Work & Energy

$\boxed{W_{net} = F_{net} \times d}$ used when (F_{net}) constant

$\boxed{W_{net} = \int F_{net} \cdot dx} \Rightarrow \boxed{W_{net} = \frac{1}{2} m (\Delta v)^2 = \Delta K}$

$\boxed{W_{net} = W_g = -mg\Delta y = -\Delta U_g}$

$\boxed{F = -\frac{dU}{dx}}$ it means that objects experience a force that pushes them toward regions of lower potential energy...

$\boxed{\Delta E = \Delta K + \Delta U = 0}$

$\boxed{W_{net} = W_g = -\Delta U = \Delta K}$

$\boxed{E_{total} = K + U = \text{constant}}$

* ((Energy conserved)) ((no change of energy))
* Unit: $kg \cdot m^2/s^2$ = Joule
* $U \leq K$
* K = always positive (velocity squared)

$\Delta E = \Delta K + \Delta U = W_{nc}$

$W_{nc} < 0$ ... negative, meaning that energy was released from the system in form of thermal energy (heat)...

Hooke's Law & Spring's Potential Energy

negative sign implies that the force of spring is opposite to the direction of the displaced object...

$\boxed{F_{Hooke} = -k(x - x_0)}$ equilibrium length: position where the spring is not streched nor compressed...

F_{Hooke}: restoring force

k: spring constant

x: position of object

$\boxed{F_{Hooke} = -\frac{dU_{Hooke}}{dx}} \xrightarrow[F_{Hooke}\text{ Substitution}]{\text{Via integration}} \boxed{U_{Hooke} = \frac{1}{2} k (x - x_0)^2 + C}$

Our universe approximately behaves like a spring when it's near the minimum potential energy
which also means energy is ((Conserved))

$$\boxed{U(x) \approx \frac{1}{2} k (x - x_0)^2, \quad \text{where } k = U''(x_0) \dots}$$

Combining Springs

Opposing connected spring:

$$\boxed{k_{(eff)} = k_1 + k_2}$$

where $k_{(eff)} > k_1$ & k_2

Directly connected spring:

$$\boxed{k_{(eff)} = \frac{k_1 . k_2}{k_1 + k_2}}$$

where $k_{(eff)} < k_1$ & k_2

Simple Harmonic Oscillator (SHO)

$x(t) = A.\cos(\omega t + \phi) + x_0$ or $x(t) = A.\sin(\omega t + \phi) + x_0$

$\boxed{v(t) = -\omega x(t)}$ * (($v_m = x_m = KE_m$)) m: maximized

$\boxed{a(t) = -\omega^2 x(t)}$ * (($a_m = x_m = U_m = F_m$))

$\boxed{\omega = \sqrt{\frac{k}{m}}}$ $\boxed{T = \frac{2\pi}{\omega}}$ $\boxed{f = \frac{\omega}{2\pi}}$ $\boxed{f = \frac{1}{T}}$ (Hz)

$E_{total} = \frac{1}{2}.k.A^2 \Rightarrow \boxed{U + KE = \frac{1}{2}.k.A^2}$

$\boxed{\omega = \sqrt{\frac{g}{l}}}$

$$\boxed{\begin{aligned} \theta(t) &= A.\cos(\omega t.\phi) + \theta_0 \\ r(t) &= A.l.\cos(\omega t.\phi) + r_0 \end{aligned}}$$

$$\boxed{v(t) = -l.\omega.\theta \text{ or } -\omega.r}$$

$$a(t) = -l.\omega^2.\theta \text{ or } -\omega^2 r$$

Damped & Driven Harmonic Oscillator

$$\frac{d^2x}{dt^2} + \left(\beta \frac{dx}{dt} + \omega_0^2 x = 0\right)$$

equation for exponential decay

$$a + \beta V + \omega_0^2 x = 0$$

$$\delta = \frac{\beta}{2\omega_0}$$

$$Q = \frac{1}{2\delta} = \frac{\omega_0}{\beta}$$

$$\frac{d^2x}{dt^2} + \beta.\frac{dx}{dt} + \omega_0^2 x = \sin(\omega_D)$$

$$x(t) = A\sin(\omega_D t) + B\cos(\omega_D t)$$

$$\sqrt{A^2 + B^2} = \frac{1}{\sqrt{(\omega_0^2 - \omega_D^2)^2 + \beta^2 \omega_D^2}}$$

(Total Amplitude)

$$x(t) = A_0 e^{-\frac{\beta}{2}t} \cos(\omega t + \phi_0) + x_0$$

$$A(t) = A_0 e^{-\frac{\beta}{2}t}$$

$$V(t) = -\left(\frac{\beta}{2} + \omega\right).x(t)$$

$$a(t) = \left(\frac{\beta^2}{4} - \omega^2 + \beta\omega\right).x(t)$$

$$\omega = \sqrt{\omega_0^2 - \frac{\beta^2}{4}}$$

Momentum & Collision

$$\boxed{\vec{P} = m.\vec{V}}$$

$$\boxed{\Delta P = \int F_{net} \, . \, dt}$$

$$\boxed{P_{01} + P_{02} = P_{f1} + P_{f2}}$$

$$\boxed{F_{net} = \frac{d\vec{P}}{dt}}$$

$$\boxed{KE_{01} + KE_{02} = KE_{f1} + KE_{f2}}$$

$$\boxed{KE = \frac{P^2}{2m}}$$

$$\boxed{V_{f2} = \frac{m_1 (V_{01} - V_{f1})}{m_2}}$$

$$\boxed{(m_1 + m_2) V_{f1}^2 - 2 m_1 V_{01} V_{f1} + (m_1 - m_2) V_{01}^2 = 0}$$

$$\boxed{V_{f2} = \frac{2 m_1}{m_1 + m_2} . V_{01}}$$

$$\boxed{V_{f1} = \frac{m_1 - m_2}{m_1 + m_2} . V_{01}}$$

Elastic Collisions

$V_{02} = 0\ m/s$

Momentum in 2D & Explosions

x - Component

$$\boxed{P_{01,x} + P_{02,x} = P_{f1,x} + P_{f2,x}}$$

y - component

$$\boxed{P_{01,y} + P_{02,y} = P_{f1,y} + P_{f2,y}}$$

$$\boxed{P_{01} + P_{02} = P_{system}}$$

$$\left[\begin{array}{l} P_{01,x} + P_{02,x} = P_{system\,(x)} \\ P_{01,y} + P_{02,y} = P_{system\,(y)} \end{array}\right\} \text{Inelastic Collisions...}$$

$$\boxed{P_{system} = P_{f1} + P_{f+2}}$$

Explosions...

Thermal Energy, Heat, & Power

$$\boxed{\begin{aligned}\Delta E_{sys} &= \Delta KE + \Delta U + \Delta E_{th} \\ &= Q - W\end{aligned}}$$

$$\boxed{P = \frac{dE}{dt} = \frac{Q}{t}}$$

unit : watt = J/s

$$\boxed{Q = C \cdot \Delta T}$$

$$\boxed{Q = m \cdot \Delta H}$$

$$\boxed{P = k \cdot A \cdot \frac{T_1 - T_2}{h}}$$

((Insulator))

mole

$$C = n \cdot \boxed{C_n} \Rightarrow \text{specific heat}$$

$$C = m \cdot \boxed{C_m}$$

mass

$$\boxed{Q_{sys} + Q_{surrounding} = 0} \Rightarrow \text{Thermal Equilibrium...}$$