Hooke's Law

negative sign implies that the force of spring is opposite to the direction of the displaced object...

$$\boxed{F_{Hooke} = -k\,(x - x_0)}$$

equilibrium length: position where the spring is not streched nor compressed...

restoring force

spring constant

position of object

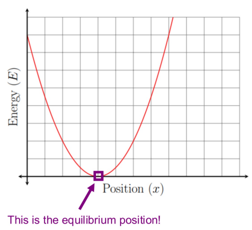
Since the force of the spring depends only on position, then the force can also be expressed as:

$$\boxed{F_{Hooke} = \frac{-dU_{Hooke}}{dx}} \xrightarrow[F_{Hooke}\text{ Substitution}]{\text{Via integration}} \boxed{U_{Hooke} = \frac{1}{2}k\,(x - x_0)^2 + C}$$

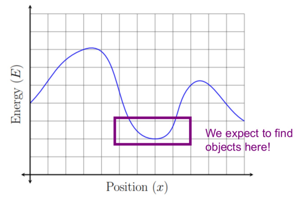
The universality of Hooke's law:

As a result of ($F = -\frac{dU}{dx}$), Our universe favors regions of lower potential energy & tend to settle in positions where potential energy is minimized reaching equilibrium state ...

Hooke's Law Pictured



The Universality of Hooke's Law



Since $F = -\frac{dU}{dx}$, the universe favors smaller U.

Based on ((Taylors expansion)), we can approximat any well behaved function around a point as follow:

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(x_0)}{n!} \cdot (x - x_0)^n}$$

, when we plug in the potential energy function of any object... we get:

$$U(x) = U(x_0) + \boxed{U'(x_0) \cdot (x - x_0)} + \frac{1}{2} U''(x_0) \cdot (x - x_0)^2 + \dots$$

((physically meaningless)) because we can choose $U(x_0)$ to be equal to Zero

Since we are looking at the behavior of an object near the minimum potential energy, then ($U'(x_0) = 0$) ((Horizontal tangent line))

this would be a good approximation for the potential energy of an object near the minimum potential energy...

Conclusion: Since [$U(x) \approx \frac{1}{2} U''(x_0) \cdot (x - x_0)^2$] looks like ($U_{Hooke}$) !!, then:

- Our universe approximately behaves like a spring when it's near the minimum potential energy which also means energy is ((Conserved))
- $U(x) \approx \frac{1}{2} k (x - x_0)^2$, where $k = U''(x_0)$ ...

Combining Springs

Opposing connected spring:

$$\boxed{k_{(eff)} = k_1 + k_2}$$

where $k_{(eff)} > k_1$ & k_2

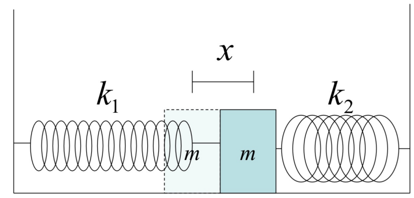
Directly connected spring:

$$\boxed{k_{(eff)} = \frac{k_1 \cdot k_2}{k_1 + k_2}}$$

where $k_{(eff)} < k_1$ & k_2

Combining Springs

- Consider the following situation:



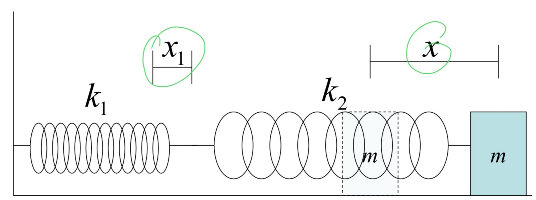
$F_1 = -k_1 x$

$F_2 = -k_2 x$

$F_T = -k_1 x - k_2 x$
$= -(k_1 + k_2)x$

Combining Springs

- Now consider this situation:



Why (F_2) has $(x - x_1)$?!
because both springs are being streched
& we only want spring 2 force on mass

