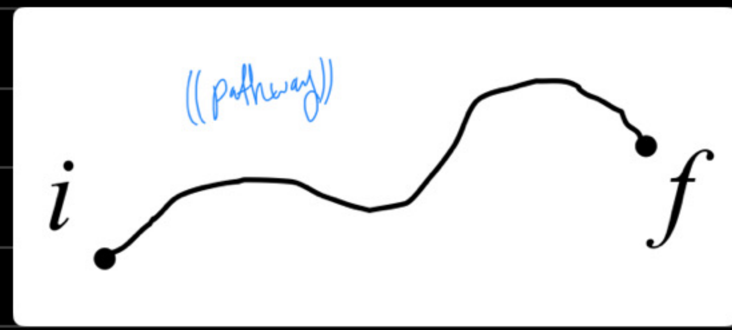
Work & Energy

$$\boxed{W_{net} = F_{net} \times d}$$



Since force can vary along pathway, then it values changes along the way. Therefore, we would need to use integral to find the work done:

$$\boxed{W_{net} = \int F_{net} \cdot dx}$$

⇒ However, this method becomes harder to deal with as we consider 2D/3D path... for us to be able to solve this we would need a good understanding of ((dot product))... Nevertheless, we would also be dealing with multiple forces...

Solution:

$$dW_{net} = \vec{F}_{net} \cdot d\vec{x}$$

$$dW_{net} = m\,\vec{a} \cdot d\vec{x}$$

$$dW_{net} = m\,\frac{d\vec{v}}{dt} \cdot d\vec{x}$$

$$dW_{net} = m\,\vec{v}\,\frac{d\vec{v}}{dt} \cdot dt$$

$$dW_{net} = \frac{1}{2}\,m\,\frac{dv^2}{dt} \cdot dt$$

$$\int_i^f dW_{net} = \int_i^f \frac{1}{2}\,m\,dv^2$$

$$W_{net} = \frac{1}{2}\,m\,(v_f^2 - v_i^2)$$

* $\vec{F}_{net} = m\vec{a}$
* $\vec{a} = \frac{dv}{dt}$
* $d\vec{x} = \frac{d\vec{x}}{dt} \cdot dt = \vec{v} \cdot dt$
* $\frac{dv^2}{dt} = \frac{d(\vec{v} \cdot \vec{v})}{dt} = \frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt}$

$$\frac{dv^2}{dt} = 2\,\vec{v}\,\frac{d\vec{v}}{dt}$$

$$\frac{1}{2}\,\frac{dv^2}{dt} = \vec{v}\,\frac{d\vec{v}}{dt}$$

$$\boxed{W_{net} = \frac{1}{2}\,m\,(\Delta v)^2 = \Delta K}$$

⇒ Kinetic energy is path independent meaning ΔK will be the same regardless of path taken as long as it has the same initial & final position...

However, this only applies if energy is conserved, which means the object is in a Conservative force field as ((gravity)), electrostatics, & Springs... ((functions of position))

$$dW_{net} = dW_g = \vec{F}_g \cdot dx = -mg \cdot dy$$

$$W_{net} = W_g = \int_i^f -mg \cdot dy = -mg\,(y_f - y_i)$$

$$\boxed{W_{net} = W_g = -mg\Delta y = -\Delta U_g}$$

$$\boxed{F = -\frac{dU}{dx}}$$ it means that objects experience a force that pushes them toward regions of lower potential energy...

$$\boxed{\Delta E = \Delta K + \Delta U = 0}$$

* ((Energy conserved)) ((no change of energy))

$$\boxed{W_{net} = W_g = -\Delta U = \Delta K}$$

* Unit: $Kg \cdot m^2/s^2$ = Joule
* $U \leq K$

$$\boxed{E_{total} = K + U = \text{constant}}$$

* K = always positive (velocity squared)

If Non-conservative forces are present as ((friction)), drag, & normal ((non-position functions))

$\Delta E = \Delta K + \Delta U = W_{nc}$

$W_{nc} < 0$ ... negative, meaning that energy was released from the system in form of thermal energy (heat)...

Springs & Oscillator

Hooke's Law & Spring's Potential Energy

negative sign implies that the force of spring is opposite to the direction of the displaced object...

$$\boxed{F_{Hooke} = -k\,(x - x_0)}$$

→ equilibrium length: position where the spring is not streched nor compressed...

