Risk, Diversification, and Competition in Trade Networks with evidence from US Natural Gas Market

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Preliminary Draft - will be updated soon

Abstract

This paper studies the effect of trade risk in networked markets, which require a physical, contractual, or relational connection for transactions between the buyer and the seller and the set of feasible trades are described by a network. When the chance that a single trade link in the network breaks down is non-trivial, the risk of some contracts not being fulfilled affects strategic behavior of participants and market outcomes. We present a model of price competition in a networked market with a two period game in which the contracts are written in first period but risk is realized in the second period. We show that the change in the contracts due to trade risk can be interpreted as effects of demand for trade diversification and trade insurance, creating additional market power for sellers. However, despite the fact that market power increases with the probability that links break the equilibrium ex-ante expected profits, initially rise with risk at low levels of risk and then falls risk. We used the continental

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US natural gas market to motivate our trade risk model and empirically test some of its implications and predictions. By combining multiple EIA datasets, we constructed a networked market between the states which are dominant producers and consumers of natural gas, while only keeping major pipelines. Since the likelihood of pipeline incidents depends on the length of the pipeline, we using the capacity weighted average length of pipelines transmitting gas to a buyer to proxy for the probability that each link to that seller breaks down. After controlling for various characteristics of buyer states that can effect their demand for natural gas, the empirical tests indicate the negative effect of the effective number of inward pipelines connected to a buyer and positive effect of the likelihood of a link breakage on equilibrium price faced by that buyer, corroborating predictions of the model. To address potential endogeneity of number of buyer connections, we utilize a few centrality measures of the buyer node in the network, such as the betweenness and closeness centrality, as successful instrumental variables. Moreover, we calibrate the parameters of our model with US natural gas market using multiple objective nonlinear least square. The main analysis considers competition in exogenous trade networks and studies the effects of network structure and risk asymmetry on size and distribution of market power among sellers. Later we considers the strategic behavior of participants when trade links are endogenously formed to harvest the gains from trade. We start by analyzing the welfare properties of the outcomes of a network formation game that represents chronological formation of an actual market. Later, we introduce an alternative formation game that improves the equilibrium efficiency that can be used for designing new markets.

1 Introduction

In recent years there has been a growing interest in addressing fundamental economic questions within networked markets [Kranton and Minehart (2000, 2003); Goyal and Joshi (2003); Kranton and Minehart (2003); Corominas-Bosch (2004); Manea (2011); Nava (2015)]. Unlike the standard Walrasian model in which all buyers can trade with all sellers at a single price, in the networked markets trade is bilateral and only takes place between linked agents. In fact, in most markets all buyers are not connected to al sellers and the transaction of goods or services between a buyer and a seller requires either a trade infrastructure or a special bilateral relationship. The significance of network structure on the strategic behavior of participants and hence the market outcome has been shown under various economic settings.

Outside the networked markets framework trade risk does not have any relevance. A product not delivered by one supplier can be obtained from another supplier. However, when each market participant has limited trade partners the potential risk of losing a trade link affects the behavior of buyer and seller agents. Especially, when buyers have concave objective function and non-fulfilled orders are costly[1], the trade risk becomes significant in determining the market equilibrium. When the buyer is the final consumer of the product like a country or a local government utility concavity originates from political satisfaction or even national security concerns. Alternatively, when a firm is purchasing a factor of production, it incurs high cost if does not obtain sufficient input. Whereas the cost of procuring extra units is bounded by their price, which leads to the profit function being concave in the amount of input orders fulfilled.

[1]The complete order usually includes the quantity, quality, and the time of delivery of the product.

Actually, there are quite a few markets in which the trade links face a nontrivial risk of being broken off. A link breakage can arise from potential damage or destruction of physical trade infrastructure, or an event that causes a delay in delivery of the product or results in the seller not fully fulfilling the contract. Examples of trade links that are physical infrastructure include gas pipelines, power lines, and fiber-optic cables, all of which have a non-zero rate of failure[2]. On the other hand, trade treaties and supply chains with substantial reputation concerns are examples of trade links that arise from special relationships. A trade link between two countries can be cut off in order to put political pressure, as a part of a trade embargo, political unrest, or one of the countries leaving a trade union, all of which are outside our model making the risk exogenous.

