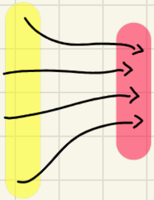
Flux and Field

- a Field Vector are everywhere tanget to the Field lines at each Point in Space

- that is the Field Vector it must be tanget to the Field lines

- Vector Fields are greater when they are closer togther

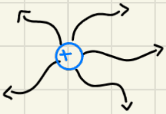


- the electric Field is smaller here
- the electric Field is greater here thats because they are Closer togther

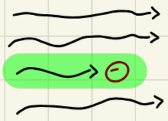
- Charges and Field lines

- electric Field lines begin on Positive Charges

☆ Field lines began and end on Charges



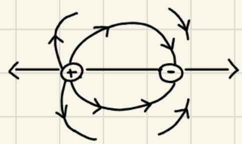
- electric Field lines end on negitive charges



- Notice how this line ends while the rest are stillgoing this happens because there is a negitive charge there

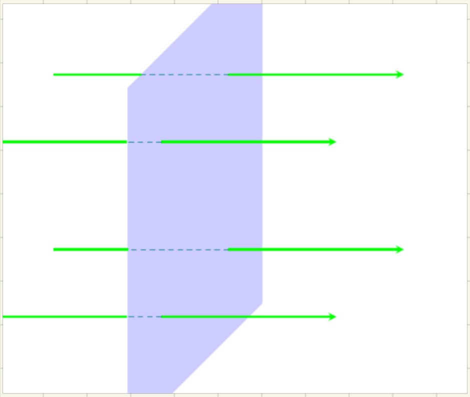
- charges and Field lines

- the Field lines give us a quanlitive picture of the Field sourced by charges



- Flux

- this can be though as a count of the number of Field lines passing through the surface

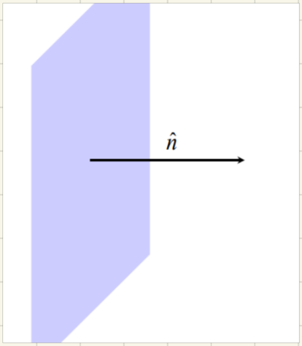


- the green lines are the Field lines

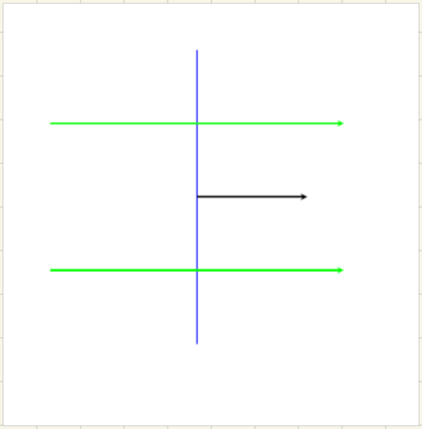
- a surface as a vector
  - in 3d space, a 2d surface is uniquely defined by the perpendicular direction
    - we call this the surface normal (which is perpendicular to the surface)

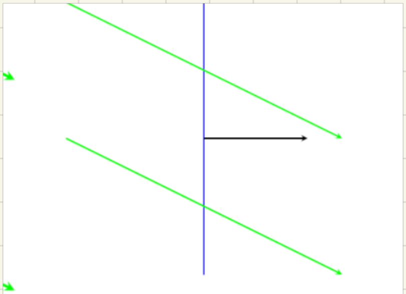
$$\vec{A} = A\hat{n}$$

- by convention, the normal of a closed surface points outward



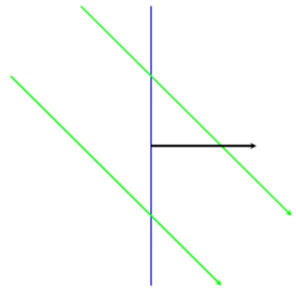
the $\hat{n}$ can be anything it can be $-A\hat{y}$ or $A\hat{x}$
the only thing that matters is which quardinate its pointing



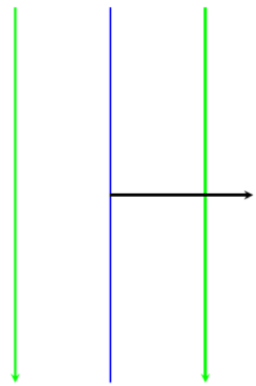


However to do so, the field lines had to get **closer together.** Hence to keep the same flux, the field had to get stronger!

