2) حال فرض کنید در حالت معدن اثر انباشت رقابت داشتیم را در نظر بگیرید. مسئله مقاله را در نظر بگیرید

قیمت به صورت $P = 100 - q - 4x + 6t$ است. و فرض کنید نرخ بهره برابر سالانه 5% است این

مسئله را حل کنید.

P is a function of $x \rightarrow q = \frac{dx}{dt}$

$$\text{Value of mine} \int_0^{\infty} P(x, q, t)\, q\, e^{-rt}\, dt = \int_0^{\infty} f(x, q, t)\, dt \quad \text{(I)}$$

$$\Rightarrow \text{(IV) Monopoly problem} \quad \frac{df}{dx} \text{ (III)} - \frac{d}{dt} \frac{df}{dq} \text{ (II)} = 0$$

$$\text{boundary conditions: } \underset{\text{for } t=0}{x = a} \quad \& \quad TC:\ f - q\frac{\partial f}{\partial q} = 0 \ \text{ or } \ q^2 \frac{\partial P}{\partial q} = 0$$

* مسئله مقاله

$$P = a - \beta q - cx + gt \longrightarrow f = pq e^{-rt} \longrightarrow (a - \beta q - cx + gt) q e^{-rt}$$

$$\text{(II)} \Rightarrow \frac{\partial f}{\partial q} = a e^{-rt} - 2\beta q e^{-rt} - cx e^{-rt} + gt e^{-rt} \rightarrow \underbrace{(a - 2\beta q - cx + gt) e^{-rt}}_{B}$$

$$0 = \frac{\partial B}{\partial t} \cdot \frac{df}{dq} \Rightarrow -ar e^{-rt} + 2\beta r \frac{dx}{dt} e^{-rt} - 2\beta \frac{d^2x}{dt^2} e^{-rt} + crx e^{-rt}$$

$$+ g e^{-rt} - grt e^{-rt} = 0 \rightarrow e^{-rt}\left(-ar + 2\beta r \frac{dx}{dt} - 2\beta \frac{d^2x}{dt^2} + crx + g - grt\right) = 0$$

$$\Rightarrow \begin{cases} e^{-rt} = 0 \\ -ar + 2\beta r \frac{dx}{dt} - 2\beta \frac{d^2x}{dt^2} + crx + g - grt = 0 \end{cases}$$

$$\Rightarrow 2\beta \frac{d^2x}{dt^2} - 2\beta r \frac{dx}{dt} - crx = -grt + g - ar$$

$\Rightarrow$ Roots are real & opposite signs ($+m$ & $-n$ are roots)

SOBHAN

DATE : / / SUB :

(IV) $\frac{\partial f}{\partial x} - \frac{\partial}{\partial t}\frac{\partial f}{\partial \dot{q}} = 0$

$$g - grt - ar = 2\beta \frac{d^2x}{dt^2} - 2\beta r \frac{dx}{dt} - crx$$

We have $m - n = r \Rightarrow x = Ae^{mt} + Be^{-nt} + \frac{gt}{c} - \frac{2\beta g}{c^2} - \frac{g}{cr} + \frac{a}{c}$

$- \frac{g}{cr} + \frac{a}{c} \Rightarrow q = Ame^{mt} + Bne^{-nt} + \frac{g}{c}$

boundary conditions: 1) $t = 0 \longrightarrow x = 0 \longrightarrow x$ is amount extracted

2) $T = t \longrightarrow q = 0 \longrightarrow x = a$

1) $Ae^{m\overset{\circ}{t}} + Be^{-n\overset{\circ}{t}} + \frac{g\overset{\circ}{t}}{c} - \frac{2\beta g}{c^2} = 0 \longrightarrow A + B + \frac{g}{c} - \frac{2\beta g}{c^2} - \frac{g}{cr} + \frac{a}{c}$ (1)

