a)

The state variable: y
The Control Variable: u

b)

$$
\begin{aligned}
&(1)\ \frac{\partial H}{\partial \lambda} = \dot{y}(t) = u(t)\\
&(2)\ \frac{\partial H}{\partial u} = 2u(t) + \lambda(t) = 0\\
&(3)\ \frac{\partial H}{\partial y} = 1 = -\dot{\lambda}(t)\\
&(4)\ \lambda(T) = \lambda(1) = 0
\end{aligned}
$$

We now deduce λ as a function of t:

$$
\begin{aligned}
\lambda(t) =& \lambda(0) + \int_0^t \dot{\lambda}(s)ds\\
\overset{(3)}{=}& \lambda(0) + \int_0^t (-1)ds\\
=& \lambda(0) - t
\end{aligned}
$$

$$
\begin{aligned}
\lambda(1) =& \lambda(0) - 1 \overset{(4)}{=} 0 \Leftrightarrow\\
\lambda(0) =& 1 \Leftrightarrow\\
\lambda(t) =& 1 - t
\end{aligned}
$$

And u as a function of t: From condition (2)

$$
\begin{aligned}
2u(t) =& -\lambda(t) = t - 1 \Leftrightarrow\\
u(t) =& \frac{1}{2}t - \frac{1}{2}
\end{aligned}
$$

And lastly y as a function of t:

$$\begin{aligned}
y(t) =& y(0) + \int_0^t \dot{y}(s)ds \\
=& 5 + \int_0^t u(s)ds \\
=& 5 + \frac{1}{2}\int_0^t (s-1)ds \\
=& 5 + \frac{1}{2}\left[\frac{1}{2}s^2 - s\right]_0^t \\
=& 5 + \frac{1}{2}\left(\frac{1}{2}t^2 - t\right) \\
=& \frac{t^2}{4} - \frac{t}{2} + 5
\end{aligned}$$