

# Thermal Energy

## ① Thermal Energy

- ↳ is the internal energy associated with an object due to its temperature ...
- ↳ When  $E_{th} = 0 \text{ J} \Rightarrow T = 0 \text{ Kelvin}$
- ↳  $E_{th} \propto T \dots$

## ② Heat ( $Q$ )

- ↳ the process of transferring energy between 2 objects due to differences in temperature ...
- ↳ Methods of heat transfer:
  - ↳ Conduction: Direct contact
  - ↳ Convection: Bulk movement of fluid (example) ↴
    - Hotter air away from heating source  $\Rightarrow$  cools down  $\Rightarrow$  colder air near heating source  $\Rightarrow$  heats up
    - (less density) (higher density)
  - ↳ Radiation: emission of light spectrum (photons)
- ↳ Conservation of energy:

$$\Delta E_{sys} = \Delta HE + \Delta U + \Delta E_{th}$$
$$= Q - W$$

$\downarrow + \Rightarrow$  work done by the system ...

$\downarrow - \Rightarrow$  work done on the system ...

↳ Sign:

$\downarrow + \Rightarrow$  Heat absorbed by the system ...

$\downarrow - \Rightarrow$  Heat released from the system ...

↳ Unit  $\Rightarrow \text{HJ or J}$

$$Q = C \cdot \Delta T$$

↳ Heat capacity [ $C = \frac{n}{m} \cdot C_n$   $\Rightarrow$  specific heat]

$$C = \frac{m}{M} \cdot C_m$$

↳ Phase Transition:

$\downarrow \Delta H_{\text{vaporization}}$

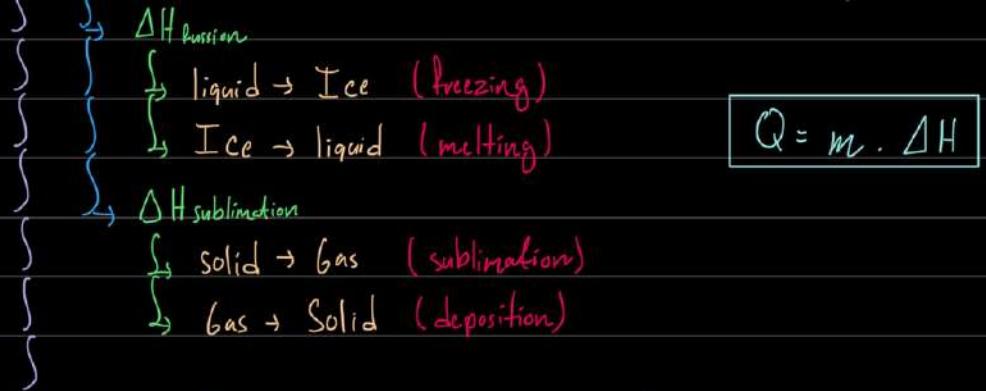
$\downarrow \text{liquid} \rightarrow \text{gas} \text{ (Evaporation)}$

$\downarrow \text{gas} \rightarrow \text{liquid} \text{ (Condensation)}$

\* Pwles (Heat transfer during phase transition)

$\downarrow$  no change in  $T$  (constant) ...

$\downarrow$  change in bond energy ...



$$Q = m \cdot \Delta H$$

Process of Heat transfer (Increase of temperature example) :



Thermal Equilibrium : ((colder)) ((Hotter))

$Q_{\text{sys}} + Q_{\text{surrounding}} = 0$  amount of energy absorbed = amount of energy released...  
 Both system & surrounding must have the same Final temperature ...

### (3) Power

is the rate at which energy is transferred

$$P = \frac{dE}{dt} = \frac{Q}{t} \quad \text{unit: Watt} = \text{J/s}$$

$$P = h \cdot A \cdot \frac{T_1 - T_2}{\text{h}} \quad (\text{Insulation})$$

thickness : How much resistance does the material provide for heat flow ?!

Area : is the surface through which heat is transferred ...

Thermal conductivity : How well does the material conduct heat ??

