Momentum & Collision

[1] Momentum : is how hard an object is traveling...

$$\boxed{\vec{P} = m.\vec{V}}$$

$$* \ F_{net} = m.\vec{a} = m.\frac{d\vec{v}}{dt} = \frac{dm\vec{v}}{dt} = \frac{d\vec{P}}{dt}$$

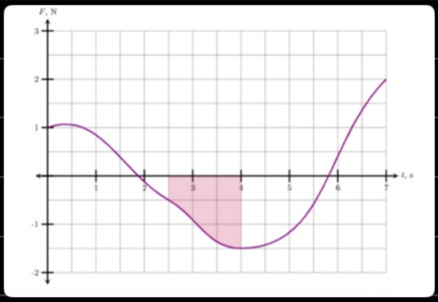
$\boxed{F_{net} = \frac{d\vec{P}}{dt}}$ ⇒ equivalent to newton's 2nd law ⇒ If $F_{net} = 0$, then $\frac{d\vec{P}}{dt} = 0$ ... Therefore

$\vec{P}$ is conserved (constant momentum)

$$* \ KE = \frac{1}{2} m v^2 = \frac{1}{2} \cdot \frac{m^2 . v^2}{m} = \frac{P^2}{2m} \Rightarrow \boxed{KE = \frac{P^2}{2m}}$$

[2] Impulse : change of momentum (ΔP) or (J)

$$\boxed{\Delta P = \int F_{net} . dt}$$

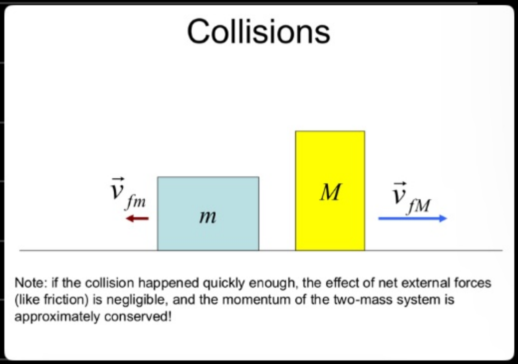


[3] Collisions : (1) If the collisions between two masses happened quickly, then :

- non-conservative forces as friction is negligible...
- Momentum of the system Conserved...

$$\boxed{P_{01} + P_{02} = P_{f1} + P_{f2}}$$

- However, even with knowing the (V_i & masses) we can't find (V_f)...
- equivalent to newton's 3rd law



Note: if the collision happened quickly enough, the effect of net external forces (like friction) is negligible, and the momentum of the two-mass system is approximately conserved!

$$P_{01} - P_{f1} = P_{f2} - P_{02} \Rightarrow -\Delta P_1 = \Delta P_2 \Rightarrow -\int \underbrace{F_{2\,on\,1}}_{constant} . dt = \int \underbrace{F_{1\,on\,2}}_{constant} . dt$$

$$\Rightarrow \boxed{-F_{2\,on\,1} = F_{1\,on\,2}}$$

2. If the collision between two masses conserve kinetic energy, then:

→ It is called ((Elastic collisions))...

→ $$\boxed{KE_{01} + KE_{02} = KE_{f1} + KE_{f2}}$$

3. Using both concepts, we can ultimately find the final velocity of both masses by substitution... However, we are assuming ($V_{02} = 0$) which means staitionary...

→ Conservation of momentum:

$$m_1 V_{01} + \underbrace{m_2 V_{02}}_{0} = m_1 V_{f1} + m_2 V_{f2} \Rightarrow \boxed{V_{f2} = \frac{m_1 (V_{01} - V_{f1})}{m_2}}$$

→ Conservation of kinetic energy:

$$\frac{1}{2} m_1 V_{01}^2 + \underbrace{\frac{1}{2} m_2 V_{02}^2}_{0} = \frac{1}{2} m_1 V_{f1}^2 + \frac{1}{2} m_2 V_{f2}^2 \quad \times 2$$

$$\Rightarrow m_1 V_{f1}^2 - m_1 V_{01}^2 + m_2 \cdot \left(\frac{m_1^2 (V_{01} - V_{f1})^2}{m_2^2} \right) \quad \begin{matrix} \times m_2 \\ / m_1 \end{matrix}$$

$$\Rightarrow m_2 V_{f1}^2 - m_2 V_{01}^2 + m_1 (V_{01}^2 - 2 V_{01} V_{f1} + V_{f1}^2) = 0$$

$$m_2 V_{f1}^2 - m_2 V_{01}^2 + m_1 V_{01}^2 - 2 m_1 V_{01} V_{f1} + m_1 V_{f1}^2 = 0$$

$$\Rightarrow \boxed{(m_1 + m_2) V_{f1}^2 - 2 m_1 V_{01} V_{f1} + (m_1 - m_2) V_{01}^2 = 0}$$

