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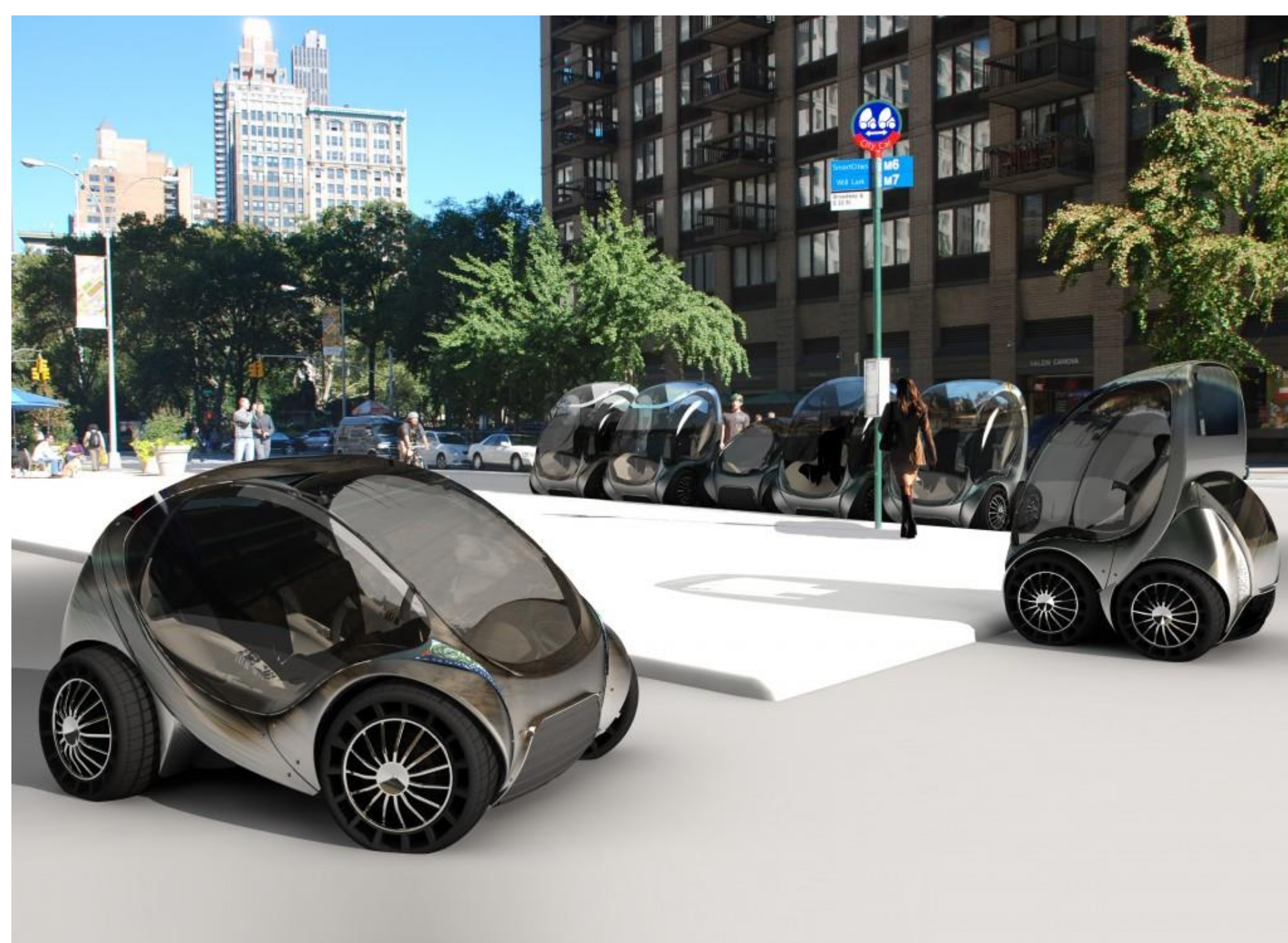
# Modeling and Experimental Analysis of Mobility-on-Demand Systems

