GNG1106 – Fundamentals of Engineering Computation
Course Project
Electrical Engineering

Design of an RLC Electrical Circuit

Electrical engineers often use Kirchhoff's laws to study transient and steady-state (not time-varying) behavior of electric circuits. This project focuses on circuits that are transient in nature where sudden changes in time take place; that is, current and voltage vary over time until they reach a steady state. Such a situation occurs following the closing of the switch in Figure 1

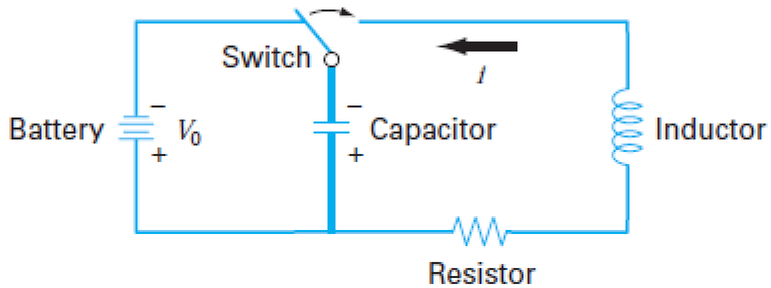

Figure 1 – RLC Circuit

In this case, there will be a period of adjustment following the closing of the switch as a new steady state is reached. The length of this adjustment period is closely related to the storage properties of the capacitor and the inductor. Energy storage may oscillate between these two elements during a transient period. However, resistance in the circuit will dissipate the magnitude of the oscillations. The flow of current, *i*, through the resistor causes a voltage drop (V_R) given by

$$V_R = iR \qquad \text{Equation 1}$$

where i = the current and R = the resistance of the resistor. When R and i have units of Ohms and amperes, respectively, V_R has units of volts. Resistor values can vary from 1 ohm to 1 MOhms (10^6 ohms).

Similarly, an inductor resists changes in current, such that the voltage drop V_L across it is

$$V_L = L\frac{di}{dt} \qquad \text{Equation 2}$$

where L = the inductance. When L and i have units of Henrys (H) and amperes, respectively, V_L has units of volts and t has units of seconds. Inductance values range from 1 nH (10^{-9} Henrys) to 10000 µH (10000 x 10^{-6} Henrys).

The voltage drop across the capacitor (V_C) depends on the charge (q) on it:

$$V_C = \frac{q}{C} \qquad \text{Equation 3}$$

where C = the capacitance. When the charge is expressed in units of coulombs, the unit of C is the Farad (F). Capacitance values range from 1 pF (10^{-12} Farads) to 10000 µF (10000 x 10^{-6} Farads).

Kirchhoff's second law states that the algebraic sum of voltage drops around a closed circuit is zero. After the switch is closed we have

$$L\frac{di}{dt}+Ri+\frac{q}{C}=0 \qquad \text{Equation 4}$$

However, the current is related to the charge according to

$$i=\frac{dq}{dt} \qquad \text{Equation 5}$$

Therefore,

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=0 \qquad \text{Equation 6}$$

This is a second-order linear ordinary differential equation that can be solved using the methods of calculus. This solution is given by

$$q(t)=q_0e^{-Rt/(2L)}\cos\left[\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t\right] \qquad \text{Equation 7}$$

where at $t = 0$, $q = q_0 = V_0C$, and V_0 = the voltage from the charging battery. The above equation describes the time variation of the charge on the capacitor. The solution $q(t)$ is plotted in Figure 2. A typical electrical engineering design problem might involve determining the proper resistor R to dissipate energy at a specified rate, for selected values for L and C. The rate of dissipation would be defined as the time, t_d, it takes for the charge on the capacitor to reach a desired percentage of its original charge, $p_c = q/q_0$. That is, $q(t_d) = p_cq_0$.

