Electromagnetic radiation and maxwells equations

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00\times10^{8}\ m/s$$

• this is the speed of light

in simple words this equation states that light travels at the speed it does because the electric and magnetic nature of empty space fixes it at $c = 1/\sqrt{\mu_0 \varepsilon_0}$

- light is an electromagnetic wave
- electromagnetic waves can travel in absense of vaccum

- These electromagnetic waves form a broad spectrum, as seen in the table below:

safe ↑

danger ↓

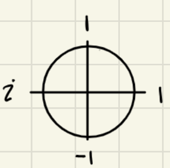
Region:	Wavelengths	Frequency
Radio	10 km to 10 cm	10^{5} to 10^{8} Hz
Microwave	10 cm to 1 mm	10^{8} to 10^{11} Hz
Infrared	1 mm to 1 µm	10^{11} to 10^{14} Hz
Visible	350 nm to 700 nm	4.5×10^{14} to 7.5×10^{14} Hz
Ultraviolet	100 nm to 10 nm	10^{15} to 10^{16} Hz
X-rays	10 nm to 10 pm	10^{16} to 10^{19} Hz
Gamma rays	10 pm to 1 pm	10^{19} to 10^{20} Hz

review and explination on complex numbers

Since EM-waves are represented by a sine wave, it is often easier to reprsents them using complex exponential

$i = \sqrt{-1} \longrightarrow i^{n}$ forms a cyclic groud of order 4

this cyclic group can only produce $[i, -1, -i, 1]$ and the cycle repeats starting at i



• and it just goes around it over and over

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Mathmatically, i^n Forms a cyclic group of order 4 so: $i^n = i^{n \bmod 4}$

Complex number:

$$z = a + bi$$

real Part

imaginary Part

- a complex number consists of a imaginary number and a real number

the complex conjugate of z denoted $\bar{z}$, is defined such that:

$$z = a + bi \longleftrightarrow \bar{z} = a - bi$$

- all we are doing here is Fliping the sign around

Notice the Product:

$$z\bar{z} = (a+bi)(a-bi)$$ ★ apply Foil method

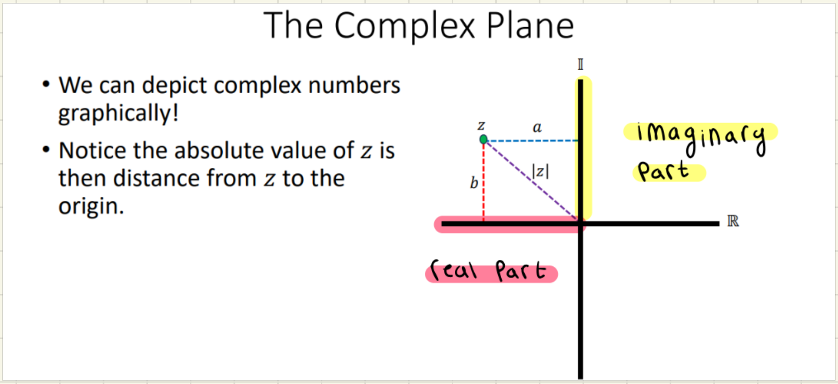
$$\longrightarrow z\bar{z} = a^2 + b^2$$

You get a Purly real solution

We define the absolute value of a complex number such:

$$|z| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$$

★ notice this looks likes the Pythagolean therom



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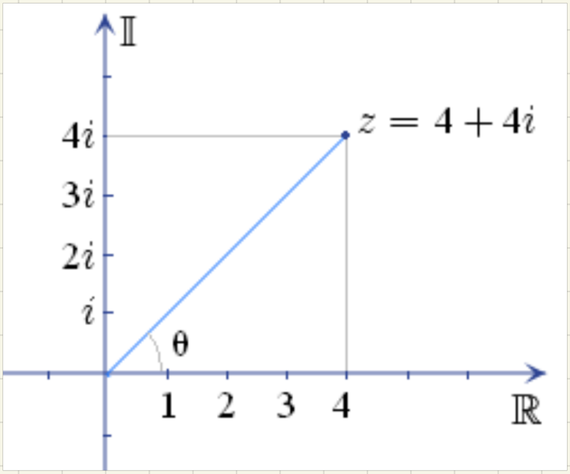


