

```

import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from matplotlib.ticker import FuncFormatter

from pandas.plotting import register_matplotlib_converters
from sklearn.ensemble import IsolationForest
from statsmodels.tsa.arima.model import ARIMA
from scipy.optimize import minimize

import warnings
warnings.filterwarnings('ignore')

df = pd.read_csv(r'C:\Users\Mohamed Fawzi\OneDrive\Área de Trabalho\
Stock Price Analysis\Stock Prices Data Set.csv')

df.head()

{"columns":[{"name":"index","rawType":"int64","type":"integer"},
{"name":"symbol","rawType":"object","type":"string"},
{"name":"date","rawType":"object","type":"string"},
{"name":"open","rawType":"float64","type":"float"},
{"name":"high","rawType":"float64","type":"float"},
{"name":"low","rawType":"float64","type":"float"},
{"name":"close","rawType":"float64","type":"float"},
{"name":"volume","rawType":"int64","type":"integer"}],"ref":"ea2a46b8-
6e20-4b8e-b21f-a71fe39d7727","rows":[["0","AAL","2014-01-
02","25.07","25.82","25.06","25.36","8998943"],["1","AAPL","2014-01-
02","79.3828","79.5756","78.8601","79.0185","58791957"],
["2","AAP","2014-01-02","110.36","111.88","109.29","109.74","542711"],
["3","ABBV","2014-01-02","52.12","52.33","51.52","51.98","4569061"],
["4","ABC","2014-01-
02","70.11","70.23","69.48","69.89","1148391"]],"shape":
{"columns":7,"rows":5}}

```

## Preparing & Cleaning Data:

```

df.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 497472 entries, 0 to 497471
Data columns (total 7 columns):
 #   Column      Non-Null Count  Dtype
---  ---
 0   symbol      497472 non-null  object
 1   date        497472 non-null  object
 2   open        497461 non-null  float64
 3   high        497464 non-null  float64

```

```

4    low      497464 non-null  float64
5    close   497472 non-null  float64
6    volume  497472 non-null  int64
dtypes: float64(4), int64(1), object(2)
memory usage: 26.6+ MB

# Statistics of the dataset
df.describe()

{"columns":[{"name":"index","rawType":"object","type":"string"},
{"name":"open","rawType":"float64","type":"float"},
{"name":"high","rawType":"float64","type":"float"},
{"name":"low","rawType":"float64","type":"float"},
{"name":"close","rawType":"float64","type":"float"},
{"name":"volume","rawType":"float64","type":"float"}],"ref":"cceb3c7b-8797-47b4-9837-fab7e05da266","rows":
[["count","497461.0","497464.0","497464.0","497472.0","497472.0"],
["mean","86.35227481611624","87.13256217193604","85.55246746216812","86.36908207456902","4253610.897777162"],
["std","101.47122779518384","102.31206175533661","100.57095703268504","101.47240737650641","8232139.235882821"],
["min","1.62","1.69","1.5","1.59","0.0"],
["25%","41.69","42.09","41.28","41.70375","1080166.5"],
["50%","64.97","65.56","64.3537","64.98","2084896.5"],
["75%","98.41","99.23","97.58","98.42","4271928.0"],
["max","2044.0","2067.99","2035.11","2049.0","618237630.0"]],"shape":
{"columns":5,"rows":8}}

#checking for null values
df.isnull().sum()

{"columns":[{"name":"index","rawType":"object","type":"string"},
{"name":"0","rawType":"int64","type":"integer"}],"ref":"570f534e-e93c-45fa-a27a-49464e3c7f4a","rows":
[["symbol","0"],["date","0"],
["open","11"],["high","8"],["low","8"],["close","0"],
["volume","0"]],"shape":{"columns":1,"rows":7}}

# filling null values with previous values
df.fillna(method='ffill', inplace=True)

# convert the 'Date' column to datetime format
df['date'] = pd.to_datetime(df['date'])

df.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 497472 entries, 0 to 497471
Data columns (total 7 columns):
#   Column      Non-Null Count  Dtype
---  -
0    symbol      497472 non-null  object

```

```

1   date      497472 non-null  datetime64[ns]
2   open      497472 non-null  float64
3   high      497472 non-null  float64
4   low       497472 non-null  float64
5   close     497472 non-null  float64
6   volume    497472 non-null  int64
dtypes: datetime64[ns](1), float64(4), int64(1), object(1)
memory usage: 26.6+ MB

```

## Exploratory Data Analysis (EDA):

- I will analyze the top 5 symbols with highest volume

```

# top 5 symbols by volume
top_5_symbols = df.groupby('symbol')
['volume'].sum().nlargest(5).index.tolist()
top_5_symbols

['BAC', 'AAPL', 'GE', 'AMD', 'F']

# mapping symbols to their respective full names
symbol_name = {
    'BAC': 'Bank of America',
    'AAPL': 'Apple Inc.',
    'GE': 'General Electric',
    'AMD': 'Advanced Micro Devices',
    'F': 'Ford Motor Company',
}

df['company name'] = df['symbol'].map(symbol_name)

# the top 5 symbols by volume
top_5_companies = df.groupby('company name')
['volume'].sum().nlargest(5).reset_index()
top_5_companies

{"columns":[{"name":"index","rawType":"int64","type":"integer"},
{"name":"company name","rawType":"object","type":"string"},
{"name":"volume","rawType":"int64","type":"integer"}],"ref":"9fe8bb91-
e29d-481e-ab53-6ef7798da955","rows":[["0","Bank of
America","89988444028"],["1","Apple Inc.","45485758169"],["2","General
Electric","41734050117"],["3","Advanced Micro Devices","33522535638"],
["4","Ford Motor Company","33144701045"]],"shape":
{"columns":2,"rows":5}}

plt.figure(figsize=(10, 4))
sns.barplot(
    data=top_5_companies,
    y='company name',
    x='volume',

```

```

    palette='Greens_r',
    edgecolor='black',
)

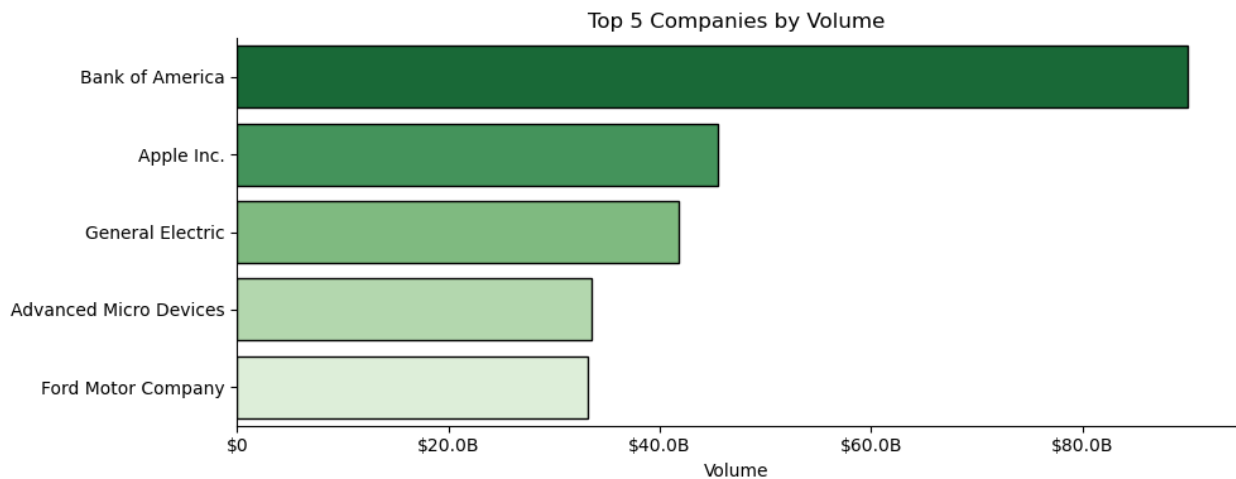
def currency(x, pos):
    if x >= 1e9:
        return f'${x*1e-9:.1f}B'
    elif x >= 1e6:
        return f'${x*1e-6:.1f}M'
    elif x >= 1e3:
        return f'${x*1e-3:.1f}K'
    else:
        return f'${x:.0f}'

plt.gca().xaxis.set_major_formatter(FuncFormatter(currency))

plt.title('Top 5 Companies by Volume')
plt.xlabel('Volume')
plt.ylabel('')

sns.despine()
plt.tight_layout()
plt.show()

