

Final Project

Capturing volatility decay in leveraged ETFs

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Overview

Volatility decay refers to the erosion of returns experienced by leveraged exchange-traded funds (ETFs) due to the compounding effects of daily price movements. Leveraged ETFs aim to amplify the returns of an underlying index or asset by a specific factor, such as 2x or 3x. For example, a 2x leveraged ETF aims to deliver twice the daily return of its underlying asset (Chen, 2024). However, this amplification only holds true on a day-to-day basis, and over longer periods, the effects of volatility can cause the ETF's returns to deviate significantly from its intended multiple.

The root cause of volatility decay lies in how leveraged ETFs rebalance their positions daily to maintain their target leverage. This rebalancing process amplifies not only the returns but also the compounding of gains and losses. When markets experience fluctuations, the leveraged ETF must adjust its holdings to maintain the desired leverage, and the compounding of these daily adjustments introduces a divergence between the ETF's performance and the long-term return of its underlying index. This divergence is exacerbated by high levels of volatility.

To illustrate this, consider a simplified example: If an index gains 5% on one day and loses 5% the next, its cumulative return would be slightly negative. However, for a 2x leveraged ETF tracking that index, the compounding effect over the same period would result in a larger negative return due to the ETF's amplified daily fluctuations. As volatility increases, these compounded daily changes lead to greater discrepancies between the ETF's actual performance and the expected long-term multiple of its underlying asset (Avellaneda & Zhang, 2010).

Investigated Securities:

Security	Long 2x	Short 2x
Gold (GC=F)	UGL	GLL
Silver (SI=F)	AGQ	ZSL
Oil (CL=F)	UCO	SCO
Natural Gas (NG=F)	BOIL	KOLD
Gold Miners (GDX)	NUGT	DUST
S&P 500 (SPX)	SSO	SDS

Data Collection and Testing

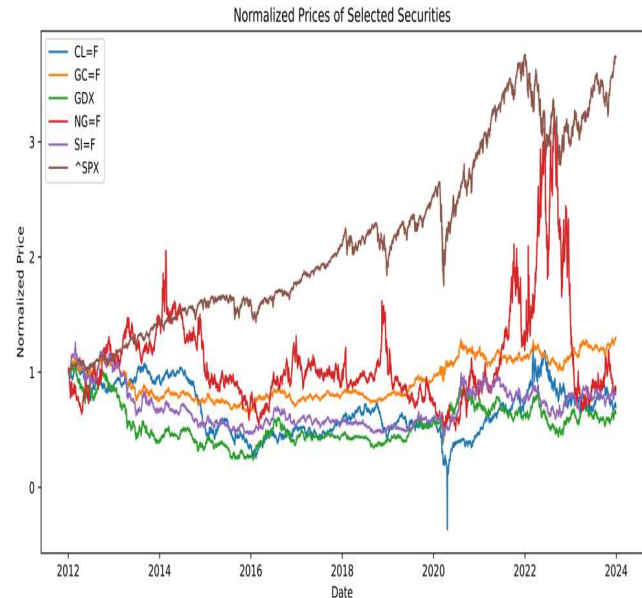
Due to data availability, a period of 12-years of historical prices is analyzed using data obtained from yahoo finance. The data sample is split for all optimization calculations into a training period from January 2012 to January 2022, and a testing period from January 2022 to January 2024. This ensures that the optimization is not overfit to the data and assumes live practice where a trader is unaware of future price returns for an underlying. Additionally, it allows for algorithm testing, wherein an algorithm will trade following a set of rules trained on previous data. The entire project has been coded in Python for scalability and the repo can be found at the following link: <https://github.com/moe1221/FNCE449>

Assumptions:

1. Fractional Contracts
2. No transaction costs
3. No borrowing costs

Analyzing Selected Assets

- **Gold (GC=F):** Gold futures represent a contract to buy or sell gold at a predetermined price and date. Gold is widely viewed as a safe-haven asset, attracting investors during market volatility or inflationary periods due to its intrinsic value and hedge potential.
- **Silver (SI=F):** Silver futures enable trading in the global market with a contract to buy or sell silver at a future date and agreed price. Silver is both an industrial and precious metal, leading to its price being influenced by economic activity, industrial demand, and investment appeal.
- **Oil (CL=F):** Crude oil futures track the price of West Texas Intermediate (WTI) crude, a global benchmark. Oil prices are driven by supply and demand factors, geopolitical events, and macroeconomic conditions, making them highly volatile and a key economic indicator.
- **Natural Gas (NG=F):** Natural gas futures involve contracts for purchasing or selling natural gas at a specified price in the future. Prices are influenced by seasonal demand, supply dynamics, and shifts in energy consumption, making it an essential commodity in energy markets.
- **Gold Miners (GDX):** The GDX ETF tracks a basket of gold mining companies, offering investors indirect exposure to gold prices through the profitability of mining operations. It is influenced by gold prices, operational costs, and geopolitical risks impacting mining regions.
- **S&P 500 (SPX):** The S&P 500 is a market-cap-weighted index representing 500 leading publicly traded companies in the U.S. It is widely considered a barometer of the U.S. stock market's performance and economic health, reflecting broad market trends.



After collecting data for the selected assets—Gold, Silver, Oil, Natural Gas, Gold Miners, and the S&P 500—from January 2012 to January 2022, the next step involves analyzing their returns to gain insights into their behavior and risk profiles over this decade-long period. We will calculate the daily returns for each asset and subsequently determine the mean and standard deviation of these returns. The mean return will provide an understanding of the average performance of each asset over the analyzed period, indicating their potential for generating returns. Meanwhile, the standard deviation will measure the volatility of these returns, offering insight into the risk associated with each asset.

Security	Daily Mean	Daily Standard Deviation
Gold (GC=F)	0.000132	0.009772
Silver (SI=F)	0.000093	0.018035
Oil (CL=F)	-0.000986	0.066178
Natural Gas (NG=F)	0.000582	0.035970
Gold Miners (GDX)	0.000154	0.024441
S&P 500 (SPX)	0.000494	0.010649

Security	Annual Mean	Annual Standard Deviation
Gold (GC=F)	0.033274	0.155128
Silver (SI=F)	0.023323	0.286295
Oil (CL=F)	-0.248377	1.050541
Natural Gas (NG=F)	0.146784	0.571009
Gold Miners (GDX)	0.038919	0.387990
S&P 500 (SPX)	0.124399	0.169050

The mean values indicate average returns, with Oil showing a negative daily and annual mean, while Natural Gas exhibits the highest annual volatility. The standard deviations highlight the risk, with Oil and Natural Gas having the most significant price fluctuations. These metrics will be central to our analysis in assessing risk-return dynamics and exploring strategies for profitability.

Rationale for Studying Shorting Strategies on Leveraged ETFs

The decision to explore shorting strategies on both long and short leveraged ETFs of selected securities—Gold, Silver, Oil, Natural Gas, Gold Miners, and the S&P 500—is motivated by the inherent nature of volatility decay associated with these financial instruments. Leveraged ETFs, designed to deliver multiples of the daily returns of an underlying asset, undergo a compounding effect that significantly impacts their value over time, especially in volatile markets. This compounding, while intended to amplify daily gains, conversely accelerates losses in value during fluctuating market conditions due to the daily rebalancing requirement. As these instruments reset their leverage on a daily basis, their value can erode quickly in the presence of frequent market swings, irrespective of the asset's longer-term price direction. The structural nature of this decay creates an opportunity for traders to generate alpha by taking short positions in both the long and short leveraged ETFs.

