A Test of the Hotelling Valuation Principle

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Time-series tests of the Hotelling r-percent rule for natural resource prices have not been strongly supportive, but the tests and the data are subject to serious difficulties. We propose here an alternative testing strategy based on another but less widely known implication of the Hotelling model. We test this implication, which we call the Hotelling Valuation Principle, by regressing the market values of the reserves of a sample of U.S. domestic oil- and gas-producing companies on their estimated Hotelling values. We find that the estimated Hotelling values account for a significant portion of the observed variations in market values and that the Hotelling measures are better indicators of the market values of petroleum properties than two widely cited publicly available alternative appraisals.

I. Introduction

The proposition that the unit price of an exhaustible natural resource, less the marginal cost of extracting it, will tend to rise over time at a rate equal to the return on comparable capital assets has come to be called the Hotelling Principle in recognition of the econo-

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mist who first derived it rigorously more than 50 years ago (Hotelling 1931).

Hotelling himself focused mainly on the normative implications of the principle. He stressed the upward trend in net prices as a hidden hand inducing the current generation of resource owners to use up just the right amount of the resource for themselves and to conserve the rest for future generations. This fundamental insight has remained the starting point for virtually all subsequent normative economic analyses of national policies on the conservation, regulation, and taxation of natural resources.

The principle's role as a proposition in positive economics is less firmly established. Until the early 1970s, in fact, the predominantly downward-trending price paths of many important minerals, notably petroleum, appeared so strongly at variance with the model's predictions that formal testing of the proposition may have seemed pointless.

Interest in the descriptive and especially the predictive ability of the Hotelling Principle was rekindled by the marked upward drift in the prices of oil and most industrial minerals that began in the early 1970s and accelerated further at the close of the decade. At the same time, theoretical extensions of the underlying Hotelling model (a notable contribution being that of Pindyck [1978]) to allow for investment in discovering new resources made the earlier era of downward-trending prices seem less obviously anomalous.[1] In the generalized model, falling output prices might well characterize the early stages of a mineral's development in which major new discoveries were adding to the stock of proven reserves faster than consumption was depleting them. But such phases were temporary interludes and, ultimately, the "exhaustibility" underlying Hotelling's analysis would reassert itself.

The recent attempts to assess the empirical relevance of the Hotelling Principle have for the most part been direct tests of the relation between interest rates and the growth rates of prices of particular minerals (see, e.g., Feige and Geweke 1979; Heal and Barrow 1980; Smith 1981). In general, the findings of these time-series studies have not been strongly supportive of the Hotelling Principle; but these rejections can hardly be considered decisive given the serious deficiencies in the tests themselves. As Feige and Geweke mildly put it: "It is well known that historical data on mineral prices are not very 'good' in the sense that there is reason to believe that there may be considerable measurement error" (1979, p. 41). And for the marginal

[1] For a survey of extensions and generalizations of Hotelling's model emphasizing contributions in the 1970s, see Devarajan and Fisher (1981).

extraction cost component of net prices, there are usually no time series at all.[2]

These obstacles in the way of time-series tests of the Hotelling Principle are not likely soon to be overcome. We propose here therefore an alternative testing strategy based on another but less widely known implication of Hotelling's analysis of the optimal time pattern of exploitation of an exhaustible resource. In particular, it can be shown that in a world in which the time path of mineral prices, less marginal extraction costs, follows Hotelling's Principle, the value of the reserves in any currently operating, optimally managed mineral deposit depends mainly on *current* period prices and extraction costs, regardless of when the reserves are extracted.

We test this proposition, which we call the Hotelling Valuation Principle, by regressing the market values of the reserves of a sample of U.S. domestic oil- and gas-producing companies on their estimated Hotelling values at several points in time during the years 1979–81. The market values are based on stock market prices for the firms' shares with adjustments for liabilities and nonpetroleum properties. The Hotelling values are based on estimates of well-head prices, extraction costs, and estimated reserves generated as a by-product of Securities and Exchange Commission (SEC) financial reporting requirements and U.S. energy regulations.

The estimated Hotelling values can account for a substantial portion of the variation in market values of the sample firms. The explanatory power of the Hotelling Valuation Principle is shown to be robust in the face of a variety of alternative specifications of the variables and test equations, and the Hotelling measures are seen to be better indicators of the stock market value of petroleum properties than two widely cited publicly available measures of the value of reserves.

The rest of this paper is divided into four main sections. Section II reviews briefly the essentials underlying the Hotelling Principle on the price path for exhaustible resources, shows the implications of the principle for the valuation of mineral properties, and restates the Hotelling Valuation Principle in testable form. Section III presents evidence of the descriptive validity of the Hotelling Valuation Principle for a sample of domestic oil and gas producers, and Section IV compares the Hotelling-based appraisals with other publicly available alternative appraisals for the sample firms. We conclude in Section V with some warnings, suggested by the analysis and the empirical results, against mechanical applications of conventional discounted-

[2] One of the rare exceptions is the recent study by Stollery (1983) of prices and extraction costs in the nickel industry.

cash-flow capital budgeting methods for valuing mineral properties.

II. The Hotelling Valuation Principle and Its Testable Implications

Although Hotelling's original model has been extended in a variety of ways in the more than half-century since its publication, the essential economic point of his analysis can be conveyed in a simple discrete-time certainty framework.

A. A Simple Restatement of the Hotelling Pricing Principle

Consider, for example, the problem confronting a profit-maximizing, price-taking owner of an exhaustible resource at time zero. The reserves may be extracted either in the current or in any of the next N periods. Let extraction costs at time t be $C_t = C_t(q_t, Q_t)$, where q_t is the current rate of extraction and $Q_t = \Sigma_{s=0}^{t} q_s$ is the cumulative level of extraction. As to the properties of C_t, we know that $\partial C_t/\partial q_t$ must be positive, since total extraction costs in period t will rise with the amount extracted; $\partial C_t/\partial Q_t$ is nonnegative and will be positive if additional reserves are increasingly expensive to extract. The discounted present value of profits is then

$$V_0 = \sum_{t=0}^{N} \frac{p_t q_t - C(q_t, Q_t)}{(1 + r)^t}, \tag{1}$$

where p_t is the exogenously given market price of output at time t (assumed, in this simplest case, to be known with certainty to the producer), r is the rate of interest (again assumed known and constant over time), and N is a known date beyond which production can safely be presumed to have ceased. V_0 is maximized subject to the constraint

$$\sum_{t=0}^{N} q_t \leq R_0, \tag{2}$$

where R_0 are total reserves. Expression (2) is written as an inequality to remind us that reserves are in principle an economic quantity, not a technological datum, and that it need not be optimal to extract all of the reserves. Assuming that it is, and thus concerning ourselves only with interior solutions, the first-order condition for profit maximization in any period is

$$(p_t - c_t)\left(\frac{1}{1 + r}\right)^t - \sum_{s=t}^{N}\left(\frac{\partial C_s}{\partial Q_s}\right)\left(\frac{1}{1 + r}\right)^s = \lambda, \quad t = 0, \ldots, N, \tag{3}$$

where $c_t = \partial C_t/\partial q_t$ is marginal extraction cost in period t and λ is the Lagrangian multiplier on the constraint (2).[3]