```



- **Key Insight**

- **Bank of America** leads significantly in trading volume, indicating high investor interest or market activity.
- **Apple Inc.** and **General Electric** follow as the next most traded stocks, but their volumes are notably lower than Bank of America's.
- **Advanced Micro Devices** and **Ford Motor Company** complete the top 5, showing moderate trading interest.

- The sharp drop after Bank of America suggests a **concentration of trading activity** in fewer stocks, potentially due to sector-specific news or broader market trends.

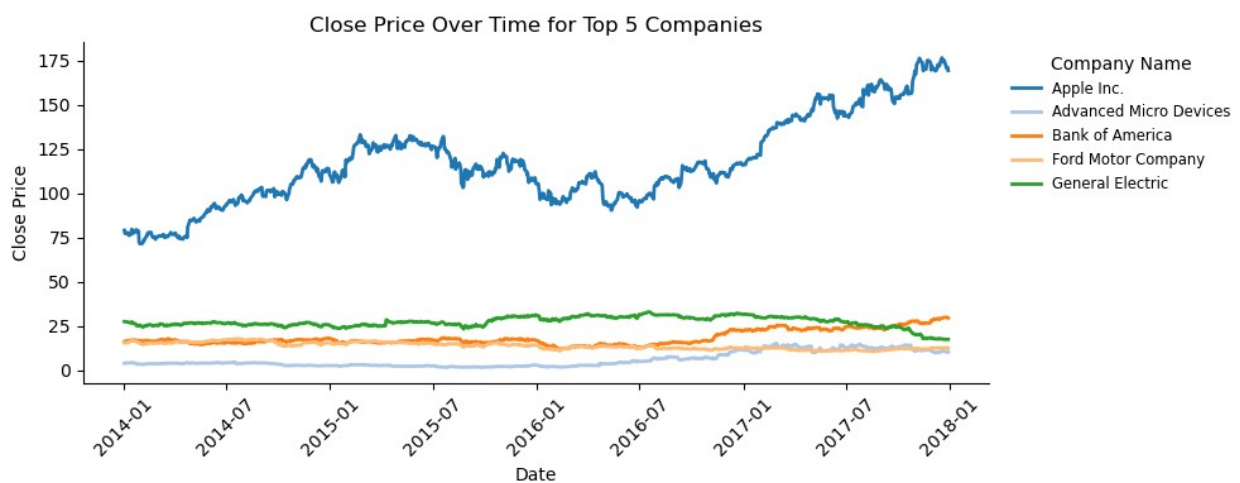
- Now, let's visualize the closing stock prices over time for the top 5 performing companies. I will filter the dataset to include only those companies.

```
top_5_companies_list = top_5_companies['company name'].tolist()
df_top_5_companies = df[df['company name'].isin(top_5_companies_list)]

plt.figure(figsize=(10, 4))
sns.lineplot(
    data=df_top_5_companies,
    x='date',
    y='close',
    hue='company name',
    palette='tab20',
    linewidth=2,
)

plt.title('Close Price Over Time for Top 5 Companies')
plt.xlabel('Date')
plt.ylabel('Close Price')
plt.legend(title='Symbol', loc='upper left', bbox_to_anchor=(1, 1))
plt.xticks(rotation=45)
plt.legend(title='Company Name', bbox_to_anchor=(1.01, 1), loc='upper left', frameon=False, fontsize='small')

sns.despine()
plt.tight_layout()
plt.show()
```



- **Key Insight**

- **Apple Inc.** shows a strong upward trend from 2014 to 2018, significantly outperforming all other companies in terms of stock price growth.
- **General Electric** remained relatively stable until a notable decline after 2017, suggesting potential structural or financial concerns.
- **Bank of America** exhibited a gradual recovery, especially post-2016, reflecting improving investor sentiment or financial performance.
- **Advanced Micro Devices** and **Ford Motor Company** maintained lower price ranges throughout the period, with AMD showing some upward momentum in the latter part of the timeline.
- Overall, the chart highlights **Apple's dominance in stock performance** compared to the other top-volume companies. \_\_\_\_\_