## Thermal Energy, Heat, & Power

$$\Delta E_{sys} = \Delta HE + \Delta U + \Delta E_{in}$$
$$= Q - W$$

$$P = \frac{dE}{dt} = \frac{Q}{t}$$

unit : watt = J/s

$$Q = C \cdot \Delta T$$

$$Q = m \cdot \Delta H$$

$$P = h \cdot A \cdot \frac{T_1 - T_2}{\textcircled{h}}$$

((Insulation))

$$C = \frac{n}{m} \cdot [C_n] \Rightarrow \text{specific heat}$$

$$C = \frac{m}{m} \cdot [C_m]$$

$$Q_{sys} + Q_{surrounding} = 0$$

$\Rightarrow$  Thermal Equilibrium...

## 2.7 Heat, Temperature, and Power

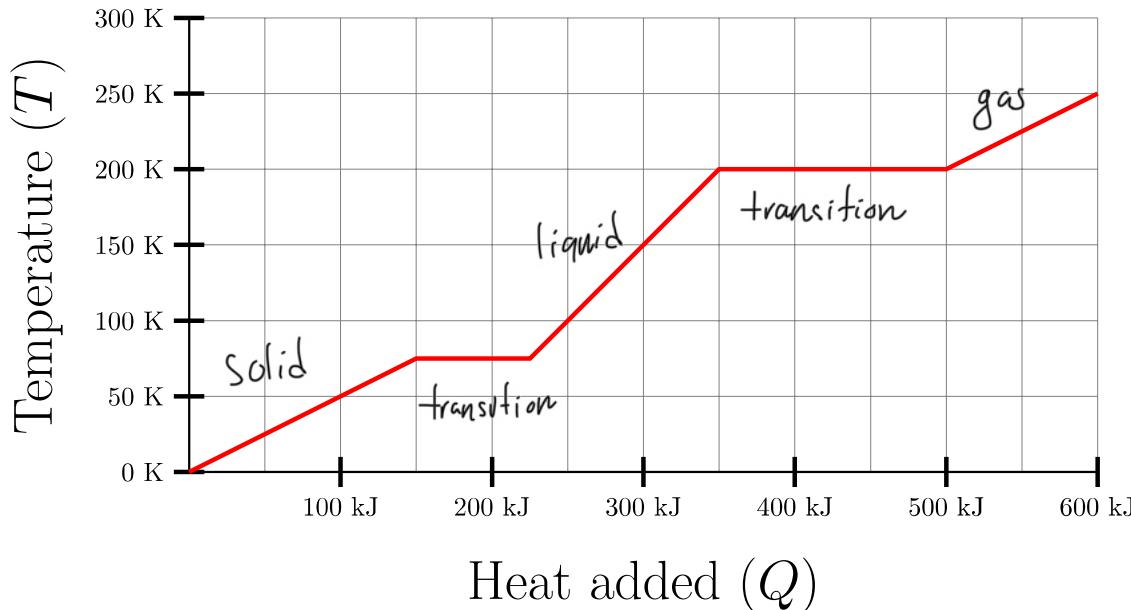
### 2.7.1 A Mixture Problem

A 50 g ice cube is taken from a freezer with temperature of  $-10^{\circ}\text{C}$  and dropped into a 250 g glass of water at  $30^{\circ}\text{C}$ . Assume the glass is fully insulated and exchanges no heat with the surroundings.

1. How much energy is required to increase the temperature of the ice to  $0^{\circ}\text{C}$ , the melting point of ice?
2. Since the energy needed in question 1 must come from the water, what is the new temperature of the water after the ice have reached  $0^{\circ}\text{C}$ ?
3. How much energy is now required to melt the entire 50 g of ice? Is this energy greater or less than the amount of energy released when the 250 g of water cools the rest of the way to  $0^{\circ}\text{C}$  (at which point the system would be in thermal equilibrium). Based on your answer, is the final state of the system in thermal equilibrium a mixed phase state at  $0^{\circ}\text{C}$ , or a pure liquid state at some temperature greater than  $0^{\circ}\text{C}$ ?
4. After transferring enough energy to melt all ice, what is the new temperature of the 250 g of water?
5. Now use that, in thermal equilibrium the temperature of all parts of the system must be the same to calculate the final temperature of the glass of water when it reaches thermal equilibrium.