$F_1 = -k_1 x_1$ $\quad F_2$

$F_2 = -k_2 \boxed{(x - x_1)}$

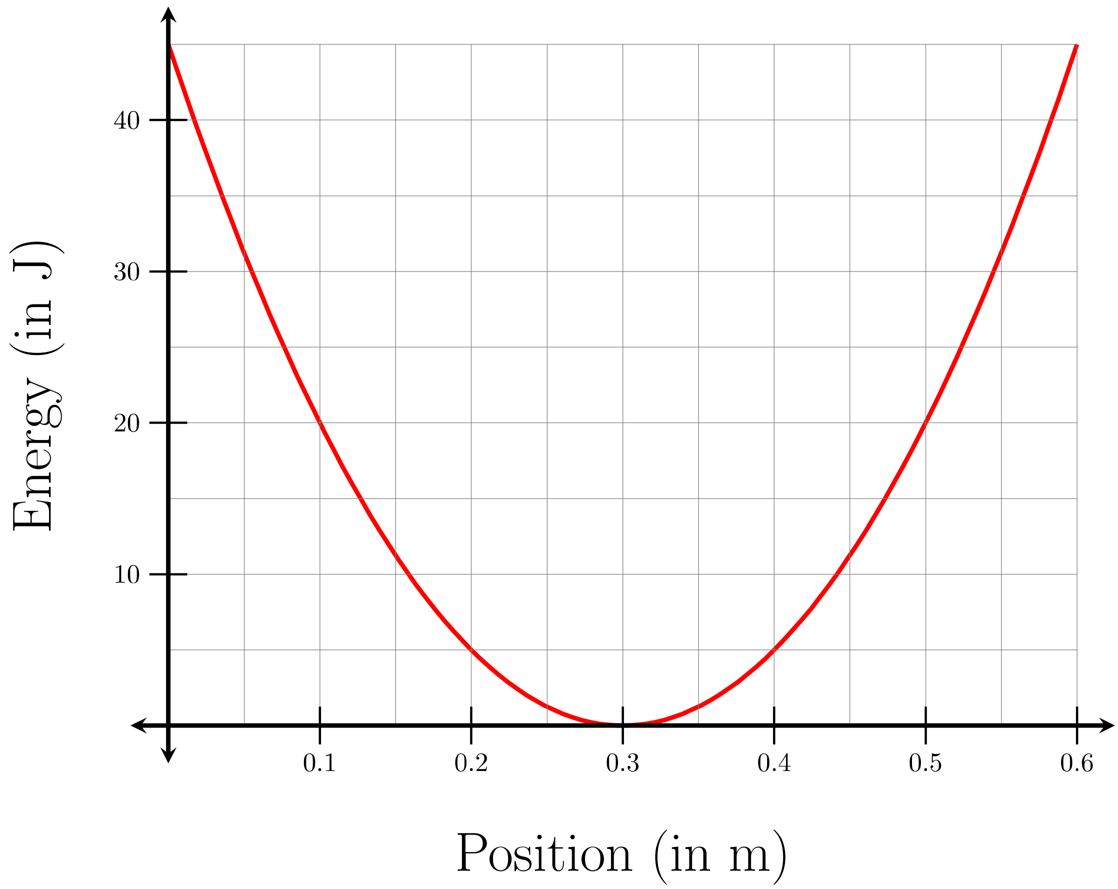
2.2 Springs and Spring-Like Potential Energies

2.2.1 A Spring Potential Energy

As we learned in lecture, the potential energy of a spring is given by

$$U = \frac{1}{2}k\left(x - x_0\right)^2 \tag{2.4}$$

where k is the spring constant, x_0 the equilibrium position of the spring, and x the length of the spring. In the graph below, I have plotted U as a function of x, the position of a mass m attached to a particular spring.



1. What is x_0, the equilibrium length of the spring?

2. If the mass is at position $x = 20$ cm, what is the potential energy of the spring, U? Use this and the equation for U given above to calculate the spring constant, k, of the spring pictured.

3. The "spring" depicted in the graph above is actually composed of two springs, one with $k_1 = 3000$ N/m. What is k_2, the spring constant of the other spring? Are the two springs both attached to the mass, as in the first example of combining springs from lecture, or are the two springs attached to each other, as in the second example? Explain.