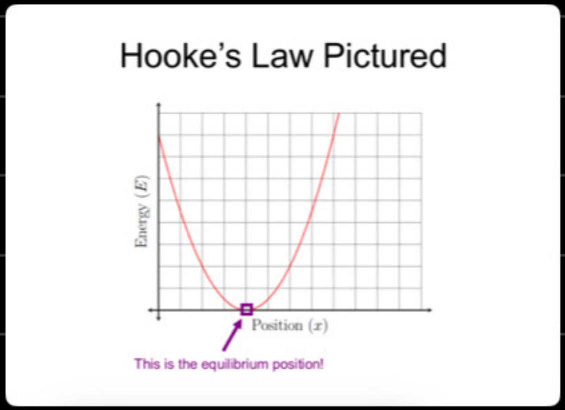
- F_{Hooke}: restoring force
- k: spring constant
- x: position of object

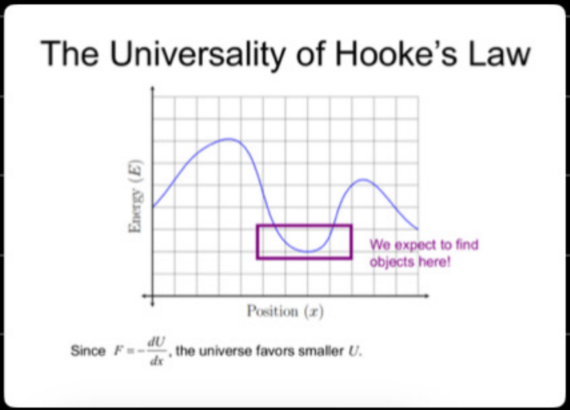
Since the force of the spring depends only on position, then the force can also be expressed as:

$$\boxed{F_{Hooke} = -\frac{dU_{Hooke}}{dx}} \xrightarrow[F_{Hooke}\text{ Substitution}]{\text{Via integration}} \boxed{U_{Hooke} = \frac{1}{2}k\,(x - x_0)^2 + C}$$

The universality of Hooke's Law:

↳ As a result of $\left(F = \frac{-dU}{dx}\right)$, Our universe favors regions of lower potential energy & tend to settle in positions where potential energy is minimized reaching equilibrium state...





↳ Based on ((Taylors expansion)), we can approximat any well behaved function around a point as follow:

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(x_0)}{n!}\cdot(x - x_0)^n}$$

, when we plug in the potential energy function of any object... we get:

$$U(x) = U(x_0) + \boxed{U'(x_0)\cdot(x-x_0)} + \frac{1}{2}U''(x_0)\cdot(x-x_0)^2 + \dots$$

((physically meaningless)) because we can choose $U(x_0)$ to be equal to Zero

Since we are looking at the behavior of an object near the minimum potential energy, then $(U'(x_0) = 0)$ ((Horizontal tangent line))

this would be a good approximation for the potential energy of an object near the minimum potential energy...

Conclusion: Since $\left[U(x) \approx \frac{1}{2}U''(x_0)\cdot(x-x_0)^2\right]$ looks like (U_{Hooke}) !!, then:

- Our universe approximately behaves like a spring when it's near the minimum potential energy which also means energy is ((Conserved))

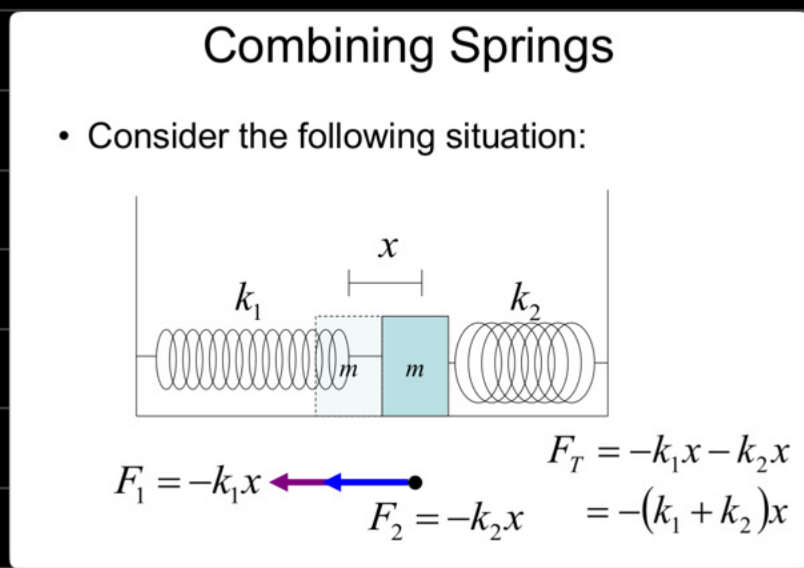
- $$\boxed{U(x) \approx \frac{1}{2}k(x-x_0)^2, \text{ where } k = U''(x_0)\dots}$$

Combining Springs

- Opposing connected spring:

$$\boxed{k_{(eff)} = k_1 + k_2}$$

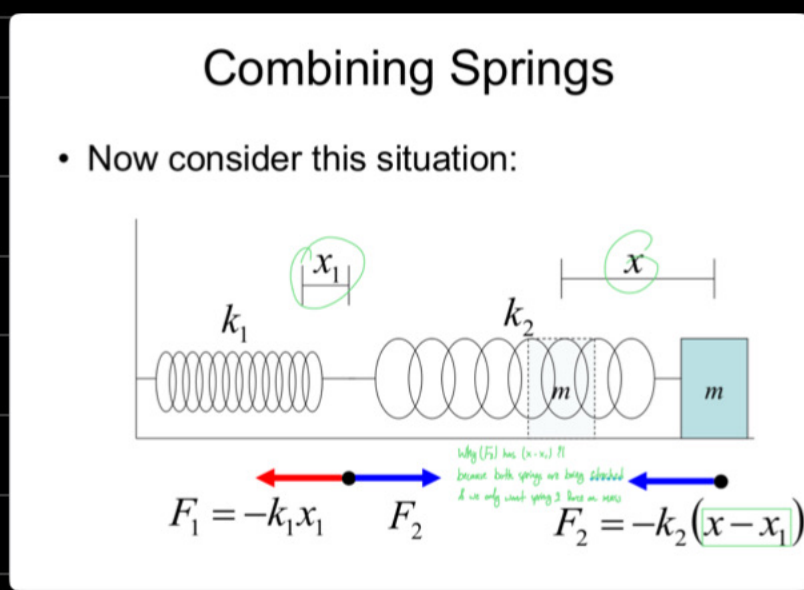
where $k_{(eff)} > k_1$ & k_2



- Directly connected spring:

$$\boxed{k_{(eff)} = \frac{k_1\cdot k_2}{k_1+k_2}}$$

where $k_{(eff)} < k_1$ & k_2



Simple Harmonic Oscillator

Simple Harmonic Oscillator (SHO) ?!

- What ?! ⇒ A system that experience a simple Harmonic motion (SHM), in which the object moves back & forth in a repeated cyclical motion...
  As long as energy is conserved, the system oscillates ((indefinitely))...
- Examples ⇒ mass attached to a ((Spring or pendulum))...
- How ?!
  - As an external force acts upon a mass attached to a spring... It streches the Spring storing potential energy in the Spring/mass system
  - Once the external force is removed, the spring releases the stored potential energy in the form of restoring force ((F_{Hooke}))... Here, U_{Hooke}, F_{Hooke}, & acceleration is ((Maximized))...
  - As the spring is being restored back to equilibrium... the attached mass increases in both velocity & kinetic energy reaching its ((Maximum)) at equilibrium length...
    At that moment, U_{Hooke}, F_{Hooke}, & acceleration equal zero ((Minimized))...
  - Since the attached mass gained kinetic energy, it will not stop at equilibrium length... instead it will continue it motion compressing the spring... During which the springs acts as a barrier storing potential energy for the system... and minimizing the mass's kinetic energy & velocity...
  - Once the attached mass - kinetic energy reaches zero ((Minimized))... the system releases its stored potential energy propelling the mass back to equilibrium...
  - The process is repeated forever only if no external force other than the first one acts on the mass attached to the Spring...

- Equation:
  - Spring:
    - The only force acting on the attached mass is ($F_{Hooke} = -kx$), $x_0 = 0$

$$F_{net} = F_{Hooke} = -kx \qquad * \; F_{net} = ma = m\frac{d^2x}{dt^2}$$

$$m \cdot \frac{d^2x}{dt^2} = -kx$$

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x} \quad (1)$$

    - SHO equation is a differential equation & physicists solve equation through trial and error:

$x(t) = A.\cos(\omega t + \phi) + x_0$ or $x(t) = A.\sin(\omega t + \phi) + x_0$

$\Downarrow$

$V(t) = \frac{dx}{dt} = -A\omega.\sin(\omega t + \phi) + x_0 = -\omega\, x(t)$

$\boxed{V(t) = -\omega\, x(t)}$ * (($V_m = x_m = KE_m$)) m: maximized

$a(t) = \frac{d^2x}{dt} = -A\omega^2.\cos(\omega t + \phi) = -\omega^2 x(t)$

$\boxed{a(t) = -\omega^2 x(t)}$ (2) * (($a_m = x_m = U_m = F_m$))

$(1 = 2) \Rightarrow -\frac{k}{m}.x = -\omega^2.x \Rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$ (3)

- Angular frequency (ω):
  - Definition: is the number of osscilation in terms of radians per second...
  - we can derive from it:
    - period: is the time required to complete a whole cycle ((whole osscilation))

      $\boxed{T = \frac{2\pi}{\omega}}$

    - frequency: is the number of osscilation per second...

      $\boxed{f = \frac{\omega}{2\pi}}$ $\boxed{f = \frac{1}{T}}$ (Hz)

- Amplitude (A):
  - peak displacement ($x_m - x_0$):
    - the difference between maximized length & equilibrium length...
  - $E_{Total} = KE_{max} = U_{max} = \frac{1}{2}.k.(x_m - x_0)^2 = \frac{1}{2}.k.A^2$

