Continous Charge distribution

- two methods

1) SuperPosition
   - will reduce problem to an integral
   - the integral may be unsolvable

2) gauss's law method
   - generally quicker than superPosition
   - require a system of high degree ($\vec{E}$ to be constant)

- the exact Form of ∂Q depends on what Short of charge distribution we have

- For 1D continous distribution (line of charge)

$$\lambda = \frac{dQ}{dx} \Rightarrow dQ = \lambda(x)\,dx$$

- For 2D continous distributions (surFace of Charge)

$$\sigma = \frac{dQ}{dA} \Rightarrow dQ = \sigma(A)\,dA$$

- For 3D Continous distributions

$$\rho = \frac{dQ}{dV} \Rightarrow dQ = \rho(V)\,dV$$

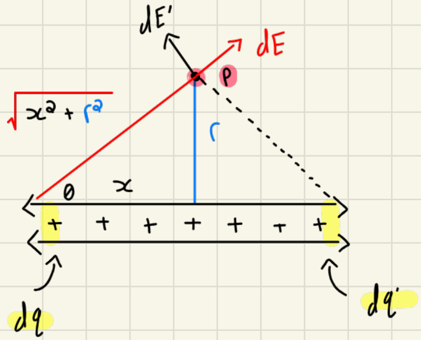
- what is SuperPosition

in Physics SuperPosition means total effect is the sum of indiviual effects

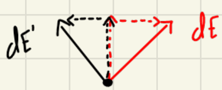
$$E_{total} = E_1 + E_2 + E_3 \ldots\ldots$$

Made with Goodnotes

For example, consider an infinite long line of charge density λ



Zoom into Point P



- they cancel in the x direction but add in the y direction

$$dE_{net} = dE_y \sin\theta$$

$$\left.\begin{aligned} dE_y &= \frac{dq}{4\pi\varepsilon_0 (x^2+r^2)} \\ \sin\theta &= \frac{r}{\sqrt{x^2+r^2}} \end{aligned}\right\}$$

- Plug into dE_{net}

$$dE_{net} = \frac{r\,dQ}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$$

this is a line of charge so $dq = \lambda(x)\,dx$

$$dE_{net} = \frac{r\lambda\,dx}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$$

- this line of charge goes from $-\infty$ to ∞

$$\int dE_{net} = \frac{r\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty} \frac{dx}{(x^2+r^2)^{3/2}}$$

☆ You cannot solve this integral we would need complex analysis but if we look at at a integral table we see that

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+r^2)^{3/2}} = \frac{2}{r^2} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}}$$

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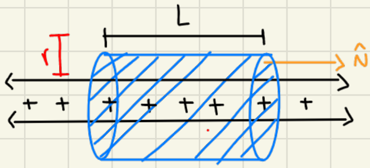
We can also use gauss's law to get the same answer also notice this system has cylinderical symmetry hence the field can only depend on r



$Q_{enclosed} = \lambda L$

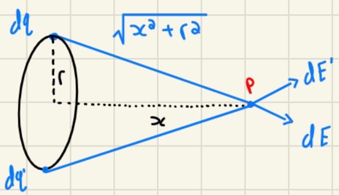
$$\frac{Q_{enclosed}}{\varepsilon_0} = \oint E(r)\, dA$$

Charge density $\dfrac{\lambda L}{\varepsilon_0} = EA \quad \Rightarrow \quad \dfrac{\lambda L}{\varepsilon_0} = E(2\pi r L)$

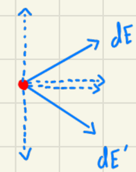
$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

• the $\hat{n}$ is perpendicular to the surface so the angle it makes is 90° the flux equation is $\Phi = EA\cos\theta \Rightarrow EA\cos(90) = 0$ so no field lines are passing through there

• now consider a slender ring of total charge Q and radius r



Zoom in to point P



• the y-component cancel

$\lambda = \dfrac{dQ}{dx} \Rightarrow dQ = \lambda\, dx$

$dE_{net} = dE_x \cos\theta$

• we do not have to add the $E_x \cos\theta$ because we are already integrating

$$dE_x = \frac{\lambda\, dx}{4\pi(x^2+r^2)\varepsilon_0}$$

square root squared

$$dE_{net} = \frac{\lambda\, dx}{4\pi\varepsilon_0(x^2+r^2)} \cdot \left(\frac{x}{\sqrt{x^2+r^2}}\right)$$

$$dE_{net} \quad \frac{x\lambda\, dx}{4\pi\varepsilon_0(x^2+r^2)^{3/2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+r^2}}$$