### 2.7.2 Three-Phase Diagrams

A *three phase diagram* offers a convenient way of depicting heat capacities and phase change enthalpies. To make a three phase diagram, we plot the temperature  $T$  of a particular amount of substance as a function of  $Q$ , the heat added. An example is shown below:



In the picture above, the curve shows the behavior of 2 kg of an unknown substance as heat is added.

1. Identify and label any phase transitions as Solid  $\rightarrow$  Liquid, Liquid  $\rightarrow$  Gas, or Solid  $\rightarrow$  Gas. What is the melting point of the unknown substance? What is the boiling point of the unknown substance?
2. What is the specific enthalpy of vaporization  $\Delta H_{vap}$  of the unknown substance? (Hint: how much of the substance is the curve above depicting?) What about the specific enthalpy of fusion of the unknown substance?
3. Notice that, in the space between the phase transitions, the substance is in a pure phase. On the graph above, label each of these pure phases as Solid, Liquid, or Gas. At room temperature, what phase is this substance in? (You might have to look up what room temperature is in Kelvin.)
4. What is the heat capacity of this sample of the unknown substance when it is in its liquid phase? What is the mass specific heat of the unknown substance in its liquid phase?

$$2.7.1 \quad m_i = 0.050 \text{ kg} \quad T = -10^\circ \quad m_w = 0.250$$

$$1 \quad q = m c \Delta T$$

$$= 0.050 \times 2.09 \times (0 + 10)$$

$$= 1.045 \text{ J}$$

$$2 \quad q = m c \Delta T$$

$$-1.045 = 0.250 \times 4.186 \times (T - 30)$$

$$T = 29 \text{ } ^\circ \text{C}$$

$$3 \quad q_{\text{fus}} = m \Delta H_{\text{fus}}$$

$$= 0.050 \times 333.6$$

$$= 16.68 \text{ J}$$

$$* \quad q_w = m c \Delta T$$

$$= 0.250 \times 4.186 (0 - 29)$$

$$= -30.3485 \text{ J}$$

$q_w$  to reach zero is higher than  $q_{\text{fus}}$ . Therefore, the ice we definitely melt... If the water wasn't enough to melt ice, then we use this energy to calculate how much mass of cube ice was melted...

$$4 \quad q_w = m c \Delta T$$

$$-16.68 = 0.250 \times 4.186 \times (T - 29)$$

$$T = 13.06 \text{ } ^\circ \text{C}$$

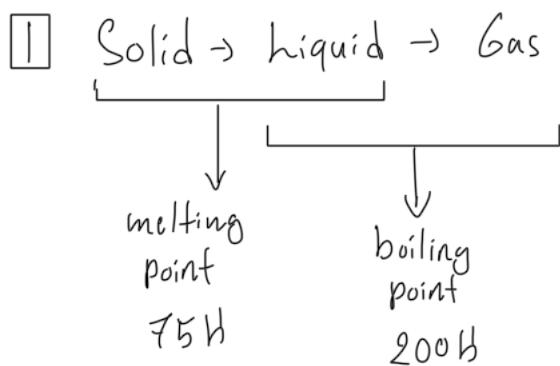
$$5 \quad q_w + q_c = 0$$

$$0.050 \times 4.186 \times (T - 0) + 0.250 \times 4.186 \times (T - 13.06) = 0$$