☆ ignore however



However to do so, the field lines had to get **closer together** once again.



✱ ignore however

✅ **Rule of thumb:**

Situation	Flux proportional to area?	Notes
Closed surface enclosing charges	❌ No	Only depends on total enclosed charge
Open surface in uniform field	✅ Yes	Also depends on angle between field and normal

So gauss law does not depend on area only depends on charges in the enclosed surface also remember gauss law only works on a closed surface So if you are looking at the rule of thumb above the first one applies to gauss law

✱ Summary:

- if the surface is open (like a flat patch) the flux does depends on area
- if the surface is closed around charges, the flux only depends on enclosed charge

* in a uniform Field the Flux and area are directley Proportional meaning if one increases the other one increases, but when you are talking about a Point Charge Field, Flux is constant that is independent of area

- So Clearly, the Flux must be directly Proportional to the area and the Magnitude of the Vector Field

What is directly Proportional?

$y = kx$

- this equation is For Sake of the explination

y and x are directly Proportional to each other if y doubles the x will also double, the k scales the relation ship it makes the equation match real-world Measurment units

- equation For Flux

$$\Phi_{\vec{c}} = \vec{C} \cdot \vec{A} = CA\cos\theta$$

- $\vec{c}$ is a Vector Field it can be any Vector Field but for us we will us $\vec{E}$ and it is a dot Product the ange θ is between the $\vec{E}$ Field and the area normal

So if the surface is curved or non-uniform then the angel (and Possibly the Field Strength) Changes From Point to Point

- if the angel Changes notice we can define a differential Flux as:

$$d\Phi_{\vec{E}} = \vec{E} \cdot d\vec{A}$$

- we integrate over the entire surface to get the total Flux

$$\Phi_{\vec{E}} = \int_A d\Phi_{\vec{E}} = \int_A \vec{E} \cdot dA$$

• gauss law

- the total electric Flux through a closed surface is directly proportional to the total charge contained within that closed surface

☆ must be a closed surface to obey gauss law

• in simpler words

- the more charges you have inside a closed surface, the more electric field lines will pass through that surface

☆ When you are talking about gauss law it can only be applied to a enclosed surface also the charge must be inside the enclosed surface it cannot be outside the enclosed surface

- gauss law Mathmatically

$$\Phi_{\vec{E}} = \oint_A \vec{E} \cdot dA \propto Q_{enclosed}$$

proportional too

means closed surface

integrating over area

So when we have a proportionality and we want to write this as a equality (equal) we need to add in a proportionality constant to help balance out units

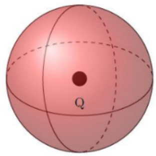
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

this constant is epsilon knot which stand for which represents premittivity of free space

• gauss law for point charge

- a point charge has spherical symmetry
- notice now our system will have spherical symmetry
- hence, the field must be purly radial and can only depend on r. Not θ or φ
- the electric field can only depend on r

- r would be how far you are from the center



The field on this red surface is therefore constant and pointing radially!

$$\vec{E} = E(r)\hat{r}$$

• the dot in the surface labeled Q is the point charge

☆ must be a closed surface to satisfy gauss law

Notice everywhere from the sphere surface the electric field has to be a constant value because everywhere on the surface is the same distance from the central point

$$\vec{E} = E(r)\hat{r}$$

radial unit vector pointing outward away from the point charges

inward would be negitive $\hat{r}$

outward would be positive $\hat{r}$

So by gauss's law:

here E is constant for sphere

$$\frac{Q}{\varepsilon_0} = \Phi_{\vec{E}} = \oint \vec{E}\cdot d\vec{A} = \oint_{\text{sphere}} E(r)\hat{r}\cdot d\vec{A} = \oint E(r)\,dA = 4\pi r^2 E(r)$$