To simplify further, consider the special case of a production function with extraction costs per unit of output independent of cumulative output so that $\partial C_s/\partial Q_s = 0$. Then the first-order condition (3) reduces to

$$(p_t - c_t)\left(\frac{1}{1 + r}\right)^t = \lambda, \quad t = 0, \ldots, N. \tag{3'}$$

In an optimal production program under the assumed cost structure, the net present value of the net price per unit of output must be the same regardless of when it is produced. Solving the system of difference equations (3′) we obtain the familiar Hotelling Principle:

$$(p_t - c_t) = (p_0 - c_0)(1 + r)^t, \quad t = 0, \ldots, N. \tag{4}$$

That is, the efficient intertemporal production of an exhaustible resource implies that the real price of the resource, net of marginal extraction costs, grows over time at a rate equal to the real rate of interest.

B. *The Hotelling Valuation Principle*

With the further assumption of constant returns to scale in current as well as cumulative extraction, the valuation expression (1) takes an extremely simple form. Marginal cost is then the same as average cost so that substitution of (4) into (1) yields as the present value of total reserves

$$V_0 = (p_0 - c_0)\sum_{t=0}^{N} q_t = (p_0 - c_0)R_0. \tag{5}$$

In words, in a world where output prices, net of extraction costs, obey the Hotelling Principle, the value of the total reserves in any mineral property depends solely on the *current* spot price per unit of the mineral, net of *current* extraction costs. True, units of production deferred to future years will earn higher net prices. But in a Hotelling world, the present value of the net price of any unit must be the same, regardless of when extracted. The growth of the net price in the numerators of the terms in the valuation summation (1) will be exactly offset by the discount factors in the denominators.

Expression (5) is a special case of what we call the Hotelling Valua-

[3] If (2) is not binding, we redefine R_0 to be the reserves that will be recovered, thus ensuring an equality constraint.

tion Principle to emphasize that it is distinct from, but still an implication of, the Hotelling Principle proper.[4] The implication can be tested, in principle, by regressing observed market values per unit of reserves for the mineral properties of particular companies at a given point in time (not necessarily the same for all the included companies) on the then current output prices net of marginal extraction costs, as in

$$\frac{V_0^{it}}{R_0^{it}} = \alpha + \beta(p_0^{it} - c_0^{it}), \tag{6}$$

where i indexes companies, t indexes calendar time, and 0 signifies the then current values as of the sample date t.[5] The test of the Hotelling Valuation Principle then hinges on the values of the coefficients. Under the constant returns assumption, for example, the Hotelling Valuation Principle implies $\alpha = 0$ and $\beta = 1$ and implies as well that additional variables such as interest rates or projected future mineral prices should contribute nothing to the explanatory power of (6).

The constant returns case, though perhaps useful as a first approximation, is extremely restrictive and unnecessarily so. Before turning to the data, therefore, we extend the analysis underlying (6) to allow for nonconstant returns to scale in current and cumulative production and for the use of average rather than marginal extraction costs in the estimating equation. We go on to consider briefly some of the consequences for valuation of cartel or government-imposed price controls as well as of uncertainty about the course of future product prices. Finally, we note some complications traceable to the treatment of mineral properties under the U.S. Internal Revenue Code.

C. *Extensions of the Valuation Principle*

1. Nonconstant Returns to Scale in Production

Diminishing returns to scale in current production will affect only the constant term in equation (6), not the slope. To see why, return to our

[4] Although the Valuation Principle is a fairly direct corollary of the Pricing Principle, it has received surprisingly little attention in the literature. Two recent studies invoking the Valuation Principle, though somewhat indirectly and not by that name, are Brown and Field (1978) and Hartwick (1978).

[5] The output prices in our empirical tests will be well-head prices. Because of differences in transportation costs and local severance taxes, these prices will in general differ from company to company at the same point in time even under otherwise strictly competitive conditions. Note also that while theoretical valuation expressions such as (5) have been, and will on occasion continue to be, expressed for convenience in terms of total values, their likely heteroscedasticity makes them less suitable for the empirical tests than per unit equations like (6).

general cost function $C = C(q_t, Q_t)$ and assume that both $\partial C_t/\partial q_t$ and $\partial^2 C_t/\partial q_t^2$ are positive. To focus sharply on the distinction between marginal and average cost, continue to ignore the cumulative cost terms and assume $\partial C_t/\partial Q_t = 0$. To simplify the derivation further, let $F_t = \bar{c}_t q_t - c_t q_t = C_t - c_t q_t$ be the difference between average and marginal cost multiplied by output. The valuation equation (1) then becomes

$$V_0 = \sum_{t=0}^{N} (p_t - c_t) q_t \left(\frac{1}{1+r}\right)^t - \sum_{t=0}^{N} F_t \left(\frac{1}{1+r}\right)^t. \qquad (7)$$

Substituting successively from the first-order condition (3′) for each term $(p_t - c_t)/(1+r)^t$ in (7), we obtain

$$V_0 = (p_0 - c_0) R_0 - \sum_{t=0}^{N} F_t \left(\frac{1}{1+r}\right)^t. \qquad (8)$$