  $E_{total} = \frac{1}{2}.k.A^2 \Rightarrow \boxed{U + KE = \frac{1}{2}.k.A^2}$

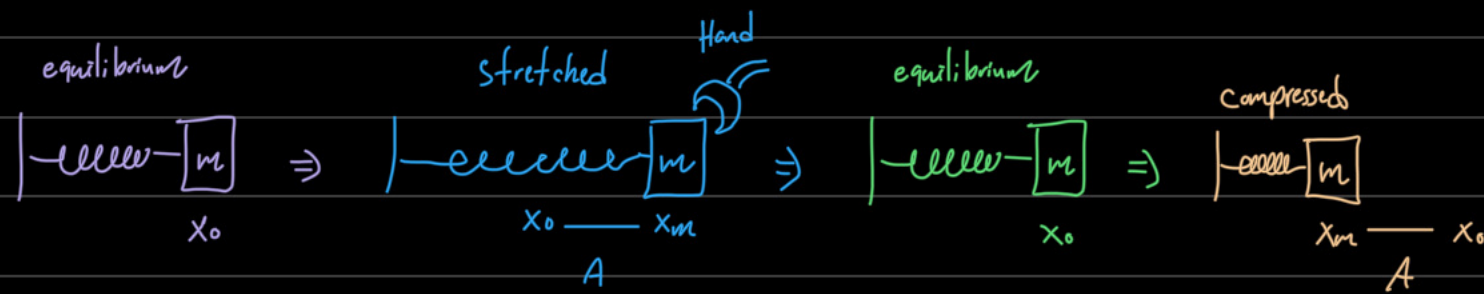
- phase Constant (ϕ): Represents starting position of osscilation in Radians

Amplitude & phase Constant :

Represent the integration constant (C) ...

They are determined by the external force ((initiator)) that set the mass/spring system into motion... not the system itself...

Remember, before setting the mass/spring system to oscillate, it was stationary, which means in ((equilibrium state))



nothing happening ((Stationary))

$F_{extend} = F_{Hooke}$

U_m & a_m

KE & $V = 0$

KE_m & V_m

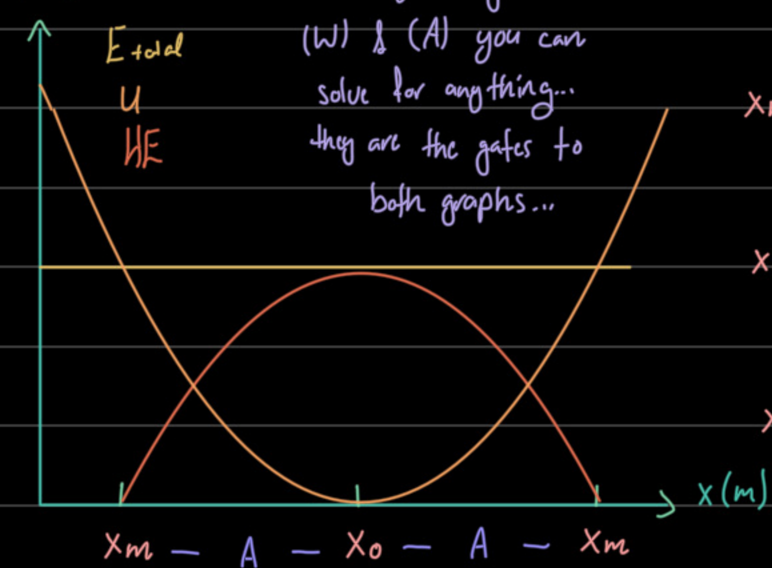
U, a, & $F_{Hooke} = 0$

U_m, a_m, $F_{Hooke\ m}$

KE & $V = 0$

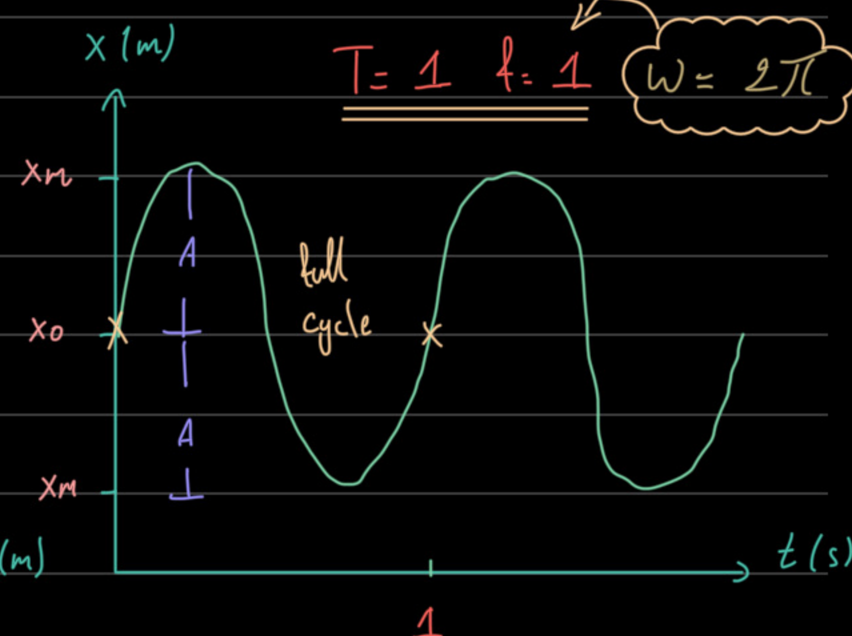
In a Conserved Energy system, once the hand is Released, the Only force is acting upon the mass/spring system is ((F_{Hooke}))...