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✫ but keep in mind that this is a ring and we only want the circumferenc

You need to use a Jacobian factor. What is a Jacobian factor? a Jacobian gives you curvature if you look at the circle it has curvature

this has curvature so you need a Jacobian factor

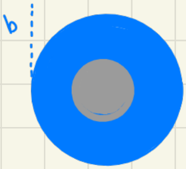
So you need $r\,d\theta$

$$\Rightarrow \frac{x \lambda r \, d\theta}{4\pi \varepsilon_0 (x^2+r^2)^{3/2}}$$

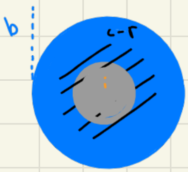
• an because we are going around a circle we will be integrating from 0 to 2π

$$E = \int_0^{2\pi} \frac{x \lambda r \, d\theta}{4\pi \varepsilon_0 (x^2+r^2)^{3/2}} = \boxed{\frac{x\, Q}{4\pi \varepsilon_0 (x^2+r^2)^{3/2}}}$$

Consider a hollow sphere of inner radius a and outer radius b



• in the gray area the $\vec{E}$ is zero



• we will be intgrating from

$$\frac{dQ}{\varepsilon_0} = \oint E \, dA$$

$$\rho = \frac{dQ}{dA} = dQ = \rho \, dA$$

$$\int_{x=a}^{x=r} \frac{\rho \, dA}{\varepsilon_0} dx = \oint E \, dA$$

• we are integrating over area keep in mind this is a hollow sphere

$$\frac{4\pi\rho}{3\varepsilon_0} \left[x^3 \right]_{x=a}^{x=r} = E(4\pi r^2)$$

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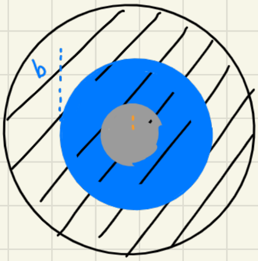
$$\frac{\cancel{4\pi}\rho}{3\varepsilon_0}\left[r^3 - a^3\right] = E(\cancel{4\pi} r^2)$$

$$\boxed{E = \frac{\rho}{3\varepsilon_0 r^2}\left(r^3 - a^3\right)}$$



$$\oint \vec{E}\cdot d\vec{A} = \frac{dQ}{\varepsilon_0}$$

$$dQ = \rho\, dA$$

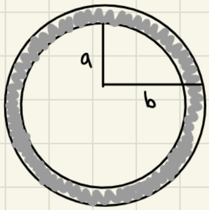
$$\oint \vec{E}\, d\vec{A} = \int_{x=a}^{x=b} \frac{\rho\, dA}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{\rho}{\varepsilon_0}\int_{x=a}^{x=b} 4\pi x^2\, dx$$

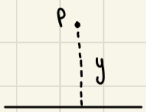
$$E(\cancel{4\pi} r^2) = \frac{\cancel{4\pi}\rho}{3\varepsilon_0}\left[(b)^3 - (a)^3\right]$$

$$\boxed{E = \frac{\rho}{3\varepsilon_0 r^2}\left(b^3 - a^3\right)}$$

More examples



Top View



Side view

Find the electric Field $\vec{E}$

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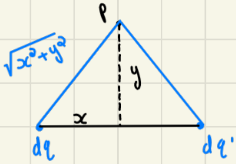
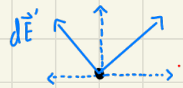


Zoom into Point P



• Notice the x components cancel out

$$dE_{net} = dE_y \sin\theta$$

$$dE_y = \frac{dq}{4\pi r^2 \varepsilon_0} = \frac{dq}{4\pi\varepsilon_0 (x^2+y^2)}$$

the square root goes away because it gets squared

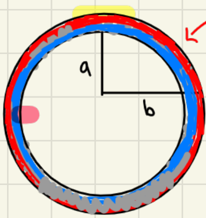
$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$dE_{net} = \left(\frac{dq}{4\pi\varepsilon_0 (x^2+y^2)}\right)\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

$$dE_{net} = \frac{y\,dq}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}}$$

$$\sigma = \frac{dq}{dA} \Rightarrow dq = \sigma\, dA$$

★ because here we are dealing with a ring with circumference we want the charge along that circumfrence so for our Jacobian Factor we use $2\pi x\, dx$