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## Objectives

Enable sustainable urban personal mobility through autonomous driving and system-level coordination of autonomous vehicles



MIT CityCar. W.J. Mitchell et al., 2010

## Motivation

**Current model of urban mobility is unsustainable**

- Over 3 trillion urban miles are driven annually in the U.S.
- Over 50% of the world population live in urban areas. This number will increase to 60% by 2030

**New models of urban mobility can benefit from system-level coordination and vehicle autonomy**

- Car-sharing services are beginning to gain popularity
- Autonomous driving and system-level coordination can ensure quality of service for Mobility-on-Demand systems

## Technical approach to studying Mobility-on-Demand Systems

**Queuing models for MOD systems**

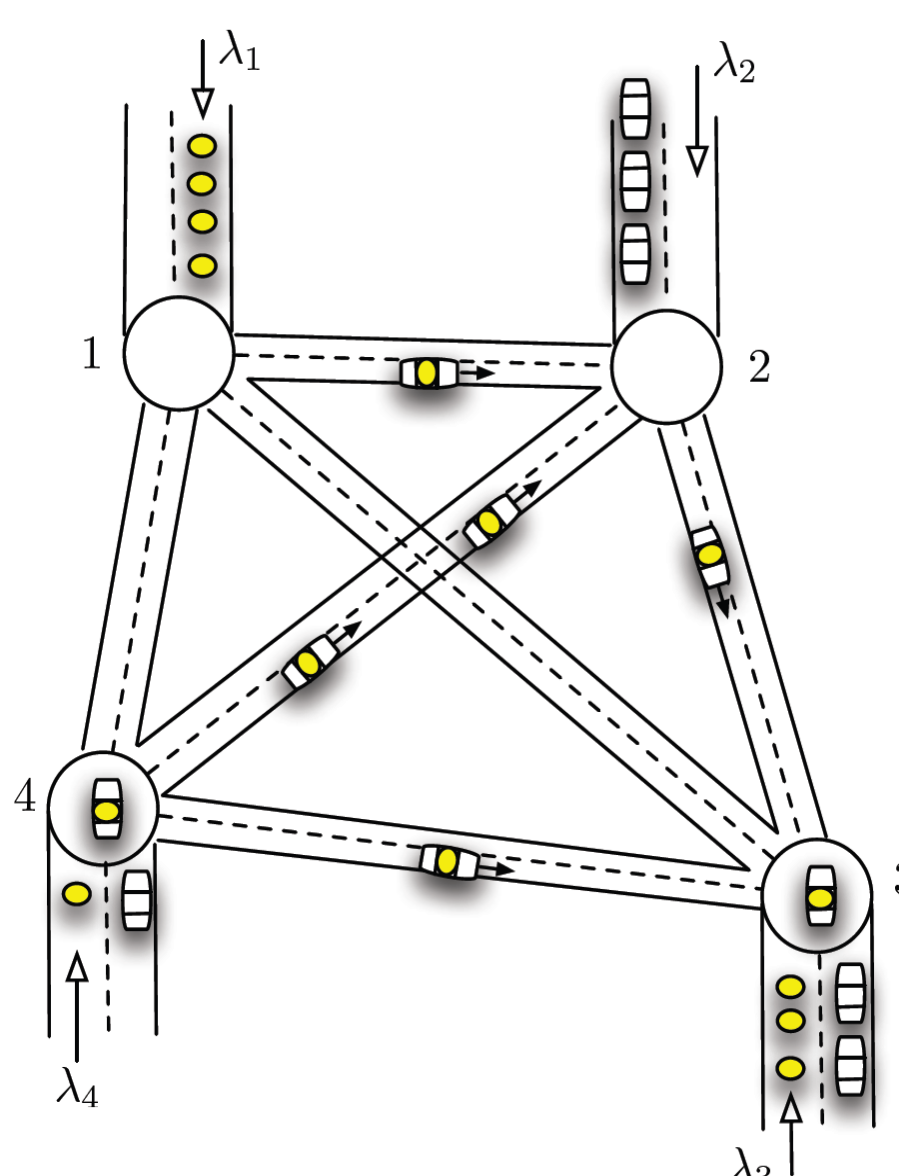
- Fully automated MOD systems
- Hybrid human/automated MOD systems
- Intermodal MOD systems