For assets like Silver, Oil, and Natural Gas, which often do not exhibit a consistent upward trend or are subject to high price volatility, both the long and short leveraged ETFs typically converge towards zero over the long term. This convergence is a product of the erosion in value caused by the daily leverage adjustments against the backdrop of market volatility, a phenomenon referred to as volatility decay. In essence, each daily reset amplifies losses during choppy market conditions, which gradually diminishes the net asset value of both leveraged ETFs. By initiating short positions in both long and short leveraged ETFs, the strategy aims to profit from this inevitable decay, exploiting the structural decline in these ETFs' prices regardless of market direction. This approach not only captures returns based on volatility but also hedges against directional market risk, as both ETFs in the pair are expected to experience similar compounding effects over time (Sackley, 2009).

However, the notable exception in this selection is the S&P 500, which tends to display a persistent upward trend over long periods. In this scenario, the long leveraged ETFs might exhibit growth, diverging from the typical decay pattern seen in other securities. This divergence occurs as the S&P 500's steady upward trajectory counters the decay effect in the long leveraged ETF, while exacerbating losses in the short ETF. Such patterns underscore the importance of understanding the interaction between volatility and directional market trends in designing an effective shorting strategy. Nonetheless, the analysis of shorting both leveraged directions remains critical to understanding and quantifying the conditions under which such a strategy can be profitable, and when it might pose risks. In particular, identifying assets with frequent price fluctuations and limited directional bias can enhance the effectiveness of shorting strategies targeting leveraged ETFs.

This study aims to provide a comprehensive framework for exploiting volatility decay through short positions, offering a strategic approach to leveraging the predictable decline in leveraged ETFs over time. By studying a variety of assets with different volatility profiles and market trends, the research seeks to establish guidelines for identifying suitable targets for shorting strategies. The focus will be on evaluating the decay patterns and profitability in different market conditions, as well as examining the impact of rebalancing thresholds and other variables on strategy performance. Additionally, this research explores the trade-offs between profitability and risk mitigation by comparing strategies that employ static short positions against those that incorporate dynamic rebalancing based on price thresholds.

For detailed charts illustrating the price behavior of both long and short leveraged ETFs, refer to Appendix A. The appendix contains visual representations of normalized price trajectories for each selected security, highlighting the compounding effect and long-term convergence patterns. These charts serve as a visual demonstration of how volatility decay affects leveraged ETFs over time, showcasing the contrasting price behaviors of leveraged ETFs for assets with consistent trends versus those with more volatile, range-bound movements. The appendix also includes key statistics and return measures for each asset, further substantiating the rationale behind employing shorting strategies on leveraged ETFs as a means to exploit predictable market inefficiencies.

Strategy 1: Shorting Both Long and Short Leveraged ETFs Without Rebalancing

The first strategy is a straightforward approach to exploiting the phenomenon of volatility decay by simultaneously shorting both the long and short leveraged ETFs of each selected asset without rebalancing. The primary assumption behind this strategy is that the structural design of leveraged ETFs, which involves daily rebalancing to achieve a multiple of the underlying asset's returns, inherently leads to a decline in value over time. This is especially true in volatile or

range-bound markets where frequent fluctuations erode the compounded returns of both the long and short ETFs. By maintaining short positions in both ETFs for an extended period, the strategy aims to capture the decline in value due to this continuous erosion.

A key advantage of this strategy is its simplicity and minimal execution requirements. With no rebalancing involved, there are significantly lower transaction costs and fewer opportunities for human error or slippage during trading. Additionally, this approach eliminates the need to constantly monitor market movements, making it easier to implement and maintain over the entire study period. However, this lack of rebalancing does introduce potential risks, particularly in scenarios where one ETF experiences a prolonged and strong directional trend, causing a significant divergence between the long and short positions.

The effectiveness of this strategy largely depends on the volatility profile of the underlying assets. Assets such as Gold, Silver, Oil, and Natural Gas, which often exhibit substantial price fluctuations without a clear directional trend over time, are ideal candidates for this approach. In these cases, both the long and short leveraged ETFs are likely to experience substantial decay, leading to potential profits from maintaining short positions. However, assets like the S&P 500, which tend to show a long-term upward trend, may pose challenges as the long leveraged ETFs could potentially gain value over time, undermining the profitability of the strategy.