Since costs are an increasing function of output in any period (i.e., $\partial^2 C_t/\partial^2 q_t > 0$), F_t is negative and the second term on the right-hand side of (8) will be positive. Note, however, that the c_0 in the first term is the *marginal* cost of extraction at output level q_0. The accounting data on which our empirical tests rely can give us only *average* extraction costs $\bar{c}_t$. With this change of variable, (8) can be rewritten in per unit form as

$$\frac{V_0}{R_0} = (p_0 - \bar{c}_0) + K_1, \qquad (9)$$

where

$$K_1 = \left(\frac{F_0}{q_0}\right) - \frac{1}{R_0} \sum_{t=0}^{N} F_t \left(\frac{1}{1+r}\right)^t.$$

The sign of K_1 is ambiguous since the first component is negative and the second positive. The intercept term α in the proposed regression (6) can thus no longer be presumed to be zero under the Hotelling Valuation Principle. But any departure is likely to be small, since the two offsetting components of K_1 are of the same order of magnitude.[6] The slope coefficient β, on the other hand, remains unity provided only that the Hotelling variables $(p_0^{it} - \bar{c}_0^{it})$ can be presumed independent of the omitted terms K_1 impounded in the intercept. Nothing guarantees this independence in any particular sample, of course. But the steps leading to (9) do show at least that for firms following the Hotelling Principle in allocating production over time,

[6] The two components would offset exactly in the special case in which (1) production was the same in every year and (2) F_t grew over time at the rate r. Under these conditions, each component would reduce to the same value, $(N + 1)F_0/R_0$.

none of the components of K_1 directly involves $(p_0 - \bar{c}_0)$ or any future net price likely to be correlated with it.

2. Extraction Costs as a Function of Cumulative Production

Although considerable simplification is achieved by assuming away the cumulative cost terms $\partial C_s/\partial Q_s$ in the first-order conditions (3), the simplification runs counter to the fundamental economic notion, tracing back at least to Ricardo, that the lowest-cost units will tend to be produced first. Hotelling himself took at least some partial steps toward bringing cumulative production into the statement of the model, and subsequent writers have furthered the task (see, e.g., Gordon 1967; Solow and Wan 1976; Hartwick 1978).

Allowing for extraction costs that rise with cumulative production has been shown to imply a time path of net prices rising at a rate below the real rate r of the simple case of the Hotelling Principle. The time path of net prices reflects the opportunity cost of deferring production—forgone interest less the saving in future production costs.

If net prices are rising at a rate slower than the r of the Hotelling Principle, then values should presumably be less than implied by the Hotelling Valuation Principle, which, it will be recalled, exploits the property that the numerators and denominators of the elements in the present value expressions will advance in parallel in a simple Hotelling world. Fortunately, as in the case of decreasing returns to current production, the effects of the user cost terms can be impounded in the intercept of the regression, leaving the presumed slope coefficient still unity.

To see why, return again to the general cost function $C(q_t, Q_t)$. V_0 is found by substituting successively from the first-order conditions (3) into the restated valuation formula (7) to obtain

$$V_0 = \lambda \sum_{t=0}^{N} q_t + \sum_{t=0}^{N} \sum_{s=t}^{N} \left(\frac{\partial C_s}{\partial Q_s} \right) q_t \left(\frac{1}{1+r} \right)^s - \sum_{t=0}^{N} F_t \left(\frac{1}{1+r} \right)^t. \tag{10}$$

But from the first equation in (3) we have

$$\lambda = (p_0 - c_0) - \sum_{s=0}^{N} \left(\frac{\partial C_s}{\partial Q_s} \right) \left(\frac{1}{1+r} \right)^s. \tag{11}$$

Substituting for λ in (10) and rearranging yields

$$V_0 = (p_0 - c_0)R_0 - \sum_{t=1}^{N} \sum_{s=0}^{t-1} \left(\frac{\partial C_s}{\partial Q_s} \right) q_t \left(\frac{1}{1+r} \right)^s - \sum_{t=0}^{N} F_t \left(\frac{1}{1+r} \right)^t. \tag{12}$$

The second term on the right-hand side of (12) is negative, while the third term is positive. However, when we substitute average extraction costs for marginal costs (as in [9]), (12) becomes, in per unit form,

$$\frac{V_0}{R_0} = (p_0 - \bar{c}_0) + K_1 + K_2, \tag{13}$$

where K_1 is the constant term from (9) and $K_2 \leq 0$ is the second term from the right-hand side of (12) divided by R_0. Although the sign of the constant term $K_1 + K_2$ is ambiguous, it seems not unlikely—given our earlier comments about K_1—that the constant term in (13) and hence the implied intercept in (6) will be negative. Note again also that whatever the sign, none of the components of either K_1 or K_2 contains $(p_0 - \bar{c}_0)$ or any future net price likely to be correlated with it. Hence the coefficient of unity for $(p_0 - \bar{c}_0)$ in (12) can be presumed to carry through to regression (6) for this extension as well.

3. Noncompetitive Output Prices

Hotelling recognized that some firms might be too large, relative to total industry output, for the effects of their production decisions on market prices to be ignored. He extended his analysis of intertemporal allocations to the case of pure monopoly and showed that the Pricing Principle carried through in a natural way, but with marginal revenue replacing price. The Valuation Principle extends as well and, in particular, the predicted slope coefficient remains unity when the independent variable is current price (average revenue) minus current average extraction costs. The difference between price and marginal revenue is impounded in the intercept in essentially the same way as the difference between average cost and marginal cost. The details, however, need not detain us further here, since the firms in our empirical sample of domestic, nonintegrated petroleum producers have negligible market power.

To say that our sample firms are price takers is not to suggest, of course, that the prices they faced were competitive. During our sample period, which runs from 1979 to August 1981, OPEC was in its heyday in the world market and the federal government was imposing price controls on domestically produced oil and gas. Such noncompetitive restrictions on the path of output prices, however, invalidate neither the Hotelling Principle, properly understood, nor our proposed tests of its corollary, the Hotelling Valuation Principle. The Hotelling Principle is a proposition about optimal production management by individual firms and not, except indirectly, industry prices. An optimizing firm will adjust its path of outputs, and hence of marginal extraction costs, until its own path of *net* prices meets the Hotelling condition, whatever the assumed path for industry market prices. The analysis leading to the tests for the valuation of price takers in a Hotelling world will thus go through exactly as before despite the actions of the price makers or price controllers. Price controls of the U.S. variety may even increase the efficiency of testing by introducing additional sources of variation in well-head prices such as between "old" oil and "new" oil and the many gradations in between.[7]