## Moving Average(MA):

- A Moving Average (MA) is a widely used indicator in stock market analysis that helps smooth out price data over a specified time period. It calculates the average closing price of a stock over a set number of days (known as the "window"), updating with each new day.
- There are two common types:
  - Short-Term Moving Average (e.g., 50-day): Reacts faster to recent price changes and shows short-term trends.
  - Long-Term Moving Average (e.g., 200-day): Slower to react, but shows the overall, long-term direction of the stock price.
- Let's prepare the data for calculating moving averages by setting the 'date' column as the index and identifying unique company names. It defines short-term (50-day) and long-term (200-day) window sizes, which will be used to compute moving averages — key indicators in identifying trends in stock prices.

```
df_companies = df[df['symbol'].isin(top_5_symbols)]
df_companies = df_companies.sort_values(by='date')

short_window = 50
long_window = 200

df_companies.copy().set_index('date', inplace=True)
unique_companies = df_companies['company name'].unique()

for company in unique_companies:
    company_data = df_companies[df_companies['company name'] ==
    company].copy()
    company_data['short_mavg'] =
    company_data['close'].rolling(window=short_window,
    min_periods=1).mean()
    company_data['long_mavg'] =
    company_data['close'].rolling(window=long_window,
    min_periods=1).mean()
```

```

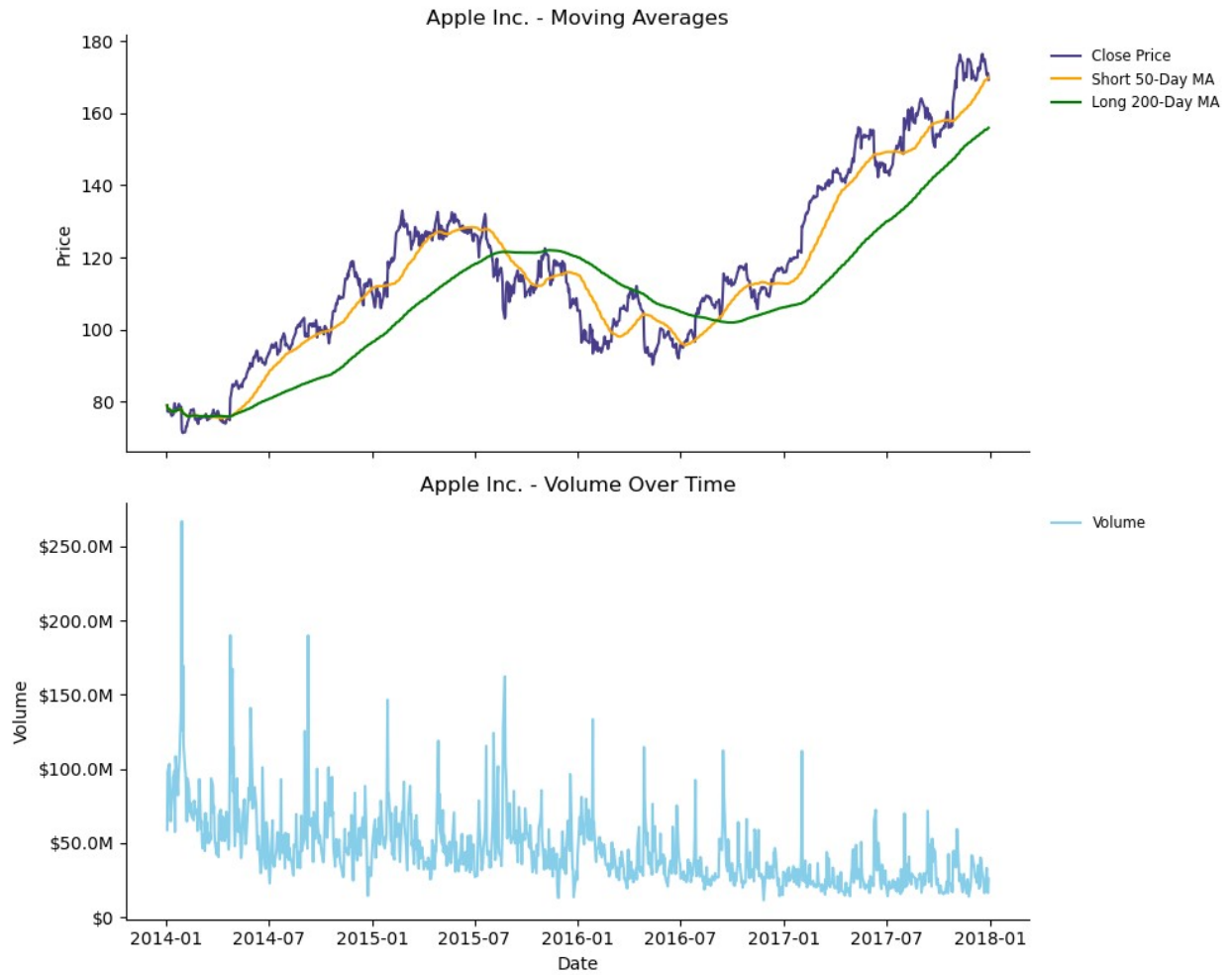
fig, axs = plt.subplots(2, 1, figsize=(10, 8), sharex=True) # 2
rows, 1 col

# First subplot: price and moving averages
sns.lineplot(data=company_data, x='date', y='close', label='Close
Price', color='darkslateblue', ax=axs[0])
sns.lineplot(data=company_data, x='date', y='short_mavg',
label=f'Short {short_window}-Day MA', color='orange', ax=axs[0])
sns.lineplot(data=company_data, x='date', y='long_mavg',
label=f'Long {long_window}-Day MA', color='green', ax=axs[0])
axs[0].set_title(f'{company} - Moving Averages')
axs[0].set_ylabel('Price')
axs[0].legend(bbox_to_anchor=(1.01, 1), loc='upper left',
fontsize='small', frameon=False)
sns.despine(ax=axs[0])

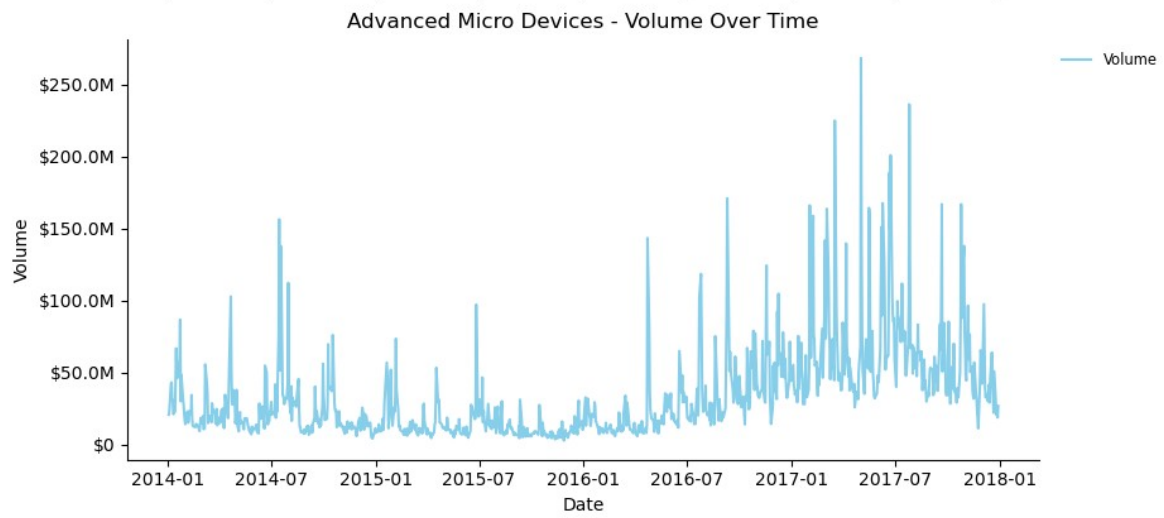
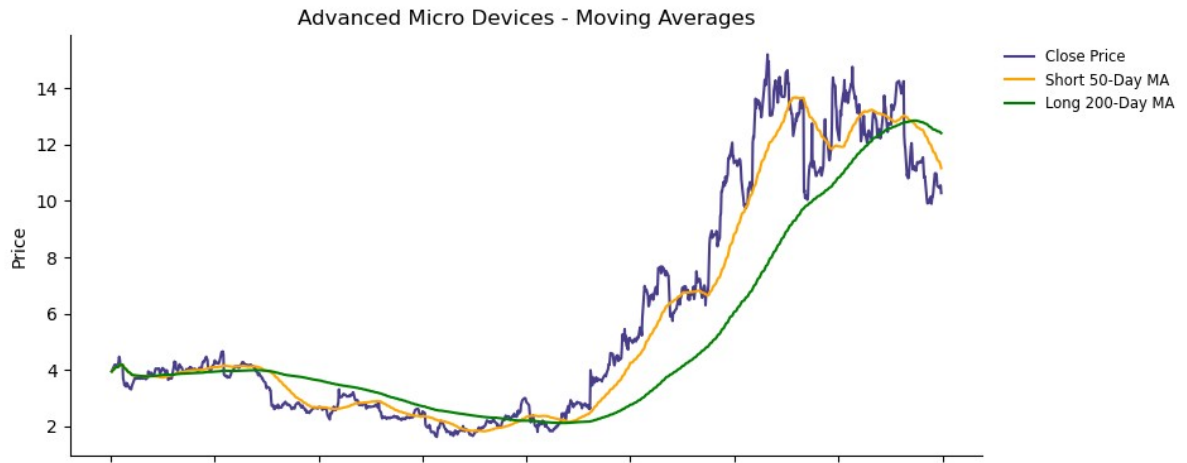
# Second subplot: volume
sns.lineplot(data=company_data, x='date', y='volume',
label='Volume', color='skyblue', ax=axs[1])
axs[1].yaxis.set_major_formatter(FuncFormatter(currency)) # your
currency formatting function
axs[1].set_title(f'{company} - Volume Over Time')
axs[1].set_xlabel('Date')
axs[1].set_ylabel('Volume')
axs[1].legend(bbox_to_anchor=(1.01, 1), loc='upper left',
fontsize='small', frameon=False)
sns.despine(ax=axs[1])

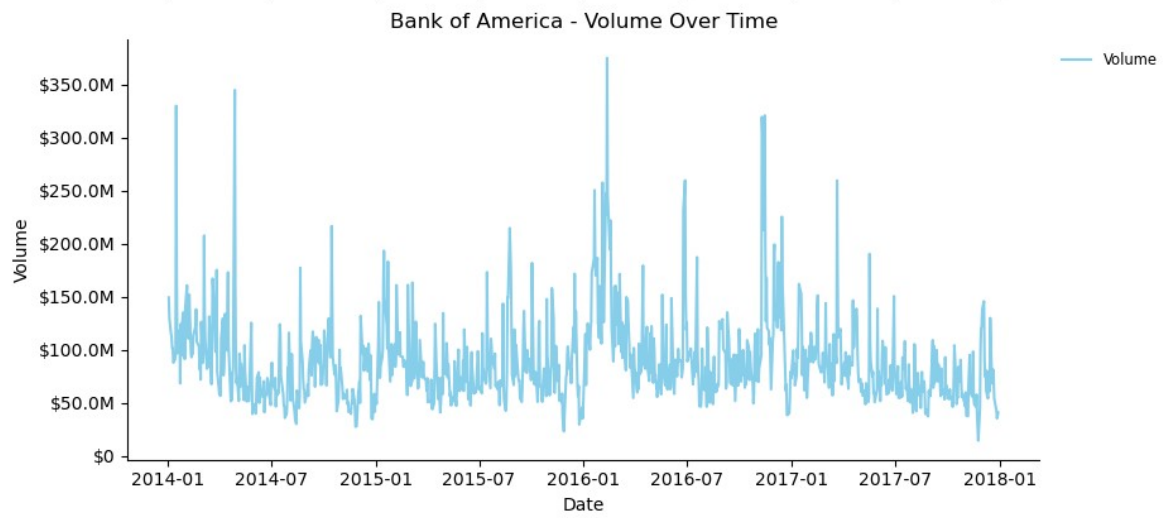
plt.tight_layout()
plt.show()

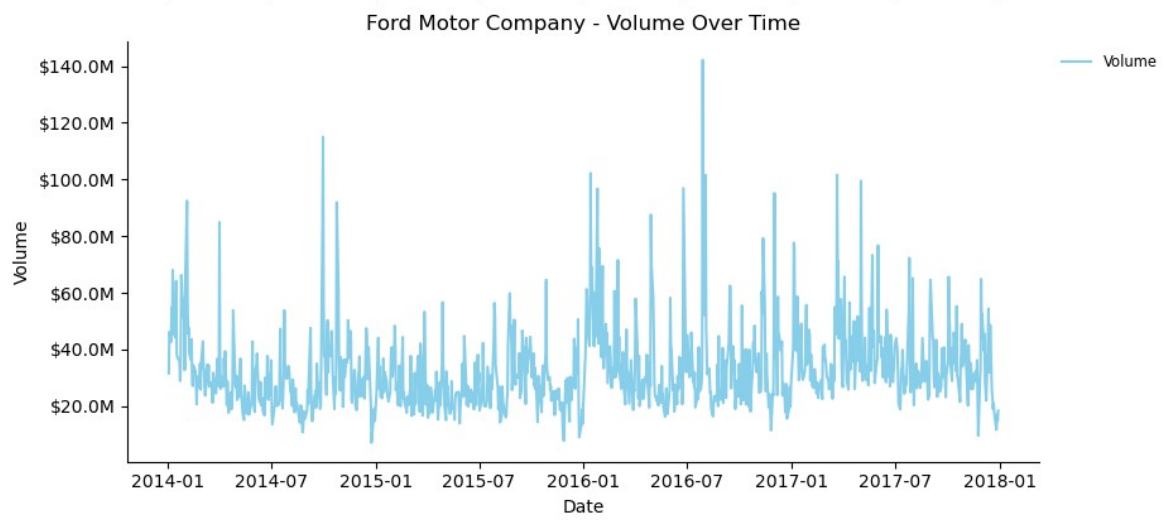
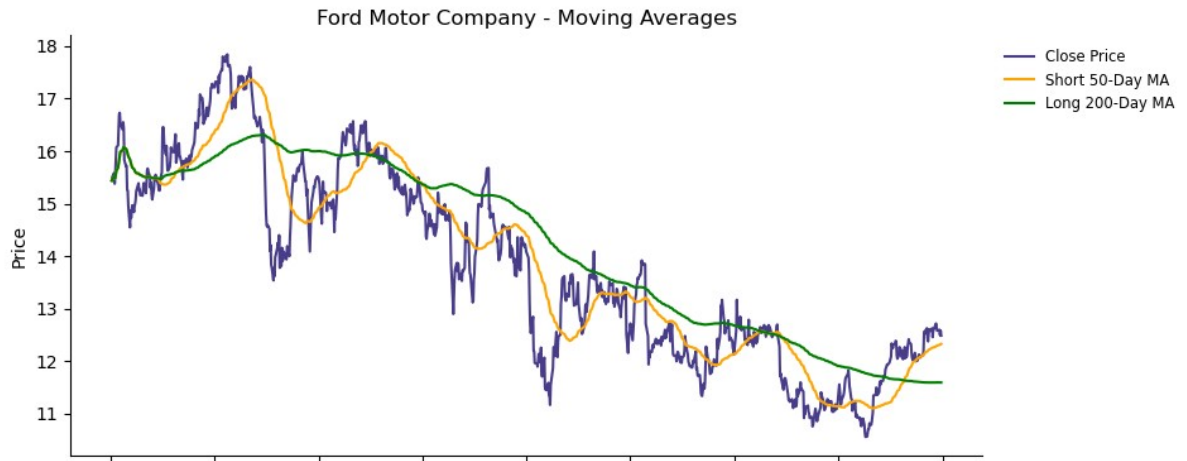
```