The goal of this work is to study the interaction of economic agents and efficiency of market equilibriums in presence of exogenous trade risk. Furthermore, we are interested in how risk and network position influence market power. The finance literature has extensively studied the effect of bankruptcy risk of an individual lender on market equilibrium and its propagation through the financial network [Acemoglu et al. (2015)]. While the focus of that literature has been studying the impact of potential loss of a node in a networked market, our emphasis is analyzing the impact of risk of an edge being removed from the trade network. The rest of the paper is organized as follows. First, we analyze the impact of risk on the behavior of a buyer in a networked market with exogenous prices. Then we study the strategic interactions between buyer and seller agents on various exogenous networks. Finally, we allow for the network to form endogenously and suggest an alternate market

[2] For instance, according to PHMSA (Pipeline and Hazardous Materials Safety Administration) over the last 20 years there has been over 300 significant pipeline incidents per year in USA natural gas pipelines. The major causes of pipeline failure are external interference, construction defects, corrosion, and ground movement.

design to achieve the most efficient outcome as market equilibrium.

2 The Model

Consider a undirected bipartite trade network with finite number of buyers and seller and one product. Let, B denote the set of buyers $\{B_1, B_2, \cdots, B_m\}$, and B denote the set of sellers $\{S_1, S_2, \cdots, S_n\}$. Each buyer has a quasilinear von Neumann-Morgenstern utility $U(q, p) = u(q) - pq$, in which $u(q)$ is both increasing and concave. While sellers are risk neutral and maximize profits. Each seller has a technology to produce the good at constant marginal cost. We express the trade network with an undirected graph $G = (V, E)$, in which $V = B \cup S$ denotes the set of all economic agents and E is a $n \times m$ matrix where the binary element e_{ij} indicates whether S_i and B_j are linked. Also, denote the flow of consumption good from S_i to B_j by q_{ij}. Each buyer j can only trade with subset of sellers $e^b_j = \{i|e_{ij} = 1\}$ which are trade partners of buyer j. Similarly, each seller i can only trade with subset of buyers $e^s_i = \{j|e_{ij} = 1\}$ which are trade partners of seller i.

We model the risk with a two-period game. In the first period, each buyer will simultaneously negotiate with all linked supplier to order the product for the next period as follows: sellers set price for each link and buyers choose quantity from that link. Each linked buyer and seller, S_i and B_j form a contract (p_{ij}, q_{ij}). In the next time period each trade link can potentially break with independent probability of η. For links that are not broken the seller delivers the contracted quantity, q_{ij}, and buyer pays its negotiated price, $p_{ij}q_{ij}$. The economy is defined as $\xi = \{G, P, Q\}$, where P and Q are $n \times m$ matrices. In the first part of the paper I analyze the competition on an exogenous network. The equilibrium concept is a *Network*

Competitive Equilibrium which refers to a pure strategy Nash equilibrium of the system,

$$\mathbf{q_j}(\mathbf{p_j}) \in \arg \max_{\mathbf{y_j} \in \mathbb{R}_+^{k_j^b}} EU_j(P, \mathbf{y_j}; \eta) \ , \quad j \in \{1, \cdots, m\} \tag{1}$$

$$\mathbf{p_i}(P_{-i}) \in \arg \max_{\rho_\mathbf{i} \in \mathbb{R}_{++}^{k_i^s}} E\pi_i(\rho_\mathbf{i}, P_{-i}; \mathbf{c}, \eta) \ , \quad i \in \{1, \cdots, n\} \tag{2}$$

where k_j^b and k_i^s denote the degree of corresponding buyer and seller nodes,

$$k_j^b = \sum_{j=1}^{m} e_{ij} \quad ; \quad k_i^s = \sum_{i=1}^{n} e_{ij}$$

The expected utility of buyer j is

$$EU_j(\mathbf{p_j}, \mathbf{q_j}; \eta) = (1-\eta)^{k_j^b} u(Q_j) + \sum_{i \in e_j^b} \eta (1-\eta)^{k_j^b - 1} u(Q_j - q_{ij}) + \cdots + \sum_{i \in e_j^b} \eta^{k_j^b - 1}(1-\eta) u(q_{ij}) - (1-\eta) \sum_{i \in e_j^b} p_{ij} q_{ij}$$

where Q_j denotes the total order size of buyer j,

$$Q_j = \sum_{i \in e_j^b} q_{ij}$$

Alternatively, we can rewrite the expected utility of buyer j as

$$EU_j(\mathbf{p_j}, \mathbf{q_j}; \eta) = \sum_{w \subseteq e_j^b} \eta^{k_j^b - k_w} (1-\eta)^{k_w} u\left(\textstyle\sum_{i \in w} q_{ij}\right) - (1-\eta) \sum_{i \in e_j^b} p_{ij} q_{ij}$$

where $k_j^b = \#e_j^b$ and $k_w = \#w$ (# representing the number of elements in a set). The ex-ante expected profit of seller i is

$$E\pi_i(\mathbf{p_i}, P_{-i}; \mathbf{c}, \eta) = (1-\eta) \sum_{j \in e_i^s} (p_{ij} - c_i) q_{ij}(\mathbf{p_j})$$