The springs keeps osscillating until the end of the world !! ☺



As long as you know (W) & (A) you can solve for anything... they are the gates to both graphs...

$T = 1$ $f = 1$ $\omega = 2\pi$

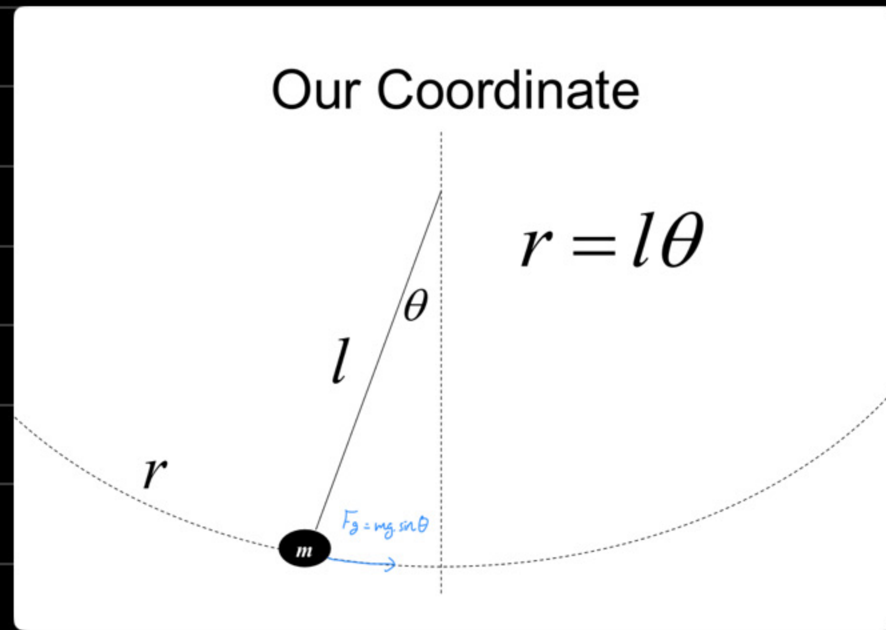


} pendulum (SP)

(distance)
$x = r = l\,\theta$

$V = \frac{dr}{dt} = l\,\frac{d\theta}{dt}$

$$\boxed{a = \frac{d^2r}{dt^2} = l\,\frac{d^2\theta}{dt^2}}$$



$F_g = -mg.\sin\theta$

$m\,a = -mg\sin\theta$

$l\,\frac{d^2\theta}{dt^2} = -g.\sin\theta$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{l}.\sin\theta}$$

negative implies that the restoring gravitational force is opposite to the displaced object...
All restoring forces in physics or chemistry are negative signaling that an object will always prefer lower potential energy reaching equilibrium...

Remember, the smaller $\theta \rightarrow$ the closer it is to lower potential... which means if we assume θ to be very small, then we can say that it behaves like a spring...
Therefore, we can steal all the formula of osscillation to use it for pendulum... By ((Taylors expansion)):

$$\boxed{\sin\theta \approx \theta}$$

$$\boxed{\frac{d^2\theta}{dt^2} = \frac{-g}{l}.\theta}$$

$$\boxed{W = \sqrt{\frac{g}{l}}}$$

SP equations:

$$\boxed{\begin{aligned} \theta(t) &= A.\cos(Wt.\varnothing) + \theta_0 \\ r(t) &= A.l.\cos(Wt.\varnothing) + r_0 \end{aligned}}$$

$V(t) = -A.l.w\sin(Wt.\varnothing) = -l.w.\theta$ or $-w.r$

$$\boxed{V(t) = -\ell \cdot \omega \cdot \theta \quad \text{or} \quad -\omega \cdot r}$$

$$a(t) = -A \cdot \ell \cdot \omega^2 \cos(\omega t \cdot \phi) = -\ell \cdot \omega^2 \cdot \theta \quad \text{or} \quad -\omega^2 r$$

$$\boxed{a(t) = -\ell \cdot \omega^2 \cdot \theta \quad \text{or} \quad -\omega^2 r}$$

Damped & Driven Harmonic Oscillator

2 Types of Harmonic Oscillator :

- Undriven Harmonic Oscillator (Initial external force is not repeated)
  - Simple Harmonic Oscillator (SHO)

$$\boxed{\frac{d^2x}{dt^2} + w^2 x = 0}$$

⇒ SHO oscillation is repeated forever if the only external force applied to the system is the initial force

$F_{(external)} = - F_{(Hooke)}$ ⇒ negative sign implies that $F_{(Hooke)}$ is in the opposite direction of $F_{(external)}$ [displaced Object]