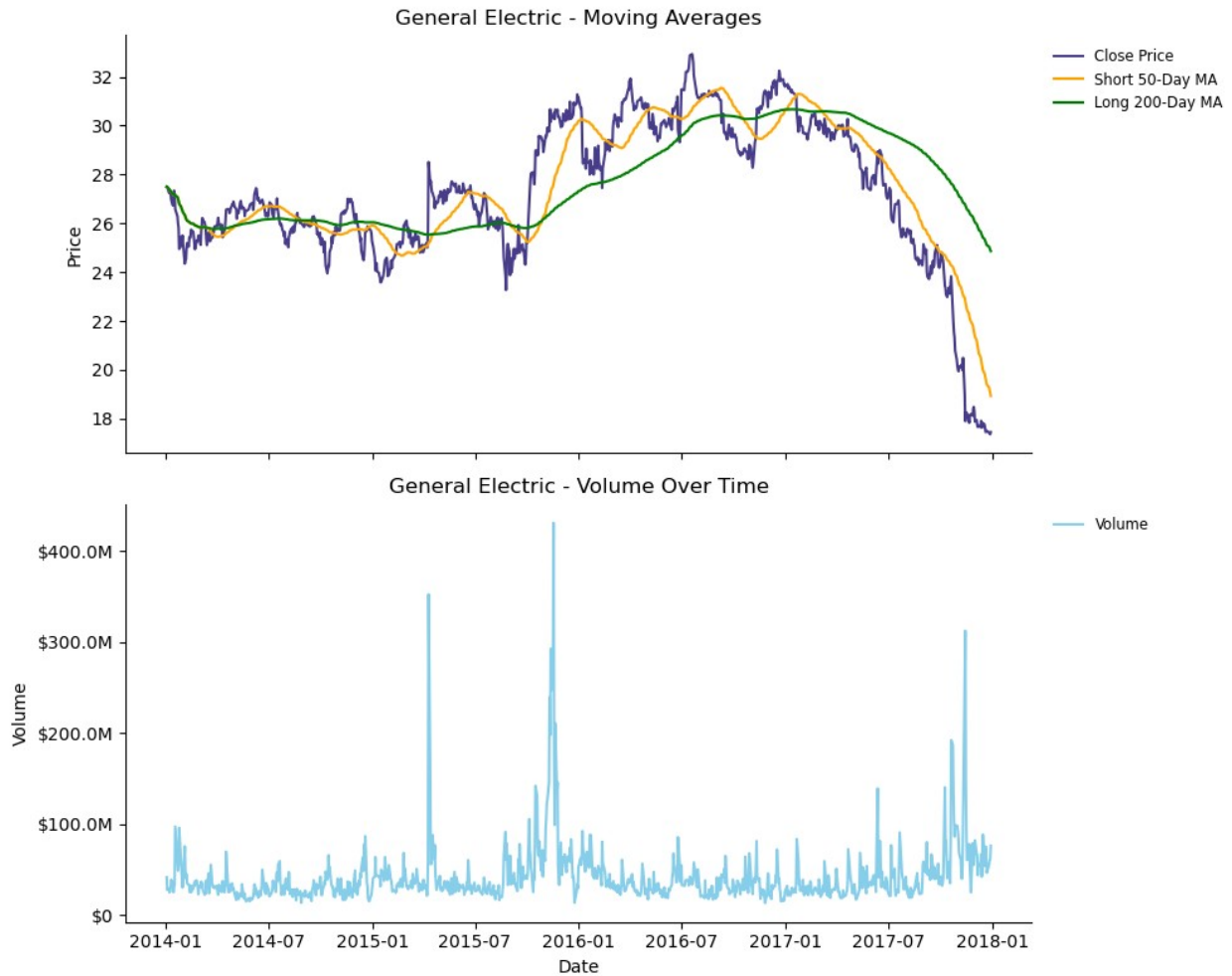












- **Key Insights**

- **Apple Inc. - Moving Averages**

- The **short-term 50-day MA** (orange line) closely tracks price movements and frequently crosses the **long-term 200-day MA** (green line), generating trading signals.
    - Notable **Golden Crosses** (where the 50-day MA crosses above the 200-day MA) occur in mid-2014 and early 2017, both followed by strong upward trends.
    - A **Death Cross** around mid-2015–2016 indicated a temporary downtrend before recovery.
    - The chart highlights how moving averages can effectively signal **trend reversals** and **confirm long-term momentum** in stock prices.

- **Apple Inc. - Volume Over Time**

- Trading **volume shows a clear declining trend** from 2014 to 2018, suggesting reduced short-term trading activity over time.

- Despite a few **sporadic volume spikes**, the overall volume became more stable and lower, possibly reflecting a shift toward longer-term investors or less speculative trading.
- The **early high-volume periods** might be linked to product launches, earnings reports, or broader market events.
- Lower volume in later years, despite rising prices, may indicate **stronger investor confidence and reduced volatility**.
- **AMD - Moving Averages**
  - From 2014 to early 2016, AMD traded sideways with low volatility, as both moving averages remained relatively flat and close.
  - A strong **uptrend began in mid-2016**, marked by a **Golden Cross**, where the 50-day MA crossed above the 200-day MA — a classic bullish signal.
  - AMD's price surged dramatically through 2017, with both MAs trending upward, reinforcing sustained bullish momentum.
  - Toward late 2017, a **Death Cross** began to form, where the 50-day MA dipped below the 200-day MA, indicating possible trend reversal or correction.
  - The chart effectively captures AMD's **transition from stagnation to high growth**, with moving averages offering valuable confirmation signals.
- **AMD - Volume Over Time**
  - **Volume remained relatively low and stable** from 2014 to early 2016, reflecting limited investor interest or market movement.
  - A **clear surge in volume** begins in mid-2016 and intensifies through 2017, aligning with AMD's strong price rally during the same period.
  - Multiple **volume spikes** in 2017 suggest periods of heightened investor activity, likely due to earnings reports, product releases, or broader semiconductor sector momentum.
  - The increasing volume supports the price trend shown in the MA chart, indicating strong **market participation during AMD's growth phase**.
- **Bank of America - Moving Averages**
  - From 2014 to mid-2016, Bank of America's price experienced **sideways movement** with modest volatility, and the moving averages frequently converged, indicating market indecision.
  - A **significant downtrend** in early 2016 is marked by the 50-day MA crossing below the 200-day MA (a bearish signal).
  - A **strong uptrend began mid-2016**, with a notable **Golden Cross** where the 50-day MA crossed above the 200-day MA, supported by a sharp price breakout.
  - From late 2016 through 2017, the gap between the short and long MAs widened, confirming **sustained bullish momentum**.
  - This moving average analysis highlights the shift from consolidation to breakout, helping to time trend entries effectively.
- **Bank of America - Volume Over Time**