$\hat{r}$ and $\hat{n}$ are parrallel they point in the same direction so they become one $\vec{E}\,d\vec{A}\cos(0) = \vec{E}\,d\vec{A}$

Q = source charge which creates the electric field

$$\vec{E} = \frac{Q}{4\pi r^2 \varepsilon_0}$$

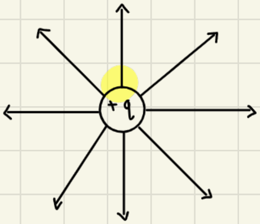
• where $\varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2\cdot s^2}{kg\cdot m^3}$

notice that the Field line density decreases as we get Further away from the central charge matching our result

$$\vec{E} = \frac{Q}{4\pi r^2 \varepsilon_0}$$

or in other words: as r (radius) increases, Field line density decreases, while Q stays Constant

the brightness per area = Field line density and it indicates the intensity = Field Strength



- and this does make sense if i increase the radius the Field lines has a larger area to spread which will will decrease the Field line density

So increasing the radius will decrease Electric Field

- Coulombs law

- to Find the force between two charges we recall our definition of electric Field

$$\vec{F} = q\vec{E}$$

- For charge q_1 placed a distance r from a second q_2, we have

$$\vec{F} = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2}\hat{r}$$

notice that one of the charges here has to produce the Field that called a source charge and another one expirencing the Field thats called a test charge

- Potential of a Point Charge

- For static situation recall that:

$$\vec{E} = -\nabla V$$

hence, For a Point Charge Q:

$$-\nabla V = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

in spherical coordinates:

$$\nabla V = \frac{\partial v}{\partial r}\hat{r} + \cancel{\frac{1}{r}\frac{\partial V}{\partial \theta}}\hat{\theta} + \cancel{\frac{1}{r\sin\theta}\frac{\partial v}{\partial \varphi}}\hat{\varphi}$$

- we only care about $\hat{r}$ the rest are simply Not needed here because they must be zero by symmetry as we stated Previosly

hence, For Point Charge Q:

$$-\nabla V = \frac{Q}{4\pi r^2 \varepsilon_0}\hat{r}$$

$$-\frac{\partial V}{\partial r}\hat{r} = \frac{Q}{4\pi r^2 \varepsilon_0}$$

- then we integrate

$$V = -\int \frac{Q}{4\pi r^2 \varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r} + c$$

- every thing was a constent except For r (radius)

- this is a scalar you don't need to break into components you can just add them up

$$V = \frac{Q}{4\pi r \varepsilon_0} + c$$

The electric Field is how Fast the electric Potential Changes

- gauge Freedom

- the Choice of c is arbitary and it does Not Change the underlying Physics
- we usally Pick: V=0 at $r \to \infty$

- this makes sense because far away from the charge it influence is negligible

  - we used to say at r is at zero but if we pick another point we are only changing the refernce point from zero potential

    - so its not the charge that changes but the point we call zero potential

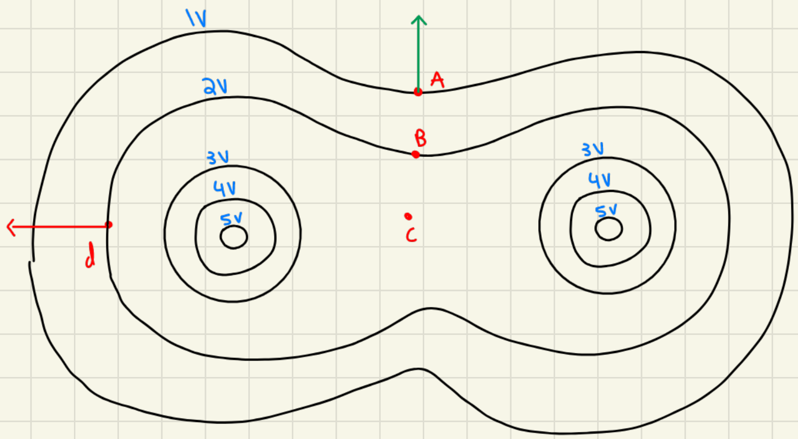
• equipotentials

- to depict electric potential (or ΔV change in voltage) we use equipotential lines so that all points on the surface have same V value