  - Damped Harmonic Oscillator (DHO)

$$\boxed{\frac{d^2x}{dt^2} + \left(\beta \frac{dx}{dt} + w_0^2 x = 0\right)}$$

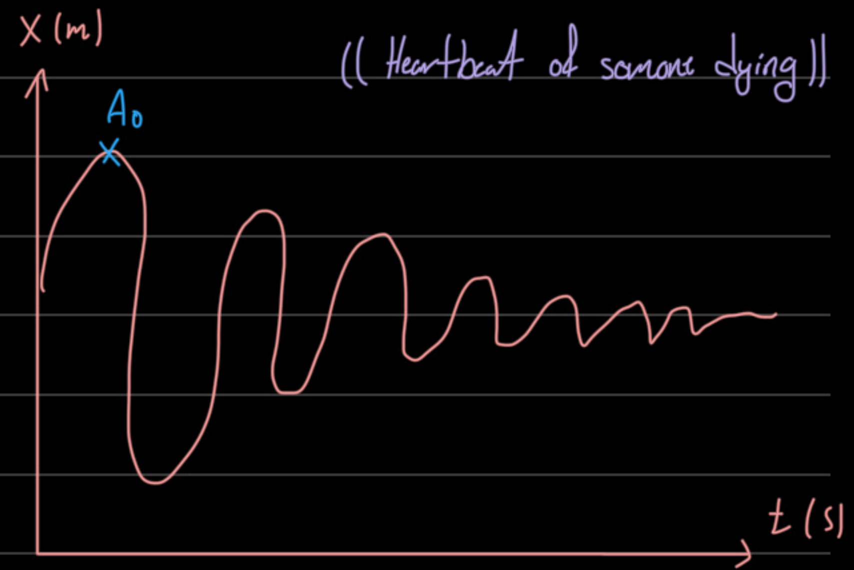
equation for exponential decay

⇒ DHO oscillation is not repeated forever and will eventually Stop ... returning back to its equilibrium state... This is due to the involvement of non-conservative forces... eating away the system total energy until it reaches zero... which means amplitude should be decreasing with time...

- Equations :

$$\boxed{a + \beta V + w_0^2 x = 0} \quad (1)$$

To find the relationship between W, W_0, & β we found the derivates of the below equation (2), (3), & (4) and plugged it back to (1) to get (5)...



(2)

$$\boxed{x(t) = A_0 e^{-\frac{\beta}{2}t} \cos(Wt + \phi_0) + x_0}$$

$$\boxed{A(t) = A_0 e^{-\frac{\beta}{2}t}}$$

The things we can interpret from graph :
- A_0 & A (Inertial A & Damped A)
- T (only Damped T not T_0)
  - using those information we can find :
    - β (damped factor)

(3)

$$\boxed{V(t) = -\left(\frac{\beta}{2} + W\right) . x(t)}$$

$$a(t) = \left(\frac{\beta^2}{4} - w^2 + \beta w\right) . x(t) \quad (4)$$

↳ W (damped angular frequency)

↳ Now we can find (W_0) which is essential to know (k) constant Hooke

$$\boxed{W = \sqrt{W_0^2 - \frac{\beta^2}{4}}} \quad (5)$$

↳ 3 types of DHO :

- ↳ underdamped ⇒ $W_0 > \frac{\beta}{2}$ ⇒ Oscillates + gradually returns to equilibrium...
- ↳ critically damped ⇒ $W_0 = \frac{\beta}{2}$ ⇒ no oscillation + quickly returns to equilibrium...
- ↳ Overdamped ⇒ $W_0 < \frac{\beta}{2}$ ⇒ no Oscillation + slowly returns to equilibrium... Imaginary slowest

↳ Damped Ratio (δ) : measures how damped is the system...

$$\boxed{\delta = \frac{\beta}{2W_0}}$$

⇒ $\delta = 0$ (undamped) $0 < \delta < 1$ (underdamped)

$\delta = 1$ (critically damped) $\delta > 1$ (Overdamped)

↳ Quality factor (Q) : measures how fast is the energy being depleted from the system...

$$\boxed{Q = \frac{1}{2\delta} = \frac{W_0}{\beta}}$$

⇒ The larger the value of (δ), the lower the value of (Q), and the faster the energy is being depleted...

This is why (Overdamped systems) is the slowest at reaching equilibrium due to its lower levels of Total energy...

↳ Driven Harmonic Oscillator (Initial external force repeated over time)

$$\boxed{\frac{d^2x}{dt^2} + \beta . \frac{dx}{dt} + W_0^2 x = \sin(W_D)}$$

↳ This equation implies that for a damped system to be able to contiously oscillate without reaching equilibrium... we would need to repeatedly applie the initial external force over time... replenishing the energy that was lost to non-conservative forces...

In DHO, we've learned that the Amplitude & phase constant change over time until it reaches equilibrium ... This change is decreasing at a constant rate of ($e^{-\frac{\beta}{2}t}$) which means it follows a predictable pattern...

However, with driven harmonic oscillator, the change of amplitude & phase constant depends on several factors:

- The repeated initial external force (Driven force)
  - How well our driven force's angular frequency (W_D) matches the system's natural frequency (W_0)... when ($W_D = W_0$), we call it ((Resonance))..
    - The energy supplied to the system by driven force is being fully absorbed ((not wasted))... maximizing our increased Amplitude...
    - It is the moment the driven force is applied to the System when it reaches its maximum amplitude... as when you push someone riding a swing that is about to swing back... It is all about ((timing))...
- How strong is our Driven Force...
- How damped is our system (β)...

