Vectors

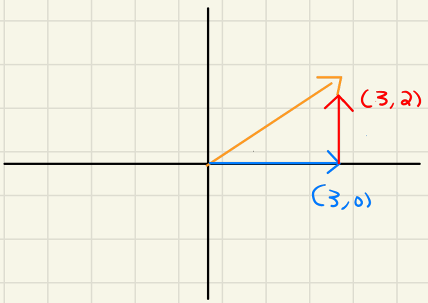
What qualifies as a vector?

you need to be fimilar with two things Vector quanties and Scalar quantities

a Scalar quantity has magnitude but No direction
example of scalar quantity is temperture it has magnitude (80°F) but it does not have direction
you cannot say the temperture is 80°F north

a Vector quantity has Magnitude and direction
example of a Vector quantity is Force it has magnitude (10n) and it also has direction (left)
you can say a Force of 10n is being applied to the right side of the car

Vector's on graph



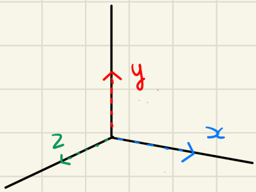
- there are many way's to repsent a vector you can write it like this [3,2], [3,0]

or you can use $\hat{i}$ and $\hat{j}$ the
$\hat{i}$ repsents movement in the x-direction
$\hat{j}$ repsents movement in the y-direction

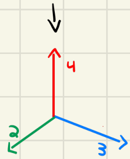
So you can write it as:

$(3,2) = 3\hat{i} + 2\hat{j}$
$(3,0) = 3\hat{i} + 0\hat{j}$

We can also do this in three dimensions (3d) imagine it like the corners of your room



[3, 4, 2]



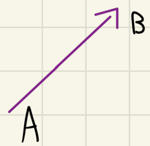
- in vector form

$3\hat{i} + 4\hat{j} + 2\hat{k}$

to indicate something is a vector we place "$\longrightarrow$" on top of it

$$\vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}$$

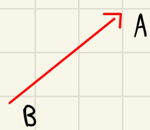
lets say we had this vector we label its head "B" and its tail "A"



We name it:

$\overrightarrow{AB}$

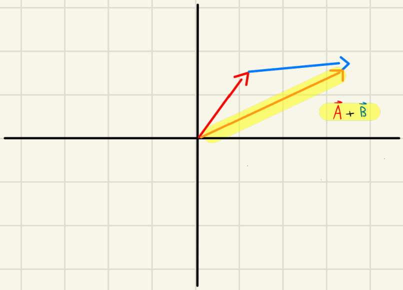
lets say we had this vector we label its head "A" and its tail "B"



We name it:

$\overrightarrow{BA}$

Basic Vector operations



$\vec{A} = 2\hat{i} + 3\hat{i}$

$\vec{B} = 4\hat{i} + 3\hat{i}$

$\vec{A} + \vec{B} = (2\hat{i} + 4\hat{i}) + (3\hat{i} + \hat{i})$

$\vec{A} + \vec{B} = 6\hat{i} + 4\hat{i}$

What is Scaling a Vector? Scaling a Vector means to double it

example:

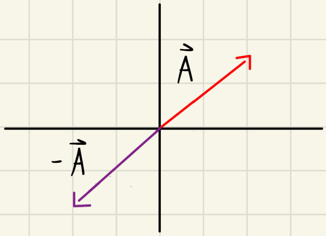
$\vec{A} = 2\hat{i} + 3\hat{i} \implies 2 \cdot \vec{A} = 2\hat{i} + 3\hat{i}$

the answer: $\vec{A} = 4\hat{i} + 6\hat{j}$

- a big number Stretches a Vector
- a small number Shrinks it

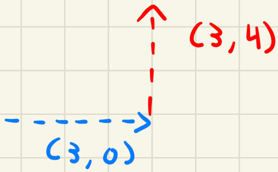
if you multiply a vector by a negitive number the Vector Flips in the oppisite direction

example:



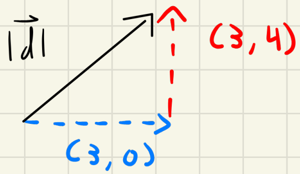
$-1 \cdot \vec{A} = -2\hat{i} - 3\hat{j}$

Calculating the Magnitude



to Find the magnitude you us the Pythagreom Formula With the distance Formula

$$|\vec{d}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



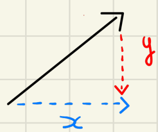
or you can use the regular Pythagreom Formula if you are given the amount of x and y

$$|\vec{d}| = \sqrt{x^2 + y^2}$$

We represent the magnitude Vector with $|\vec{d}|$ can be any letter

breaking vectors into components

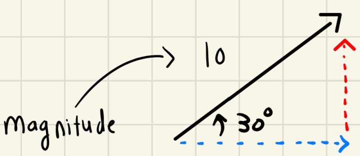
You can break a vector into x and y components



