Universal gravitation

elliptical orbits = in elliptal orbits the orbital radius which is the semi-major axis a, but in simple terms its a orbit in the shape of a ellipse, not a perfect circle

unless a problem says otherwise, orbital radius in a elliptical usally means the sem-major axis

- semi-major axis is $\frac{r_p + r_a}{2} = a$

r_p = perpheral axis (closet to the sun in a orbit)

r_a = aphelion radius (farthest from the sun in orbit)

the semi-major axis is what you are using in $T^2 = \frac{4\pi^2}{GM} r^3$

this is "r" in the equation the letter does not matter
the only thing you need to know what it represents

For elliptical orbits

Phyical radius = the size of a body, the distance from the center to the surface

Orbital radius = the distance between the centers of the orbiting object and the center body

gravational Force

- two masses m and M seperated by a distance r feel a gravational Force given by:

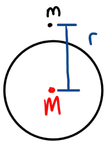
Mass thats orbiting the larger mass

mass thats being orbited (larger mass)

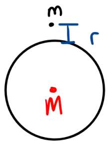
$$F = G\frac{mM}{r^2}$$

$G = 6.67 \times 10^{-11}$

- Use orbital radius unless you you are asked to use physical radius



this is how r is supposed to be calculated



this is incorrect

- the radius you want to use on this is the distance between the two masses sometimes you will be given two seperate radiuses and to get the orbital radius you need to add them both otherwise you will only have radius of one of the masses

- r is the distance between two masses

Potential gravational energy

$$F = -\frac{du}{dx} \quad \text{and} \quad F = -G\frac{mM}{r^2}$$

$$\int \cancel{-}\frac{du}{dr} = \int \cancel{-} G\frac{mM}{r^2}$$

$$u = GmM\int\frac{1}{r^2} \quad \approx> \quad u = -\frac{GmM}{r}$$

- G, m, M, are all constant you can take them out the integral

$$g = \frac{GM}{r^2}$$

- this equation is mostly used to find the gravity on a planet

★ to find gravity on a surfaces you need to use the Mass of that surface and the radius of that surface (Physical radius)

Deriving the equation

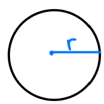
$$F = ma \Rightarrow a = \frac{F}{m}$$

- return acceleration to gravity

$$g = \frac{F_{gravity}}{m} \Rightarrow \frac{\frac{G\cancel{m}M}{r^2}}{\cancel{m}} \Rightarrow \frac{GM}{r^2}$$

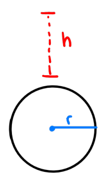
For gravation field strength on the surface of a planet you can use the the physical radius which is from the center to the surface

$$g_{surface} = \frac{GM}{r_{Planet}}$$



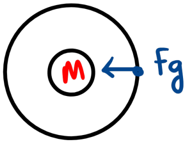
For gravational field strength above the surface you must use the orbital radius orbital radius $r_{Planet} + h$ (where h is the height above the surface)

$$g_{above} = \frac{GM}{(r_{Planet} + h)^2}$$



Circular Velocity

- this equation was derived From the gravational force and Centripetal force the reasoning is the orbiting mass has a force that Pointed to the Center of mass



$$\cancel{m} \cdot \frac{v^2}{\cancel{r}} = \frac{G\cancel{m}M}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r} \implies v = \sqrt{\frac{GM}{r}}$$

- Notice how the little m cancels out and you are only left with big M meaning if you wanted to get the velocity of a Satellite you don't use its mass but mass of whatever its orbiting

- when using this equation you will use the Mass thats being orbited and the orbital radius which is $r = r_{Plant} + r_{neight}$ unless you were only given orbital radius as apart of the question

Keplers third law

- Planets that are further from the sun take longer take longer to go around it

$$VT = 2\pi r \implies V = \frac{2\pi r}{T} \implies \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = \frac{4\pi r^2}{T^2} \implies T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

tells you how long it take to complete one full orbit

- r is the radius of two mass (orbital radius)
- M is the Mass thats being orbited

- notice how mass and G "universal gravity" are constant meaning it does not change Mass will vary for different problems but there will be any Δ changes but G will be the same all of this is important for the next equation

the only thing that is changing is r while $\frac{4\pi^2}{GM}$ is constant

$$T^2 \propto r^3 \approx \frac{T^2}{r^3} = \text{constant} \approx> \frac{T^2}{r^3} = \frac{T^2}{r^3}$$

$$T^2 \propto r^3 \approx> \frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

- this equation is used when two planets are orbiting the same mass they must be orbiting the same mass

Kepler's Third Law Example

Jupiter has a mass that is about 318 times the mass of the earth and orbits the sun at a radius that is 5.2 times the size of the earth's orbital radius. Approximately how many years does it take Jupiter to complete a single orbit around the sun?

If we then plug in that $r_J = 5.2r_E$, we have:

$$\frac{T_E^{\,2}}{r_E^{\,3}} = \frac{T_J^{\,2}}{(5.2r_E)^3} \Rightarrow T_J^{\,2} = \frac{(5.2r_E)^3}{r_E^{\,3}} T_E^{\,2}$$

- this example should give you a greater reasoning on why the two masses need to orbit the same mass

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We now do a little algebra, canceling the r_E^3 in the numerator and the denominator, and then take a square root:

$$T_J = \sqrt{5.2^3}\,T_E \approx 11.9T_E$$

Summary of equations

$$T^2 \propto r^3 \Rightarrow \frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$g = \frac{GM}{r^2}$$

$$u = -\frac{GmM}{r}$$

$$F = G\frac{mM}{r^2}$$