4. Allowing for Uncertainty

The consequences for our test equation of weakening the assumption of certainty depend to a considerable extent on the assumptions about extraction costs. For the constant-returns-to-scale case, the value per unit of the reserves remains the current spot price, net of extraction cost, since the Hotelling Valuation Principle under those cost conditions really says little more than that the value of the reserves is independent of where they happen to be stored, above the ground or below the ground. No rational buyer would pay more for the reserves and no rational owner would take less. For that special cost case, moreover, direct analogues to the Hotelling price growth Principle will hold. Pindyck (1980) has shown, for example, that where the uncertainties characterizing market demands and reserve levels can be adequately represented as diffusions (Ito processes), the *expected* rate of change of the net price will equal the opportunity cost of funds, exactly as in the deterministic case. Sundaresan (in press) has shown that the same conclusion holds even when the process governing new reserves is a jump process (i.e., one admitting the possibility of occasional major new discoveries).[8]

[7] Remember in this connection that unless a group of fields is operated as a single unit nothing in the Hotelling analysis requires net prices or marginal extraction costs to be equal across fields (see Gordon 1967).

[8] Pindyck's analysis of the price path assumes risk-neutral producers; Sundaresan considers both risk neutrality and constant relative risk aversion. For the valuation issues that are our main concern here, the risk adjustments are not of great moment because they enter both the price growth terms in the numerators and the offsetting discount factors in the denominators of the terms in the value summation.

Even under certainty, the simple r-percent rule was seen not to hold for nonconstant returns to scale in extraction. Uncertainty per se may cause additional departures from the simple rule under those conditions. Pindyck (1980) has shown, for example, that for nonlinear production-cost functions uncertainty about aggregate future reserves implies a path for net prices that rises more steeply than r. If so, valuations computed solely from current spot prices will tend to understate true market values.

Uncertainty can also affect valuation through the optionlike elements, noted in Brennan and Schwartz (1983), that arise from stochastic fluctuations in output prices. The owners of the property have the right to close the property down if current spot prices fail to cover current extraction costs; and more important, they can exercise the option of starting it up again when and if profit prospects unexpectedly become more favorable. Fortunately, however, as the numerical simulations of Brennan and Schwartz suggest, these options bulk large only for properties not currently producing. For companies already in production, such as those in our sample, the option to close is essentially a deep out-of-the-money put and the departure from the strict Hotelling Valuation Principle is small enough to be safely neglected.[9]

5. The Effect of Income Taxes

Up to this point we have entirely ignored the effect of corporate income taxes on the production and valuation of exhaustible resources. We are at least in good company in that respect; in the vast theoretical literature on the Hotelling Principle surveyed by Devarajan and Fisher (1981) there are rarely even passing references to income taxes, let alone detailed analyses of how such taxes affect the r-percent rule. The public finance literature, naturally enough, has shown greater concern with the possible nonneutralities of the U.S. tax treatment of mineral properties, but even here, interest seems to have waned after the ending of percentage depletion for corporations in the early 1970s. (For a recent brief survey of the public finance literature see Gaudet [1977] and the references there cited.)

Although neglect of this complication is certainly understandable when the focus is on basic theoretical issues, the requirements change when the concern is, as here, with empirical testing of actual against theoretical valuations. Since the real-world-observed valuations can

[9] As a precautionary check, we computed Hotelling values under the same assumptions as used by Brennan and Schwartz (1983) in the numerical approximations to their valuation formula in their table 1. For firms in production the values under the two approaches were virtually identical.

be presumed to take income taxes into account, some assumption must be made as to how such taxes enter the predicted values.

It is tempting to follow the practice common in other settings of merely reinterpreting all the variables as after-tax magnitudes. Thus the Hotelling Pricing Principle would read "net prices, after taxes, grow at the after-tax interest rate"; and the Hotelling Valuation Principle would imply a value for β in our estimating equation (6), not of unity, but of unity minus the tax rate.

Such a solution, however, would give only a lower bound for the value of reserves because it neglects the many offsets to income taxes available to mineral producers under the U.S. Internal Revenue Code. One major offset is the depletion allowance. For our sample corporations during our sample period, the depletion deduction allowed per unit of production was so-called cost depletion defined as "the adjusted basis of the property" (essentially its original cost less accumulated previous depletion deductions) divided by estimated total reserves remaining. The present value of the future tax shields from such cost depletion will add a positive term to the intercept in the valuation equation (6).

But this is only the beginning of the story. Suppose the firm finds hitherto unsuspected (or at least unreported) reserves on its property; or suppose its output prices unexpectedly surge upward. Then its projected corporate income tax payments will also surge. The law, however, offers a simple remedy: sell out! The property is worth more to a buyer than to the seller because the buyer can step up the basis. The buyer's basis for depletion will be not the low original cost basis of the seller, but the higher price actually paid for the property. Systematic reshuffling of properties of this kind can have substantial impact on the effective corporate income tax rate and hence on the implied slope coefficient. For example, with a 10 percent discount rate, production declining exponentially at 20 percent per year over an assumed 20-year economic life, and resale with stepped-up basis at the Hotelling value every 5 years, a statutory corporate income tax rate of 46 percent falls to an effective rate of only a bit more than 10 percent.

This incentive to keep selling out old properties is dampened by transactions costs and, in principle, also by the tax on the seller's gain. But at most, that tax is at the lower corporate capital gains rate; and during our sample period, even that lower rate tax could have been avoided, without triggering other recapture provisions, by such devices as tax-free partial liquidation of the property or by spinning off the low basis property to shareholders as a royalty trust.[10]

[10] For an account of the potential tax savings in properly handled sales and liquidations of mineral properties see the account in Brown (1982) of the tax angles in the U.S.

As an alternative to selling off low-basis properties, many firms in our sample chose to offset the taxable income against the expenses of drilling new wells. The permitted 1-year write-off of so-called intangible drilling costs—currently typically from two-thirds to three-fourths of the cost of digging the well—not only reduces the effective rate of corporate tax on the successful properties but creates strong incentives to keep plowing earnings back into the firm, thereby also reducing the tax bite on the owners under the personal income tax.

On balance, then, we conclude that taxes push the implied slope coefficient in the Hotelling valuation equation below unity, but probably not substantially so given the tax benefits, intended and unintended, to oil and gas producers during our sample period.