- notice the field lines will be perpendicular to the equopotential surfaces

- the closer togther the equipotentials, the greater the electric field (this implies V is changing more rapidly)

- recall $\vec{E}$ points in the direction of decreasing potential



* if you move Point A to Point B then return it back to Point A the Net Change is Zero because you ended at the same Place

if an electron if moved from Point B to D what is the change in energy? There will be no change in energy because they are on the same equoPotential line so there will be no change in energy

What is the direction of the electric Field at Point A? The direction is going to be Perpendicular to the equoPotential line and in the direction of decreasing Potential

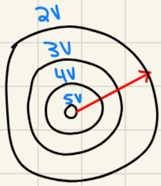
What is the direction of electric Force on a Proton Placed at Point D? The electric electric Force Points in the direction of the electric Field for Positive Charges we also know that the electric Field goes in the direction of decreasing Potential electric so it goes left

is the electric Field Stronger or larger at Point C or Point D? The electric is larger at Point D because there are more equoPotential lines at Point D

how many (and what type) of charges are Pictured? Here we are looking for

and we see we have two that means we have two charges but to Find what kind of Charge they are we need to look at their electric Potential

- if you have

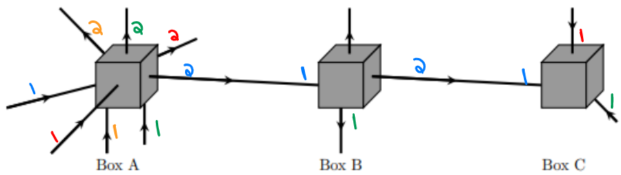


- remember $\vec{E}$ Points in the direction of decreasing v So Following the decreasing Potential we get a outward line which means the charge is Positive because the Field is going away From the charge

if you have

- as we said the $\vec{E}$ Points in the direction of decreasing electric Potential Following the decreasing Potential we get a inward line which means the Charge is Negitive because the Field is going towards it

While the formal definition of Gauss's law is somewhat opaque, the qualitative idea is fairly straightforward: since electric field lines can only begin on positive electric charges and end on negative charges, if we count all the field lines entering and exiting a closed surface, the net number of field lines must be proportional to the net charge contained within that surface. Consider the pictures below, which show three mystery boxes and the electric field lines going into and out of them.



1. Order from least to greatest **the total number of discrete charges** contained inside each of the mystery boxes. If that cannot be determined from the information given, explain why it cannot be determined.
2. Order from least to greatest the **magnitude of the net charge** contained inside of the the three mystery boxes. If that cannot be determined from the information given, explain why it cannot be determined.
3. Is the total net charge contained inside **all three mystery boxes combined** positive or negative? If that cannot be determined from the information given, explain why it cannot be determined.

1) You cannot answer the First Problem because gauss law because it only gives you the Net charge inside a closed surface not the number of discrete charge (how many Positive charges and how many negitive particles individually)

2) looking at the First box all the Field lines go in and come out, the second box has one Field line going in and coming out but there are two Field line coming out as if they are repeling some, the third box there air three Field lines all going inward the question asked for net charges contained in each box From least to great so a, b and c

3) the First box has a net charge of zero because field lines go into the surface and come out the second box has the one Field line going in and coming out but two Field lines are being repelled by something so the second box has a total of +2 the last box has three Field all going in something must attract them so the third box has total of -3

$$0+2-3=-1$$

Super Position and dipoles

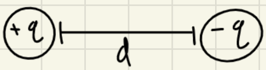
- the total Field expirenced by a Particular charge could be Found by adding togther the Fields produced

- For $\vec{E}$ this is Vector addition
- For Finding V, we Pick the gauage where V=0 or Very Far away From Charges

When you are asked about electric Potential remember its a scalar so you can just add it up also trust the sign whatever sign you get when you do the math that sign will be correct because its a scalar