the red can be $2\pi x$ as circumfrence

the blue is change in dx as we can all agree that here and here are Not the same because the x is changing in length

$$dq = 2\pi x\, dx$$

$$dE_{net} = \frac{y\, 2\pi x\, dx}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}} \Rightarrow \int_a^b \frac{y\sigma 2\pi x}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}}\, dx$$

$$\frac{\sigma y}{2\varepsilon_0} \int_a^b \frac{x}{(x^2+y^2)^{3/2}}\, dx$$

★ use u-substitution

$u = x^2+y^2$ — y is constant

$du = 2x \cdot dx$

$\frac{du}{2x} = dx$

$$\frac{\sigma y}{4\varepsilon_0} \int_a^b \frac{\cancel{x}}{(u)^{3/2}} \frac{du}{\cancel{x}}$$

$$-\frac{\sigma y}{2\varepsilon_0} \left[(u)^{-1/2}\right]_a^b$$

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$$-\frac{\sigma y}{2\varepsilon_0}\left[(u)^{-1/2}\right]_a^b$$

$$\boxed{E = \frac{-\sigma y}{2\varepsilon_0}\left[\frac{1}{\sqrt{b^2+y^2}} - \frac{1}{\sqrt{a^2+y^2}}\right]}$$

We can now use the equation we got to modify and get other equation For other Shapes

Consider a disk charge of radius R and uniform charge density σ and a distance d above the center of the disk what is the electric Field

* notice this only has one radius so From the last problem we know that we had two radiuses on ending at the end (b) and one ends a bit more earlier (a) in this we can assume we have a regular radius R like b so $a = 0$ and $b = R$ you can also replace $y = d$

$$\vec{E} = \frac{-\sigma d}{2\varepsilon_0}\left[\frac{1}{\sqrt{R^2+d^2}} - \frac{1}{d}\right]$$

$$\boxed{\vec{E} = \frac{-\sigma d}{2\varepsilon_0\sqrt{R^2+d^2}} + \frac{\sigma}{2\varepsilon_0}}$$

Consider the Case of a infinite Plane of Charge of constant Surface charge density σ Find Find the electric Field a distance z above the Plane you can also use gauss's law

• goes on For infinite

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$$E = \frac{-\sigma y}{2\varepsilon_0}\left[\frac{1}{\sqrt{b^2+y^2}} - \frac{1}{\sqrt{a^2+y^2}}\right]$$

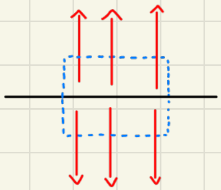
b will go to infinite

- anything thats divided by infinite is 0
- b goes to infinite because if you look back b is the longer radius we treat b as as if its ∞ but a was the shorter radius a infinite sheet has no radius so a=0

$$\vec{E} = -\frac{\sigma y}{2\varepsilon_0}\left[-\frac{1}{y}\right]$$

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}}$$

✩ you can also use gauss law to get the same answer



the black is the infinitely charged surface

gaussian surface

- electric field lines they can only go in a striagnt line due to symmetry if they move in a curved path they break that symmetry

$$\sigma = \frac{dQ}{dA} = dQ = \sigma\, dA$$

$$\oint \vec{E}\, d\vec{A} = \frac{dQ_{enclosed}}{\varepsilon_0}$$

$$\oint E\, dA = \int \frac{\sigma\, dA}{\varepsilon_0}$$

we have two dA or areas that the electric field line is passing through

$$E\, 2\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

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Using the Previous answer Find the electric Potential V a distance Z above a infinite Sheet of uniform charge density σ assume the Potential on the Plane is V_0

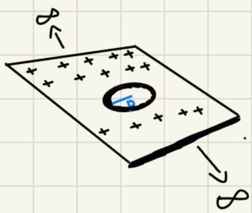
$$\vec{E} = -\nabla V$$

$$\int \frac{\sigma}{2\varepsilon_0}\hat{z} = -\int \frac{\partial V}{\partial z}\hat{z}$$

$$\frac{\sigma}{2\varepsilon_0} + C = -V$$

$$\boxed{V = V_0 - \frac{\sigma}{2\varepsilon_0}}$$

Now consider a infinte Sheet of charge with uniform surface surface density σ and has a circular hole of radius R What is the $\vec{E}$ at height h



- the Sheet is going to infinite
- The blue line in the hole of the Paper is R (radius) while b is going to infinite because b was the bigger radius and a was the smaller radius