3 The Demand Response

One can obtain the Network Competitive Equilibrium in two steps. First, find the buyer demand functions, $\mathbf{q_j}(\mathbf{p_j})$, for any exogenous price vector, $\mathbf{p_j} \in \mathbb{R}_{++}^{k_j^b}$. Second, obtain the pure Nash equilibrium(s) of the system in 2 using those demand function. In this section, we introduce the general demand model and elaborate its main characteristics. Then, we analyze the demand function when the buyer faces two sellers. We use this model to illustrate some intuitions.

Buyer j has k_j^b trade partners from which it can order the product. It faces a price vector, $\mathbf{p_j} \in \mathbb{R}_{++}^{k_j^b}$. Assume without loss of generality that the $\mathbf{p_j}$ is ordered in descending manner, $p_{hj} \geq p_{lj}$ for $h < l$. With some abuse of notation lets rename the elements of price vector $\mathbf{p_j}$ to $\{p_1 j, \cdots, p_{k_j^b j}\}$. (Assume without loss of generality that B_j is connected S_1 to $S_{k_j^b}$ and the price vector $\mathbf{p_j}$ is ordered in descending manner, $p_{1j} \geq \cdots \geq p_{k_j^b j}$.)

Proposition 1 *The buyer has zero demand from first $\tilde{k}$ sellers and strictly positive demand from the remaining ones uniquely determined by the function $q(p_{ij}, \mathbf{p_{-ij}}; \eta)$ if and only if,*

$p_{\tilde{k}j} \geq G(\tilde{k})$ *and* $p_{\tilde{k}+1,j} < G(\tilde{k}+1)$ *where*

$$G(\tilde{k}) \equiv (1-\eta)^{k_j^b-\tilde{k}} u'\left(\sum_{i>\tilde{k}} q_{ij}^*\right) + \eta(1-\eta)^{k_j^b-\tilde{k}-1} \sum_{l>\tilde{k}} u'\left(\sum_{\substack{i>\tilde{k}\\ i\neq l}} q_{ij}^*\right) + \cdots + \eta^{k_j^b-\tilde{k}-1}(1-\eta)\sum_{i>\tilde{k}} u'\left(q_{ij}^*\right) + \eta^{k_j^b-\tilde{k}} u'(0) \quad (3)$$

and $q_{ij}^* = q(p_{ij}, \mathbf{p_{-ij}}; \eta)$ *for* $\tilde{k} < i \leq k_j^b$.

Now, we analyze a case in which the buyer has only two trade partners to clarify some additional intuitions from the model.

3.1 A Model of one Buyer and two Sellers

This is the simplest trade network providing buyer with the choice of placing orders form multiple suppliers. As shown in Figure 1, the buyer B_1 has two trade links to sellers S_1 and S_2, with per unit production costs of c_1 and c_2, respectively. The demand function $q(\cdot)$ is characterized by

$$(1-\eta)u'(Q_1) + \eta u'(q_{i1}) \leq p_{i1} \quad , \quad i = 1,2 \quad (4)$$

where $q_{i1}^* = q(p_{i1}, p_{-i1}; \eta)$, with equality as long as a change in price from seller i affects the demanded quantities (i.e. when $q_{i1}^* > 0$). Assume without loss of generality that $p_{i1} \geq p_{-i1}$. Note that as long as there is some positive q_{i1} that satisfies

$$p_{i1} < \eta\left(\frac{u(q_{i1}) - u(0)}{q_{i1}}\right) + (1-\eta)\left(\frac{u(q_{i1} + q_{-i1}^*) - u(q_{-i1}^*)}{q_{i1}}\right) \quad (5)$$

the buyer has positive trade volume with both sellers. When the condition in 5 holds, the

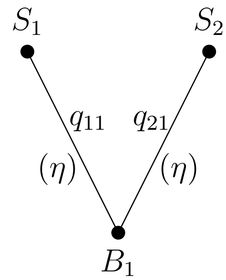

Figure 1: The trade network in section 3.1

prices lead to an interior solution and we have the following two comparative statics with respect to prices and risk.

