# L'hospitals rule

L'hospitals rule is a method for evaluating limits of indetermine form which means when you solve the limit you get an answer like $\frac{0}{0}$ or $\frac{\infty}{\infty}$ we apply L'hospitals rule by taking the derivative of the top and the bottom do not apply the quotient rule

$$\lim_{x\to 0} \frac{f(x)}{(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty} \Rightarrow \lim_{x\to 0} \frac{f'(x)}{g'(x)}$$

if you apply L'hospitals rule and your still are getting a indeterminte form feel free to apply it again until you get a reasonable answer L'hospital rule can be applied more then once on the same problem you must show that you got a indeterminte form first before applying L'hospitals rule

## types of indeterminate forms

- $\frac{0}{0}$, $\frac{\infty}{\infty}$
- $0\cdot\infty$, $1^{\infty}$, $0^{0}$, $\infty\cdot\infty$ : when you have a limit that results in any one of these you must transform it to become $\frac{\infty}{\infty}$ or $\frac{0}{0}$ or in other words return the entire limit to a fraction if this is not done you cannot apply L'hospitals rule

## examples

$$\lim_{x\to\infty} \frac{2x^2-5x+1}{3+x+6x^2} = \frac{2(\infty)^2-5(\infty)+1}{3+(\infty)+6(\infty)^2} = \frac{\infty}{\infty}$$

apply L'hospital rule

$$\lim_{x\to\infty} \frac{\frac{d}{dx}\left[2x-5x+1\right]}{\frac{d}{dx}\left[3+x+6x^2\right]} = \frac{4x-5}{1+12x}$$

$$\lim_{x\to\infty} \frac{4(\infty)-5}{1+12(\infty)} = \frac{\infty}{\infty}$$

apply L'hospital rule again

L'hospital rule again

$$\frac{\frac{d}{dx}\left[4x-5\right]}{\frac{d}{dx}\left[1+12x\right]} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

the qutient rule was not applied here we only took the derivative of the numerator if the product rule need to be applied you can apply but you cannot treat the as if its a quotient rule

$$\lim_{x\to 0} \csc x - \cot x$$

$$\lim_{x\to 0} \csc(0) - \cot(0) = \infty - \infty$$

you cannot apply L'hospitals rule
you need to get this into $\frac{a}{b}$ form

$$\lim_{x\to 0} \left[\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right] \Rightarrow \lim_{x\to 0} \left[\frac{1-\cos x}{\sin x}\right]$$

take the derivative

$$\lim_{x\to 0} \frac{\sin x}{\cos x} = \frac{0}{1} = \boxed{0}$$

this is not in $\frac{\infty}{\infty}$ or $\frac{0}{0}$ so this will be our answer