Therefore, to account for such factors we use the following equation:

$$\boxed{x(t) = A\sin(W_D t) + B\cos(W_D t)}$$

$$\boxed{\underset{\text{(Total Amplitude)}}{\sqrt{A^2 + B^2}} = \frac{1}{\sqrt{(W_0^2 - W_D^2)^2 + \beta^2 W_D^2}}}$$

	SHO	underdamped	critically damped	overdamped	Driven harmonic oscillator
Initial force	(All 4 cars have driven over one speed bump)				multiple speed bumps
car's suspension system	Bounces forever	Bounces in a certain time frame	only compresses & quickly returns to its initial state	only compresses & slowly returns to its initial state	Oscillation never stop as SHO... However, how it oscillates depends on how damped the system...

Momentum & Collision

1. Momentum : is how hard an object is traveling...

$$\boxed{\vec{P} = m.\vec{V}}$$

* $F_{net} = m.\vec{a} = m.\frac{d\vec{v}}{dt} = \frac{d m\vec{v}}{dt} = \frac{d\vec{P}}{dt}$

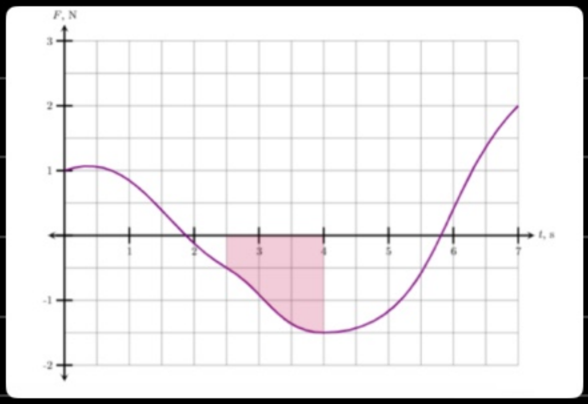
$\boxed{F_{net} = \frac{d\vec{P}}{dt}}$ ⇒ equivalent to newton's 2nd law ⇒ If $F_{net} = 0$, then $\frac{d\vec{P}}{dt} = 0$ ... Therefore

$\vec{P}$ is conserved (constant momentum)

* $KE = \frac{1}{2} m v^2 = \frac{1}{2} \cdot \frac{m^2 \cdot v^2}{m} = \frac{P^2}{2m}$ ⇒ $\boxed{KE = \frac{P^2}{2m}}$

2. Impulse : change of momentum (ΔP) or (J)

$$\boxed{\Delta P = \int F_{net} \cdot dt}$$

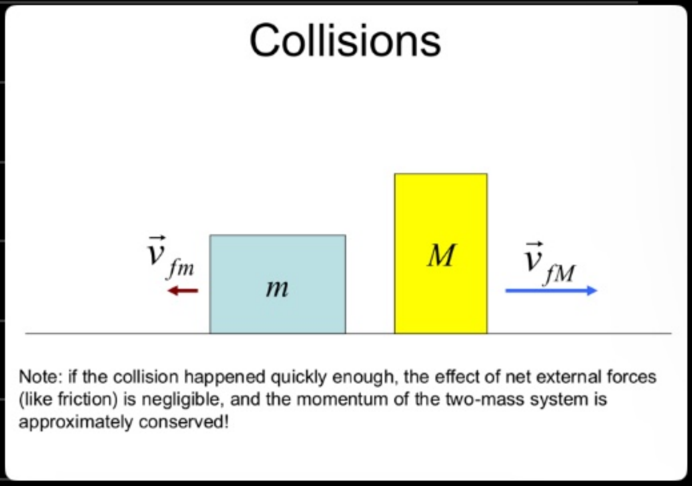


3. Collisions : (1) If the collisions between two masses happened quickly, then :

- non-conservative forces as friction is negligible ...
- Momentum of the system Conserved ...

$$\boxed{P_{01} + P_{02} = P_{f1} + P_{f2}}$$

- However, even with knowing the (V_i & masses) we can't find (V_f) ...
- equivelent to newton's 3rd law



$$P_{01} - P_{f1} = P_{f2} - P_{02} \Rightarrow -\Delta P_1 = \Delta P_2 \Rightarrow -\int \underbrace{F_{2\,on\,1}}_{\text{constant}} \cdot dt = \int \underbrace{F_{1\,on\,2}}_{\text{constant}} \cdot dt$$

$$\Rightarrow \boxed{-F_{2\,on\,1} = F_{1\,on\,2}}$$

(2) If the collision between two masses conserve kinetic energy, then:

It is called ((Elastic collisions))...

$$\boxed{KE_{01} + KE_{02} = KE_{f1} + KE_{f2}}$$

(3) Using both concepts, we can ultimately find the final velocity of both masses by substitution... However, we are assuming ($V_{02} = 0$) which means staitionary...