**System-wide coordination architectures**

- Dynamically route vehicles by taking into account safety, demand uncertainty, and charging constraints

**Validate results**

- Simulations
- Experimental testbed of a mock urban environment with a fleet of vehicles
- Driverless shuttles providing MOD service on Stanford Campus



## Modeling MOD systems as queuing networks

**Approaches to modeling MOD systems**

- Fluidic model
- Dynamic vehicle routing
- Jackson network model**

**Vehicles form a closed Jackson network**

- Roads between pairs of stations form infinite server queues
- Congestion effects are not taken into account

**Key Quantities:**

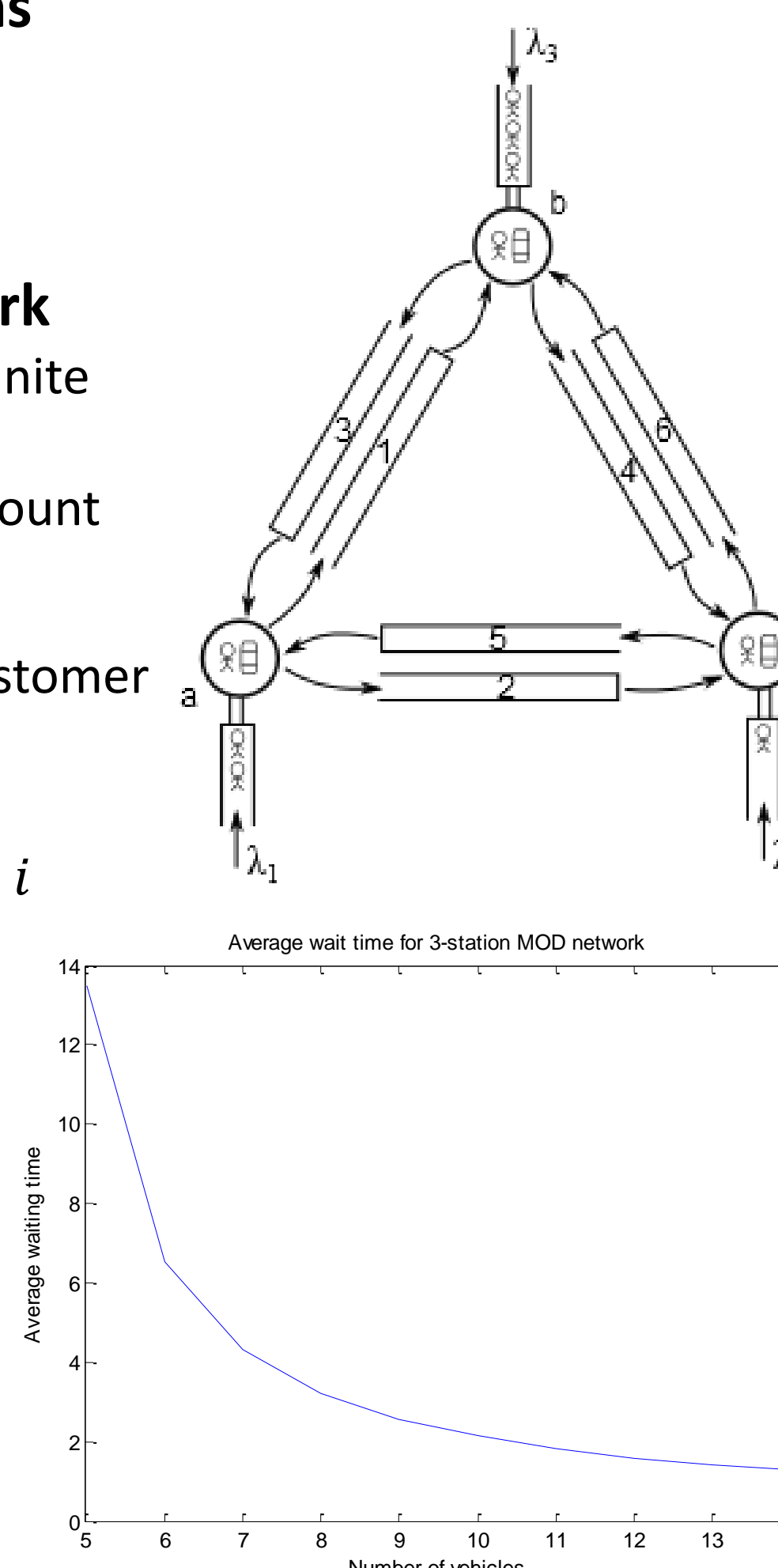
- $\alpha_{jk}$  = rate of empty vehicles + rate of customer carrying vehicles
- $\rho_i$  = Throughput of station  $i$
- $\lambda_{ci}$  = Arrival rate of customers to station  $i$
- $\bar{Q}_i$  = Probability station  $i$  is empty