- Trading volume fluctuates, with **peaks at various points**, suggesting heightened investor activity.
  - The highest spikes occur **in mid-2016 and early 2017**, possibly due to major financial events or market shifts.
  - Volume activity does not show a clear trend but reflects **periodic surges in interest**.
  - **Ford Motor Company - Moving Averages**
    - The stock price shows **fluctuations** between **\$11 and \$18** from 2014 to 2018.
    - The **50-day moving average (orange line)** follows the stock price closely, reflecting short-term trends.
    - The **200-day moving average (green line)** smooths out volatility, highlighting long-term movements.
    - When the stock price crosses the moving averages, it can signal **potential buying or selling opportunities**.
  - **Ford Motor Company - Volume Over Time**
    - Trading volume ranges from **\$0M to \$140M**, with **significant fluctuations** over time.
    - Peak trading activity appears at **various points**, possibly indicating major news or investor sentiment shifts.
    - The volume trends don't show a clear uptrend or downtrend, suggesting **variable market interest** in Ford.
  - **General Electric – Moving Averages & Volume Insight**
    - From 2014 to 2016, GE's price remained relatively stable with **short fluctuations** around the 50-day and 200-day moving averages.
    - A **clear bullish phase** emerged mid-2016, when the 50-day MA crossed above the 200-day MA and the stock climbed toward \$32.
    - However, in **early 2017**, the price reversed sharply, and the 50-day MA **dropped below** the 200-day, confirming a **bearish trend**.
    - The stock experienced a **steep decline throughout 2017**, closing near \$18 by the end of the year.
    - Volume spikes—especially during sharp declines—suggest **panic selling or institutional unloading**, supporting the downtrend signal.
    - Overall, the trend shows a **strong bearish reversal** after a short-lived rally, confirmed by both moving averages and volume surges.
- 

## Daily Return:

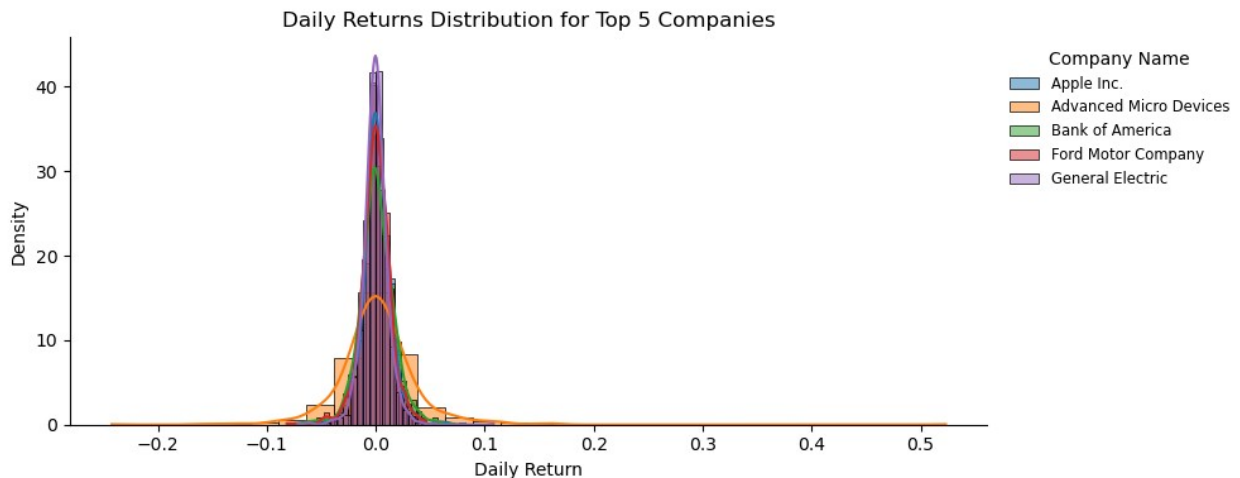
- Daily Return is the percentage change in a stock's price from one day to the next. It tells you how much the stock gained or lost in value each day, expressed as a percentage.
- Now, I will calculate the daily percentage return for each company's stock by computing the percentage change between consecutive closing prices. This helps in analyzing the stock's volatility and performance over time.

```

df_companies['daily_return'] = df_companies.groupby('company name')
['close'].pct_change()

plt.figure(figsize=(10, 4))
for company in unique_companies:
    company_data = df_companies[df_companies['company name'] ==
company]
    sns.histplot(
        data=company_data,
        x='daily_return',
        label=company,
        kde=True,
        stat='density',
        bins=30,
        edgecolor='black',
        linewidth=0.5,
        palette='Set1',
        fill=True
    )
plt.title('Daily Returns Distribution for Top 5 Companies')
plt.xlabel('Daily Return')
plt.ylabel('Density')
plt.legend(title='Company Name', bbox_to_anchor=(1.01, 1), loc='upper
left', frameon=False, fontsize='small')
sns.despine()
plt.tight_layout()
plt.show()

```



## • Key Insights

- Most Returns Are Clustered Around Zero
  - The peak of each distribution is centered around **0% daily return**, indicating that most days, the stock prices of these companies do not

experience significant gains or losses.

- This suggests that small price fluctuations are common, while extreme movements are rare.
  - Symmetry (or Lack Thereof) in Distributions
    - Some stocks show **symmetrical distributions** (e.g., Apple Inc.), suggesting balanced volatility.
    - Others show **skewness**:
      - **Right-skewed**: More frequent small losses but occasional large gains.
      - **Left-skewed**: More frequent small gains but occasional large losses.
    - Example: **Advanced Micro Devices** shows a slight right skew.
  - Volatility Differences Between Companies
    - The **width** of the distribution reflects **volatility**:
      - **Wider distributions** = higher volatility (e.g., General Electric, Advanced Micro Devices)
      - **Narrower distributions** = lower volatility (e.g., Bank of America)
  - Outliers and Extreme Returns
    - **Longer tails** indicate a higher probability of extreme returns:
      - **General Electric** has long tails → more risk/reward potential
      - **Ford Motor Company** has shorter tails → fewer extreme events
  - Risk Assessment
    - **High volatility stocks**: Riskier investments with potential for larger gains/losses.
    - **Low volatility stocks**: Safer investments with steadier returns.
- 