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$$E = \frac{-\sigma y}{2\varepsilon_0}\left[\frac{1}{\sqrt{b^2+y^2}} - \frac{1}{\sqrt{a^2+y^2}}\right]$$

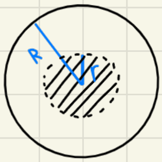
$b = \infty \qquad a = R \qquad y = h$

$$\boxed{\vec{E} = \frac{\sigma h}{2\varepsilon_0}\cdot\left[\frac{1}{\sqrt{R^2+h^2}}\right]}$$

Find the Force using Consider a $-q$ the Previous question.

$$F = qE \implies \boxed{\vec{E} = \frac{-\sigma h q}{2\varepsilon_0}\cdot\left[\frac{1}{\sqrt{R^2+h^2}}\right]}$$

Consider a spherical gaussian surface of radius $r<R$ centered within the sphere



★ this answer is For the inside surface that shadded

$$\rho = \frac{dQ}{dV} \implies dQ = \rho\, dV$$

$$\oint \vec{E}\, dA = \frac{dQ_{enclosed}}{\varepsilon_0}$$

$$\oint \vec{E}\, dA = \int \frac{\rho\, dV}{\varepsilon_0}$$

$$E A = \frac{\rho V}{\varepsilon_0}$$

$$E(\cancel{4\pi r^2}) = \frac{\rho(\cancel{4\pi r^2}\, r)}{3\varepsilon_0}$$

$$\boxed{E = \frac{\rho r}{3\varepsilon_0}}$$

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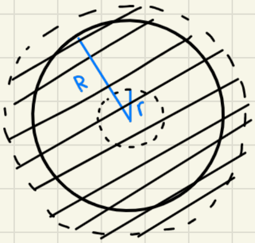
Now consider a spherical gaussian surface of radius $r > R$



$$\rho = \frac{dq}{dV} \Rightarrow dq = \rho \, dV$$

$$\oint \vec{E} \, dA = \frac{dq}{\varepsilon_0}$$

$$\oint \vec{E} \, dA = \int \frac{\rho \, dV}{\varepsilon_0}$$

$$E A = \frac{\rho V}{\varepsilon_0}$$

$$E(\cancel{4\pi} r^2) = \frac{\rho(\cancel{4\pi} R^3)}{3 \varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho R^3}{3 \varepsilon_0 r^2}}$$

☆ here we take the surface area of the gaussian surface but the volume of R

Now Find V, the electric potential, at all points in space assuming $V \to 0$ in a region very from the charged sphere hint: start in the region outside the sphere and work inward also notice that V must be continous on the boundry between solution $r = R$

$$E = -\nabla V$$

$$\int \frac{\rho R^3}{3 \varepsilon_0 r^2} \, dr = -\int \frac{\partial V}{\partial z} \, dr$$

$r > R$

$$\boxed{\frac{\rho R^3}{3 \varepsilon_0 r} + C = V}$$

• this is for $r > R$

$V \to 0$ as $r \to \infty$, we conclud $C = 0$ for the region $r < R$

$$\frac{\rho r}{3 \varepsilon_0} = -\frac{\partial V}{\partial r} \Rightarrow V = \frac{-\rho}{3 \varepsilon_0} \int r \, dr = \frac{-\rho r^2}{6 \varepsilon_0} + C$$

We said it must be continous on $r = R$

$$\frac{-\rho R^2}{6 \varepsilon_0} + C = \frac{\rho R^2}{3 \varepsilon_0 R} \Rightarrow C = \frac{\rho R^2}{3 \varepsilon_0} + \frac{\rho R^2}{6 \varepsilon_0} = \frac{\rho R^2}{2 \varepsilon_0}$$

• we said $r = R$ so we can replace r with R

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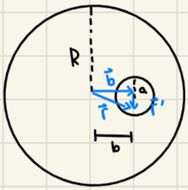
$$\boxed{V = \frac{\rho R^2}{2\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0}}$$

- this is for $r < R$

Finding the electric Field of the hole?



$$E_{tot} = \frac{\rho \vec{r}}{3\varepsilon_0} - \frac{\rho \vec{r}'}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{r}')$$

$$E_{tot} = \frac{\rho}{3\varepsilon_0}\vec{b} \Rightarrow \boxed{|\vec{E}| = \frac{\rho b}{3\varepsilon_0}}$$

- this Points in the direction given by vector $\hat{b}$

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