its primal are as follows :

$$\begin{aligned}
\text{(D)}\ \ \max\ & \sum_{j \in J} \pi_j - \sum_{i \in I} \phi_i, \\
\text{s.t.}\ & \sum_{j \in J} \delta_k^j \pi_j - \phi_i \leq c_k^i, \quad \forall k \in K_i, i \in I, \\
& \pi_j \leq \pi'_j \quad \forall j \in J_2 \\
& \pi_j \geq 0, \quad \forall j \in J, \\
& \phi_i \geq 0, \quad \forall i \in I.
\end{aligned}$$

$$\begin{aligned}
\text{(P)}\ \ \min\ & \sum_{i \in I} \sum_{k \in K} c_k^i x_k^i + \sum_{j \in J_2} \pi'_j y_j, \\
\text{s.t.}\ & \sum_{i \in I} \sum_{k \in K_i} \delta_k^j x_k^i \geq 1, \quad j \in J_1, \\
& \sum_{i \in I} \sum_{k \in K_i} \delta_k^j x_k^i + y_j \geq 1, \quad j \in J_2, \\
& - \sum_{k \in K_i} x_k^i \geq -1, \quad \forall i \in I, \\
& x_k^i \geq 0, \quad \forall k \in K_i, i \in I, \\
& y_j \geq 0, \quad \forall j \in J.
\end{aligned}$$

Stabilization

$$
\begin{aligned}
(\tilde{P})\ \ \min\ & \sum_{i\in I}\sum_{k\in K} c_k^i x_k^i + \sum_{j\in J_2} \pi'_j y_j + \sum_{j\in J} \delta_j(\gamma_j^+ - \gamma_j^-) + \sum_{i\in I} \phi_i (y_i^+ - y_i^-),\\
\text{s.t.}\ & \sum_{i\in I}\sum_{k\in K_i} \delta_k^j x_k^i + \gamma_j^+ - \gamma_j^- \geq 1, \quad j \in J_1,\\
& \sum_{i\in I}\sum_{k\in K_i} \delta_k^j x_k^i + y_j + \gamma_j^+ - \gamma_j^- \geq 1, \quad j \in J_2,\\
& -\sum_{k\in K_i} x_k^i + y_i^+ - y_i^- \geq -1, \quad \forall i \in I,\\
& \gamma_j^+ \leq \epsilon,\ \gamma_j^- \leq \epsilon, \quad \forall j \in J,\\
& y_i^+ \leq \epsilon,\ y_i^- \leq \epsilon, \quad \forall i \in I,\\
& x_k^i \geq 0, \quad \forall k \in K_i, i \in I,\\
& y_j \geq 0, \quad \forall j \in J.
\end{aligned}
$$