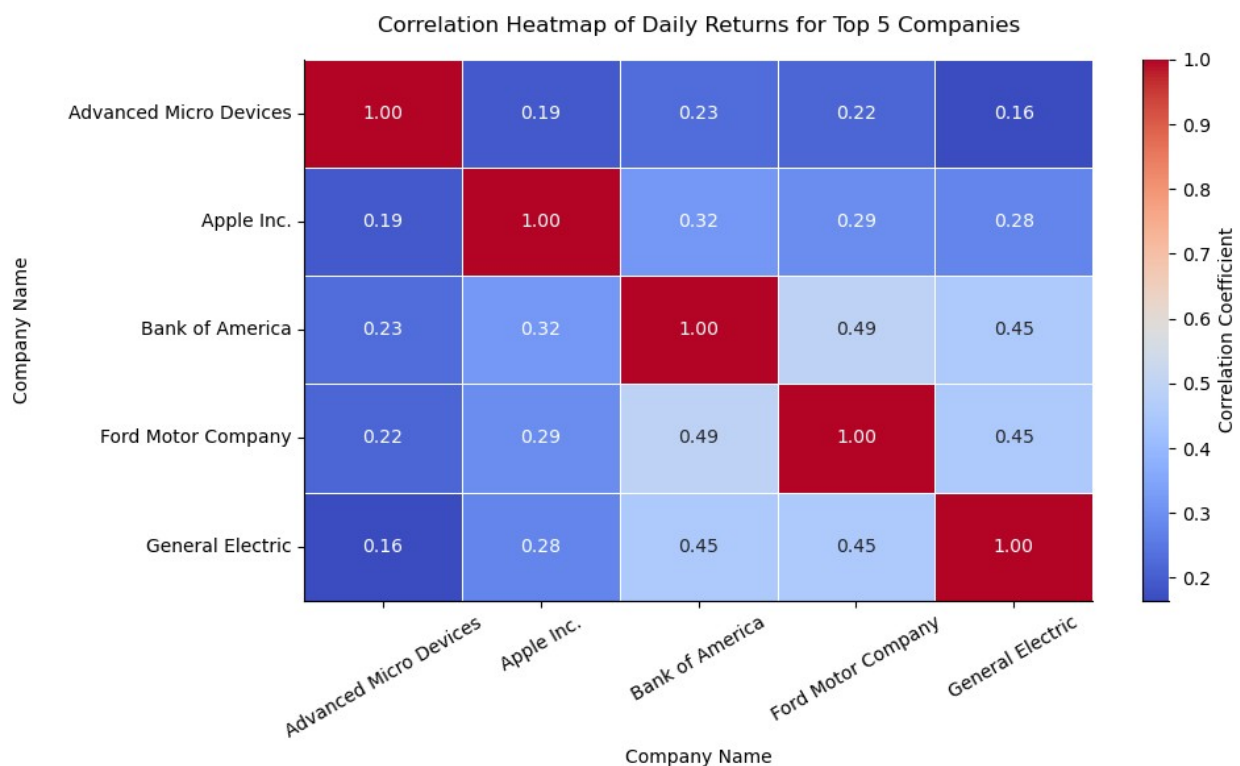
## Correlation:

- A high correlation (close to 1) means two stocks tend to move in the same direction, while a low or negative value indicates they move independently or in opposite directions. This helps in understanding diversification opportunities and portfolio risk.
- Now, let's visualize the **correlation matrix** of daily returns among the top 5 companies.

```
daily_returns = df_companies.pivot_table(  
    index='date',  
    columns='company name',  
    values='daily_return'  
)  
daily_returns = daily_returns.dropna()
```



```
plt.figure(figsize=(10, 6))
sns.heatmap(
    daily_returns.corr(),
    annot=True,
    cmap='coolwarm',
    fmt='.2f',
    linewidths=0.5,
    cbar_kws={'label': 'Correlation Coefficient'}
)
plt.title('Correlation Heatmap of Daily Returns for Top 5 Companies',
pad=15)
plt.xlabel('Company Name')
plt.ylabel('Company Name', labelpad=20)
plt.xticks(rotation=30)
sns.despine()
plt.tight_layout()
plt.show()
```



- **Key Insights**

- Self-Correlation (Diagonal Values)
  - Each company has a perfect correlation with itself (**1.00**), as expected.
  - This confirms that each stock's daily return is perfectly aligned with itself.
- Positive Correlations
  - **Strongest Pair: Bank of America ↔ Ford Motor Company (0.49)**

- These two stocks tend to move together, suggesting similar exposure to market or sector factors.
  - **General Electric ↔ Ford Motor Company (0.45)**
    - Also shows a strong positive relationship.
  - **Apple Inc. ↔ Bank of America (0.32)**
    - Indicates moderate co-movement between tech and finance sectors.
  - Weak or Low Correlations
    - **Advanced Micro Devices** shows low correlations with all other companies:
      - **Apple Inc. (0.19)**
      - **Bank of America (0.23)**
      - **Ford Motor Company (0.22)**
      - **General Electric (0.16)**
    - **Apple Inc. ↔ General Electric (0.28)**
      - Suggests limited shared movement between these two.
  - Diversification Opportunities
    - **Low-correlation assets** like **Advanced Micro Devices** are valuable for diversifying a portfolio.
    - **Highly correlated pairs** (e.g., Bank of America and Ford Motor Company) may not significantly reduce risk if held together.
  - Market Behavior Insights
    - The heatmap reveals mixed behavior — some stocks move together due to common influences (industry, macroeconomic factors), while others behave independently.
    - Understanding these patterns helps identify which stocks respond similarly to market events.
- 

## Optimization:

- I will calculate the **annualized expected returns** and **volatility (risk)** for each stock based on historical daily returns.
  - **\*\*Expected\_Returns\*\***: The average daily return is multiplied by 252 to annualize it (assuming 252 trading days in a year).
  - **\*\*Volatility\*\***: The standard deviation of daily returns is multiplied by the square root of 252 to annualize it.
  - These values are then combined into a DataFrame showing each company's expected return and associated risk.

```
expected_returns = daily_returns.mean() * 252 # annualize the returns
volatility = daily_returns.std() * np.sqrt(252) # annualize the volatility

stock_stats = pd.DataFrame({
    'Expected Return': expected_returns,
```

```

    'Volatility': volatility
}))

stock_stats.round(4)

{"columns":[{"name":"company", "rawType":"object", "type":"string"}, {"name":"Expected Return", "rawType":"float64", "type":"float"}, {"name":"Volatility", "rawType":"float64", "type":"float"}], "ref":"f9c7b770-882b-4734-b2dd-e34976422f4d", "rows":[["Advanced Micro Devices", "0.4245", "0.6207"], ["Apple Inc.", "0.2166", "0.2271"], ["Bank of America", "0.1846", "0.2558"], ["Ford Motor Company", "-0.0288", "0.2201"], ["General Electric", "-0.0963", "0.1881"]], "shape":{"columns":2, "rows":5}}

```

- **Insights:**

- **Advanced Micro Devices** offers the **highest expected return (42.45%)**, but it also comes with the **highest volatility (62.07%)**, making it the riskiest investment.
- **Apple Inc.** and **Bank of America** show **positive returns** with **moderate volatility**, indicating a more balanced risk-return profile.
- **Ford Motor Company** and **General Electric** have **negative expected returns**, suggesting declining trends in their stock prices over the period analyzed.
- All companies exhibit varying levels of risk, which should be considered alongside return expectations when constructing an optimized portfolio.

```

# function to calculate portfolio performance
def portfolio_performance(weights, returns, cov_matrix):
    portfolio_return = np.dot(weights, returns)
    portfolio_volatility = np.sqrt(np.dot(weights.T,
np.dot(cov_matrix, weights)))
    return portfolio_return, portfolio_volatility

# number of portfolios to simulate
num_portfolios = 10000

# arrays to store the results
results = np.zeros((3, num_portfolios))

# annualized covariance matrix
cov_matrix = daily_returns.cov() * 252

np.random.seed(42)

for i in range(num_portfolios):
    weights = np.random.random(len(unique_compnies))
    weights /= np.sum(weights)

    portfolio_return, portfolio_volatility =
portfolio_performance(weights, expected_returns, cov_matrix)