**Solving for customer wait time**

- $\rho_i$  represents an effective customer service rate
- Stable if  $\lambda_{ci} < \rho_i$
- Waiting time at station  $i$  is  $\frac{1}{\rho_i - \lambda_{ci}}$
- $\bar{Q}_i = 1 - \frac{\lambda_{ci}}{\rho_i}$
- $\alpha_{jk}$  depends on  $\bar{Q}_i$ . If we knew  $\bar{Q}_i$ , we can solve for the waiting time

**An iterative procedure to solve for  $\bar{Q}_i$**

- Make an initial guess for  $\bar{Q}_i^0$
- Solve the closed Jackson network to find throughput  $\rho$
- Use Little's theorem to determine  $\bar{Q}_i^1$
- Iterate until convergence
- Solve for the waiting time at each station

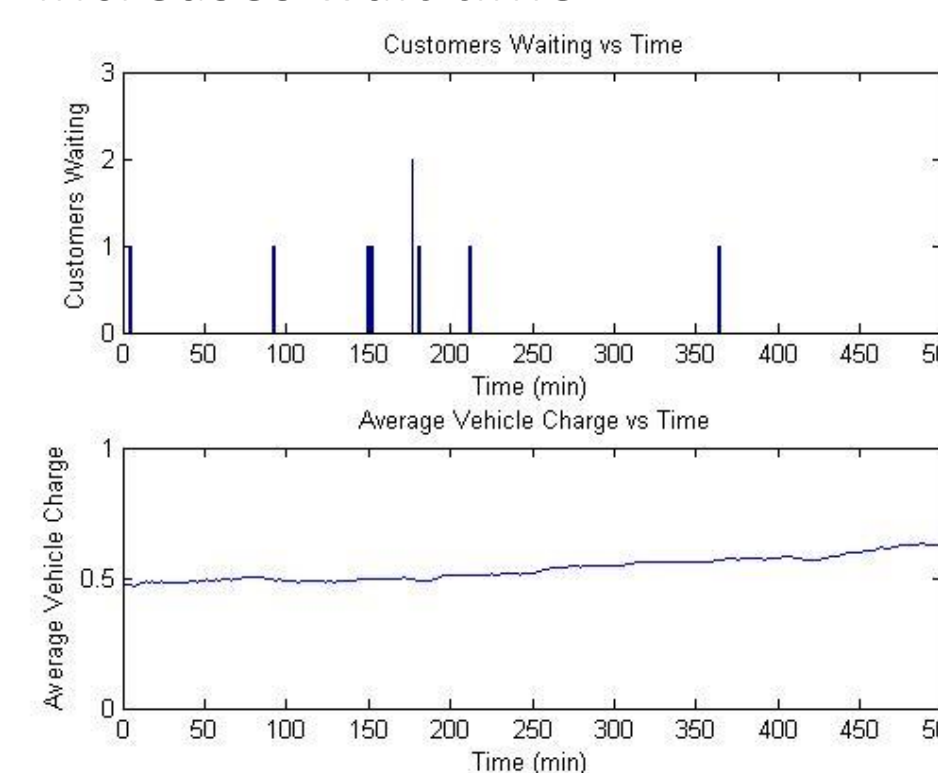


For a given system, we can determine the number of vehicles required to achieve a desired wait time