4. What direction does the Hooke's Law force point at position $x = 0.05$ m? What about at position $x = 0.4$ m? Draw approximate scaled vectors depicting these forces on the graph above?

5. Imagine now that the mass and spring system is given a total energy of 20 J.

   (a) On the graph above, draw a line corresponding to this total energy. Label this line E_{tot}.

   (b) Assuming total mechanical energy is conserved, what is the kinetic energy at position $x = 0.2$ m? What about at position $x = 0.3$ m?

   (c) We call positions where $K = 0$ *turning points*. Given a total energy of 20 J, where are the turning points located on the graph above? Draw dots at each of these locations on the graph. (Hint: if $K = 0$, how are U and E_{tot} related?) How many turning points are there?

(d) Find the turning point located farthest to the right (that is, at largest x). When it is at this location, what is the mass's speed v? What direction does the force point? What direction would you therefore expect the mass to travel in if time were to continue forward?

(e) Find the turning point located farthest to the left (that is, at smallest x). When it is at this location, what is the mass's speed v? What direction does the force point? What direction would you therefore expect the mass to travel in if time were to continue forward?

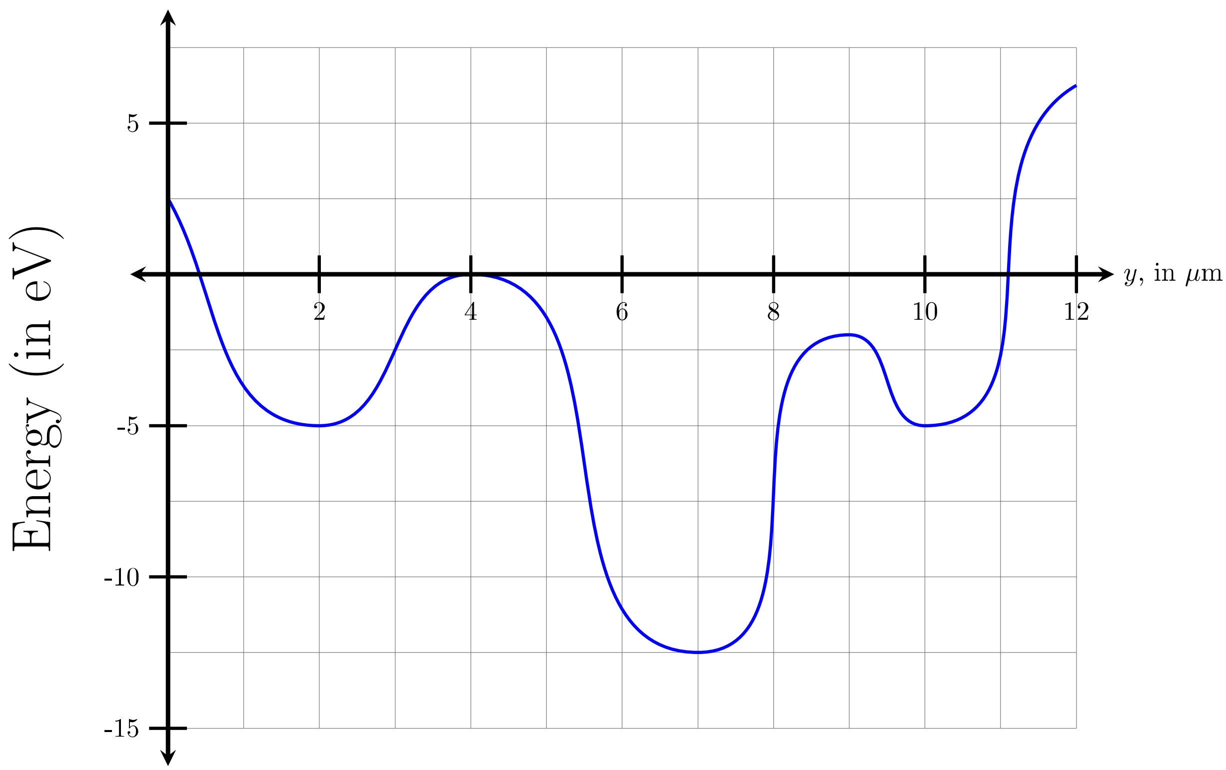
(f) Use your answers to (d) and (e) to describe in words the motion of the mass and spring system with 20 J of total energy as a function of time.