Proposition **2** *The quantity demanded from each seller is strictly increasing with own price, $\frac{\partial q_{i1}}{\partial p_{i1}} < 0$, and strictly decreasing with price of the competing seller, $\frac{\partial q_{i1}}{\partial p_{-i1}} > 0$. Moreover, the total order volume increases with any of the bilateral prices, $\frac{\partial Q_1}{\partial p_{i1}} < 0$.*

We provide these expressions and elaborate these results for a buyer that faces two sellers in the Appendix. However, these results are general. $\blacksquare$

Now let us define the two fundamental demand responses to the presence of risk in networked markets.

Trade Diversification Moving a portion of the total purchase from the cheaper trade partners to the more expensive trade partners in response to risk.

Trade Insurance Increasing the total order volume in response to risk to insure against the states of world in which at least one of the links is broken.

Proposition **3** *The buyers optimal response to the risk of losing a trade link is "trade diversification" and "trade insurance".*

Bilateral trade volumes are affected by the risk of losing a trade link as follows,

$$\frac{\partial q_{i1}}{\partial \eta} = \frac{(1-\eta)u''(Q_1)\left[u'(q_{-i1}) - u'(q_{i1})\right] + \eta u''(q_{-i1})\left[u'(Q_1) - u'(q_{i1})\right]}{\eta(1-\eta)u''(Q_1)\left[u''(q_{11}) + u''(q_{21})\right] + \eta^2 u''(q_{11})u''(q_{21})} \tag{6}$$

The first term of the demand response to risk in equation 6 represents the trade diversification in response to the risk. Let us denote this term as D_{i1}. One can observe that the D_{i1} terms add up to zero. Therefore, this term represents the net substitution between demanded quantities across sellers (from other sellers to seller i) due to risk, keeping the aggregate demand constant. That is why it is regarded as trade diversification. Since $p_{11} \geq p_{21}$, we have $u'(Q_1) < u'(q_{21}) \leq u'(q_{11})$. Hence, when the buyer faces different prices D_{i1} is positive for seller 1, and negative for seller 2. While, there is no trade diversification when the buyer faces the same price from all sellers.

On the other hand, the second terms of the partial derivative of the demand from each seller with respect to risk add up to the sensitivity of total order volume to risk. Let us denote the second term in equation 6 as I_{i1}. In fact, I_{i1} indicates the share of the total trade insurance that is purchased from seller i. The allotment of these share depend on the specific utility functions. For quadratic utility function and CRRA utility with $\rho < 1$, more insurance is purchased from relatively more expensive sellers. On the other hand, CRRA utility with $\rho > 1$ more insurance is purchased from relatively less expensive sellers. Buyers with log utility purchase equal amount of insurance from all linked sellers.

Note that the trade insurance term of the demand response to risk, I_{i1}, is positive for all sellers. Therefore, higher risk of losing a trade link will always induce the buyer to order more from the seller with higher price, $\frac{\partial q_{11}}{\partial \eta} > 0$. However, the effect of risk on the order from the seller with lower price, $\frac{\partial q_{21}}{\partial \eta}$, is negative for sufficiently small values of η and positive for sufficiently large values of η.

Example **1** *A buyer with utility function $u(q) = log(q)$ can order products from two trade partners, charging per unit prices of $p_{11} = 4$ and $p_{21} = 3$. If both trade links are secure and do not entail the risk of breaking in future, the buyer would only order from S_2, the seller who offers the product at a lower price. Correspondingly, the optimal quantities orders are $q_1 = 0$ and $q_2 = \frac{1}{3}$. Nevertheless, when each trade link could be broken with probability of $\eta = \frac{1}{4}$ before trade takes place, the optimal quantities orders become $q_1 = \frac{1}{8}$ and $q_2 = \frac{1}{4}$. One can observe that the buyer reacts to the risk of losing trade opportunities by increasing the total order size(trade insurance) and buying some quantity from the seller of the more expensive product (trade diversification). In fact, despite the Bertrand nature of the competition, the risk generates some degree of market power among sellers. Figure 2 illustrates how the demanded quantities from each seller and the total demand are affected by varying degrees of trade risk. Note that the demand from the more expensive seller and the total demand increase with risk. However, the demand from the less expensive seller is U-shaped. The trade diversification term, D_{21}, dominates the trade insurance term, I_{21}, for risk levels up to a threshold, $\hat{\eta}$. But, for risk levels above $\hat{\eta}$ threshold D_{21} is dominated by I_{21}.*