- the electric dipole

- a electric dipole consists of two charges of equal magnitude q and opposite signs spaced a small distance d apart



the dipole moment is a vector quantity that measures the strength and orientation of a dipole

$$p = q d$$

q= the magnitude of one of the charges
d= the displacement vector Pointing From negitive charge to Positive charge
p= the dipole moment (in C·m)

the Formula $p = qd$ is strictly For a ideal dipole Just two equal and opposite Charges Seperated by a distance

if you had two Charges that were acting on each other but had $\hat{i}$ and $\hat{j}$ Components you would break them into Vectors

example:

$+3.0\mu$ at $(0.02, 0.04)$

-3.0μ at $(1,2)$

$$\vec{p}_1 = (3.0 \times 10^{-6})(0.02, 0.04) \qquad \vec{p}_2 = (-3.0\times 10^{-6})\cdot(1,2)$$

$$\vec{p}_1 = (6.0\times 10^{-8}, 1.2\times 10^{-7}) \qquad \vec{p}_2 = (-3.0\times 10^{-6}, -6.0\times 10^{-6})$$

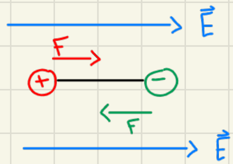
$$p = \vec{p}_1 + \vec{p}_2$$

$$p = (6\times 10^{-8}, 1.2\times 10^{-7}) + (-3.0\times 10^{-6}, -6.0\times 10^{-6})$$

$$\boxed{p = (-2.94\times 10^{-6}, -5.88\times 10^{-6})}$$

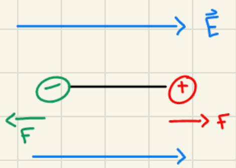
- if we Place the dipole in a electric Field

☆ Net Force is zero

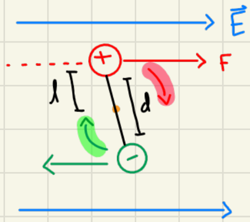


- the Positive dipole will go with the electric Field while Negitive will go away From the electric Field

☆ Net Force is zero



- the Positive goes with the electric Field and the negitive goes away From the electric Field



- When they are vertical the positive will go with the electric field and negitive will go away from the electric field. Notice this will cause it to rotate therefore creating a torque

this force will be created will go clockwise call this τ_1

- this force will be created will also go be clockwise call this τ_2

$\tau_{net} = \tau_1 + \tau_2$

- $\tau_1 = Fl$
- $\tau_2 = Fl$

the distance between the two charges is d which means $l = d/2$

So:

$$\tau_{net} = F \cdot \frac{d}{2} + F \cdot \frac{d}{2} = F \cdot d$$

$\frac{d}{2}$ and l are the same thing

Now the force acting inside the electric field is $F = qE$ we can take this equation and plug it into $\tau_{net} = F \cdot d$

So: $\tau_{net} = Eqd$

• review on what torque looks like

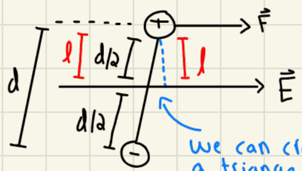
- So torque is $\tau = F \times r$ r is the leverarm here

• a Force is applied to the leverarm Causing in to rotate

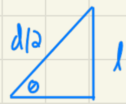
• going back to our dipole we want to know where the "lever arm" is

as time passes by the electric dipole is going to rotate

• Notice l is the same on both sides



we can create a triange with angle θ



$\sin\theta = \frac{l}{d/2} = l = \frac{d}{2}\sin\theta$

$\tau = F \cdot \frac{d}{2}\sin\theta$

• this is just τ_1 but τ_2 is equal

$$F\left(\frac{d}{2}\right)\sin\theta + F\left(\frac{d}{2}\right)\sin\theta = Fd\sin\theta$$

We also know $F = Eq$ So: $\tau_{net} = Eqd\sin\theta$

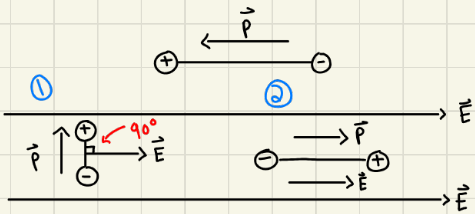
also $P = qd$ So we can rewrite it as: $\boxed{\tau_{net} = PE\sin\theta}$

in cross product form: $\tau_{net} = \vec{P} \times \vec{E}$

if the dipole is Parallel to Field (p in the same direction as E) $\theta = 0$ $\tau = 0$ No torque

if the dipole is Perpendicular to the Field, $\theta = 90°$ you have $\tau_{max} = PE$