Despite its passive nature, this strategy's core strength lies in its ability to exploit the predictable decay caused by the inherent structure of leveraged ETFs. By holding short positions without rebalancing, the strategy aims to capture a gradual decline in both the long and short ETFs over the entire period, leveraging the market's tendency to experience periods of volatility and price reversals. This approach is particularly suitable for traders who seek to minimize complexity and rely on the structural inefficiencies of leveraged ETFs to generate consistent returns over the long term.

Strategy 2: Shorting Both Leveraged ETFs with a Dynamic Rebalancing Approach

The second strategy takes a more dynamic approach by introducing a threshold-based rebalancing mechanism. In this strategy, short positions are taken in both the long and short leveraged ETFs of each selected asset, but with a key difference: the positions are adjusted whenever the price of either ETF crosses predefined upper or lower thresholds. The rationale behind this approach is to provide an adaptive framework that allows for greater flexibility in responding to market movements and mitigating potential risks from strong directional trends in either ETF.

This strategy operates under the assumption that while leveraged ETFs are prone to volatility decay, sudden and sustained price trends could temporarily boost the value of one of the ETFs, leading to potential losses in an underbalanced short position. By setting threshold levels for rebalancing, the strategy aims to reduce the risk of significant losses during such market conditions. For example, if a long leveraged ETF surpasses an upper threshold due to a strong upward trend in the underlying asset, rebalancing would involve adjusting the short position to limit exposure to further gains. Conversely, if an ETF falls below a lower threshold, indicating substantial decay, rebalancing would aim to lock in profits and maintain a balanced short position.

The dynamic nature of this strategy allows it to better withstand prolonged market trends and sharp price movements, which may otherwise undermine a passive shorting strategy. However, this increased adaptability comes at the cost of complexity and higher transaction costs due to more frequent trading. It also requires careful selection of threshold levels to strike the right balance between mitigating risk and capturing the decay effect. Setting thresholds too wide could reduce the effectiveness of risk management, while overly narrow thresholds could result in excessive rebalancing and erode profits through trading costs.

Overall, this strategy offers a more active and controlled approach to profiting from volatility decay. It provides the flexibility to adapt to market conditions while still leveraging the predictable decline in leveraged ETFs due to their structural design. Traders implementing this strategy must be prepared to manage the increased complexity and transaction costs, but the potential benefits of improved risk management and higher returns in volatile markets could outweigh these challenges. This strategy is particularly well-suited for assets that exhibit sharp price swings or prolonged trends, where a passive approach might not be as effective.

Variables Affecting the Profitability of Strategy 1

In Strategy 1, where short positions are held in both the long and short leveraged ETFs without rebalancing, the primary variable influencing profitability is the asset's volatility relative to its price movements. More specifically, we employ a modified version of the Sharpe Ratio as the key metric to assess and select assets for this strategy. The traditional Sharpe Ratio measures the risk-adjusted return by dividing the excess return over the risk-free rate by the standard deviation (volatility). However, in the context of this strategy, a more relevant approach involves focusing on the relationship between absolute returns and volatility, which is crucial for understanding decay behavior in leveraged ETFs.

For this strategy, the modified Sharpe Ratio is defined as the ratio of the absolute return of the underlying asset divided by its volatility. Mathematically, this is expressed as:

$$\text{Modified Sharpe Ratio} = \frac{|\text{Mean Return}|}{\text{Volatility}}$$

Where:

- **|Mean Return|** represents the absolute value of the mean return of the underlying asset over a given period.
- **Volatility** is the standard deviation of returns over the same period.

This adjustment emphasizes selecting assets with high volatility in relation to changes in price in any direction. By focusing on this ratio, we aim to identify securities that exhibit significant price fluctuations without consistently trending in one direction. This approach is essential for pinpointing assets where both long and short leveraged ETFs experience consistent volatility decay over time, due to frequent price swings and compounding losses (Lu et al., 2009).