4 Competition on Exogenous Trade Networks

In this section we study price formation on a given trade network with risky links. We begin by analyzing the competition on a network with one buyer and n sellers. Then we study the competition on general bi-regular trade networks. Finally, we use an asymmetric net-

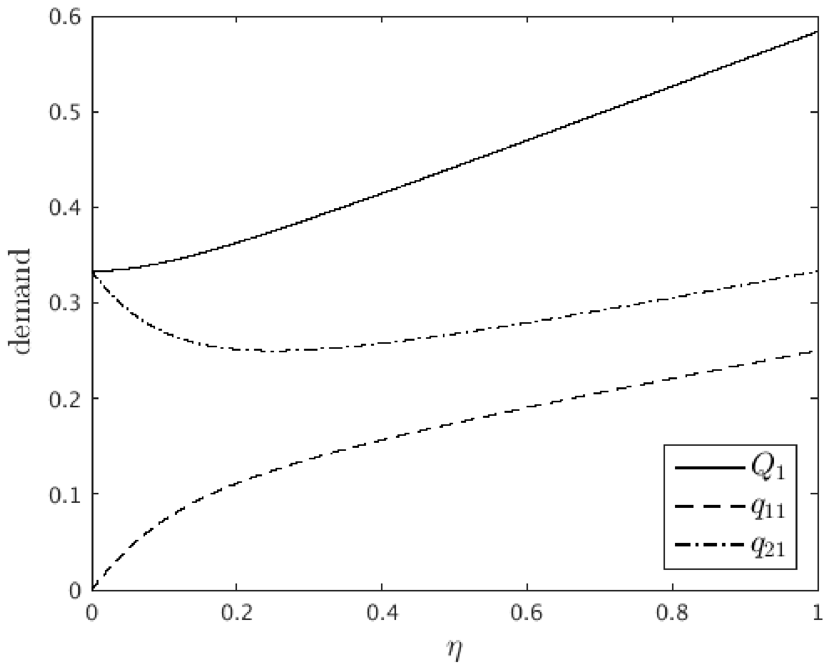

Figure 2: The shape of demand for different degrees of trade risk when B_1 is faced with two sellers charging $p_{11} = 4$ and $p_{21} = 3$. (Example 1)

work with three buyers and three sellers to provide the intuition for competition on general bipartite trade networks. Note that we only analyze connected bipartite trade networks, because in any disconnected network the Network Competitive Equilibrium for each connected segment is independent of other segments of the trade networks.

4.1 A Model of one Buyer and n Sellers

Let us start with analyzing the competition on a network with one buyer and n sellers, represented in figure 3. Each seller sets its price to maximize its expected profits. Optimal prices are described by the Nash Equilibrium of first order conditions of equation 2,

$$q(p_{i1}, \mathbf{p_{-i1}}; \eta) + \frac{\partial q(p_{i1}, \mathbf{p_{-i1}}; \eta)}{\partial p_{i1}}(p_{i1} - c_i) \leq 0 \quad , \quad i = 1, \ldots, n$$

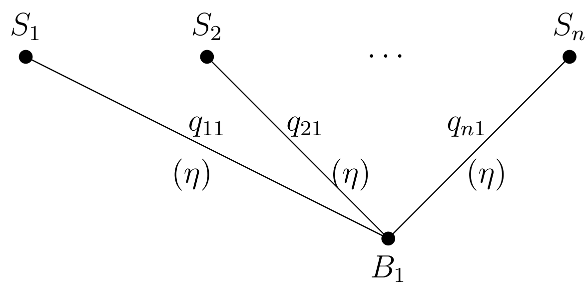

Figure 3: The network for section 4.1.

Proposition 4 *In a network with one buyer facing n symmetric sellers, with equal marginal costs of production, the equilibrium price and hence market power always increase with risk. However, the equilibrium ex-ante expected profits initially increase with η for small levels of risk and then decrease with η afterwards.*

Denote the equilibrium price, demand, and ex-post realized profits in a network with one buyer facing n symmetric sellers as, $p_{(n)}$, $q_{(n)}$, and $\pi_{(n)}$. Also, define the equilibrium Lerner index as $\mu_{(n)} = 1 - \frac{c}{p_{(n)}}$ and equilibrium ex-ante expected profit as $\pi_{[n]}$.

