improper integrals

a improper integral is one that goes to infinte either on the top limit or lower limit also a improper integral can take the denominator to zero

• type one

Continous from $[a, \infty)$ or $(-\infty, a]$ or $(-\infty, \infty)$

• $\int_a^{\infty} f(x)\,dx = \lim_{m \to \infty} \int_a^m f(x)\,dx \Rightarrow \lim_{m \to \infty} [f(m) - f(a)] =$ exist and finite then it converges

• if they don't converge they diverge

• $\int_{-\infty}^a f(x)\,dx = \lim_{m \to -\infty} \int_m^a f(x)\,dx \Rightarrow \lim_{m \to \infty} [f(m) - f(a)] =$ exist and finite then it converges

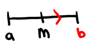
• $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^{\infty} f(x)\,dx \Rightarrow \lim_{m \to -\infty} \int_m^c f(x)\,dx + \lim_{m \to \infty} \int_c^{\infty} f(x)\,dx$

• when you are dealing with an integral like the one above going from negitive infinte to positive infinite you break it up and choose a convient number as c

• you might have to use l'hospitals rule to solve you might also need to use previous integration techniques

• type two

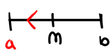
• $\int_a^b f(x)\,dx \Rightarrow \lim_{m \to b^-} \int_a^m f(x)\,dx$ continous on $[a, b)$, its continous on a but not continous on b



: m is approaching b from the left side or the negitive side b is greater then m

m is a number in the middle

• $\int_a^b f(x)\,dx \Rightarrow \lim_{m \to a^+} \int_m^b f(x)\,dx$ continous on $(a, b]$, its not continous on a but it is continous on b

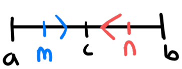


: m is approaching a from the right side positive side m is bigger then a

m is a number in the middle

• $\int_a^b f(x)\,dx \Rightarrow \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \Rightarrow \lim_{m \to c^-} \int_a^m f(x)\,dx + \lim_{n \to c^+} \int_n^b f(x)\,dx$

• the integral above pops up when a eqution is continous through a to F but is not continous at b



• m is approaching c from the left (negitive)

• n is approacning c from the right (positive)

examples

5) $\int_{-\infty}^{0} x e^{5x}\, dx$

STEP1: $\lim\limits_{M \to -\infty} \int_{M}^{0} x e^{5x}\, dx$

STEP2: $\lim\limits_{M \to -\infty} \left[\frac{5x e^{5x}}{5 \cdot 5} - \frac{e^{5x}}{25} \right]_{M}^{0}$

Simplified.

$\lim\limits_{M \to -\infty} \left[\frac{5x e^{5x} - e^{5x}}{25} \right]_{M}^{0} = \lim\limits_{M \to -\infty} \left[\frac{e^{5x}(5x-1)}{25} \right]_{M}^{0}$

STEP3: $\lim\limits_{M \to -\infty} \left[\frac{-1}{25} - \frac{e^{5M}(5M-1)}{25} \right]$

STEP4: $-\frac{1}{25} - \frac{1}{25} \lim\limits_{M \to -\infty} \left[e^{5M} (5M-1) \right]$

LH Rule

$= \frac{-1}{25} - \frac{1}{25} \left[0 \cdot (-\infty) \right]$ IDF

Rewrite

$= \frac{-1}{25} - \frac{1}{25} \lim\limits_{M \to -\infty} \left[\frac{(5M-1)}{e^{-5M}} \right]$

$\stackrel{LH}{=} \frac{-1}{25} - \frac{1}{25} \lim\limits_{M \to -\infty} \left[\frac{5}{-5e^{-5M}} \right]$

$= -\frac{1}{25} - \frac{1}{25} \lim\limits_{M \to -\infty} \left[-1 e^{5M} \right]$

$= -\frac{1}{25} - \frac{1}{25} (0) = -\frac{1}{25}$

STEP 5: $\boxed{\int_{-\infty}^{0} x e^{5x}\, dx = -\frac{1}{25} \text{ Converge}}$

sidework1.

$\int x e^{5x}\, dx$ Int. by parts

$u = x \quad dv = e^{5x}\, dx$

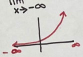
$du = dx \quad v = \frac{e^{5x}}{5}$

$= uv - \int v\, du$

Side work 2

$\lim\limits_{x \to \infty} e^{x} = \infty$

$\lim\limits_{x \to -\infty} e^{x} = 0$



sidework 3

$\lim\limits_{x \to -\infty} \frac{(5M-1)}{e^{-5M}}$

$= \frac{-\infty}{e^{\infty}} = \frac{-\infty}{\infty}$

$=$ IDF

Side work 4

$\lim\limits_{M \to -\infty} \left[-1 e^{5M} \right]$

$= -1 e^{-\infty} = 0$

4) $\int_{1}^{\infty} \frac{1}{(3x+5)^4}\, dx$

STEP1: $\lim\limits_{M \to \infty} \int_{1}^{M} \frac{1}{(3x+5)^4}\, dx$

STEP2: $\lim\limits_{M \to \infty} \left[-\frac{1}{9(3x+5)^3} \right]_{1}^{M}$

STEP3: $\lim\limits_{M \to \infty} \left[\frac{-1}{9(3M+5)^3} + \frac{1}{9(3+5)^3} \right]$

STEP4: $\left[\frac{-1}{\cancel{\infty}}^{0} + \frac{1}{9(8)^3} \right] = \frac{1}{9(8)^3}$

STEP5: $\boxed{\int_{1}^{\infty} \frac{1}{(3x+5)^4}\, dx = \frac{1}{9(8)^3} \text{. Converge}}$

Sidework : Anti Derivative

$\int \frac{1}{(3x+5)^4}\, dx$ <substitution>

$u = 3x+5$

$du = 3dx \rightarrow dx = \frac{du}{3}$

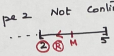
$\int \frac{1}{u^4} \cdot \frac{du}{3}$

$\frac{1}{3} \int u^{-4}\, du = \frac{1}{3} \frac{u^{-3}}{-3}$ $+C$

$= -\frac{1}{9u^3} = -\frac{1}{9(3x+5)^3}$

8) $\int_{5}^{2} \frac{x}{x-2}\, dx$

STEP1: Type 2 Not continuous at $x=2$



$\lim\limits_{M \to 2^+} \int_{5}^{M} \frac{x}{x-2}\, dx$

STEP2: $\lim\limits_{M \to 2^+} \left[x + 2\ln|x-2| \right]_{5}^{M}$

STEP3: $\lim\limits_{M \to 2^+} \left[M + 2\ln|M-2| - 5 - 2\ln(3) \right]$

STEP4: $2 + \cancel{2\ln|0|}^{-\infty} - 5 - 2\ln(3) = -\infty$

STEP5: $\boxed{\int_{5}^{2} \frac{x}{x-2}\, dx = \text{Diverge}}$

Sidework 1

$\int \frac{x}{x-2}\, dx$

Rational function with Num.Deg = Deno.Deg

$\int \frac{x-2+2}{x-2}\, dx$

$\int \left[1 + \frac{2}{x-2} \right] dx$

$x + 2\int \frac{1}{x-2}\, dx$

$u = x-2$

$du = dx$

$x + 2\int \frac{1}{u}\, du$

$x + 2\ln|u|$

$x + 2\ln|x-2|$

Sidework 2