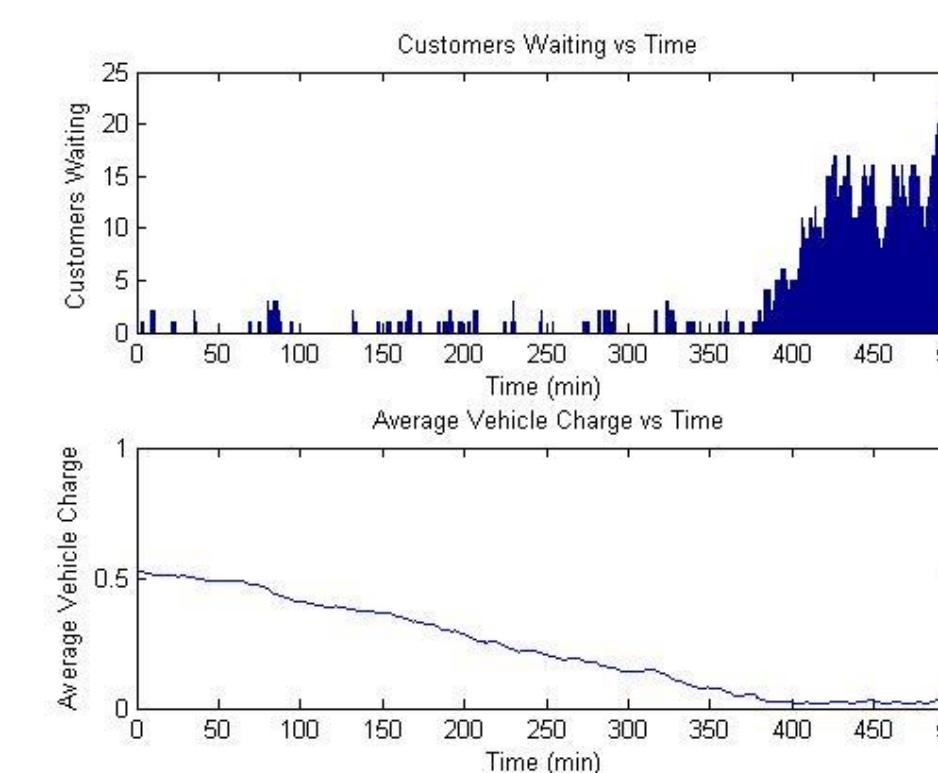
## Charging problem for electric vehicles

**Current battery technology takes longer to charge than to discharge**

- One of the main issues of current car-sharing services
- Developing real-time rebalancing algorithms for electric vehicles
- Can only rebalance vehicles with sufficient charge
- Adding a "safety threshold" on the amount of charge required significantly increases wait time