(g) If the the system has 20 J of total energy, could the mass ever be at position $x = 0.6$ m? (Hint: what would the kinetic energy have to be at that point? Is this a valid kinetic energy?) If yes, how fast would the mass be traveling at this point? If not, how much energy would the mass and spring system require in order to reach position $x = 0.6$ m?

(h) On the energy vs. position graph on the first page, sketch a graph of K as a function of x assuming a total energy of 20 J. What is the domain of K, given your answer to (g) above?

2.2.2 A More Complicated Potential Energy

In lecture, we learned that around the minima of complicated potential energy functions, the Hooke's law potential still gave us a good approximation of the object's behavior. The graph below shows the potential energy U as a function of position y experienced by a particular electron due to the presence of a number of nearby charged particles. (Note: the electronvolt, or eV, is a unit of energy in particle physics, defined such that 1 eV $\approx 1.6 \times 10^{-19}$ J. One μm, or micrometer, is equal to 1×10^{-6} m.)



1. Order from least to greatest the strengths of the forces experienced by the electron at positions $y = 3\mu$m, $y = 8\mu$m, and $y = 10\mu$m.

2. Consider the situation where the electron has -10 eV of total energy.

   (a) Draw a total energy curve on the graph on the previous page corresponding to $E_{tot} = -10$ eV. Label this E_{tot}.

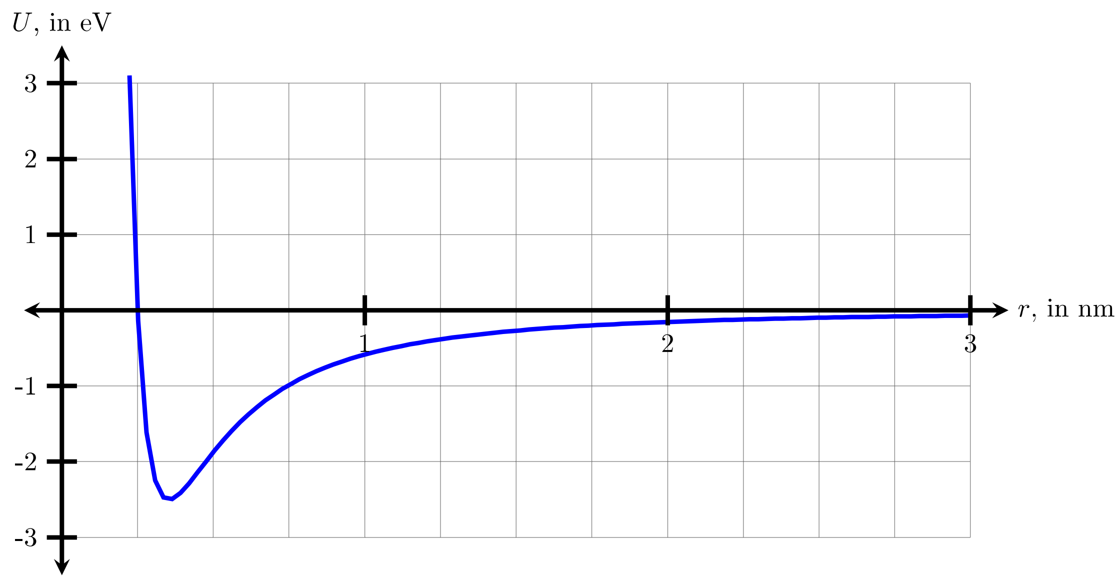
   (b) How many turning points are there in this case? Estimate the allowed range of y values for the electron's location.

   (c) On the graph on the last page, sketch a picture of the kinetic energy K as a function of the electron's position y. What is the maximum K that the electron can have, in eV? At what position will the electron have this K?