Assets with higher levels of volatility in relation to changes in price are expected to be the most profitable for Strategy 1. The rationale behind this expectation is that greater volatility, regardless of directional movement, increases the likelihood of capturing the decay effect in both leveraged ETFs. A lower modified Sharpe Ratio in this context indicates relatively higher volatility compared to absolute returns, suggesting a greater propensity for simultaneous decay in the long and short leveraged ETFs. This focus on assets with higher volatility helps to mitigate the risk of one ETF significantly diverging from the other due to strong directional trends.

The Modified Sharpe Ratio thus provides a systematic framework for filtering and choosing securities that align with the core objective of Strategy 1: exploiting volatility decay through passive short positions in both long and short leveraged ETFs. This emphasis on assets with high relative volatility ensures that the strategy can capture the decay effect effectively, regardless of the underlying asset's directional price movement.

Security	Mean	Standard Deviation	Modified Sharpe
Gold (GC=F)	0.033274	0.155128	0.163640
Silver (SI=F)	0.023323	0.286295	0.060914
Oil (CL=F)	-0.248377	1.050541	0.273359
Natural Gas (NG=F)	0.146784	0.571009	0.289402
Gold Miners (GDX)	0.038919	0.387990	0.086333
S&P 500 (SPX)	0.124399	0.169050	0.889133

Based on the table, we expect Silver (SI=F) to yield the highest returns in Strategy 1 due to its low Modified Sharpe Ratio of 0.060914. This suggests that Silver exhibits significant volatility relative to its returns, increasing the likelihood of capturing consistent decay in both the long and short leveraged ETFs. Conversely, the S&P 500 (SPX), with the highest Modified Sharpe Ratio of 0.889133, is anticipated to have the lowest expected returns from this strategy.

Variables Affecting the Profitability of Strategy 2

In Strategy 2, where positions are dynamically adjusted based on threshold breaches, two primary variables play a critical role in determining the strategy's profitability: the rebalancing threshold and the volatility of returns. Both of these variables influence how effectively the strategy can capture decay while minimizing exposure to large directional movements.

The first variable, volatility of returns, is essential because the strategy relies on shorting both long and short leveraged ETFs to capitalize on their inherent decay. Higher volatility in the underlying asset leads to more frequent price swings, which exacerbate the compounding effect and accelerate decay in both ETFs. Thus, in this strategy, a higher level of volatility is expected to result in higher returns, as the decay becomes more pronounced with increased market fluctuations. This reinforces the importance of selecting assets with elevated volatility to maximize the profitability of this approach.

The second variable is the rebalancing threshold, which dictates when short positions are adjusted based on price movements. This threshold is crucial because it determines the balance between capturing gains from decay and limiting exposure to prolonged trends. If the threshold is set too wide, the strategy may fail to rebalance adequately, resulting in substantial losses when one ETF experiences a prolonged price trend. Conversely, if the threshold is set too narrow, excessive rebalancing could lead to high transaction costs and erode potential profits. To optimize this variable, a sensitivity analysis will be conducted to evaluate how different threshold levels impact the strategy's overall returns and risk exposure. This analysis will help identify the optimal range for the rebalancing threshold to achieve a balance between mitigating risks and capturing gains from decay.

Results

The results of Strategy 1, which involved shorting both long and short leveraged ETFs without rebalancing, are summarized in the final values table for each selected security. The goal of this strategy was to exploit the inherent volatility decay in leveraged ETFs over time. The table below displays the final values of the M2M Portfolio for each asset at the end of the study period:

Ticker	GC=F	SI=F	CL=F	NG=F	GDX	^SPX
Final Value	0.089	0.523	0.487	1.525	0.734	0.21

The figures accompanying this section illustrate the normalized prices of the long and short leveraged ETFs, as well as the M2M Portfolio values over the study period for each selected asset. These charts highlight how the prices of the long and short ETFs evolved over time, and how the strategy's M2M Portfolio value fluctuated in response (appendix A).