Not to be able to pin down the slope coefficient more precisely than "one or somewhat less" is disappointing. But the model has other testable restrictions (e.g., that the coefficient on the extraction cost variable be the negative of that on output prices), and other predictors of value exist against which to judge the Hotelling valuations.

III. The Empirical Tests of the Hotelling Valuation Principle

Previous tests of the Hotelling Principle have for the most part been time-series tests of the price-path predictions. Our tests, by contrast, are pooled, cross-sectional tests of the relation between observed market values for oil- and gas-producing properties and the values implied by the corollary to the Hotelling Principle that we have dubbed the Hotelling Valuation Principle.

A. *Sources of the Data*

For many natural resources it is difficult if not impossible to obtain reliable estimates either of recoverable reserves, R, or of the current price net of extraction costs, $p_0 - \bar{c}_0$. In the case of oil and gas proper-

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Steel–Marathon merger. During our sample period distribution of corporate reserves in a royalty trust occasioned no taxable gain or loss to the corporation, but the fair market value of the trust interests received were treated as dividends by the shareholders. If larger than the firm's accumulated retained earnings, the dividend was a nontaxable return of capital under the personal income tax. When not a return of capital, taxable individual shareholders could sell their shares cum the trust dividend (i.e., after the dividend is declared, but before the stock goes ex dividend), thereby converting the dividend to a capital gain. The cum dividend shares were attractive to taxable domestic corporations, which could exclude 85 percent of the dividends received and then sell the shares ex dividend, claiming the cum-ex differential as a capital loss. The net negative tax on a corporation could run over 20 percent, though some of the benefits were presumably shared with the original selling shareholders. Many of these loopholes have since been closed, particularly tightly, by the Tax Reform Act of 1984.

ties, however, publicly held companies, since 1978, have been required by the SEC, pursuant to Regulation F-X, rule 4-10, to publish annually estimates of "proved reserves," economically recoverable at current prices, and of actual net prices received for production during the month prior to the date of the report. Additional details on prices, costs, production, and reserves have become available as a by-product of federal energy legislation during the middle and late 1970s.

The specific estimates of reserves, sale prices, and operating costs we use are those presented in the periodical *Oil Industry Comparative Appraisals*, published by John S. Herold, Inc. The Herold compilations are readily available and widely used references based on public sources such as the SEC-mandated company statements and reports to state and federal regulatory agencies. The Herold figures for operating costs also allow for the estimated excise taxes payable after March 1, 1980, under the Crude Oil Windfall Profit Tax Act of 1980.

Herold lists some 60 firms as U.S. oil- and gas-producing companies (as opposed to producing and/or refining companies), but we have had to pare that number back to 39, eliminating mainly firms whose oil and gas reserves were too small in absolute size or relative to their nonpetroleum activities to get a reliable fix on their values.[11] For each of the remaining companies, stock price data were obtained from the *Bank and Quotation Record* for the dates corresponding to the dates of the Herold estimates of reserves, usually, but not always, the date of the closing of the company's fiscal year. For 19 of the companies, there were three reserve evaluation dates during our sample period from December 1979 to August 1981, and for 17, there were two, making 94 sample observations in all.[12]

B. *Definitions of the Variables*

Recall that the variables in our basic test equation (6) are V, the total market value of the property, R, the total recoverable reserves, and $(p_0 - c_0)$, the current net price per unit of reserves.

All the companies in our sample own both oil and gas reserves. Since the Hotelling Valuation Principle presumably applies with equal force to each, we begin by pooling the two types into a single composite aggregate reserve of R barrels of oil or oil equivalents. The conversion factor is the conventional one in the industry (corre-

[11] Two of the firms dropped appear to have been mainly coal companies; three had what appear to be significant undisclosed reserves of uranium; one was essentially a contract driller; one appears to have been a subsidiary of a company already in the sample. The remainder were firms with less than two million barrels of oil.

[12] The names of the sample firms are available from the authors.

sponding to approximate Btu equivalence) of 5,700 cubic feet of gas for each barrel of oil. The net price $(p_0 - \bar{c}_0)$ then becomes the weighted average of the values for oil and gas defined as

$$p_0 - c_0 = \left(\frac{R_{\text{oil}}}{R}\right)(p_{\text{oil}} - c_{\text{oil}}) + \left(\frac{R_{\text{gas}}}{R}\right)(p_{\text{gas}} - c_{\text{gas}}), \quad (14)$$

where R = aggregate current reserves in barrels, R_{oil} = oil reserves in barrels, R_{gas} = gas reserves in oil-equivalent barrels, and $(p_{\text{oil}} - \bar{c}_{\text{oil}})$ and $(p_{\text{gas}} - \bar{c}_{\text{gas}})$ are the net prices for each barrel of oil and oil-equivalent gas.

For V, the value of the reserves, the ideal measure would be actual transaction prices for fields. Not enough publicly recorded cases are yet available, however, to permit formal statistical testing. Absent transaction prices for the petroleum reserves on the asset side, we shall instead follow the standard practice in finance of substituting the values of the claims on the liability side—a practice that for our sample industry has come to be called prospecting for oil on the floor of the New York Stock Exchange.

Were it a matter only of measuring the value of the equity claims, the calculations would be straightforward (price per share times number of shares outstanding). But virtually all companies in our sample had creditor claims outstanding, and valuing these is not so simple. Typically, debts were of three types: short-term obligations (mostly trade accounts payable); bank loans (normally at floating rates); and long-term, fixed-rate bonds and notes. We took the first two at book value. For long-term debt, we used actual market values where publicly traded; and where not, we used estimates based on prices for publicly traded issues of comparable rating and maturity.

Further complications arise because most companies own assets other than the oil and gas reserves of our primary concern and these other assets must be netted out. For some nonpetroleum assets such as investments in the securities of other firms, market prices could be obtained. The others, perforce, were taken at book value. That approximation is unlikely to be far off for assets such as cash or accounts receivable, but is more problematic for assets like plant, equipment, tank trucks, inventories, and in a few cases "intangibles."