Complex numbers in Polar Form:

- this gives us a equivalent way of writing a generic complex number

$$z = Re^{i\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta \Big\} \rightarrow \text{Euler's Formula}$$

$$\underbrace{Re^{i\theta}}_{z} = \underbrace{R\cos\theta}_{a} + i\underbrace{R\sin\theta}_{b}$$

$$e^{ix} = 1 + ix + \frac{x^2 i^2}{2!} + \frac{x^3 i^3}{3!} + \frac{x^4 i^4}{4!} + \frac{x^5 i^5}{5!} + \frac{x^6 i^6}{6!} + \frac{x^7 i^7}{7!} + \cdots$$

$$= 1 + ix - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} + i\frac{x^5}{5!} - \frac{x^6}{6!} - i\frac{x^7}{7!} + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \cos x$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \sin x$$

$$e^{ix} = \cos x + i \sin x$$

So now $\bar{z}$ will be:

$$Re^{-i\theta} = R\cos(-\theta) + Ri\sin(-\theta)$$

rewrite

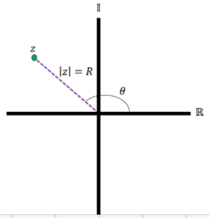
$$\bar{z} = Re^{-i\theta} = R\cos(\theta) - iR\sin(\theta)$$

hence:

$$|z| = \sqrt{z\bar{z}} = \sqrt{Re^{i\theta}\,Re^{i\theta}} = \sqrt{R^2} = R$$

Polar Form

- Notice polar form is analogous to using polar coordinates when graphing z in the complex plane.



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- EM-Plane waves

For convenience, it is common to Pick coordinates such that the wave is traveling along the Positive z axis

$\boxed{v = \lambda f}$ $\boxed{v = \frac{\omega}{k}}$

$\boxed{k = \frac{2\pi}{\lambda}}$ $\boxed{\omega = \frac{2\pi}{T}}$

$$\vec{A} = \vec{R}(\vec{x} \cdot \hat{z} - vt) = \vec{R}e^{i(kz-\omega t)}$$

- represents a traveling wave

R = the amplitude of the wave

$e^{i(kz-\omega t)}$ = describes the oscillation of the wave in both space and time

- k = wave number
- z = Position along the waves Path (usally the direction wave is traveling)
- w = angular Frequency
- t = time

- We choose to write our wave equation solution in this way for two reasons:

Physics Reason: In general, the velocity of a wave is a function of the frequency (a phenomenon called *dispersion*).

Math Reason: Any reasonably well-behaved function can be built by summing together complex exponentials (a process called *Fourier decomposition*).

- Wave equation For Electric Field

$$\vec{E}(\vec{x}, t) = \vec{E}_0 e^{i(kz-\omega t)}$$

- $\vec{E}_0$ = the amplitude of the electric Field
  - k = wave number
  - z = Position along the waves Path (usally the direction wave is traveling)
  - w = angular Frequency
  - t = time

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• Wave equation for magnetic Field

$\vec{H}(\vec{x},t) = \vec{H}_0 e^{i(kz-wt)}$

• in many EM-waves treatment espically in Free Space B and H ale essientially intercnangeable

- H_0 = amplitude
- k = wave number
- z = Position along the waves Path (usally the direction wave is traveling)
- w = angular Frequency
- t = time

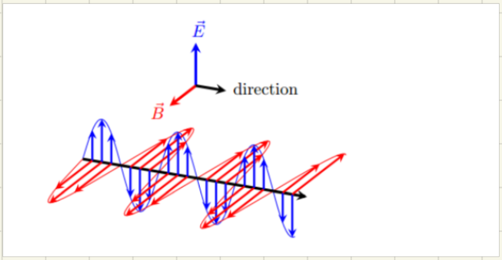
hence the Source - Free maxwell equations in linear media becomes

$$\left.\begin{aligned} 0 &= ik\hat{z}\cdot\vec{E} \\ 0 &= ik\hat{z}\cdot\vec{H} \end{aligned}\right\}$$