$$0.050 T + 0.250 T = 3.265$$

$$T = 10.883 \text{ } ^\circ \text{C}$$

2.7.2



2  $Q_{\text{vap}} = m \Delta H_{\text{vap}}$

$$150 = 2 \Delta H_{\text{vap}}$$

$$\Delta H_{\text{vap}} = 75 \text{ kJ/kg}$$

4  $Q_L = C \Delta T$

$$125 = C (200 - 75)$$

$$C = 1 \text{ kJ/kg}$$

$$m C_{\text{mL}} = 1$$

$$2 C_{\text{mL}} = 1$$

$$C_{\text{mL}} = 0.5 \text{ kJ/kg}$$

\*  $Q_S = m C_m \Delta T$

$$150 = 2 C_{\text{mS}} (75)$$

$$C_{\text{mS}} = 1 \text{ kJ/kg}$$

$$Q_{\text{fus}} = 2 \Delta H_{\text{fus}}$$

$$75 = 2 \Delta H_{\text{fus}}$$

$$\Delta H_{\text{fus}} = 37.5 \text{ kJ/kg}$$

5  $Q_S = m C \Delta T$

$$= 4 \times 1 \times (75 - 50)$$

$$= 100 \text{ kJ}$$

$$Q_{\text{fus}} = m \Delta H_{\text{fus}}$$

$$= 4 \times 37.5$$

$$= 150 \text{ kJ}$$

$$Q_h = m C \Delta T$$

$$= 4 \times 0.5 \times (200 - 75)$$

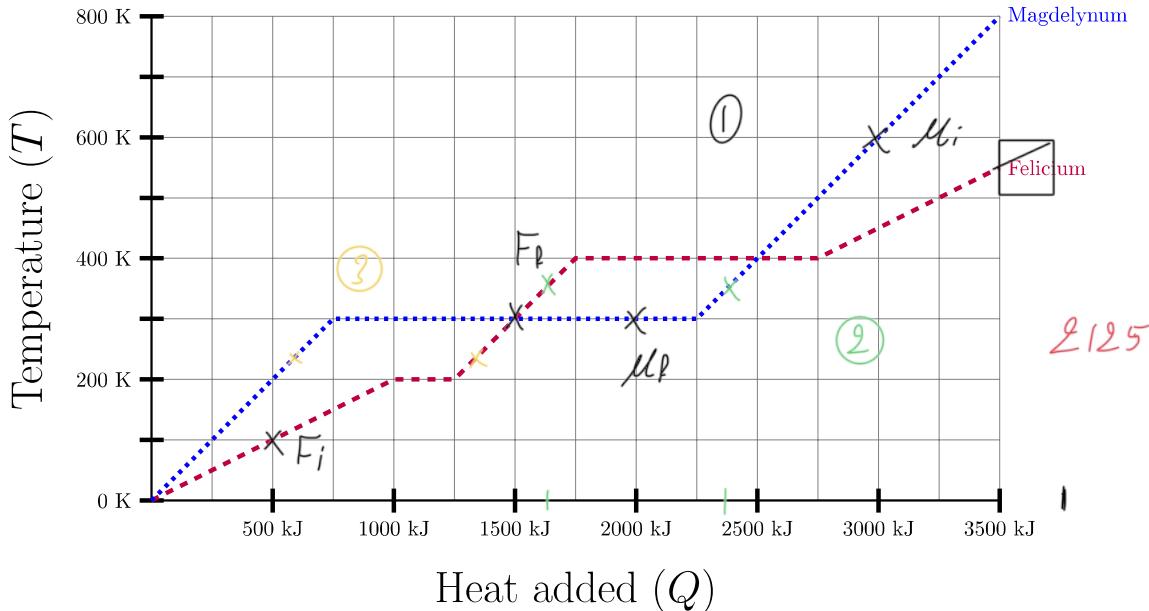
$$= 250$$

Instead of doing this look at the graph and multiply heat by 2 since mass increased by 2

- How many kilojoules of heat must be added to **4 kg** of the substance to increase the temperature from 50 K to 200 K?
- Lillian has 2 kg of the unknown substance at a temperature of 100 K. If she adds 150 kJ of heat, what is the final temperature of the substance? What phase will the substance be in after the 150 kJ of heat are added? Will it be in a pure phase or a mixed phase? If the substance is in a mixed phase, what two phases are mixed? About how much of the 2 kg total mass is in each phase?

### 2.7.3 A Double Phase Diagram

Now consider the diagram below, which shows (in a dotted line) the behaviors of one kilogram samples of two substances: Felicium and Magdelynium. Note that Magdelynium does not have a liquid phase at atmospheric pressure.