Stable system

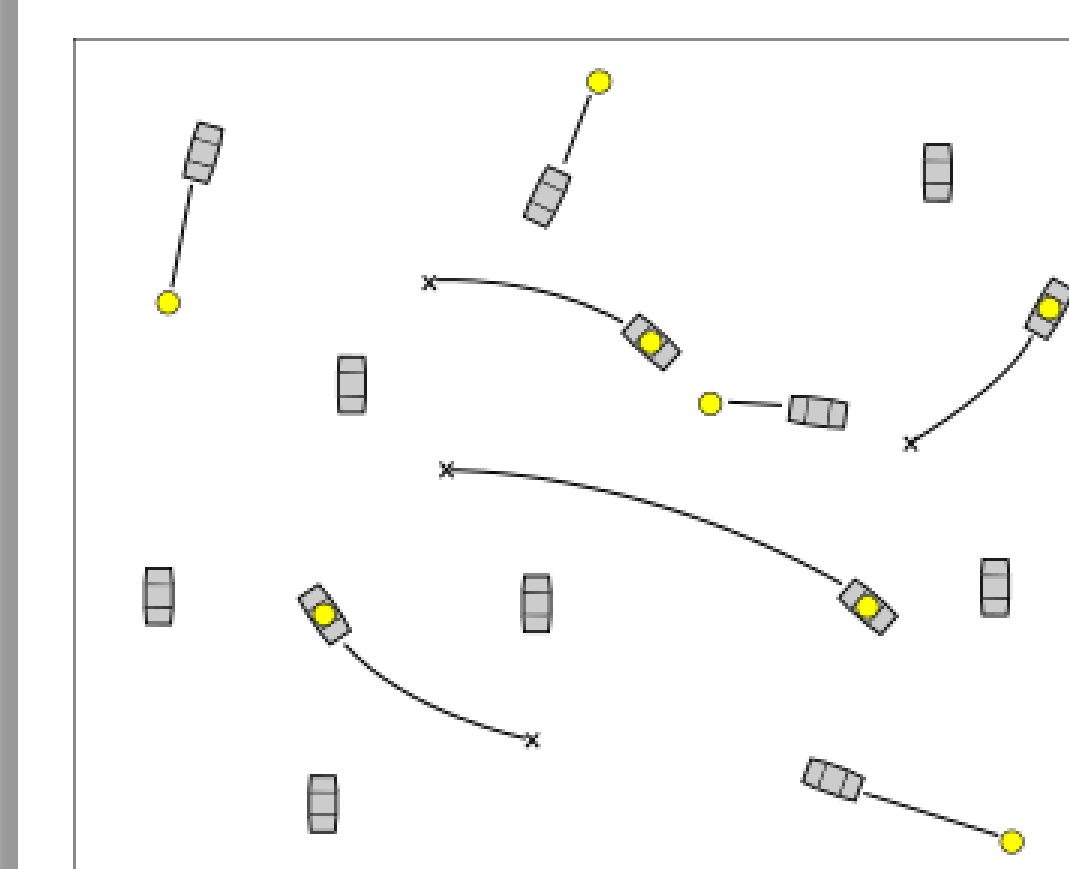


Instability due to charge. System load too high for vehicles to stop and charge

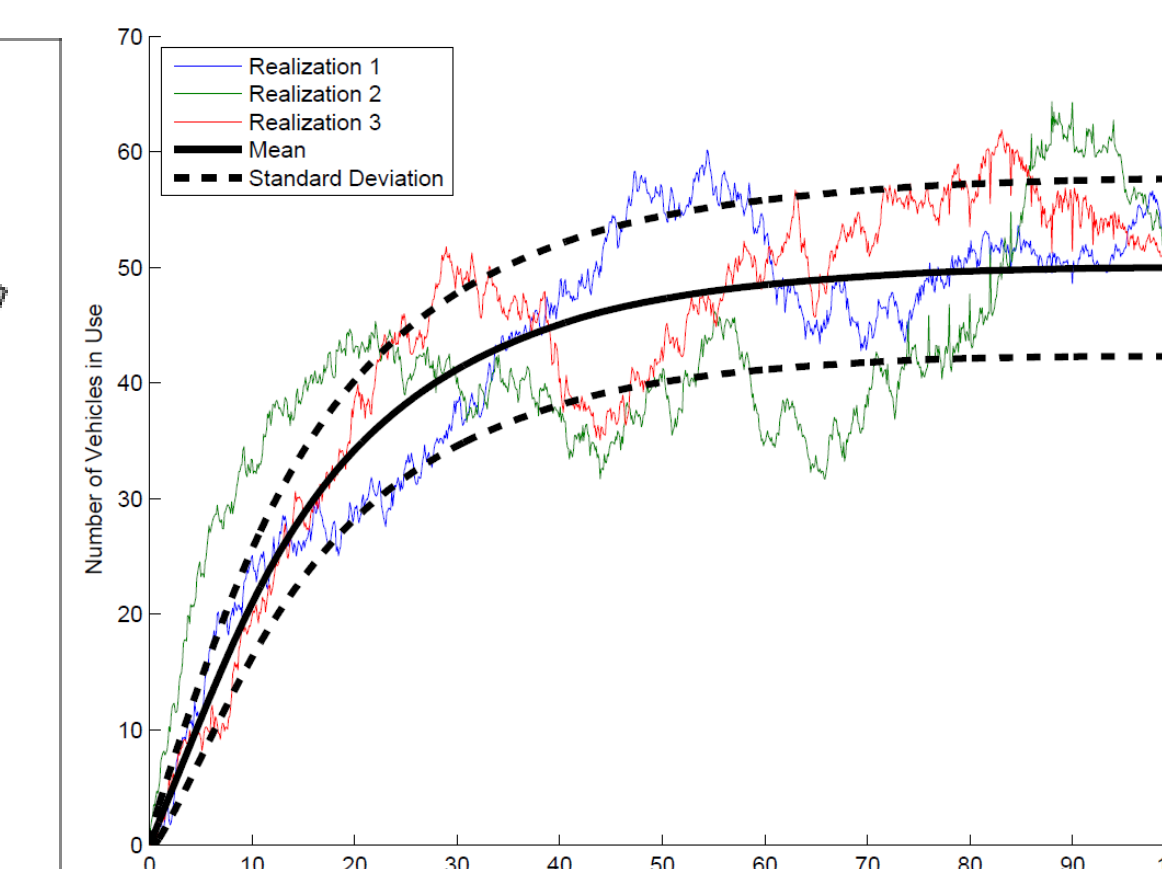
## Routing algorithms with provable performance guarantees