Conservation of momentum:

$$m_1 V_{01} + \underline{m_2 \overset{0}{V_{02}}} = m_1 V_{f1} + m_2 V_{f2} \Rightarrow \boxed{V_{f2} = \frac{m_1 (V_{01} - V_{f1})}{m_2}}$$

Conservation of kinetic energy:

$$\frac{1}{2} m_1 V_{01}^2 + \underline{\frac{1}{2} m_2 \overset{0}{V_{02}^2}} = \frac{1}{2} m_1 V_{f1}^2 + \frac{1}{2} m_2 V_{f2}^2 \quad \times 2$$

$$\Rightarrow m_1 V_{f1}^2 - m_1 V_{01}^2 + m_2 \cdot \left(\frac{m_1^2 (V_{01} - V_{f1})^2}{m_2^2} \right) \quad \begin{matrix} \times m_2 \\ / m_1 \end{matrix}$$

$$\Rightarrow m_2 V_{f1}^2 - m_2 V_{01}^2 + m_1 (V_{01}^2 - 2 V_{01} V_{f1} + V_{f1}^2) = 0$$

$$\underline{m_2 V_{f1}^2} - \underline{m_2 V_{01}^2} + \underline{m_1 V_{01}^2} - 2 m_1 V_{01} V_{f1} + \underline{m_1 V_{f1}^2} = 0$$

$$\Rightarrow \boxed{(m_1 + m_2) V_{f1}^2 - 2 m_1 V_{01} V_{f1} + (m_1 - m_2) V_{01}^2 = 0}$$

Momentum in 2D & Thermal Energy

Momentum in 2D

(1) Momentum Conservation in 2D

x - Component

$$\boxed{P_{01,x} + P_{02,x} = P_{f1,x} + P_{f2,x}}$$

y - component

$$\boxed{P_{01,y} + P_{02,y} = P_{f1,y} + P_{f2,y}}$$

(2) When 2 objects collide to form one system, it is called ((Inelastic Collisions))...

$$\boxed{P_{01} + P_{02} = P_{system}}$$

$$\left[\begin{array}{l} P_{01,x} + P_{02,x} = P_{system(x)} \\ P_{01,y} + P_{02,y} = P_{system(y)} \end{array}\right.$$

(3) Explosions

- Oppisite of inelastic collisions...
- A whole system shatters into pieces due to internal forces...
- Since explosions happens quickly, Momentum is also conserved...

$$\boxed{P_{system} = P_{f1} + P_{f+2}}$$

Thermal Energy

① Thermal Energy

- is the internal energy associated with an object due to its temperature...
- when $E_{th} = 0\ J \Rightarrow T = 0$ Kelvin
- $E_{th} \propto T$ ...

② Heat (Q)

- the process of transfering energy between 2 objects due to differences in temperature...
- Methods of heat transfer:
  - Conduction: Direct contact
  - Convection: Bulk movement of fluid (example) ⇓
    - Hotter air away from heating source ⇒ cools down ⇒ colder air near heating source ⇒ heats up
    - (lesser density) (Higher density)
  - Radiation: emition of light spectron (photons)
- Conservation of energy:

$$\boxed{\begin{aligned}\Delta E_{sys} &= \Delta KE + \Delta U + \Delta E_{th} \\ &= Q - W\end{aligned}}$$

- W:
  - + ⇒ work done done by the system ...
  - − ⇒ work done on the system ...
- Sign:
  - + ⇒ Heat absorbed by the system ...
  - − ⇒ Heat released from the system ...
- Unit ⇒ KJ or J

$$\boxed{Q = C \cdot \Delta T}$$

→ Heat capacity [$C = n \cdot \boxed{C_n}$; $C = m \cdot \boxed{C_m}$] ⇒ specific heat

(n: mole; m: mass)

- Phase Transition:
  - $\Delta H_{vaporization}$
    - liquid → Gas (Evaporization)
    - Gas → liquid (condensation)

* Rules (Heat transfer during phase transition)

- no change in T (constant) ...
- Change in bond energy ...

ΔH fussion
- liquid → Ice (freezing)
- Ice → liquid (melting)

$$\boxed{Q = m \cdot \Delta H}$$

ΔH sublimation
- solid → Gas (sublimation)
- Gas → Solid (deposition)

Process of Heat transfer (Increase of temperature example):

$$\text{Solid} \xrightarrow{+T} \text{Melting point} \xrightarrow{\Delta H_{fus}} \text{liquid} \xrightarrow{+T} \text{Evaporation point} \xrightarrow{\Delta H_{vap}} \text{Gas}$$

Thermal Equilibrium:

- $Q_{sys} + Q_{surrounding} = 0$ — amount of energy absorbed ((colder)) = amount of energy released ((Hotter))...
- Both system & surrounding must have the same Final temperature ...

3 Power

- is the rate at which energy is transfered

$$\boxed{P = \frac{dE}{dt} = \frac{Q}{t}}$$

unit: watt = J/s

$$\boxed{P = k \cdot A \cdot \frac{T_1 - T_2}{h}}$$

((Insulator))

- h → thickness: How much resistance does the material provide for heat flow?!
- A → Area: is the surface through which heat is transfered...
- k → Thermal conductivity: How well does the material conduct heat?!