- $x = \cos(\theta)|\vec{F}|$
- $y = \sin(\theta)|\vec{F}|$

→ equations For breaking Vector into its x and y components

- examples:



$x = \cos(30) \cdot |10|$

$y = \sin(30) \cdot |10|$

Dot Product

the dot Product is Scalar its Just a number
but has No direction

to find their dot Product

$\vec{A} = 10\hat{i} + 2\hat{j}$
$\vec{B} = 5\hat{i} + 10\hat{j}$

$\vec{A}\cdot\vec{B} = (\text{multiply all } \hat{i}) + (\text{multiply } \hat{j})$

$\vec{A}\cdot\vec{B} = (10\cdot 5)\hat{i} + (2\cdot 10)\hat{j}$

$\vec{A}\cdot\vec{B} = 70$

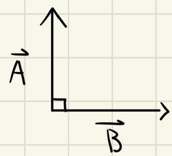
if we extend this to three dimensions

$\vec{A} = 10\hat{i} + 2\hat{j} + 2\hat{k}$
$\vec{B} = 5\hat{i} + 10\hat{j} + 2k$

$\vec{A}\cdot\vec{B} = (\text{multiply all } \hat{i}) + (\text{multiply } \hat{j}) + (\text{mutiply } k)$
$\vec{A}\cdot\vec{B} = (10\cdot 5)\hat{i} + (2\cdot 10)\hat{j} + (2\cdot 2)\hat{k}$

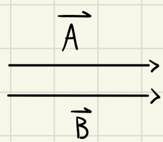
$\vec{A}\cdot\vec{B} = 74$

the dot Prodot For Perdenicular Vectors is Zero



$\vec{A}\cdot\vec{B} = 0$

the dot Product For Parallel Vector is the Product of their magnitude



$\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|$

Properties of dot Vectors

- **Commutativity**
  - which means the order of multiplication does not matter

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

- **Distributivity**
  - which means you can distribute

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

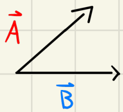
- **Dot Product with itself**
  - multipiying a Vector with itself we are squaring it

$$\vec{A} \cdot \vec{A} = |\vec{A}|^2$$

- **Perpendicular Vectors**
  - if the Vectors are perpendicular meaning they form a right angle the dot product is always zero

$$\vec{A} \cdot \vec{B} = 0$$

Finding the angel between two Vectors



$\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{B} = 3\hat{i} + 2\hat{j}$

$\vec{A} \cdot \vec{B} = (3 \cdot 2)\hat{i} + (3 \cdot 2)\hat{j}$

You do not need to include "$\hat{i}$" and "$\hat{j}$" I did that to show how to do it but you would just add them

$\vec{A} \cdot \vec{B} = 12$

$$\left.\begin{aligned} \vec{A} \cdot \vec{B} &= |\vec{A}| \cdot |\vec{B}| \cdot \cos(\theta) \\ \vec{A} \cdot \vec{B} &= (\text{multiply all } \hat{i}) + (\text{multiply } \hat{j}) \end{aligned}\right\} \longrightarrow \text{these two are the same}$$

$|\vec{A}| = \sqrt{(2\hat{i})^2 + (3\hat{j})^2} = \sqrt{13}$

$|\vec{B}| = \sqrt{(3\hat{i})^2 + (2\hat{j})^2} = \sqrt{13}$

You do not need to include "$\hat{i}$" and "$\hat{j}$" I did that to show how to do it

$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos(\theta)$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$$

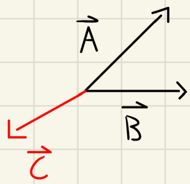
→ get the angel or what the $\cos\theta$ is equal to

$$\cos(\theta) = \frac{12}{\sqrt{13} \cdot \sqrt{13}} = \boxed{\frac{12}{13}}$$

Cross Product

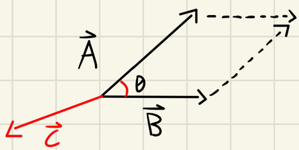
the cross product is a vector it has magnitude and direction

$$\vec{C} = \vec{A} \times \vec{B}$$



- the cross product of two vectors $\vec{A}$ and $\vec{B}$ results in another vector that is perpendicular to both of the originals

the magnitude or length of this vector is equal to the area of the parallelogram



$$|\vec{C}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin(\theta)$$

note: the cross product itself is a vector not an area but its magnitude gives the area

the direction of the resulting vector $\vec{C}$ can be by using the right hand rule