**Large fleets of autonomous "taxis" could revolutionize urban transit, but optimal routing of vehicles to customers is NP hard**

- Service time for customers depend on many **geographically distributed components**, which are statistically dependent
- With many vehicles and many customers, the **spatial** element of the queue becomes **less important**
- Tractable algorithms can be given which are **asymptotically optimal** for systems with **many customers and many vehicles**



A spatially constrained MOD system with many customers and many vehicles



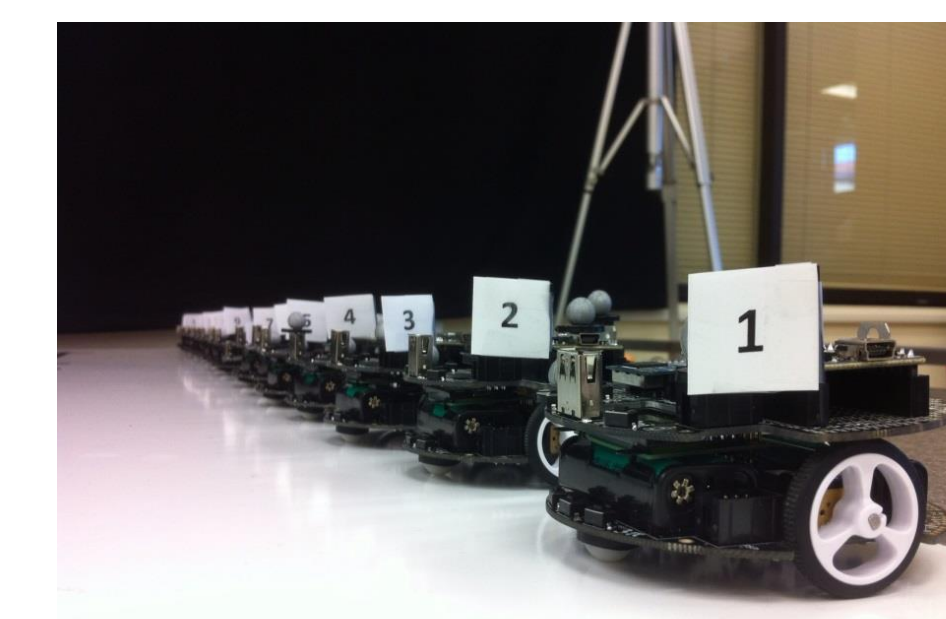
For large systems, we can approximate the number of vehicles in use at any time

## Experimental testbed

**Experimentally evaluate rebalancing policies for MOD systems**

**Hardware**

- 18 Pololu m3pi Robots
- Vicon Motion Capture System



**Software**