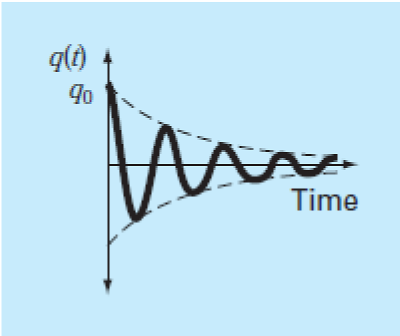

Figure 2 – Decreasing Charge on Capacitor C

By substituting $q(t_d) = p_cq_0$ into Equation 7, the value of R must be selected such that the following holds:

$$q(t_d)=p_cq_0=q_0e^{-Rt_d/(2L)}\cos\left[\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t_d\right]$$

$$p_c=e^{-Rt/(2L)}\cos\left[\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t_d\right] \qquad \text{Equation 8}$$

Finding the value of R from Equation (8) using analytical means is far from trivial. Numerical methods can provide very good results by using a root finding method, that is, finding the root of the function g(R) defined as

$$g(R) = e^{-Rt_d/(2L)}\cos\left[\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}\right)t_d\right] - p_c \qquad \text{Equation 9}$$

will give the value of R that satisfies Equation 8.

The *bisection method* for finding roots, which is alternatively called binary chopping, interval halving, or Bolzano's method, is one type of incremental search method in which the interval is always divided in half. If a function changes sign over an interval, the function value at the midpoint is evaluated. The location of the root is then determined as lying at the midpoint of the subinterval within which the sign change occurs. The process is repeated to obtain refined estimates. A simple algorithm for the bisection calculation to finding the root of the function *f(x)* consists of the following steps.

Step 1: Choose lower x_L and upper x_U guesses for the root such that the function changes sign over the interval. This can be checked by ensuring that $f(x_L)f(x_U) < 0$.

Step 2: An estimate of the root x_R is determined by $x_R = (x_L + x_U)/2$.

Step 3: Make the following evaluations to determine in which subinterval the root lies:

(*a*) If $f(x_L)f(x_R) < 0$, the root lies in the lower subinterval. Therefore, set $x_U = x_R$ and return to step 2.

(*b*) If $f(x_L)f(x_R) > 0$, the root lies in the upper subinterval. Therefore, set $x_L = x_R$ and return to step 2.

(*c*) If $|f(x_L)f(x_R)|$ < some small tolerance value (that is, can be considered equal to 0), the root equals x_R; terminate the computation.

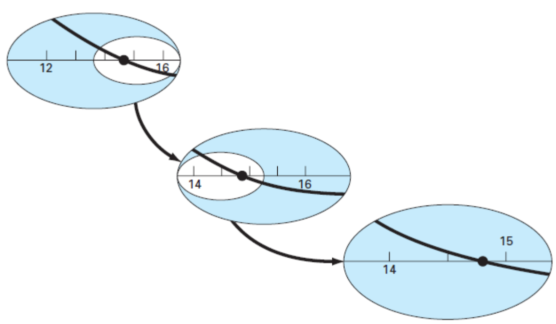

Figure 3 – Bisection Method for Finding Roots

In Figure 3, the searching for the root is started on the interval between the values of 12 and 16 where the function changes sign.

- Therefore, the initial estimate of the root x_R lies at the midpoint of the interval $x_R = (12+16)/2 = 14$.
- Next the product of the function value is computed at the lower bound and at the midpoint: $f(12)\,f(14) = 6.067(1.569) = 9.517$ which is greater than zero, and hence no sign change occurs between the lower bound and the midpoint. Consequently, the root must be located between 14 and 16.
- Therefore, we create a new interval by redefining the lower bound as 14.
- A revised root estimate is determined as $x_R = (14+16)/2 = 15$.

- The process can be repeated to obtain refined estimates. For example, $f(14)\,f(15) = 1.569(-0.425) = -0.666$. Therefore, the root is between 14 and 15.
- The upper bound is redefined as 15, and the root estimate for the third iteration is calculated as $x_R = (12+16)/2 = 14$ which represents a percent relative error of $\varepsilon = 1.9\%$.
- The method can be repeated until the result is accurate enough.

A design tool is required to determine the value of a resistor R in an RLC circuit shown in Figure 1 to dissipate the charge on the capacitor to a percentage of its original charge within a dissipation time.

The software will allow the user to determine the resistor value from the component values L and C, the battery voltage, the dissipation time t_d, and the percentage of original charge p_c to reach within the dissipation time.

When new input values are given, the user is given the option to save them into a file; up to five sets of values can be stored. Thus when the software starts the user can elect to use one of the five stored values or enter new values.

A bisection root finding method is used to find the Resistor. To support the method, the user must provide an interval in which to search for a root. The software first calculates an upper bound for the resistor using the square root term in the function g(R) and plots the function for root finding to allow the user to select an appropriate interval for searching the root of the function. The user can then repeatedly provide the upper and lower bounds to replot g(R) until satisfied with the bounds.