

Poynting Vectors

describes the Flow of energy in electric and magnetic Fields

$$\alpha = \sqrt{\frac{\mu \epsilon \omega^2}{2}} \sqrt{1 + \sqrt{1 + D^2}}$$

- α (alpha) is the attenuation constant it measures how fast an electromagnetic wave amplitude decays as it travels through material

$$\beta = \sqrt{\frac{\mu \epsilon \omega^2}{2}} \sqrt{-1 + \sqrt{1 + D^2}}$$

- β (Beta) controls how fast the phase changes it also controls wavelength inside the material

$$D = \frac{\sigma}{\omega \epsilon}$$

- D measures how conductive a material is compared how capacitive it is

if $D \ll 1 \rightarrow$ good insulator

if $D \gg 1 \rightarrow$ good conductor

in the case of good Dielectric, D is small because the conductivity σ must be small for an insulator

$$\alpha = \sqrt{\mu \epsilon \omega^2}$$

$$\beta = \frac{D \sqrt{\mu \epsilon \omega^2}}{2}$$

For good conductor D is very large, as σ is large

$$\alpha = \sqrt{\frac{\mu \epsilon \omega^2 D}{2}}$$

$$\beta = \sqrt{\frac{\mu \epsilon \omega^2}{2}}$$

When dealing with a good conductor alpha and Beta will be the same

$$\vec{S} = \vec{E} \times \vec{H}$$

Poynting Vector electric Field magnetic Field

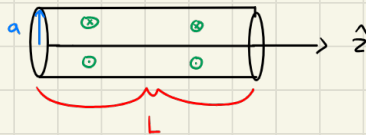
units: W/m^2

if you have surface, the total electromagnetic Power Flowing through it is:

$$P = \iint \vec{S} \cdot d\vec{A}$$

Where $\vec{S} = \vec{E} \times \vec{H}$ and the units for $P = \text{W}$

- Consider a cylindrical wire of radius a , length L , and total resistance R carrying a constant current I . You may assume the material comprising the wire is Ohmic and has dielectric constant K and relative permeability μ_r , and that the current density is uniform within the wire.
 - Assuming the current travels in the \hat{z} direction of a cylindrically polar coordinate system, what is \vec{J} , the current density inside the wire?
 - What is the electric field \vec{E} inside the wire? (Hint: notice you can solve for the conductivity σ from the information given in the problem.)
 - What is the magnetic field \vec{H} as a function of the radial cylindrical coordinate r for the region inside the wire? (Hint: use Ampère's law!)
 - What is the Poynting vector \vec{S} inside the wire? Which direction does \vec{S} point? Does this make sense based on what you know about the flow of energy inside a resistor? (Recall that resistors dissipate electromagnetic energy.)
 - To find the total power dissipated by the resistor, we can find the total flux of the Poynting vector $\Phi_{\vec{S}}$ through the surface of the resistor. Find this total flux of \vec{S} . Does this result make sense to you, based on what you know from unit #1 about resistors?



\vec{B} Field remember that \vec{B} and \vec{H} point in the same direction

a) remember the current density is the current divided by Area $J = \frac{I}{A}$

$$J = \frac{I}{\pi a^2} \hat{z}$$

2) you are told that its Ohmic so by Ohm's law in material form or in microscopic form you can plug in the current density from previous problem and isolate for \vec{E}

$$\vec{J} = \sigma \vec{E} \Rightarrow \frac{I}{\pi a^2} = \sigma E$$

$$\vec{E} = \frac{I}{\pi a^2 \sigma} \hat{z}$$

You can also use: $R = \frac{L}{\sigma A}$ which is Ohm's law for resistance in material form to solve for σ you can use this because you were told it's Ohmic

$$R = \frac{L}{\sigma A} \Rightarrow \sigma = \frac{L}{R A} \Rightarrow \frac{L}{R \pi a^2}$$

$$\vec{E} = \frac{I}{\pi a^2 \sigma} \Rightarrow \frac{I R \pi a^2}{\pi a^2 L} = \frac{I R}{L}$$

$$E = \frac{I R}{L} \hat{z}$$

3) We know the \vec{B} for a cylindrical long wire: $B = \frac{\mu_0 J_r}{2}$
We use this because we will plug in the current density we got

in magnetic material $\mu_0 \rightarrow \mu \rightarrow \mu_r \mu_0$

$$\vec{J} = \frac{I}{\pi a^2} \Rightarrow B = \frac{\mu_0 \mu_r I}{2 \pi a^2} \hat{\theta} \Rightarrow B = \mu_0 \mu_r \vec{H} \Rightarrow$$

$$\vec{H} = \frac{B}{\mu_0 \mu_r}$$

we are in linear material so we can use

$$\vec{H} = \frac{1}{\cancel{\mu_0} \cancel{\mu_r}} \left(\frac{\cancel{\mu_0} \cancel{\mu_r} I}{2 \pi a^2} \right) = \boxed{\vec{H} = \frac{I}{2 \pi a^2} \hat{\theta}}$$