3. Now consider the situation where the electron has total energy $E_{tot} = -2.5$ eV.
   (a) How many possible turning points are there in this case?
   (b) If the electron starts at position $y = 2\mu$m, qualitatively describe its motion. What is, approximately, the maximum and minimum positions the electron reaches?
   (c) Now answer the previous question for the case that the electron starts at position $y = 6\mu$m.
   (d) If the electron starts at position $y = 10\mu$m with -2.5 eV of total energy, could it ever reach position $y = 7\mu$m, assuming the physics that you know?
   (e) It turns out quantum mechanics actually changes things here due to a phenomenon called "quantum tunneling" which allows particles to use their quantum mechanical wave-like natures to "tunnel" through energetically forbidden regions. Using your phone or some other similar device, look up quantum tunneling online. How cool is this concept? Are you excited to study quantum mechanics in some future physics class?

2.2.3 A Generalized Interaction Potential

The graph below gives the general shape of the potential energy of two molecules based on their separation distance, r. and therefore gives us a rough picture of a chemical bond. Note that 1 eV $\approx 1.601 \times 10^{-19}$ J is a unit of energy, and 1 nm $= 1 \times 10^{-9}$ m.



1. Consider the situation where E_{tot} = -2 eV. Sketch both $E_{tot}(x)$ and $K(x)$ on the curve above for this situation. How many turning points are there?

2. How does your $K(x)$ curve drawn above compare to the $K(x)$ curve you drew in activity #2 for the mass and spring system? Are they qualitatively similar? Would you say that modeling the chemical bond as a spring connecting the two bonded molecules is a good approximation in this case?

3. Now consider the situation where $E_{tot} = -0.5$ eV. How well does the model of the chemical bond as a spring do now?

4. We can think of increasing the E_{tot} as corresponding to increasing the *thermal energy* of the system, and therefore to increasing the temperature of the system. As the temperature increases, what happens to the molecule-molecule bond? Does this match what your physical intuition tells you? What state of matter corresponds to the case where E_{tot} is negative? What state of matter corresponds to positive E_{tot}?

$\boxed{2.2.1}$

$\boxed{1}$ $\boxed{0.3 \text{ m}}$

$\boxed{2}$ $5 J$, $U = \frac{1}{2} k (x - x_0)^2$

$$5 = \frac{1}{2} k (0.2 - 0.3)^2$$

$$\boxed{k = 1000 \ J/m^2}$$

$\boxed{3}$ since the value of $k_1 = 3000$ N/m & $k_{eff} = 100$ N/m,

then $k_{eff} = \dfrac{k_1 k_2}{k_1 + k_2}$ where it match it lower value than k_1 ...

((k can never be negative))

$$1000 = \frac{3000 k_2}{3000 + k_2} \Rightarrow 3 \times 10^6 + 1000 k_2 = 3000 k_2$$

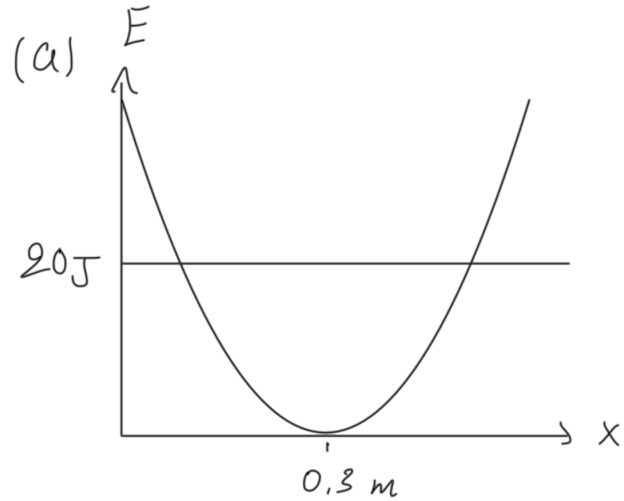
$$k_2 = \frac{3 \times 10^6}{2000} = \boxed{1500 \ N/m}$$

$\boxed{4}$ $\xrightarrow{0.05}$ $F = 250N$ $\xleftarrow{0.04}$ $F = 100N$

$$F = U'(x)$$
$$= k(x - x_0)$$
$$= \boxed{1000 (x - x_0)}$$

$\boxed{5}$ $E_{total} = 20\,J$

(a)



$\boxed{b}$ $E_{total} = K + U$

$U(0.2) = 5$

$U(0.3) = 0$

$20 = K + 5 \Rightarrow \boxed{K(0.2) = 15\,J}$

$20 = K + 0 \Rightarrow \boxed{K(0.3) = 20\,J}$

$\boxed{c}$ $20 = 0 + U \Rightarrow \boxed{U = 20\,J} \Rightarrow \boxed{x = 0.1m \ \& \ 0.5m}$ turning point...