Em waves are transverse (meaning $\vec{E}$ and $\vec{H}$ have No component along propgation → $\vec{E}$ and $\vec{H}$ $\perp$ $\hat{z}$)

$$\left.\begin{aligned} \vec{H} &= \frac{1}{\mu v}\hat{z}\times\vec{E} \\ E &= -\frac{1}{\varepsilon v}\hat{z}\times\vec{H} \end{aligned}\right\}$$

$\vec{E} \perp \vec{H}$



Using Eulers relation our Plane wave solution can be though as real valued sinusoid

$\vec{E} = \vec{E}_0 \cos(kz-wt)$, $\vec{H} = \vec{H}_0 \cos(kz-wt)$

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Conclusions

due to $\vec{E}$ being $\perp$ $\vec{H}$, we can uniquely specify as an EM wave by specifying how $\vec{E}$ behaves in 2 linearly independent directions

these independent directions are termed: Polarization

an EM wave that oscillates in only one such direction (and not the other) is said to be: Polarized

Polarizers

Material that only transmits one Polarization direction:

- Polymer chains absorb Polarization aligned with their direction (allowing Polarization $\perp$ with their direction to pass) $I_T = I_I / 2$

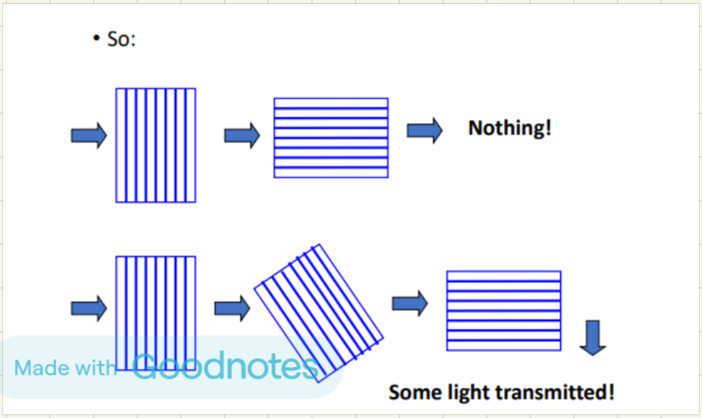
Passing light through 2 Polarizers oriented 90° blocks all light

rotated Polarizers transmit varing intensity $I_T = I_I \cos^2\theta$

I_T: intensity of transmitted light

I_I: intensity of incident light

the equation is for transmitted light intensity after a Polarizer



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impedance (Z)
and units of Ω (ohms)

impedance is a measure of how much a circut resists the flowing of alternating current (AC)

due to Euler's relation, we can model circuts with sinusodial varying emfs using complex exponentials

$$I = |I| e^{i(\omega t + \phi_I)}$$

$$\Delta V = |\Delta V| e^{i(\omega t + \phi_V)}$$

$$Z = \frac{|\Delta V| e^{i(\phi_V - \phi_I)}}{|I|}$$

$Z_R = R$ · resistor

$Z_C = \frac{1}{i\omega C}$ · capacitor

$Z_L = i\omega L$ · inductor

Equivalent impedance

Parallel

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \cdots$$

Series

$$Z_{eq} = Z_1 + Z_2 + \cdots$$

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inductance (L)
(L is in henries H)

If 2 current loops are placed next to each other, putting a current through one will induce a current in the other by Faraday's law

$$\mathcal{E} = -\frac{d\Lambda}{dt} \qquad \Lambda = MI$$

$$\mathcal{E} = -M\frac{dI}{dt}$$ - induced emf due to mutual inductance

$$\mathcal{E} = -L \cdot \frac{dI}{dt}$$ - induced emf due to inductors self inductance

$$\Delta V_{total} = L_1 \cdot \frac{dI}{dt} + M_2 \frac{dI}{dt}$$

equivalent inductance

Parallel

$$\frac{1}{L_{eq}} = \frac{L_1 + L_2 - 2M}{L_1 L_2 - M}$$

Series

$$L_{eq} = L_1 + L_2 + 2M$$

important notes

$M_1 = M_2 = M$ (for simplicity)

1. A *leaky* capacitor is one in which the charge can flow through the dielectric medium separating the two capacitor plates. Such a capacitor can be modeled by an ideal capacitor and a resistor placed in parallel with one another. Consider a 50 μF leaky capacitor with an effective resistance of 300000Ω.