4) the equation for a Poynting Vector is $\vec{S} = \vec{E} \times \vec{H}$ by using the right rule where index = \vec{E} , middle finger = \vec{H} and thumb = \vec{S} we can see that the \vec{S} is going inward if \vec{S} points outward energy is radiated away if \vec{S} is pointing inward energy is flowing inward

$$\vec{S} = \left(\frac{IR}{L} \right) \left(\frac{I}{2\pi a^2} \right) (\hat{z} \times \hat{\phi}) = \boxed{-\frac{I^2 R}{2\pi L a^2} \hat{r}}$$

5) we will need a double integral on calculating in \hat{z} and other in the $\hat{\phi}$ because are losing energy in \hat{z} and $\hat{\phi}$

$$\int_0^L \int_0^{2\pi} -\frac{I^2 R}{2\pi L a^2} r d\phi dz$$

Polar coordinates
so add r

$$= \frac{-2\pi L a^2 I^2 R}{2\pi L a^2} = \boxed{-I^2 R}$$

2. At time $t = 1.06$ seconds, the current through a 800 turn, 0.10 m long solenoid of radius 0.02 m is 4.5 A and decreasing at a rate of 0.5 A/s. As viewed from the front, the current is going around the solenoid coils in the clockwise direction. Feel free to assume that the region inside the solenoid is vacuum.

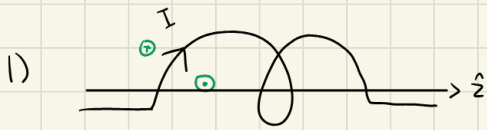
- Sketch a picture of this solenoid, given the qualitative directions of the current, the magnetic field \vec{H} , and the electric field \vec{E} .
- What is the magnetic field \vec{H} inside the solenoid at time $t = 1.06$ seconds? Please use the convention that the \hat{z} direction points outward towards an observer view the solenoid from the front.
- What is the electric field, \vec{E} , as a function of the radial coordinate r for the region inside the solenoid?
- What is the Poynting vector \vec{S} as a function of the radial coordinate r inside the solenoid?
- What is the total flux of the Poynting vector field through the outer surface of the solenoid? Is energy flowing into or out of the solenoid at this moment of time?
- Notice this total flux should correspond to the total power flowing into or out of the inductor. Recall from Class 21 that the energy stored in an inductor was given by

$$U = \frac{1}{2} L I^2 \quad (3.36)$$

Using that the power is the time rate of change of the energy, and that, for a long solenoid

$$L = \frac{\mu_0 \pi R^2 N^2}{l} \quad (3.37)$$

find the total power. Does this answer match the answer you found in part e?



2) remember that \vec{B} For a solenoid is: $B = \frac{\mu_0 N I}{L}$ and for linear material you can:

$$\vec{B} = \mu_0 \mu_r H \Rightarrow H = \frac{\vec{B}}{\mu_0 \mu_r} \Rightarrow \left(\frac{\mu_0 N I}{L} \right) \left(\frac{1}{\mu_0 \mu_r} \right) = \frac{N I}{L}$$

★ We are in vacuum $\mu_r = 1$

$$H = \frac{N I}{L} = \frac{(800)(4.5)}{(0.1)} = \boxed{36000 \text{ A/m}}$$

3) We can use Faradays law because we already know what \vec{B} For a solenoid

Faradays law: $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \Rightarrow \frac{1}{r} \frac{\partial (r E_\theta)}{\partial r} = -\frac{\mu_0 N I}{L}$$

$$\int \frac{\partial (r E_\theta)}{\partial r} = - \int \frac{\mu_0 N I r}{L} dr \Rightarrow r E_\theta = -\frac{\mu_0 N I r^2}{2L}$$

$$E_\theta = -\frac{\mu_0 N I r}{2L} = \frac{(4\pi \times 10^{-7})(800)(-0.5)}{2(0.1)}$$

$$= \boxed{-0.00251 \frac{\text{N}}{\text{Cm}}}$$

4) remember that the Poynting Vector \vec{S} is: $\vec{S} = \vec{E} \times \vec{H}$

$$\vec{S} = (0.0251)(36000)(\hat{\theta} \times \hat{z}) = \boxed{90.5 \text{ W/m}^2}$$

5) the total Flux on a Poynting vector is: $\Phi = \int \vec{S} \cdot d\vec{A}$

$$\int \vec{S} \cdot d\vec{A} \Rightarrow S \int dA = SA \Rightarrow S(2\pi r h)$$

$$(90.5)(2\pi(0.02)(0.1)) = \boxed{1.1 \text{ W}}$$

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