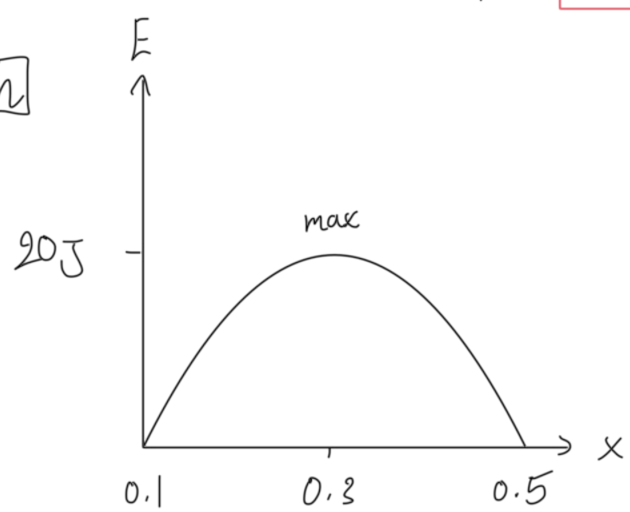
$\boxed{d}$ Since $K = 0$, then $V = 0\,m/s$, slope positive $\Rightarrow F = -$ slope $=$ Left $(-x)$

$\boxed{E}$ Since $K = 0$, then $V = 0\,m/s$, slope negative $\Rightarrow F = -$ slope $=$ Right $(+x)$

$\boxed{f}$ It would go back and forth like a zigzag until it reaches equilibrium

$\boxed{g}$ No, E_{total} should equal $\boxed{45\,J}$

$\boxed{h}$

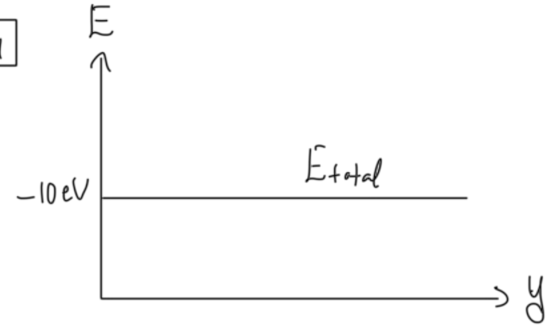


$\boxed{2.2.2}$ $1\ eV \approx 1.6 \times 10^{-19}$ Joule & $\mu m = 1 \times 10^{-6}\ m$

$\boxed{1}$ $\boxed{10\ \mu m,\ 3\ \mu m,\ 8\ \mu m}$

$\boxed{2}$

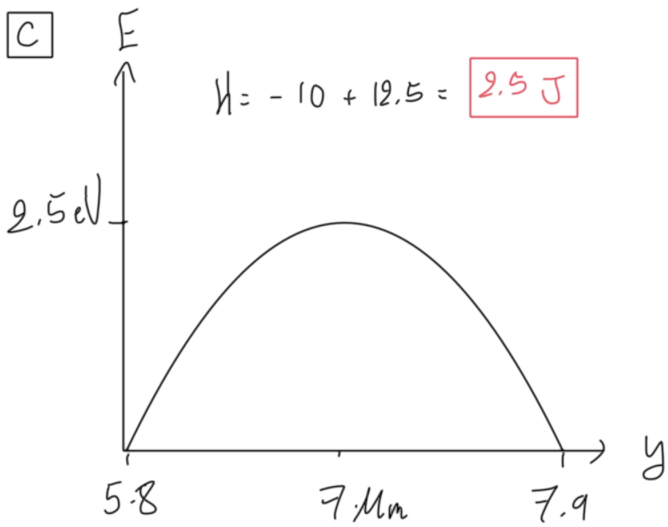
$\boxed{a}$



$\boxed{b}$ 2 turning points, $\boxed{y = 5.8,\ 7.9}$

$\boxed{c}$

$K = -10 + 12.5 = \boxed{2.5\ J}$



Potential energy & total energy has no meaning in the sign of energy because when we do integration (C) cancels out so we can choose any position to be our $U = 0$