   (a) If this leaky capacitor is being driven by an AC voltage source with a frequency $f = 60$ Hz (i.e., the same as a typical US wall outlet), then what is the absolute value of the total impedance, Z?

   (b) If this leaky capacitor is brought to Europe (where the AC voltage sources operate at 50 Hz), would the absolute value of the total impedance increase or decrease? Explain briefly.

a) You have a capacitor and a resistor in Parrallel when you have a situation like this you use this equation:

$$\frac{1}{Z} = \frac{1}{R} + i\omega C$$

if the resistor and capacitor are in series use this equation

$$Z = R - \frac{i}{\omega C}$$

$$\frac{1}{Z} = \frac{1}{R} + i\omega C \longrightarrow \frac{1}{Z} = \frac{1 + i\omega C R}{R} \longrightarrow Z(1 + i\omega C R) = R$$

$$Z = \frac{R}{(1 + i\omega C R)} \cdot \frac{(1 - i\omega C R)}{(1 - i\omega C R)}$$

★ Never keep i in the denominator So you will need to rationalize it

$$Z = \frac{R}{1 + \omega^2 R^2 C^2} - i\frac{\omega R^2 C}{1 + \omega^2 R^2 C^2} \longrightarrow Z = \sqrt{\left(\frac{R}{1 + \omega^2 R^2 C^2}\right)^2 - \left(\frac{\omega R^2 C i}{1 + \omega^2 R^2 C^2}\right)^2}$$

★ square both side to get rid of i $(i^2 = -1)$

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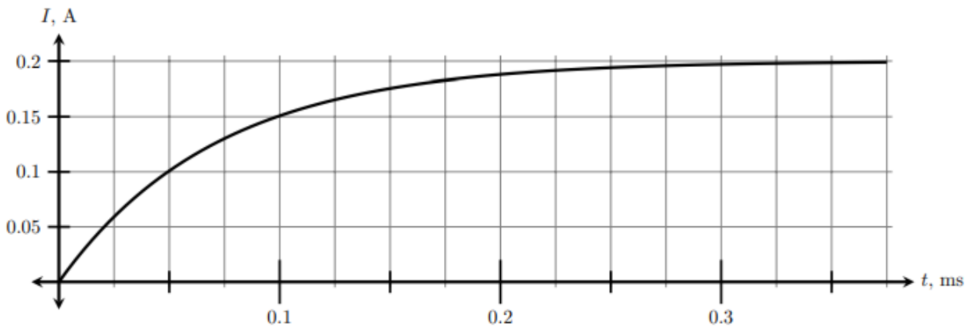
3. Two 0.25 mH inductors are wired together in series and then connected (in series) to a real 2.4 V battery with an internal resistance of 1.8Ω. The current as a function of time through the inductors is then plotted, as shown in the graph below.



(a) (*Draws on Class #21*) Assuming inductors have identical internal resistances, what is the internal resistance, R, of one of the inductors?

(b) What is the mutual inductance between the two 0.25 mH inductors?

a)

$$\mathcal{E} = (IR + IR + 1.8I)$$

$$\mathcal{E} = 2IR + 1.8I$$

$$R = \frac{\mathcal{E} - 1.8I}{2I} \longrightarrow R = \frac{2.4 - 1.8(0.2)}{2(0.2)} = \boxed{5.1\,\Omega}$$

if they are all in series you can apply a Kircoffs rule around the circut where you spin around the circut and get the equation

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b)

For series:

$$L = L_1 + L_2 + 2M$$

now you isolate for M (mutual inductance) remember L = self inductance and M = mutual inductance

$$\tau = \frac{t_{1/2}}{LN(2)} = \frac{0.0005}{LN(2)} = 7.21\times10^{-4}$$

$$\tau = \frac{L}{R} \longrightarrow \tau R = L \longrightarrow (7.21\times10^{-4})(5.1) = 3.67\times10^{-3}\,H$$

$$= 3.67\,mH$$

$$M = \frac{L - L_1 - L_2}{2} = \frac{3.67 - 0.25 - 0.25}{2} = \boxed{1.58\,MH}$$

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