Conducters

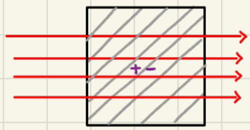
- a conducter is a material in which charges are able to move Freely
- this is as opposed to insulator or dielectric which are materials where charges are Fixed and cannot move

- inside a conducter there is NO electric Field when you have a conducter the charges go to the surface where the charges inside the conducter produce electric Field and can cel the incoming Field

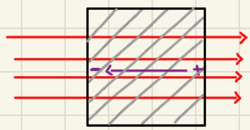
- consider a conducter in a electric Field

electric Field

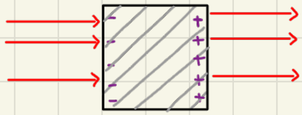
Positive and negitive Charges



the Positive charge go with the electric Field while the negitive charge will take the oppisite side of the electric Field



the Positive charge produces a electric Field to the Negitive charge this will cancel the incomming electric Field



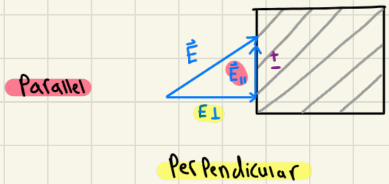
Notice as long as there is Field inside the conducter charges will move opposing the Field

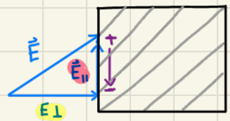
- this observation gives us three results

- the electric field inside the conducter must
- Since $E = -\nabla V$, $\vec{E} = 0$ every in the Conducter implies V is constant (which means conducters are equopotentials)

- Charges on a conducter must lie on the surface on the conducter

- now consider a field near the surface of a conducter



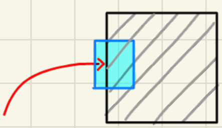


- a electric field is produced by the positive to negitive which cancels out this electric field

- the local electric field must be perpendicular to the surface (ie, parallel to the surface normal) of a conducter

- this is consistent with the conducter being a equopotential

- we create a gaussian surface and making exit the conducter a bit



$$Q = \sigma A$$

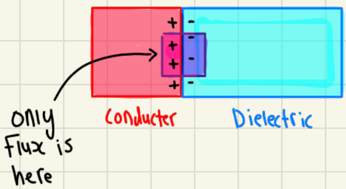
$$\oint E\, dA = \frac{Q}{\varepsilon_0}$$

$$E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

- you can use this equation
- the field inside the conducter is zero
- excess charges reside on the surface
- the field just outside the conducter is perpendicular to the conducter surface

• Now consider a boundary between a dielectric and a charged conducter



✗ Opposite charges occur when when a electric Field exists between the two medias

Notice this gives through the right side surface (Not left because $E=0$ in conducter)

$$\oint E\,dA = \frac{Q}{\varepsilon_0} \implies E\cancel{A} = \frac{\cancel{A}(\sigma_f+\sigma_b)}{\varepsilon_0}$$

$Q = (\sigma_F + \sigma_b)A$

(No dividing because its a surface)

and

$$\int D\,dA = Q \implies \vec{D}\cancel{A} = \cancel{A}\sigma_f$$

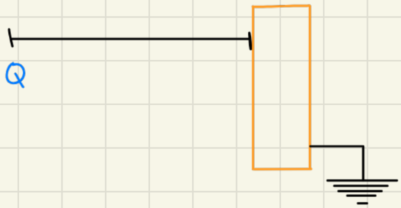
• Now using $\vec{D} = \varepsilon_0 K \vec{E}$ we can solve for bound surface charge density

$$\sigma_b = -\left(1 - \frac{1}{K}\right)\sigma_f$$

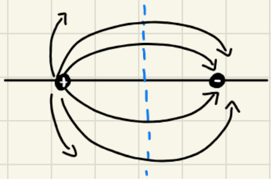
• the bound charge density on the surface of a dielectric with the free charge density on the surface of a conducter that is in contact with that dielectric

✗ in conducter all charges are free there are no bound charges in a conducter

• Consider a charge Q Place before an infinte conducting Plate that is grounded (think of grounded as a infinite supply of charge) (i.e at a constant Potential)

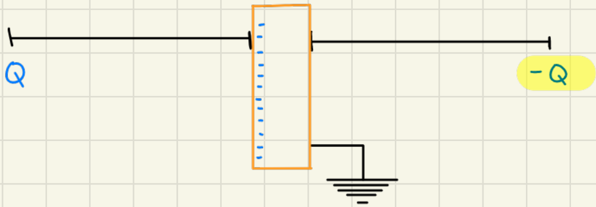


if we recall the electric dipole



• the equopotential surface between the two charges is a infinite flat plane and specifally the equopotential surface that has the same potential as infinitely far away

hence we observed that the problem we want to solve is equivalent to the electric dipole field region to the left of the plate