C. *The Basic Regression*

Line 1 of table 1 shows the results of fitting the regression equation (6) to our sample of producing companies. The average net current price (denoted as HOTEL) has a slope coefficient of 0.910 with a standard error of 0.114 and is thus in close agreement with the predicted "one, or a bit less" of the tax-adjusted Hotelling Valuation Principle. The

TABLE 1

BASIC REGRESSIONS

Regression	Constant (β_0)	HOTEL (β_1)	$1/R$ (β_2)	OIL (β_3)	GAS (β_4)	OILP (β_5)	OILC (β_6)	GASP (β_7)	GASC (β_8)	PRICES (β_9)	COSTS (β_{10})	R^2	σ_ϵ^2
(1)	−2.240	.910										.408	3.187
	(1.035)	(.114)											
(2)	−2.252	.896	1.174									.417	3.181
	(1.061)	(.115)	(1.006)										
(3)	−3.078			.844	1.134							.426	3.155
	(1.138)			(.120)	(.174)								
(4)	−2.233									.982	−1.117	.414	3.188
	(1.035)									(.137)	(.244)		
(5)	−3.469					.944	−1.041	1.088	−.836			.432	3.174
	(1.268)					(.177)	(.320)	(.184)	(.437)				
(6)	3.486			.560								.159	3.799
	(.636)			(.134)									
(7)	2.241				.687							.113	3.902
	(1.054)				(.201)								
(8)	−2.693					.423		.814				.342	3.380
	(1.310)					(.070)		(.147)					
(9)	−.150									.433		.279	3.518
	(1.026)									(.073)			
(10)	3.641										.418	.082	3.970
	(.791)										(.146)		

NOTE.—Dependent variable: market value of oil and gas/barrel equivalent of oil and gas reserves. Independent variables defined in text. $N = 94$. Standard errors in parentheses.

negative intercept of -2.240 with a standard error of 1.035 is consistent with the predictions derived in the previous section for the Hotelling Valuation Principle under conditions of decreasing returns to scale in current and cumulative production.[13]

Not only are the regression coefficients in accord with prior expectations, but the net price variable HOTEL accounts for a substantial proportion of the variation ($R^2 = .408$) of V/R (denoted hereafter as VALUE). A glance at the scatter plot in figure 1 shows the relation to be a pervasive one and not attributable merely to a few outliers. Nor is either the strength of the relation or the near-unity value of the slope coefficient merely an artifact of the correlation of errors in the total reserves, R, that appears in the denominators of both VALUE and HOTEL. Row 2 of table 1 shows that adding $1/R$ as a separate variable leaves both intercept and the slope coefficient of HOTEL virtually unchanged. The term $1/R$ itself appears to have little marginal explanatory power in the presence of the key variable HOTEL.[14]

D. *The Basic Regression Further Disaggregated*

Additional tests of the restrictions implied by the Hotelling Valuation Principle, as well as checks on the validity of the pooling assumptions underlying the basic regression, are presented in rows 3–9 of table 1. Rows 3, 6, and 7 show the effect of separating HOTEL into its two components, OIL and GAS. Note from rows 6 and 7 the dramatic drop in explanatory power when the components are entered singly. Clearly market values reflect, as they should, both kinds of reserves. As to the relative weights on each, the Hotelling Valuation Principle presumably applies equally to reserves of both types, and hence the coefficients should be the same. As it turns out, the coefficient of GAS is somewhat higher than that of OIL but not by more than could be accounted for by normal sampling fluctuations. A formal F-test confirms that the restriction of equal coefficients on OIL and GAS cannot be rejected at conventional levels of significance.

Disaggregation of HOTEL into its output price and extraction cost components provides another, and in some respects even more strin-

[13] Since our R measures "proved" reserves rather than "proved and developed" reserves, the negative intercept may also reflect the market's anticipation of future development costs.

[14] We also reran the basic regression in ratio form (i.e., regressing VALUE/HOTEL on 1/HOTEL and a constant) partly as a further check on any problems caused by a common R and partly as a simple correction for the slight heteroscedasticity in the basic regression detected in a Park-Glejser test. The standard errors dropped somewhat, as would be expected, but the coefficients themselves were virtually unchanged. The results were also essentially the same when the regression was run in terms of total value rather than on a per barrel basis.

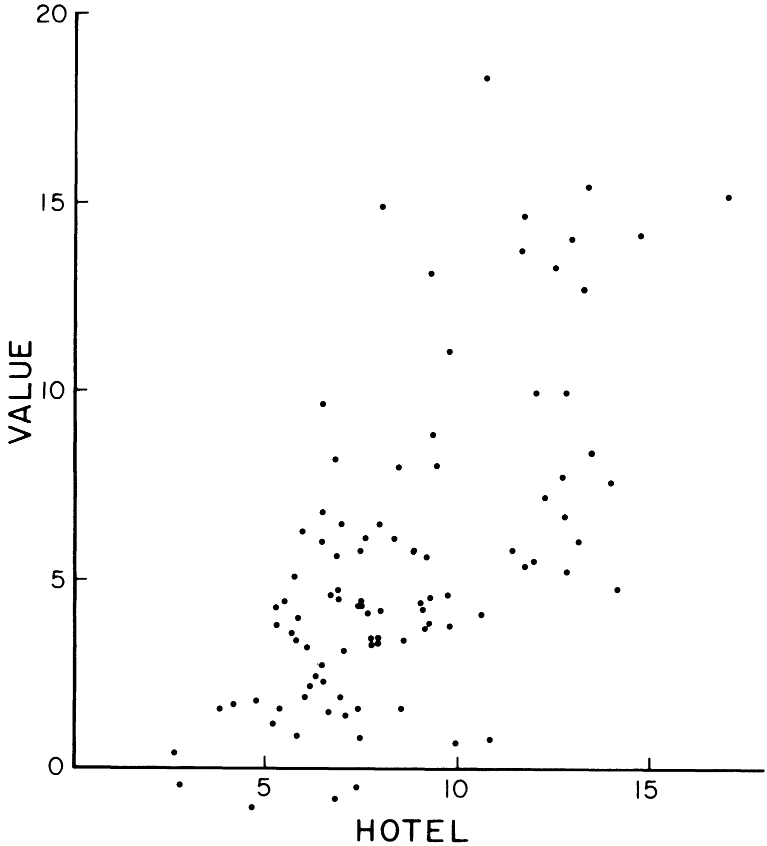

FIG. 1.—Plot of VALUE versus HOTEL. Variables measured in dollars per barrel of reserves.