For the S&P 500 (^SPX): The persistent upward trend in the S&P 500 (^SPX) led to the divergence between the long and short ETFs, limiting the portfolio's ability to capture decay effectively. The final M2M Portfolio value remains relatively flat, reflecting the challenges faced in exploiting volatility decay in this case.

For Crude Oil (CL=F): The price movements in Crude Oil led to moderate success in capturing volatility decay, as seen by the modest rise in the M2M Portfolio value.

For Gold (GC=F): The relatively low volatility in Gold resulted in a flatter M2M Portfolio curve, highlighting the limited decay effect.

For Natural Gas (NG=F): The high volatility in Natural Gas is evident from the significant fluctuations in the ETF prices, which contributed to the sharp rise in the M2M Portfolio value over time.

For Silver (SI=F) and Gold Miners (GDX): Both assets exhibited patterns of moderate volatility, leading to steady gains in the M2M Portfolio values.

The results of Strategy 1 demonstrate that the effectiveness of capturing volatility decay varies significantly across different assets. Natural Gas emerged as the most favorable asset for this strategy, while Gold and the S&P 500 presented challenges due to their relatively stable or upward-trending price behavior. The visual and numerical analysis supports the hypothesis that assets with higher levels of volatility without directional movements are more conducive to capturing decay, validating the approach taken in Strategy 1.

In Strategy 2, which involved rebalancing positions when price thresholds were breached, the objective was to exploit volatility decay in leveraged ETFs while dynamically managing exposure. The results of Strategy 2 are summarized in the final values table for each selected security, as shown below:

Ticker	GC=F	SI=F	CL=F	NG=F	GDX	^SPX
Final Value	-0.090881	0.047612	0.298948	1.308237	0.306374	0.025169

The figures accompanying this section illustrate the M2M Portfolio values and the ETF positions for each selected asset during the study period (Appendix C):

- S&P 500 (^SPX): The portfolio value graph demonstrates a gradual rise initially, but losses accumulate as the persistent upward trend in the S&P 500 makes it difficult to capture decay consistently. The price positions for the ETFs indicate frequent rebalancing, which incurs costs without delivering proportional gains.
- Crude Oil (CL=F): The Portfolio graph shows steady growth throughout the period, supported by consistent fluctuations in the ETF prices, which enabled the strategy to capture decay effectively.
- Gold (GC=F): The declining Portfolio value highlights the ineffectiveness of the strategy for Gold, where the stable price behavior resulted in net losses despite rebalancing efforts.
- Gold Miners (GDX): The Portfolio graph indicates steady growth for Gold Miners, reflecting the asset's moderate volatility, which facilitated decay capture through rebalancing.
- Natural Gas (NG=F): The graph demonstrates a clear upward trend in the M2M Portfolio value, indicating the effectiveness of the strategy in capitalizing on the asset's volatility.
- Silver (SI=F): The portfolio value graph shows volatility and fluctuations, but with limited overall gains due to the rebalancing strategy being less effective in capturing decay for this asset.

Strategy 2, which involved dynamic rebalancing based on price thresholds, was most successful with assets exhibiting high volatility, such as Natural Gas. While the strategy yielded moderate gains for Crude Oil and Gold Miners, it struggled with assets like Gold and the S&P 500 due to their stable or trending behavior. The visual and numerical analysis suggests that dynamic rebalancing is beneficial when the underlying asset displays significant volatility, reinforcing the importance of asset selection based on volatility characteristics.

The frequency of rebalancing significantly impacts the performance of this strategy, especially when targeting decay in leveraged ETFs. High-volatility assets, such as Natural Gas, benefit from more frequent rebalancing since rapid price movements lead to consistent decay. For assets like Crude Oil and Gold Miners, moderate rebalancing frequency also proves beneficial as their volatility patterns support steady decay capture. Conversely, low-volatility assets like Gold and Silver suffer from high rebalancing frequency, as costs accumulate without sufficient returns. This analysis emphasizes

that optimal rebalancing frequency should be asset-specific, with a high frequency for highly volatile assets, moderate frequency for mid-volatility assets, and reduced frequency for low-volatility assets to minimize unnecessary costs and maximize decay capture efficiency.