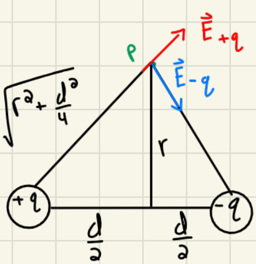
★ the dipole moment goes From Negitive Charge to the Positive Charge



① So as stated before $\sin(90°) = 1$ which means torque is at the Maximum Value at $\tau_{max} = PE$

② here it has reached its most stable Position but the angel is $\theta = 0$ which τ is at its minimum value

• The electric dipole



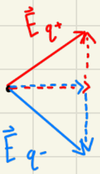
$$|\vec{E}_{+q}| = |\vec{E}_{-q}| = \frac{q}{4\pi\varepsilon_0 (r^2 + \frac{d^2}{4})} = \frac{q}{\pi\varepsilon_0 (4r^2 + d^2)}$$

square root got squared

$$\cos\theta = \frac{d}{2\sqrt{r^2 + \frac{d^2}{4}}} = \frac{d}{\sqrt{4r^2 + d^2}}$$

Zoom in on Point P

★ we Place the two in the square root



• Notice the x component cancel

$$E_{net} = \vec{E}_{\bar{x}} \cos\theta + \vec{E}_{+} \cos\theta = 2E_q \cos\theta\, \hat{x}$$

$$2\left(\frac{q}{\pi\varepsilon_0 (4r^2 + d^2)}\right)\left(\frac{d}{\sqrt{4r^2 + d^2}}\right) = \frac{2dq}{\pi\varepsilon_0 (4r^2 + d^2)^{3/2}}$$

$$\vec{E} = \frac{2dq}{\pi \varepsilon_0 (4r^2 + d^2)^{3/2}}$$

• notice that in the limit we get very far a ay away away from the dipole ($r \gg d$) we have

$$\vec{E}_{total} (r \to \infty) \frac{dq}{4\pi \varepsilon_0 r^3} \hat{x}$$

$r \gg d$ mean r is much greater then d

You can only use $E = \dfrac{dq}{4\pi \varepsilon_0 r^3}$ when you have $r \gg d$ (r is much greater then d) else you cannot use it you need to use $\vec{E} = \dfrac{2dq}{\pi \varepsilon_0 (4r^2 + d^2)^{3/2}}$

equation summary

• Flux and Fields

$$\Phi_{\vec{E}} = \vec{E} \cdot \vec{A} = \vec{E} A \cos\theta \quad (N/C)$$

$$\Phi_{\vec{E}} = \oint \vec{E}\, dA = \frac{Q_{enclosed}}{\varepsilon_0} \quad (N/C)$$

$$\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \quad (N/C)$$

$$\vec{F} = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2} \quad (N)$$

$$V = \frac{Q}{4\pi \varepsilon_0 r} + C \quad (V)$$

$$F = q E \quad (N)$$

$$\Delta u = q \Delta V \quad (J \text{ Joules})$$

• dipoles and Superposition

$$\vec{P} = q d \quad (C \cdot m)$$

$$\tau_{net} = \vec{p} \times \vec{E} \Rightarrow \tau_{net} = \vec{E}\vec{P} \sin\theta \quad (N \cdot m)$$

$$u = -\vec{P} \cdot \vec{E} \Rightarrow -P E \cos\theta$$ maximum u: $\theta = 180°$ (J Joules)
minimum u: $\theta = 0°$

(this was not disscused by i though it would be smart to add it
this is the Potential energy of a electric dipole in a uniform electric Field)

$$E = \frac{2qd}{\pi \varepsilon_0 (4r^2 - d^2)^{3/2}} \quad (N/C)$$

$$E = \frac{dq}{4\pi \varepsilon_0 r^3} \text{ For } r >> d \quad (N/C)$$