• this is acting like the mirror image of the other charge but the only thing that is different is it has oppisite sign because its a dipole

When ever you go draw a mirror image you need to give it a oppisite charge

So inside of picking the left side that has a bunch of weird charges you pick the right side and treat it as its only one charge

Made with Goodnotes

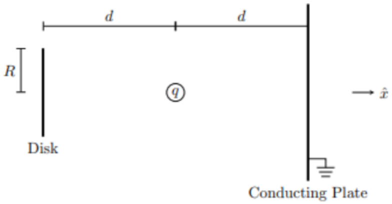
hence if we wanted to Find the Force on Q would Just apply Coloumbs law We apply coloumbs law For the Charge and its image

although the -Q is Not real and Just a image Charge the affects on the +Q Caused by it are thats Why We take it in account

$$|F| = \left| \frac{(Q)(-Q)}{4\pi \varepsilon_0 (2x)^2} \right|$$

- and notice r is $2x$ because its the distance is x From the Charge to the Plate and another x From the Plate to the image Charge

Problem 3 (35 points total): A positive point charge q is placed a distant d away from an infinite grounded conducting plate, as shown in the picture below. A distance d from the charge in the opposite direction is the center of a negatively charged disk of radius R and uniform surface charge density $-\sigma$, where σ is a positive constant. The disk is oriented so that it is parallel to infinite conducting plate, as shown. In this problem, please use a coordinate system where $\hat{x}$ points to the right, as pictured.



in terms of the Variables given, What is the electric Field $\vec{E}$ expirenced by the Point charge due to the Charged disk

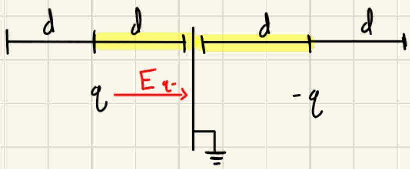
recall the Formula of a charged disk: $-\frac{\sigma}{2\varepsilon_0}\left[\frac{d}{\sqrt{R^2+d^2}}-1\right]$

the Problem is simply asking for the Field due to the Charged disk

the answer Should be in the Variable given

$$\boxed{\vec{E} = \frac{-\sigma}{2\varepsilon_0}\left[\frac{d}{\sqrt{R^2+d^2}} - 1\right]}$$

in terms of the variables given What is the electric Field $\vec{E}$ expirenced by the Point Charge q due to the infinite grounded conducting Plate



$r = 2d$

$$E_{net} = \frac{q}{4\pi r^2 \varepsilon_0} = \frac{q}{4\pi (2d)^2 \varepsilon_0}$$

$$\boxed{E_{net} = \frac{q}{16\pi d^2 \varepsilon_0}}$$

What is the net electric Force $\vec{F}$ acting on the Point Charge q?

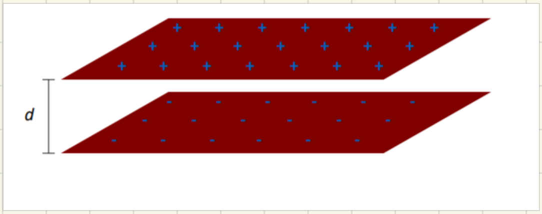
✱ use columbs law: $\vec{F} = \frac{q_1 q_2}{4\pi r^2 \varepsilon_0}$

$$\vec{F} = \frac{|q(-q)|}{4\pi (2d)^2 \varepsilon_0} = \frac{q^2}{16\pi d^2 \varepsilon_0}$$

$$\boxed{\vec{F} = \frac{q^2}{16\pi d^2 \varepsilon_0}}$$

Capacitors

let us Place a total charge $+Q$ on the top Plate and $-Q$ on the lower Plate Since the Plates are conducters the charges will spread evenly across



assuming the Plates have area A, the surface charge denisites are given as:

$$\sigma_+ \; \frac{Q}{A} \qquad \sigma_- = -\frac{Q}{A}$$

and the electric Field of a flat Plane is:

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}$$

now using superposition, the Field between the Plates is found to be

$$E_{total} = \vec{E}_+ + \vec{E}_- \implies \frac{-Q\hat{z}}{2A\varepsilon_0} - \frac{Q\hat{z}}{2A\varepsilon_0} = -\frac{Q\hat{z}}{A\varepsilon_0}$$

$$E = \frac{-Q\hat{z}}{A\varepsilon_0}$$

hence the Voltage of the two Plates is:

$$E = -\frac{dV}{dz} \Rightarrow E = -\frac{\Delta V}{\Delta z} \Rightarrow E\,\Delta z = -\Delta V$$

• where Δz becomes d

$$\Delta V = \frac{Qd}{A\varepsilon_0}$$

Notice the Charge is diretly Proportional to the Voltage difference (Which Just Means if you double the Voltage the Charge doubles and if you cut the Voltage in half the charge is halfed)

$$Q = C|\Delta V| \quad \text{For} \quad C = \frac{A\varepsilon_0}{d}$$

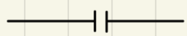
all we did over here is Plug the Previous ΔV into the $Q = C|\Delta V|$ and Solved For C

units of Capacitance:

F (Farad)

you will most likly Not see a Farad but you will be seeing nanoFarad and maybe microFarad the reasoning behind that is a Farad is way too big

When Placed in eletric circuits, capacitor are given the Following symbol



- Potential energy inside a capacitor

Consider a capacitor that carries charges q and $-q$ on the two Plates then the energy required to move an additional dq From the negitive to the Positive is

$$dU = \Delta V\, dq = \frac{q}{C}\, dq$$

if we integrate From zero to Q because we want to get total Charge we get:

$$U = \frac{Q^2}{2C}$$

but we rewrite this using our definition $Q = C\,\Delta V$

$$U = \frac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2} Q (\Delta V)$$

$$U = \frac{Q^2}{2C}$$

• if we pour dielectric between plates we would increase C which would therefore decrease U

• these equations will give you the same equation but you can use each one depending on the situation

We know that $|\vec{D}| = \sigma_f$

We can find the voltage difference between the plates

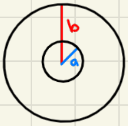
$$\vec{E} = \frac{D}{\varepsilon_0 K} = \frac{\sigma_f}{K \varepsilon_0} \implies |\vec{E}| = \frac{\sigma_f}{K \varepsilon_0}$$

Constant between the plates, the voltage is just

$$\Delta V = |\vec{E}|\, d = \frac{d\, \sigma_f}{K \varepsilon_0} = \frac{d Q}{K \varepsilon_0 A}$$

• all these equations are the same the reasoning on why there are three is because you can use any one depending on the situation

Cylinderical Capacitors



$$E = \frac{Q}{2\pi \varepsilon_0 r L} \Rightarrow E = -\frac{dV}{dr}$$

$$\int_a^b \frac{Q}{2\pi r \varepsilon_0 L} dr = \int -\frac{dV}{dV} \Rightarrow \frac{Q}{2\pi L \varepsilon_0} \Big[LN(r) \Big]_a^b = -V$$

$$\frac{Q}{2\pi L \varepsilon_0} \cdot LN(b) - LN(a) = \Delta V = \frac{Q}{2\pi L \varepsilon_0} \cdot LN\left(\frac{b}{a}\right)$$

$$Q = C|\Delta V| \Rightarrow \Delta V = \frac{Q}{C}$$

$$\frac{\cancel{Q}}{C} = \frac{\cancel{Q}}{2\pi L \varepsilon_0} \cdot LN\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{2\pi L \varepsilon_0}{LN(b/a)}}$$

- this equation works when you don't have a dielectric

When you have a dielectric you replace $\varepsilon_0 = K\varepsilon_0$

$$\boxed{C = \frac{2\pi L K \varepsilon_0}{LN(b/a)}}$$

Sphere

$$V = \frac{Q}{4\pi \varepsilon_0 r}$$

$$\Delta V = V(a) - V(b) = \frac{Q}{4\pi \varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{4\pi \varepsilon_0 ab}$$

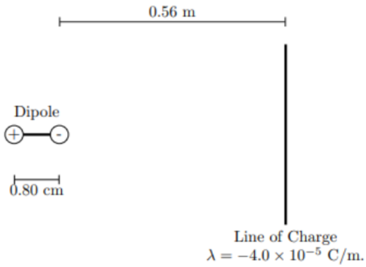
$$\boxed{C = \frac{4\pi \varepsilon_0 ab}{(b-a)}}$$

- this works when you don't have dielectrics

if you do have a dielectric you can replace the ε_0 with $\varepsilon_0 K$

$$\boxed{C = \frac{4\pi \varepsilon_0 K ab}{(b-a)}}$$

Problem 13 (25 points): An electron dipole consisting of a $+2.5 \times 10^{-6}$ C charge and a -2.5×10^{-6} C charge separated by a fixed distance $d = 0.80$ cm is placed such that the positive charge is a distance of 0.56 m away from an infinite line of charge with constant linear charge density $\lambda = -4.0 \times 10^{-5}$ C/m, as shown in the picture below. (Note: the figure is not drawn to scale!)



a). (15 points) What is the net force on the electric dipole? Please specify both the magnitude and the direction of the net force.