- 1 kg of Magdelynium at a temperature of 600 K is mixed with 1 kg of Felicium at a temperature of 100 K in an insulated container.
  - Locate the initial states of both the samples on the three phase diagram. Label the Felicium starting point  $F_i$  and the Magdelynium starting point  $M_i$
  - By stepping each system forward or backward one grid line at a time, identify the final states of both the Felicium and the Magdelynium. Label these as  $F_f$  and  $M_f$ . Hint: the system is in thermal equilibrium when the two substances are at the same temperature, not necessarily where the two three phase diagrams intersect.
  - What is the final temperature of the system?
  - What is the final state of the Felicium? What is the final state of the Magdelynium? If either substance is in a mixed state, what fraction of the substance is in each phase?
  - How much heat is exchanged between the Magdelynium and the Felicium in coming to thermal equilibrium?
  - How would your answers to (c), (d), and (e) change if, instead of 1 kg of each substance, we mixed together 2 kg of each substance at the same starting temperatures?
- If the initial temperatures in problem one were reversed (that is, if the 1 kg of Magdelynium were at 100 K and the 1 kg of Felicium was at 600 K), what would the final temperature of the system be when it reached thermal equilibrium? What would be the final states of both substances? (Hint: notice you will have to extrapolate the diagram!)
- Imagine now that I mix **2 kg** of solid Magdelynium at 100 K with **1 kg** of liquid Felicium at 300 K in an insulated container. What is the temperature when the system reaches equilibrium? What is the final state of both substances?
- Consider the sample of 1 kg of Felicium at 300 kg from the last problem. Assuming the Felicium has no thermal energy and no bond energy at 0 K, how much thermal energy does it have at 300 K? How much bond energy does it have at 300 K? How much heat does it have at 300 K? Hint: one of the three is a trick question!

6 200 K, liquid & gas, mixed phases,  $\frac{2}{3}$  liquid,  $\frac{1}{3}$  gas

2.7.3

1 Both samples should reach thermal equilibrium, meaning

$$Q_{\text{absorbed}} = -Q_{\text{released}}$$

Also both sample should end up with the same final temperature...

c)  $M_f \rightarrow T = 300K$

$F_f \rightarrow T = 300K$

d)  $M \rightarrow \text{solid} \& \text{gas} \dots \frac{1}{6} \& \frac{5}{6}$

$F \rightarrow \text{liquid}$

e) 1000 kJ

f) c & d same only the amount of heat exchanged increased to double

2000 kJ

2  $T_f = 350K$ ,  $M \rightarrow \text{gas}$   
 $F \rightarrow \text{liquid}$

3

$$2Q_m = Q_F$$

$$2 \text{ block} = 1 \text{ block}$$

$T_f = 233 \text{ Kelvin}$ ,  $M \rightarrow \text{solid}$

$$1.33 = 0.66$$

$F \rightarrow \text{liquid}$

4 Thermal energy = 1500 kJ Heat = 1500 kJ

Bond = 0 kJ

Since it began with zero then there is no bond to break

## 2.7.4 Hot Coffee

A closed thermos containing 500 g of hot coffee, with an initial temperature of 60°C, is left out on a counter in a room with an ambient air temperature of 25°C. The thermos is approximately cylindrical, with a height of 14 cm and a radius of 4.5 cm, and is made from a 0.75 cm thick insulating material with thermal conductivity  $k \approx 0.068 \text{ W/m}\cdot\text{K}$ . Assume coffee has the same specific heat as water.

1. What is the final temperature of the coffee after it has reached thermal equilibrium?
2. In order to reach thermal equilibrium, how much energy must be exchanged between the coffee and the air? Does heat flow from the air to the coffee, or from the coffee to the air?
3. Initially (i.e., when the coffee is still at 60°C), how much heat per unit time is being transferred through the walls of the thermos?
- Temperature not constant* 4. You might naively expect we could get the time it would take the thermos to reach thermal equilibrium by dividing our answer in question 2 by our answer in question 3, following the example that we did in the example problem in our lecture slides. Explain briefly while this method will not work in this situation!
5. To develop a more accurate model for how the coffee cools, we start by observing that a differential transfer of heat  $dQ$  must correspond to a differential temperature change  $dT$  such that

$$dQ = -CdT \quad (2.15)$$

where  $C$  is the heat capacity of the coffee. Why must this negative sign be included?