gent, test of the Hotelling Valuation Principle. The theory says that both terms are required to explain value and that they enter the regression with equal magnitude but opposite sign. Note then once again the substantial drop in explanatory power when each component is entered singly—for example, in regressions (9) and (10) as compared to regressions (1) and (4) where both are present. (These results should also serve as a useful reminder of the dangers of focusing solely on product prices to the neglect of extraction costs—a failing, we suspect, that is at least partly responsible for the popular journalistic belief that the stock market value of oil is always cheap relative to market prices and exploration costs.[15] Extraction costs

[15] Note also the negative intercept in the valuation equation. Even after allowing for current production costs, the current price gives only an upper bound to the value per barrel because user costs (and, in our sample, future development costs) must be taken

TABLE 2

TESTS OF HOTEL VERSUS TIME

Regression	Constant	HOTEL	DATEST	R^2	$\hat{\sigma}_\epsilon$
(1)	2.886		.0091	.289	3.495
	(.572)		(.0015)		
(2)	−1.645	.750	.0026	.420	3.174
	(1.122)	(.165)	(.0026)		

NOTE.—Dependent variables: market value of oil and gas/barrel equivalent of oil and gas reserves. Independent variables defined in text. N = 94. Standard errors in parentheses.

must be included along with prices to account for differences in the market value of reserves.) And, more to our main concern, the results in regressions (4) and (5) are consistent, up to sampling error, with the prediction that the coefficients of prices and costs are equal but of opposite sign.

E. *A Check for Common Time Trends*

The seeming explanatory power of our HOTEL variable may perhaps be reflecting only the common upward trend in both stock prices and petroleum prices during our sample period of December 1979–August 1981. We have already seen some indirect evidence that such is not the case: regression (9) of table 1 has lower explanatory power than regression (1). But a more direct test is provided in table 2, which adds to the basic regression an additional variable, DATEST, representing the date for which the reserves and market values were estimated (December 31, 1978 = 1, etc.). As can be seen from the first regression in table 2, market value is indeed positively correlated with DATEST. But the explanatory power of the time-trend variable DATEST is substantially less than that of HOTEL alone; and both the regression coefficient of HOTEL and the correlation coefficient of the regression remain essentially the same whether or not the time-trend variable is included.[16]

F. *Correcting for Serial Dependence of Individual Company Residuals*

In a sample such as ours, with multiple observation dates for each company, the standard regression assumption of independent resid-

out as well. Recall also the earlier discussion in Sec. II*C*5 above of the tax benefits from sales of reserves and from drilling.

[16] As an additional check on the validity of the pooling of observations at different times we reran the basic regression separately for each half of the sample period. We could not reject the restriction of equal slopes and intercepts in the two subperiods.

uals cannot be taken for granted. Indeed, if capital markets are efficient—and we have, of course, been relying on that efficiency implicitly in our use of security prices as the relevant measure of the value of reserves—the individual company residuals from our basic regression cannot be serially independent. To see why, suppose that $\epsilon_{i,t+1}$, the residual difference between company i's market value at time $t + 1$ and its regression prediction $\hat{\alpha} + \hat{\beta}\text{HOTEL}_{i,t+1}$, were independent of the previous residual $\epsilon_{i,t}$. Then for any firm with a positive residual at t, the expected *change* in residual is negative, implying an expected fall in value relative to the regression line; and for a firm with a negative residual, the expected change in residual and in value relative to the line is positive. Thus positive expected profits could be earned on a zero-investment portfolio formed at t, long in the shares of the "undervalued" stocks with negative $\epsilon_{i,t}$ and short in the "overvalued" stocks with positive $\epsilon_{i,t}$.

To guard against any mistaken inferences traceable to this autocorrelation of individual firm residuals, we have reestimated the basic equation in first-difference form with the following result:

$$\text{VALUE}_{i,t+1} - \text{VALUE}_{i,t} = \underset{(0.794)}{0.185} + \underset{(0.226)}{0.978}(\text{HOTEL}_{i,t+1} - \text{HOTEL}_{i,t}), \tag{15}$$

$$R^2 = .262, \; \hat{\sigma}_\epsilon = 3.747.$$

The relation between HOTEL and VALUE found earlier thus appears to persist even in the face of differencing.[17]

G. *Other Tests of the Specification*

Two other tests of the robustness of the specification are perhaps also worth noting. As a test of the predicted linearity of the basic relation, we added a variable $(\text{HOTEL})^2$ to the equation and found that it made no material contribution. As a further check on distortions introduced by errors or omissions in our estimates of creditor claims or of other, nonreserve assets, we also reran the basic equation with stock market value rather than total net value as the dependent variable and with creditor claims and nonreserve assets on the right-hand side as separate independent variables. These variables came in with correct signs and reasonable magnitudes, suggesting at least that they are not hopeless proxies for what they seek to measure. And, more to the immediate point, the coefficient of HOTEL was little affected.

[17] The first differences are taken as absolute differences rather than percentage or logarithmic differences because of negative values in a few cases for our variable VALUE (defined, it will be recalled, as the sum of equity plus debt minus the estimated value of other, nonreserve assets).

H. Summary

For our sample of domestic petroleum-producing firms, the relation between the market value of reserves and current output prices net of extraction costs appears to conform closely to that predicted by the Hotelling Valuation Principle. This closeness stands up, moreover, in the face of a variety of standard and special diagnostic checks. Before awarding any palms to the Hotelling Valuation Principle, however, it is important to check out the alternatives. The net current price, after all, is likely to play a substantial role in any "discounted cash flow" valuation formula, though only the Hotelling Valuation Principle identifies it as the critical determinant of value. A more stringent test of the principle is thus whether it can predict more accurately than reasonable alternatives, and it is to these comparisons we now turn.

IV. Comparison with Alternative Valuations

As noted earlier, we are fortunate in having two publicly available valuations of the reserves for the companies in our sample. One is the annual statement of Estimated Present Value of Estimated Future Net Revenues from Oil and Gas Activities mandated since 1978 by the SEC pursuant to Regulation F-X, rule 4-10, for inclusion in the financial reports of registered oil and gas producers. The other is the regularly published appraisals of John S. Herold, Inc., from whose raw data we have constructed our own Hotelling measures.

A. The SEC Valuations

Under SEC regulations, estimated future net revenues are computed by applying, for conservatism, current oil and gas prices net of extraction costs to estimated future production from net proved reserves, less estimated future development expenditures. The present value of the estimated net revenue stream is then determined by discounting at a prescribed rate of—what else?—10 percent.