Proposition 5 *In a network with one buyer facing n symmetric sellers, with equal marginal costs of production, we have the following comparative statics with respect to risk of a trade link breaking, η, and number of linked seller, n.*

1. i *The equilibrium unit price, $p_{(n)}$, and hence market power always increase with risk.*

   ii *The equilibrium quantity demanded from each seller, $q_{(n)}$, always increase with risk.*

   iii *The equilibrium ex-ante expected profits, $\pi_{[n]}$, initially increase with risk η for small levels of risk and then decrease with η afterwards.*

2. *When $\eta > 0$,*

   i *$p_{(n)}$ and $\mu_{(n)}$ decrease with number of linked sellers, n.*

ii The equilibrium ex-ante expected profits, $\pi_{[n]}$*, decreases with* n*.*

4.2 Competition on a General Bi-regular Trade Network

Until now we have analyzed situations in which all suppliers only catered to one buyer. Since the risk of losing trade links does not inherently change the behavior of the buyers with only one trade partner, we only explore networks in which each buyer has at least two trade partners. Consider a network of m identical buyers and n symmetric sellers in which degree of buyer vertices is $k_b > 1$ and degree of seller vertices is $k_s > 1$. Due to the symmetry, all sellers will charge the same price p to all buyers, and buyers will demand the same amount, q, through each link in the Nash equilibrium. Therefore, solution to the Network Competitive Equilibrium is described by

$$\sum_{i=0}^{k_b-1} \eta^i (1-\eta)^{k_b-1-i} \binom{k_b-1}{i} u'\left([k_b-1-i]q\right) = p$$

$$q + \frac{\partial q(p_{lj} = p, \mathbf{p_{-lj}} = p\mathbf{1_{k_b-1}}; \eta)}{\partial p_{lj}}(p-c) = 0$$

where $\mathbf{p_{-lj}} \equiv \{p_{ij} | i \in b_j^b / \{l\}\}$. Note that the equilibrium price of this network is the same as the network in section 4.1.

Proposition **6** *The nature of the competition on any* (k_b, k_s) *bi-regular trade network with identical buyers and sellers, it is the same as the competition on a network in which one buyer faces* k_b *sellers.*

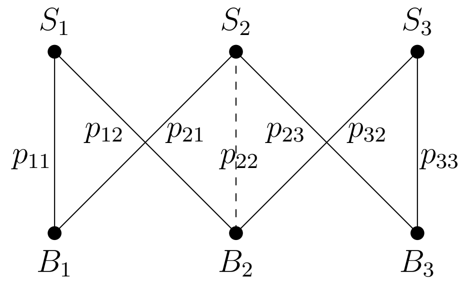

Figure 4: The trade network in Example 2

4.3 Competition on a General Bipartite Trade Network

In section 4.2 we described the Network Competitive Equilibrium on fully symmetric trade networks. To extend the analysis to any connected bipartite trade network, we study competition on the simplest buyer-seller network that conveys the intuition. In addition, this example provides some insight for the network formation games in next section.

EXAMPLE **2** *Lets consider the network illustrated in Figure 4 by solid lines. Assume all sellers have access to an the same technology and incur a per unit production cost of c. This is a* $(2, 2)$ *Bi-regular network with 3 buyers and 3 seller. From section 4.2 we know that the equilibrium prices and quantities in this network are the same as its sub-network in which* B_1 *is linked with* S_1 *and* S_2*. Therefore, the Network Competitive Equilibrium in this trade network can be described by* $\{p_{ij} = p_{(2)}(c, \eta), q_{ij} = q(p_{(2)}, p_{(2)}; \eta), E\pi_i = 2\pi_{[2]}\}$ *Now, assume a new opportunity arises for buyer* B_2 *and supplier* S_2 *to create a trade link. The new link is only formed in case a mutual interest exists. To identify who benefits and who is harmed by the new partnership, we need to find the Network Competitive Equilibrium for the new* *network. The first order conditions of profit maximization for S_1 are*

$$[p_{11}]: \quad q(p_{11}, p_{21}; \eta) + \frac{\partial q(p_{11}, p_{21}; \eta)}{\partial p_1}(p_{11} - c) = 0 \tag{7}$$

$$[p_{12}]: \quad q(p_{12}, p_{22}, p_{32}; \eta) + \frac{\partial q(p_{12}, p_{22}, p_{32}; \eta)}{\partial p_1}(p_{12} - c) = 0 \tag{8}$$

Because of the symmetry in the network we know that in the equilibrium,

$$p_{11} = p_{33} \quad , \quad p_{12} = p_{32} \quad , \quad p_{21} = p_{23}$$

So, the profit maximization problem faced by S_2 can be simplified to

$$\max_{p_{21}, p_{22}} 2q(p_{21}, p_{11}; \eta)(p_{21} - c) + q(p_{22}, p_{12}, p_{12}; \eta)(p_{22} - c)$$