2) $\begin{cases} q : Ae^{mT} + Be^{-nT} + \frac{g}{c} = 0 & (2) \\ x : Ae^{mT} + Be^{-nT} + \frac{gT}{c} - \frac{2\beta g}{c^2} - \frac{g}{cr} + \frac{a}{c} = a & (3) \end{cases}$

system of equations :

steps :

(I) Isolate r in (1) & $A + B + \frac{g}{c} - \frac{2\beta g}{c^2} - \frac{g}{cr} + \frac{a}{c} = 0$

$$A + B + \frac{g}{c} - \frac{2\beta g}{c^2} + \frac{a}{c} = \frac{g}{cr} \longrightarrow cr\left(A + B + \frac{g}{c} - \frac{2\beta g}{c^2} + \frac{a}{c}\right) = g$$

$$cr = \frac{g}{A + B + \frac{g}{c} - \frac{2\beta g}{c^2} + \frac{a}{c}} \longrightarrow r = \frac{g}{AC + BC + g - \frac{2\beta g}{c} + a}$$

$r \neq \frac{gc}{AC^2 + BC^2 + gC - 2\beta g + aC} \wedge C \neq 0$

→ substitute x in (3):

$$Ae^{m\tau} + Be^{-n\tau} + \frac{g\tau}{c} - \frac{2\beta g}{c^2} - \frac{g}{c\left(\frac{gc}{Ac^2+Bc^2+gc-2\beta g+ac}\right)} + \frac{a}{c} - a = 0$$

$$\Rightarrow Ae^{m\tau} + Be^{-n\tau} + \frac{g\tau}{c} - \frac{2\beta g}{c^2} - \frac{Ac^2+Bc^2+gc-2\beta g+ac}{c^2} + \frac{a}{c} - a = 0$$

$$\rightarrow Ae^{m\tau} + Be^{-n\tau} + \frac{g\tau}{c} - \frac{(Ac+Bc+g+a)}{c^2} + \frac{a}{c} - a = 0$$

$$\rightarrow Ae^{m\tau} + Be^{-n\tau} + \frac{g\tau - Ac - Bc - g}{c} - a = 0 \quad (4)$$

→ Isolate m in (3):

$$Ae^{m\tau} + Be^{-n\tau} + \frac{g}{c} = 0 \rightarrow \ln A + m\tau + \ln B - nt + \ln g - \ln c = 0$$

$$m\tau = n\tau - \ln A - \ln B - \ln g/c \rightarrow m = \ln\left(\frac{-bce^{-n\tau}+g}{Ac}\right)/\tau$$

→ substitute m in (4):

$$ae^{\tau\left(\frac{\ln\left(\frac{-Bce^{-n\tau}-g}{Ac}\right)}{\tau}\right)} + be^{-n\tau} + \frac{g\tau - g - ac - Bc}{c} - a = 0$$

↳ Isolate $t \rightarrow \boxed{t = \frac{2ac+Bc+2g}{g}}$ (5)

↳ $m = g\ln\left(\frac{-bce^{-n\left(\frac{2ac+Bc+2g}{g}\right)} - g}{Ac}\right)/2ac+bc+2g$ (6)

$$P = 100 - q - 4x + 16t$$

$$r = \frac{gc}{AC^2 + BC^2 + gc - 2\beta g + \alpha C} \longrightarrow g = 16,\ \alpha = 100,\ \beta = 1,\ C = +4$$

$$\Rightarrow 0.05 = \frac{16(4)}{16A + 16B + 16(4) - 2(1)(16) + (100)(4)}$$

$$\rightarrow 0.05 = \frac{64}{16A + 16B + 64 - 32 + 400}$$

$$\rightarrow 64 = 0.8A + 0.8B + \underbrace{3.2 - 1.6 + 20}_{21.6} \longrightarrow A + B = 53 \checkmark$$

From 1: $A + B + \frac{g}{C} - \frac{2\beta g}{C^2} - \frac{g}{cr} + \frac{\alpha}{C} = 0$

$$A + B + \frac{16}{4} - \frac{32}{16} - \frac{16}{0.2} + \frac{100}{4} \longrightarrow A + B = 53 \checkmark$$

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