In using these SEC-mandated values as standards for comparison it is not our intention to suggest that they are or ever were used by anyone to appraise reserves. The SEC valuations rather should be thought of as analogues to the "naive models" against which more elaborate forecasts are calibrated.

B. The Herold Appraisals

The Herold appraisals, by contrast, are intended to be taken seriously and are, in fact, widely cited in the financial press. The appraisals themselves are representive of the "discounted-cash-flow" calculations that have become standard in the literature of finance. Herold has, it is true, finessed the problems connected with estimating an appropriately risk-adjusted "cost of capital," opting instead, like the SEC, for the conventional 10 percent. But unlike the SEC rules, Herold's estimates of future cash flows do try to allow for what the firm regards as the likely future course of petroleum prices and costs. Herold (November 1980), for example, assumes "that the current average price received by each individual company will increase to a decontrolled world oil price of $35.00 per barrel by 1982, at which time the price will increase at an annual rate of 6% until the year 2000 and remain constant thereafter." Over the same horizon, "unit operating costs are escalated at a rate for each individual company which is dependent on the number of years to exhaust the reserves and the unit price of oil or gas at the time of exhaustion" (p. 706).

C. *Comparison of the Alternative Valuations*

Table 3 shows the correlation coefficients between the three alternative valuation measures and between each and VALUE, our measure of the market value of reserves. Note that the three alternative valuations are indeed highly correlated, as would be expected from the important information about current prices and costs they share in common. Note also that even the naive SEC measure appears to account for a nontrivial fraction of the variation in VALUE, though substantially less so than its better-grounded rivals, HOTEL and HEROLD. Of the three, the strongest association with VALUE is found for HOTEL.

Not only is HOTEL the best single predictor of market value in our sample, but neither of the alternatives, despite their seemingly high correlations with VALUE, appears to have significant information about VALUE not already subsumed in HOTEL. Table 4, for example, presents the results of Davidson-MacKinnon (1981) tests of

TABLE 3

Correlation Coefficients between Alternative Valuation Measures*

	VALUE	HOTEL	SEC	HEROLD
VALUE	1.0	.644	.547	.628
HOTEL		1.0	.751	.897
SEC			1.0	.701
HEROLD				1.0

* Based on the 92 observations for which SEC valuations were available.

TABLE 4

TESTS OF ALTERNATIVE MODELS OF MARKET VALUES FOR RESERVES*

H_0	Alternative	$\hat{\alpha}$	$t_{\hat{\alpha}}$
HOTEL	SEC	.265	1.192
HOTEL	HEROLD	.409	1.420
SEC	HOTEL	.831	4.395
SEC	HEROLD	.765	4.239
HEROLD	HOTEL	.642	2.284
HEROLD	SEC	.384	1.854

* Based on the 92 observations for which SEC valuations were available.

regression specification in the presence of alternative models purporting to explain the same phenomenon.

For testing a linear model H_0: $y_i = f_i(X_i, \beta_1) + \epsilon_{0i}$ against an alternative nonnested model H_1: $y_i = g_i(Z_i, \beta_2) + \epsilon_{1i}$ (where β_1 and β_2 are vectors of parameters to be estimated), Davidson and MacKinnon propose the regression

$$y_i = (1 - \alpha)f_i(X_i, \beta_1) + \alpha\hat{g}_i + \epsilon_i, \tag{16}$$

where $\hat{g}_i = \hat{g}(Z_i, \hat{\beta}_2)$, and $\hat{\beta}_2$ is the maximum likelihood estimate of β_2. If H_0 is true, then the true value of α is zero and the t-statistic provides a test of that hypothesis. Generalizations of (16) are possible if f is nonlinear, but they need not concern us here.

To implement the Davidson-MacKinnon test we first fit a relation between VALUE and SEC of the form

$$\text{VALUE}_i = \gamma_0 + \gamma_1\text{SEC}_i + \epsilon_i \tag{17}$$

and then use the fitted values to compute the regression estimate $\hat{g}$. We then estimate an equation of the form

$$\text{VALUE}_i = (1 - \alpha)(a + b\text{HOTEL}_i) + \alpha\hat{g}_i + \eta_i \tag{18}$$

and test for $\alpha = 0$. Similar steps are followed for testing HOTEL against HEROLD. For completeness, we also test SEC and HEROLD as the maintained hypothesis against each of the alternatives.

Columns 3 and 4 of table 4 show the values of α and $t(\alpha)$ for various combinations of maintained and alternative hypotheses. In the first two rows, the Hotelling model prediction is the standard and SEC and HEROLD are the alternatives. The low t-values for α imply that $\alpha = 0$ cannot be rejected at conventional significance levels, which is to say that SEC and HEROLD *can* be rejected as alternatives to HOTEL. On the other hand, when the roles are reversed, as in the subsequent rows 3 and 5, we cannot reject HOTEL as an alternative to either SEC or HEROLD.

V. Conclusion

The role of the Hotelling Principle or "r-percent rule" and its natural extensions as the central propositions of resource economics is unlikely to be much affected by "mere" empirical testing. No viable alternative paradigm exists. Still, economists should be comforted to learn, given the generally poor results to date of direct time-series tests, that at least some of the restrictions implied by the Hotelling model of optimal intertemporal production can indeed be detected in real-world data. In particular, we have seen that an interesting but little-cited corollary of the Hotelling Principle, the Hotelling Valuation Principle, provides not only reasonably good descriptions of the structure of actual market values of a sample of U.S. petroleum-producing companies, but substantially better descriptions than the publicly available alternative appraisals based on much the same underlying raw data.

The relative robustness and predictive success of the Hotelling-based appraisals provide additional evidence, if that is needed, that more complicated models are not always better. The Hotelling Valuation Principle could hardly be simpler: it says, essentially, that the value of a unit of reserves in the ground is the same as its current value above the ground less the marginal costs of extracting it. Things work out this neatly because in a Hotelling world, the two key components of the valuation formula—the expected trend of future net product prices and the appropriate discount rate—are not independent, but are two sides, as it were, of essentially the same coin. Conventional valuation procedures that try to estimate the pieces separately will thus ignore the important restrictions imposed by the theory of optimal exploitation.

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