* this is a tricky one if you don't know what you are doing because you were not required to apply mirror images the only time you need to apply mirror images: when you have a conducting surface thats grounded or at potential 0V (or anything else)

$$E = \frac{\lambda}{2\pi r \varepsilon_0} \implies F = qE$$

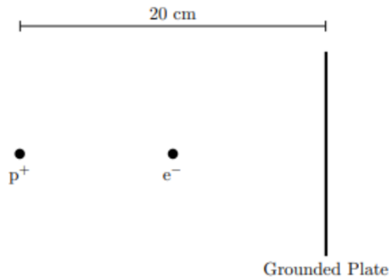
$$F = q_1 \frac{\lambda}{2\pi r \varepsilon_0} + q_2 \frac{\lambda}{2\pi r \varepsilon_0}$$

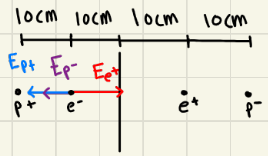
$$F_{net} = (2.5\times10^{-6})\left(\frac{-4.0\times10^{-5}}{2\pi\varepsilon_0(0.56)}\right) + (-2.5\times10^{-6})\left(\frac{-4.5\times10^{-5}}{2\pi\varepsilon_0(0.568)}\right)$$

$$-3.612 + 3.56 = \boxed{-0.052}$$

In the following two examples, we will use the method of images to solve problems involving infinitely large, grounded conducting plates.

1. A proton is located 20 cm from an infinite grounded conducting plate. An electron is then placed midway between the plate and the proton, as shown in the picture below.



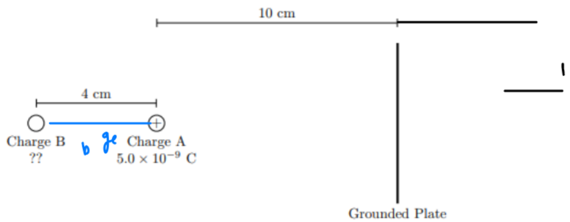


$$E_{total} = |E_{e^+}| - |E_{p^-}| + |E_{p^+}|$$

$$\frac{1.6\times10^{-19}}{4\pi\varepsilon_0(0.2)^2} - \frac{1.6\times10^{-19}}{4\pi\varepsilon_0(0.3)^2} + \frac{1.6\times10^{-19}}{4\pi\varepsilon_0(0.1)^2}$$

$$= \boxed{1.24\times10^{-7}\ N/C}$$

2. Charge A, a positive point charge of magnitude 5.0×10^{-9} C charge is placed 10 cm from an infinite grounded conducting plate. Charge B, a point charge of unknown magnitude and sign, is then placed 4 cm away from charge A, in the opposite direction as the infinite plate, as shown in the picture below. As a result, the net electrostatic force on the first charge is zero.



the key is they told you the net electrostatic Force is zero by mirroring we already the Positive charge becomes negitive but For it to be electrostaticly zero charge b would need to be zero so the mirror image would be positive

Find charge b?

we will Find charge b by using the first charge (A) has net electrostatic OF zero

$$E_{net} = E_- - E_{\text{charge } b} - E_{\oplus} = 0$$

• notice we got the electric of charge b and image of charge b they will have the same q

$$E_- = E_{\text{charge } b} + E_{\oplus}$$

$$\frac{5.0 \times 10^{-9}}{4\pi\varepsilon_0(0.2)^2} = \frac{q}{4\pi\varepsilon_0(0.04)^2} + \frac{q}{4\pi\varepsilon_0(0.24)}$$

$$\frac{5.0 \times 10^{-9}}{4\pi\varepsilon_0(0.2)^2} = q\left(\frac{1}{4\pi\varepsilon_0(0.04)^2} + \frac{1}{4\pi\varepsilon_0(0.24)^2}\right)$$

$$1123 = q(5.78 \times 10^{12})$$

$$\boxed{q = 1.95 \times 10^{-10}\ C}$$