6. Now use our equation for the power

$$P = \frac{dQ}{dt} = \frac{kA}{L}(T - T_f) \quad (2.16)$$

and our equation for  $dQ$  given in equation 2.15 to write a differential equation of the form

$$\frac{dT}{dt} = -\lambda(T - T_f) \quad (2.17)$$

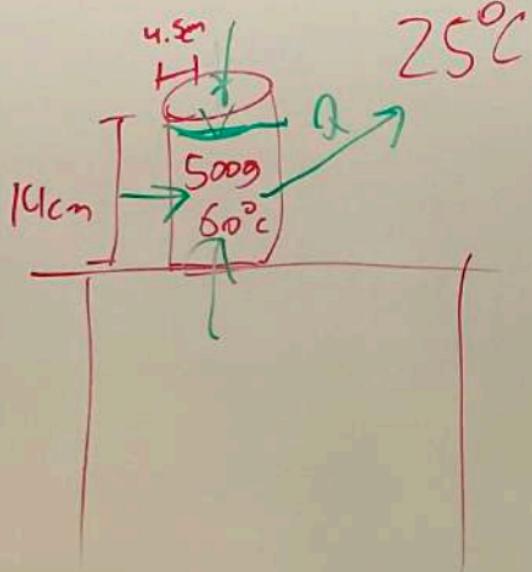
for the temperature. What is the appropriate constant  $\lambda$  for this problem? What units does this  $\lambda$  (which is a lower case Greek letter lambda) have?

7. The differential equation form in 2.17 is well known in physics and chemistry; it is the differential equation for an exponential decay to a nonzero asymptotic value. Its solution is

$$T(t) = Ae^{-\lambda t} + C \quad (2.18)$$

for  $A$  an arbitrary constant. Plug this solution form into your version of equation 2.17 and show that it solves the differential equation, provided a specific value is chosen for  $C$ . What is the correct value for  $C$  that will work to solve your differential equation?

8. Notice since the temperature function you have found for  $T(t)$  has a horizontal asymptote at the equilibrium temperature, the coffee never actually reaches room temperature (though it does get arbitrarily close). How much time does it take the coffee to get within 0.5° C of the equilibrium value?
9. Another way to parameterize an exponential decay is the *half-life*; that is, how long it takes for the function to decay halfway to its equilibrium value. What is the half-life for this coffee thermos?



$$1). \quad 25^\circ\text{C}$$

$$2). \quad Q = m \cdot c \cdot (\Delta T)$$

$$Q = (0.5 \text{ kg}) (4.186 \frac{\text{J}}{\text{kg}\cdot\text{K}}) (35^\circ) =$$

$$3). \quad P = \frac{Q}{A \cdot \Delta T}$$

$$0.0075 \text{ m}^2$$

$$= ?$$

$$2\pi r L + 2\pi r^2$$

$$(0.045)(0.14 \text{ m}) \quad (0.045)$$

$$\approx 0.0523 \text{ m}^2$$

5).

$$dQ = -C dT$$

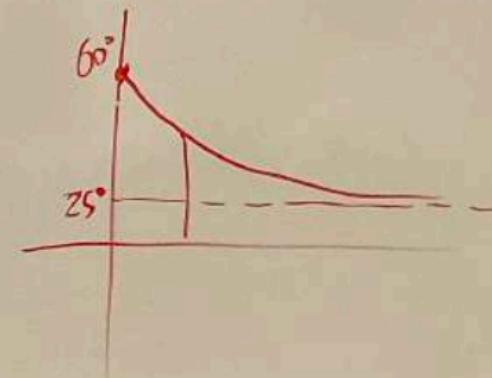
$$P = -\frac{dT}{dt} = \frac{kA}{L}(T_i - T_f)$$

$$\Rightarrow \frac{dT}{dt} = -\frac{kA}{LC}(T - T_f)$$

$$T(t) = Be^{-\lambda t} + D$$

~~$$\frac{dT}{dt} = -\lambda e^{-\lambda t} = -\lambda(Be^{-\lambda t} + D - T_e)$$~~

$$T(t) = 35e^{-\lambda t} + 25^\circ$$



$$9. \quad 175^\circ - 35^\circ e^{-\lambda t \frac{1}{2}} + 25^\circ$$

$$\ln\left(\frac{1}{2}\right) = k\left(-\lambda t \frac{1}{2}\right)$$

$$+\ln 2 = +\lambda t \frac{1}{2}$$

$$\frac{\ln 2}{\lambda} = +\frac{1}{2}$$

$$\frac{\ln 2 L C}{K A} = +\frac{1}{2}$$