```

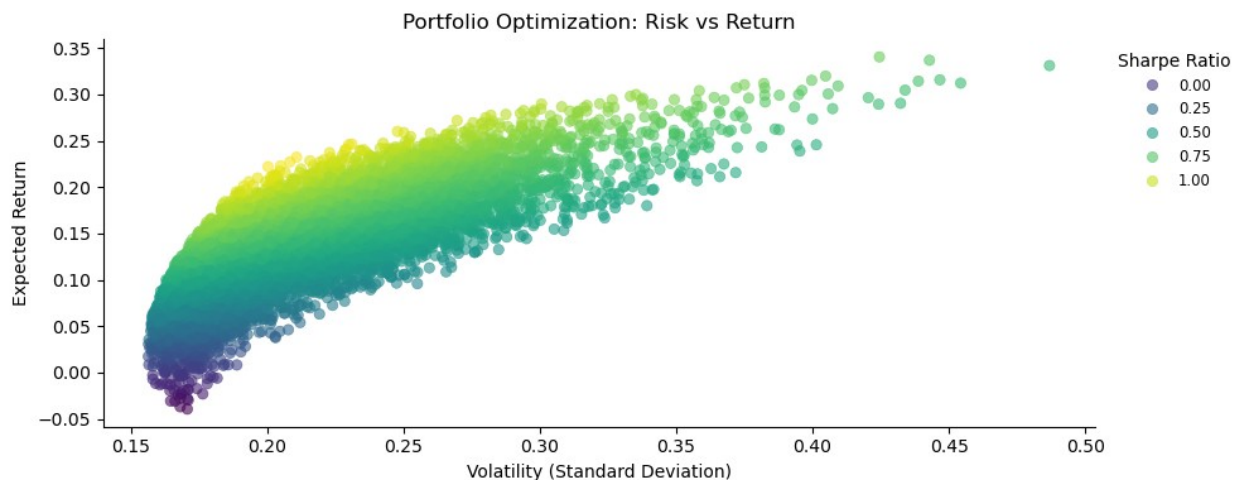
```

    results[0,i] = portfolio_return
    results[1,i] = portfolio_volatility
    results[2,i] = portfolio_return / portfolio_volatility # Sharpe
Ratio

plt.figure(figsize=(10, 4))
sns.scatterplot(
    x=results[1,:],
    y=results[0,:],
    hue=results[2,:],
    palette='viridis',
    alpha=0.6,
    edgecolor=None
)

plt.title('Portfolio Optimization: Risk vs Return')
plt.xlabel('Volatility (Standard Deviation)')
plt.ylabel('Expected Return')
plt.legend(title='Sharpe Ratio', bbox_to_anchor=(1.01, 1), loc='upper
left', frameon=False, fontsize='small')
#plt.grid()
sns.despine()
plt.tight_layout()
plt.show()

```



### Key Insights:

- Risk-Return Tradeoff
  - The plot shows a clear **tradeoff between risk and return**:
    - **Left side**: Lower volatility, lower expected returns
    - **Right side**: Higher volatility, higher expected returns

- This aligns with the principle that **higher returns typically come with higher risks.**
  - Efficient Frontier
    - The **upper boundary** of the scatter plot represents the **efficient frontier** — the set of portfolios that offer the **highest expected return for a given level of risk.**
    - These portfolios are **optimal**, maximizing returns without unnecessary risk.
  - Sharpe Ratio as a Measure of Efficiency
    - The **color gradient** indicates the **Sharpe Ratio**, which measures **return per unit of risk**:
      - **Darker colors (purple/blue)** = lower Sharpe Ratios (poor risk-adjusted performance)
      - **Lighter colors (yellow/green)** = higher Sharpe Ratios (better risk-adjusted performance)
    - The **green-yellow region** along the efficient frontier contains the most desirable portfolios.
  - High-Risk, High-Return Portfolios
    - On the **top-right corner**:
      - Portfolios with **high volatility (>0.45)** and **high returns (>0.30)**
      - Suitable for **aggressive investors** who can tolerate high risk for potential reward
  - Low-Risk, Low-Return Portfolios
    - On the **bottom-left corner**:
      - Portfolios with **low volatility (<0.20)** and **low returns (<0.10)**
      - Ideal for **risk-averse investors** who prioritize stability over growth
  - Optimal Portfolio Region
    - The **region with the highest Sharpe Ratio** (lightest green/yellow) is where the **optimal portfolios** lie:
      - These balance **risk and return** effectively
      - Investors should focus on these when constructing their portfolio
  - Outliers and Extreme Points
    - Some portfolios appear as **outliers**:
      - **Very high-risk, very low-return portfolios** → inefficient, avoid them
      - **Very low-risk, very high-return portfolios** → likely unrealistic or anomalous
- 
- Now, let's identify the **portfolio with the highest Sharpe Ratio** from the 10,000 simulated portfolios.

```

max_sharpe_idx = np.argmax(results[2])
max_sharpe_return = results[0, max_sharpe_idx]
max_sharpe_volatility = results[1, max_sharpe_idx]
max_sharpe_ratio = results[2, max_sharpe_idx]

max_sharpe_return, max_sharpe_volatility, max_sharpe_ratio
(0.2219682245380699, 0.20037189025807617, 1.107781257401814)

```

- **Expected Annual Return:** 22.20%
- **Annual Volatility (Risk):** 20.04%
- **Sharpe Ratio:** 1.11

This portfolio offers the **best risk-adjusted return** among all simulated portfolios. With a Sharpe Ratio of **1.11**, it delivers a strong balance between return and risk, making it a strong candidate for investment based on modern portfolio theory.

- Let's identify the **portfolio weights** that correspond to the **maximum Sharpe Ratio** found earlier.

```

max_sharpe_weights = np.zeros(len(unique_compnies))

for i in range(num_portfolios):
    weights = np.random.random(len(unique_compnies))
    weights /= np.sum(weights)

    portfolio_return, portfolio_volatility =
portfolio_performance(weights, expected_returns, cov_matrix)

    if results[2, i] == max_sharpe_ratio:
        max_sharpe_weights = weights
        break

portfolio_weights_df = pd.DataFrame({
    'Ticker': unique_compnies,
    'Weight': max_sharpe_weights
})

portfolio_weights_df.round(4)

{"columns":[{"name":"index","rawType":"int64","type":"integer"},
{"name":"Ticker","rawType":"object","type":"string"},
{"name":"Weight","rawType":"float64","type":"float"}],"ref":"16f78a12-
0708-4870-9904-7db5922dca32","rows":[["0","Apple Inc.,"0.2996"],
["1","Advanced Micro Devices","0.1751"],["2","Bank of
America","0.0669"],["3","Ford Motor Company","0.2287"],["4","General
Electric","0.2296"]],"shape":{"columns":2,"rows":5}}

```

- This table shows the **optimal allocation of funds** across the top 5 companies that yields the **highest Sharpe Ratio**, meaning the best return per unit of risk.
  - **Apple Inc.** has the highest weight (**~30%**), indicating it contributes significantly to balancing risk and return.
  - **Advanced Micro Devices** is allocated **~17.5%**, reflecting its higher return but also higher volatility.
  - **Ford Motor Company** and **General Electric** receive similar weights (**~23% each**), suggesting they contribute to diversification.
  - **Bank of America** receives the smallest weight (**~6.7%**) due to its moderate return and relatively higher volatility compared to others.

This optimal portfolio provides a data-driven strategy for investors aiming to maximize returns while managing risk effectively.

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