Conclusion

In Strategy 1, the approach of holding short positions in both long and short leveraged ETFs without rebalancing capitalizes effectively on volatility decay in the short term. The strategy thrives when both leveraged ETFs experience consistent decay due to frequent price fluctuations. However, it carries a significant risk: if one ETF in the pair diverges sharply from the other due to a strong directional market trend, it can lead to substantial losses. This divergence risk is particularly prominent in assets with persistent trends or low volatility, making Strategy 1 vulnerable to market movements that favor one direction over the other.

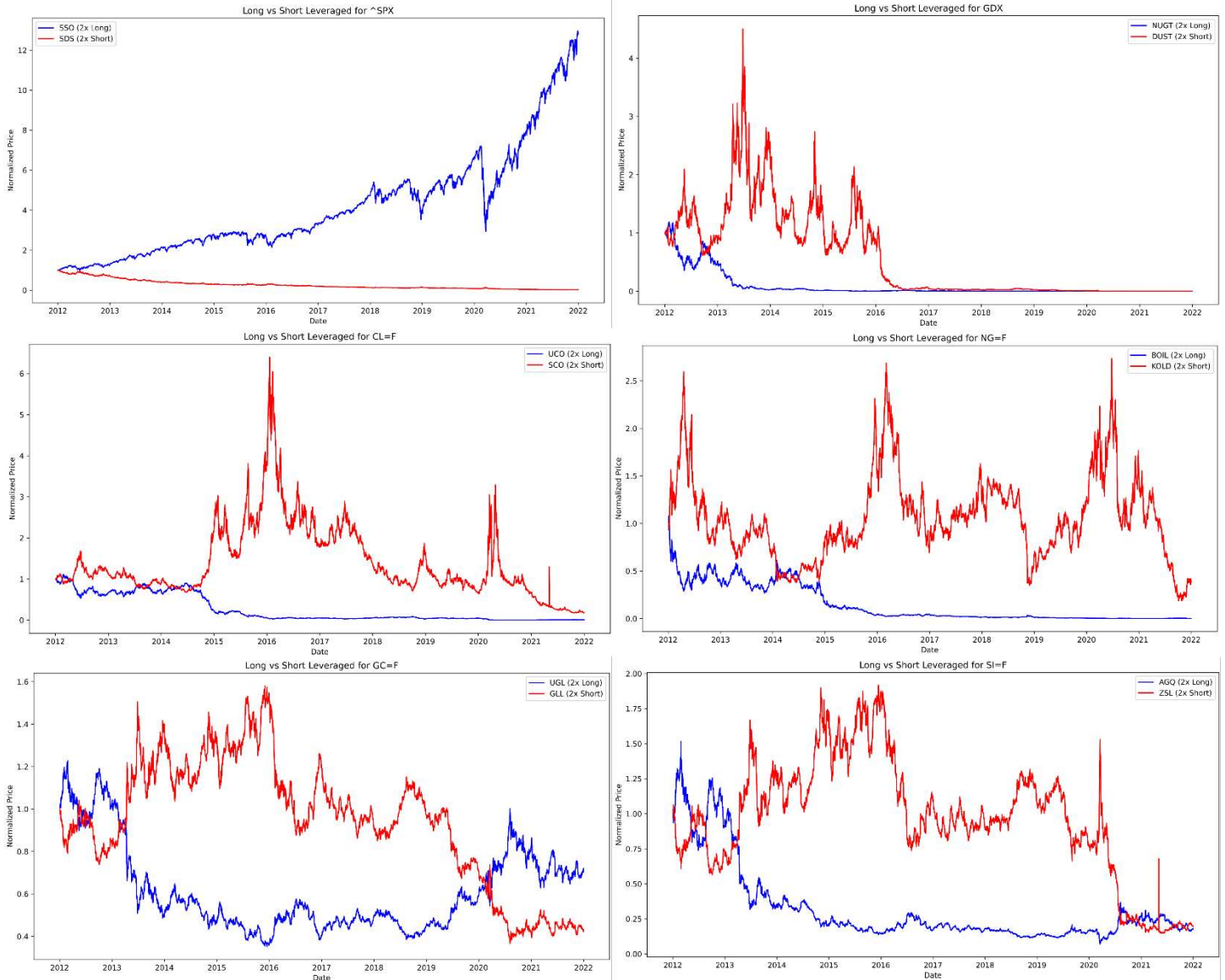
In contrast, Strategy 2 introduces a more risk-averse approach by dynamically rebalancing positions based on preset price thresholds. This method actively mitigates the risk of divergence between the two leveraged ETFs, allowing for more consistent performance across varying market conditions. However, the success of Strategy 2 is highly dependent on the chosen rebalancing thresholds. To optimize profitability and risk management, these thresholds should be evaluated and adjusted more frequently based on prevailing market volatility and price behavior. By refining these thresholds, Strategy 2 can better capture volatility decay while maintaining tighter control over potential losses, making it a more resilient long-term strategy.