Hence, the first order conditions of profit maximization for S_2 are

$$[p_{21}]: \quad q(p_{21}, p_{11}) + \frac{\partial q(p_{21}, p_{11})}{\partial p_1}(p_{21} - c) = 0 \tag{9}$$

$$[p_{22}]: \quad q(p_{22}, p_{12}, p_{12}; \eta) + \frac{\partial q(p_{22}, p_{12}, p_{12}; \eta)}{\partial p_1}(p_{22} - c) = 0 \tag{10}$$

Therefore, the equilibrium is characterized only by 4 equations. However, p_{11} and p_{21} only appear in equations 7 and 9. Also, p_{21} and p_{22} only appear in equations 8 and 10. By solving equations 7 and 9 we find that $p_{11} = p_{21}$ and equal to the price set by two symmetric suppliers facing only one buyer, $p_{(2)}$. In addition, by solving equations 8 and 10 we find that $p_{12} = p_{22}$ and equal to the price set by three symmetric suppliers facing only one buyer,

$p_{(3)}$. In Appendix 1, we explicitly derive the Network Competitive Equilibrium for the trade network in section 4.1 in which the buyers have quadratic utility of the form, $u(q) = -(q-d)^2$. The equilibrium prices and quantities for buyers with quadratic utility are as follows

$$
\begin{array}{ll}
p_{11} = p_{21} = p_{23} = p_{33} = \dfrac{2\eta d + c}{1+\eta} & , \quad p_{12} = p_{22} = p_{32} = \eta d + \dfrac{2-\eta}{2} c \\
q_{11} = q_{21} = q_{23} = q_{33} = \dfrac{2d - c}{2(1+\eta)(2-\eta)} & , \quad q_{12} = q_{22} = q_{32} = \dfrac{(2-\eta)(2d-c)}{4(3-2\eta)}
\end{array}
$$

One can observe that the new trade partnership will benefit both B_2 and S_2. But, it will decrease the profits of S_1 and S_3. On the other hand, the other buyers are not affected by establishment of the new trade link.

PROPOSITION **7** *To analyze the competition on any general bipartite trade network with identical buyers and sellers, it is sufficient to break up the problem to networks in which each buyer j faces k_j^b sellers that are only connected to that buyer.*

5 Competition on Endogenous Trade Networks

In this section we study simultaneous price and network formation. Network formation not only depends on the cost of forming new links versus their value for the economics agents who have a say in forming them, but also it depends on the order in which those economics agents will get to take their action. Therefore, it is important to assume links are formed sequentially. At this stage we assume that sellers initiate the link formation process. If the targeted buyer benefits from the new link too then the seller pays the cost and the trade link will form next period. Denote the Nash equilibrium of the formation games in which seller pays the full cost as *Seller-Oriented Equilibrium*. Later we can analyze the *Buyer-Oriented Equilibrium* in which buyers propose and pay for the links. Note that We do not allow sellers and buyers to compensate each other for the cost of link formation. Because then for sufficiently low transaction costs, by Coase Theorem the Nash equilibrium of the formation game would always be the efficient equilibrium (i.e. a link will from if and only if it is efficient to be formed).

5.1 Fixed Link Formation Cost

Assume forming any new link would cost a fixed amount F. Also, a new link is formed only if it is in the interest of both parties involved. As discussed earlier, the intensives of new link formation depends on the current network structure at the time of the decision. Therefore, the *Seller-Oriented Equilibrium* of the network formation game depends on the order by which sellers get to make their decisions. Now let us define an important potential choice ordering for sequential network formation games.

Equitable Formation Game This formation game consists of multiple time periods. At each time period t, sellers receive some random priority ranking. Then, each seller gets to propose forming one new link at its turn, during that time period. Formation continues until no seller proposes a new link. Since the priority ordering of decisions is not an enforced tangible rule but merely a model for actual external forces that lead to some ordering of decisions, we assume that the economic agents only know the current structure of the network at their turn but not the priority order of agents who will deciding after them. Moreover, when links have different costs creating less expensive trade links has strict priority over more expensive trade links. Note that when sellers are proposing, this game consists of m time periods with $n!$ potential rank orderings within each time period.

