angular momentum

refresher: angular means "rotation" things with angular motion are moving in a circular Path but the object does not need to be round

- linear momentum

$\vec{P} = m\vec{v}$

- angular momentum

$\vec{L} = I\vec{\omega}$

I = moment of interia
ω = angular velocity

torque (τ)

torque is the change in angular momentum

$$\tau = \frac{d\vec{L}}{dt}$$

rotational second law

$$\tau = I\alpha + \frac{dI}{dt}\cdot\omega$$

- this is a more accurate version of $\tau = I\alpha$

- look at the next Page for a explanation of the equation

angular momentum conservation

angular momentum is conserved if the net torque is zero

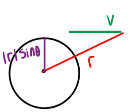
$$\tau_{net} = 0 \Rightarrow \frac{dL}{dt} = 0$$

- remember that τ is $\frac{dL}{dt} \approx \frac{\Delta L}{\Delta t}$ but when $\frac{dL}{dt} = 0$ there is No change in angular momentum overtime so it just becomes $L_{Final} - L_{initial}$
- the relationship between moment of interia and angular velocity is an inverse relationship meaning when one increase the other one decreases this typically happens when No external force (τ) is acting on the system
- when you decrease moment of interia (like pulling your arms in) your angular velocity increases and vice versa this occur when there is No external external torque

Point mass angular momentum

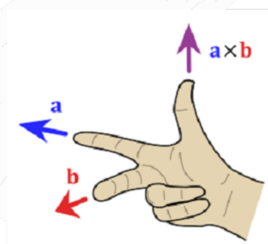
$\vec{L} = \vec{r} \times \vec{P} \Rightarrow |\vec{L}| = |\vec{r}|\sin\theta|\vec{P}|$

this works even for a straight-line motion as long as your looking at the angular momentum relative to a point or axis which means you looking at it from a point thats not changing



- r: (radius) from the pivot to velocity
- v: (velocity) will mostly be an object that has mass meaning it can be treated as momentum due to it having velocity and mass

$$\vec{L} = \vec{r} \times \vec{P} \approx> |\vec{L}| = |\vec{r}|\sin\theta|\vec{P}|$$



- thumb ($\vec{L}$) (total)
- index ($\vec{r}$) (radius)
- middle ($\vec{P}$) (Potential)

$$\tau = I \cdot \alpha + \frac{dI}{dt} \cdot \omega$$

this equation is used when the Moment of interia is changing overtime

$I\alpha$ = Changing spin speed

$\frac{dI}{dt} \cdot \omega$ = how the distribution of mass is changing with time when there is angular velocity

★ when r (radius) is changing the moment of interia is also changing

- this equation has a momentum of interia of a disk but the same thing gets done for other shapes

ex:

$$I_{disk} = \frac{1}{2} m R^2 \xRightarrow{\text{derivative}} \frac{dI}{dt}(disk) = m \cdot R \cdot \frac{dR}{dt}$$

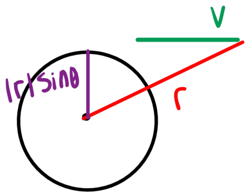
$$\tau = I \cdot \alpha + \frac{dI}{dt} \cdot \omega$$

this tells you how fast the radius is changing overtime its unit are in (m/s)

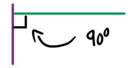
$$\tau = I\alpha + \left(m \cdot R \cdot \frac{dR}{dt}\right) \cdot \omega$$

that is just the radius (m)

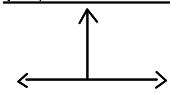
mass is a constant the only thing that is changing is R



$|\vec{r}| \sin\theta$ = represents perpendicular distance from the axis of rotation to the line of motion of the object



Perpendicular lines



- the two lines are perpendicular lines they are at a angle of 90° when you plug in 90° into $\sin\theta$ you get $\sin(90°) = 1$

Statics

- a system is in static equalibrium if is neither moving or changing how its moving
  - this means that

$$\vec{F} = 0$$

$$\vec{\tau} = 0$$

- For the system to be zero it has to be torque going in and a torque going out

$$\tau_{in} = \tau_{out}$$

- r is measured from the axis of rotation (which is often the pivot) to where the force is applied

- when you see uniform density it is the center of mass which is the gravity location so if the object has uniform density then the center of mass is the location where gravity acts

Summary of equations

$$\vec{L} = I\vec{\omega}$$

$$\tau = \frac{d\vec{L}}{dt}$$

$$\tau = I\alpha + \frac{dI}{dt}\cdot\omega$$

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow |\vec{L}| = |\vec{r}| \sin\theta |\vec{p}|$$

$$\tau_{net} = 0 \Rightarrow \frac{dL}{dt} = 0 \Rightarrow L_{final} - L_{inital}$$