$\boxed{3}$ $E_{total} = -2.5\ eV$

$\boxed{a}$ $y = (0.8,\ 3,\ 5.2,\ 8.4,\ 9.2,\ 11)\ \mu m$

$\boxed{b}$ $y : (0.8,\ 3)\ \mu m$

$\boxed{c}$ $y = (5.2,\ 8.4)\ \mu m$

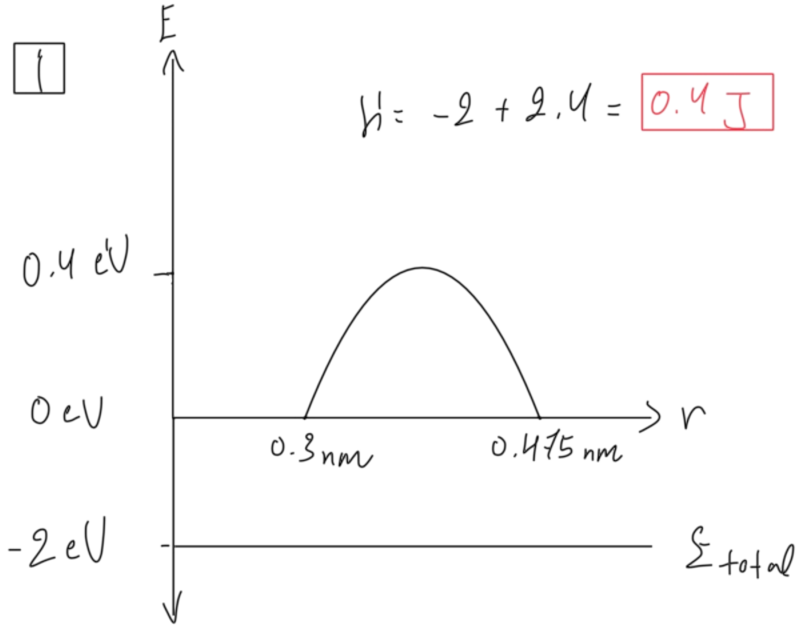
$\boxed{d}$ $y \neq 7$ because K can never be negative

$\boxed{e}$ IdK about that

$\boxed{2.2.3}$ $eV \approx 1.601 \times 10^{-19} J$ & $nm = 1 \times 10^{-9} m$

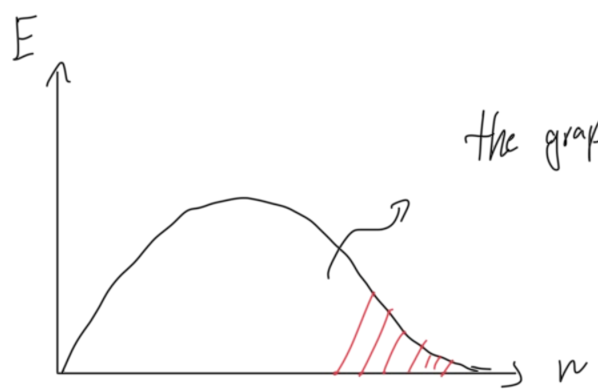
$\boxed{1}$

$K = -2 + 2.4 = \boxed{0.4 J}$



$\boxed{2}$ It has the same graph. It would be a good approximation...

$\boxed{3}$ It would not be a good approximation:



the graph doesn't match a spring

$\boxed{4}$ As energy is absorbed by the system, E_{total} is positive & the Temp increases along with the kinetic energy until the bond between molecule break...

E_{total} = negative (Solid)

E_{total} = positive (gas)