Proposition 8 *In a network with n buyers and n sellers, the only Seller-Oriented Equilibrium of the Equitable Formation Game is a $\tilde{k}$-regular network such that $\pi_{[\tilde{k}+1]} \leq F < \pi_{[\tilde{k}]}$.*

5.2 Variable Link Formation Cost

In this section we are interested in the effect of actual distance of buyer and seller agents on the Nash equilibrium of the network formation game. It is reasonable to assume that the cost of creating a trade link increases with the distance between buyer and seller. For physical trade links this distance describes the geographical proximity of the two partners. For now we assume the geographical positioning of the partners can be expressed by their location the trade network as follow: A link between S_j and B_k costs $F_{|i-j+1|} \equiv f\left(|i-j+1|\right)$, where $|i-j+1|$ denotes the length of the trade link and $f(\cdot)$ is an increasing function. One special case that we analyze is a linear cost function $f(\Delta) = \frac{\Delta}{n}$. Here, we introduce another important potential choice ordering for sequential network formation games.

Historical Formation Game In this formation game the proposing agents are randomly selected one at a time and get the option to propose forming a new link. If that agent proposes a link to another interested party the link would be formed. Otherwise, another random proposing agent gets the option to propose a new link. The proposing agent are drawn randomly with limited repetition. In fact, an agent only get the option to propose a new link equal to the number of non-proposing agents. Note that when sellers are proposing, this game consists of with $(m \times n)!$ potential rank orderings. Although, we assume random ordering we are not making any assumptions about the probability distribution of potential rank orderings, which is determined by forces outside this model.

A trade network E is among the *Stable Trade Networks* of a formation game if no agent is willing to propose a new link that would be accepted by other involved party and no agent benefits from breaking up one of the established trade links.

CONJECTURE **1** *For any relative ordering of the formation cost for trade links of various length and the ex-ante expected profits of seller facing different levels of competition, a trade network is a Seller-Oriented Equilibrium of the Historical Formation Game if and only if it Pareto dominates other Stable Trade Networks of the Historical Formation Game.*

A trade network E_1 is *Cost Efficient Trade Network* relative to some other networks with same buyer degree distribution of buyers, if it has lowest cost of link formation among them.

CONJECTURE **2** *For any relative ordering of the formation cost for trade links of various length and the ex-ante expected profits of seller facing different levels of competition,any Seller-Oriented Equilibrium of the Equitable Formation Game is a Stable Trade Network and is Cost Efficient relative to other Stable Trade Networks that are not Seller-Oriented Equilibrium of the Equitable Formation Game.*

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Appendix

Proof of Proposition 1:

The expected utility of buyer j that is connected to n sellers can be written as

$$(1-\eta)^n u(Q_j)+\eta(1-\eta)^{n-1}\textstyle\sum_{i=1}^{n} u(Q_j-q_{ij})+\cdots+\eta^{n-1}(1-\eta)\sum_{i=1}^{n} u(q_{ij})+\eta^n u\,(0)-(1-\eta)\sum_{i=1}^{n} p_{ij}q_{ij}$$

Since the second order condition of utility maximization is satisfied for any concave utility function, the first order condition of utility maximization for buyer j with respect to q_{ij} yield

$$(1-\eta)^{n-1}u'\left(Q_j^*\right)+\eta(1-\eta)^{n-2}\sum_{l\neq i} u'\left(Q_j^*-q_{lj}^*\right)+\ \cdots+\eta^{n-1}u'\left(q_{ij}^*\right)-p_{ij}\leq 0$$

with equality as long as a change in price from seller i affects the demanded quantities (i.e. when $q_{ij}^*>0$). Assume without loss of generality that the price vector $\mathbf{p_j}$ faced by buyer j is ordered in descending manner, i.e. $p_{1j}\geq\cdots\geq p_{nj}$.

For instance, if $p_i\geq p_j$ then q_j^* is always positive. Moreover, $q_i^*=0$ merely occurs when there is no positive q_i that satisfies

$$p_i<\eta\left(\frac{u(q_i)-u(0)}{q_i}\right)+(1-\eta)\left(\frac{u(q_i+q_j^*)-u(q_j^*)}{q_i}\right)$$

Of course, the second condition is irrelevant when $\tilde{k}=k_j^b$ and the first condition is irrelevant when $\tilde{k} = 0$. $\blacksquare$

Proof of Proposition 3: One can note, the second order condition of utility maximization with respect to q_i is satisfied for any concave utility function.

$$(1-\eta)^2 u''(Q) + \eta(1-\eta)u''(q_i) < 0 \tag{11}$$

The first order conditions of utility maximization yield

$$(1-\eta)u'(Q) + \eta u'(q_i) \leq p_i \qquad , \qquad i = 1,2 \tag{12}$$

$\blacksquare$