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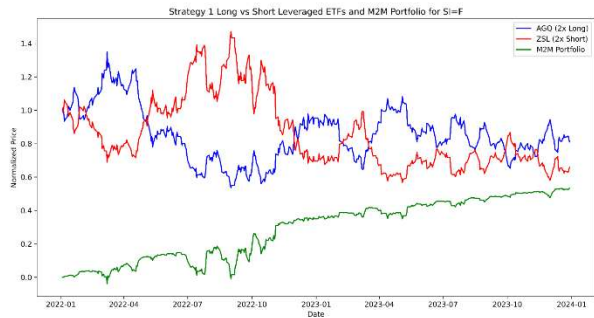
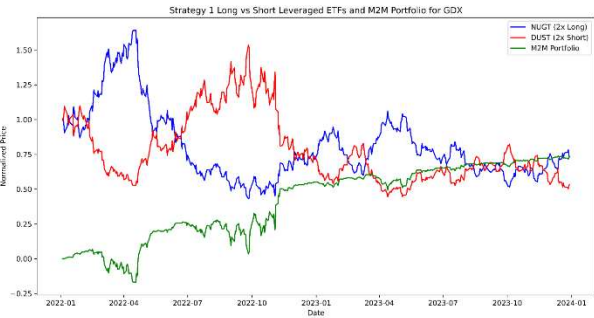
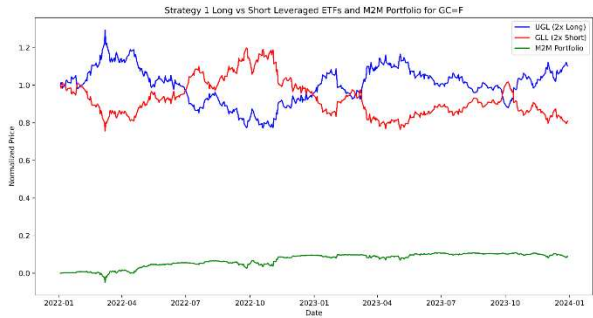
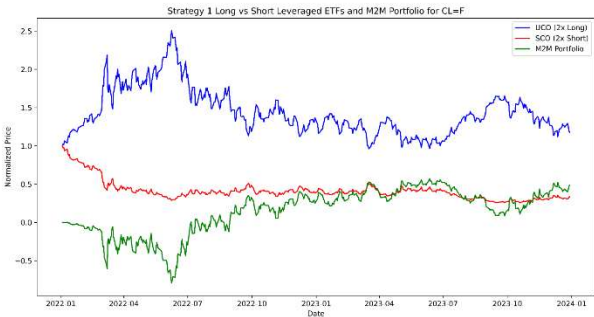
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Appendix

Appendix A: Historical prices for leveraged ETFs



Appendix B: Strategy 1 Results



Appendix C: Strategy 2 Results