$$\vec{p} = m.\vec{V}$$

$$\Delta p = \int F_{net} \, . \, dt$$

$$p_{01} + p_{02} = p_{f1} + p_{f2}$$

$$F_{net} = \frac{d\vec{p}}{dt}$$

$$KE_{01} + KE_{02} = KE_{f1} + KE_{f2}$$

$$KE = \frac{p^2}{2m}$$

$$V_{f2} = \frac{m_1 (V_{01} - V_{f1})}{m_2}$$

$$(m_1 + m_2) V_{f1}^2 - 2 m_1 V_{01} V_{f1} + (m_1 - m_2) V_{01}^2 = 0$$

$$V_{f2} = \frac{2 m_1}{m_1 + m_2} \, . \, V_{01}$$

$$V_{f1} = \frac{m_1 - m_2}{m_1 + m_2} \, . \, V_{01}$$

Elastic Collisions

$V_{02} = 0 \, m/s$

2.5 Momentum in One Dimension

2.5.1 Impulse and Forces in Car Accidents

In this activity, we will examine impulse in the context of how seat belts work in cars. Consider the case of a car traveling at 20 m/s (about 45 m.p.h) that runs straight into a thick stone wall and stops.

1. If there is a 70 kg person seated within the car, what is the total impulse the person experiences during the collision with the wall? Does this value depend on the details of the collision, such as whether or not he is wearing his seat belt?

2. Injuries to a person are caused by the forces a person experiences. In order to decrease the total force experienced by the person, what must happen to the time Δt over which the collision occurs? Should we want Δt to be larger or smaller?

3. Assume now the person is wearing his seat belt, which has the effect of ensuring that he is always co-moving with the car. Notice that modern cars are designed to crumple when they are involved in collisions, which has the effect of increasing the time over which the collision occurs.

   (a) Estimate the amount of distance you would expect a car to be able to crumple.

   (b) Use this distance and a kinematics equation to estimate Δt, the time over which the collision occurs, assuming the acceleration is constant.

   (c) Use your answer to (b) above to estimate the magnitude of the force experienced by the 70 kg person in the car.

4. Now consider the situation where the person is not wearing a seat belt. In this case, the person will come to a stop when he smashes into the front of his car.

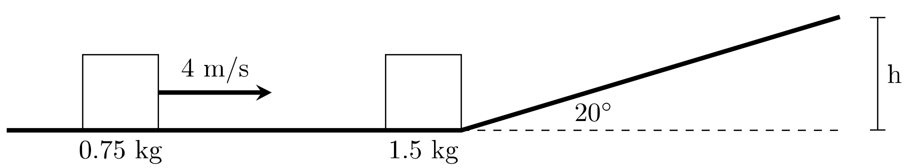
   (a) Estimate the amount of distance over which a person could safely "crumple" when they smashed into their windshield.

   (b) Use this distance and the same kinematics equation as in part (b) of question 3 above to find the time Δt over which the person's collision with the front of his car occurs.

   (c) Now use your answer to (c) to find the force on the person now, in the case where he did not wear his seat belt. How does this force compare to the force you found in 3(c)? Is he more likely to survive the collision when wearing his seat belt or when he is not?

2.5.2 An Elastic Collision Example

A 0.75 kg block sliding along a frictionless surface with an initial speed of 4 m/s collides elastically with a stationary 1.5 kg block at the base of a frictionless incline, as shown in the picture below.

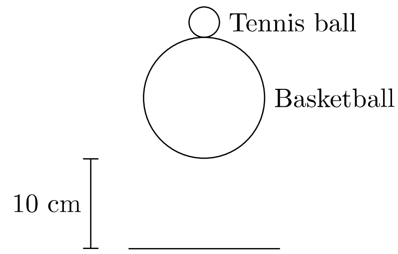


1. What is the final velocity of the 0.75 kg block after the collision? Be sure to specify both the speed and direction the block is traveling in.

2. What is the maximum height h up the incline that the 1.5 kg block will reach after the collision?

2.5.3 The Galilean Cannon

A 0.6 kg basketball and 0.06 kg tennis ball, stacked one on top of the other, are dropped from a height of 10 cm. Assume all collisions (the basketball with the ground and the tennis ball with the basketball) are elastic.



1. Using either kinematics or conservation of energy, determine how fast the basketball and tennis ball are moving the instant the basketball strikes the ground.

2. Now consider the elastic collision between the basketball and the earth. In the lecture notes when we solved the problem of an elastic collision between a moving object (here, the basketball) and a stationary object (here, the earth), we found the following equation for the final velocity $v_{f,1}$ of the moving object

$$0 = (m_1 m_2 + m_1^2)v_{f,1}^2 - 2m_1^2 v_{f,1}v_{i,1} + (m_1^2 - m_1 m_2)v_{i,1}^2 \tag{2.14}$$

   (a) For the case of the basketball and the earth, what can you say about the relationship between m_1 and m_2? Is one significantly bigger than the other?

   (b) Using the relationship between m_1 and m_2 found above, which two terms in equation 1 (after distributing everything out) are much, much bigger than the other three terms?

   (c) Notice that, since two terms in equation 1 are so much bigger than the others, equation 1 is well approximated if we keep only these two terms. Use this approximation to find the final velocity of the basketball after colliding with the earth. Which direction is the basketball traveling in after the collision? Does this match your physical intuition?

3. Now consider the elastic collision between the basketball and the tennis ball, which happens just after the collision between the basketball and the ground.

   (a) Based on your answer to questions 1 and 2(c) above, what is the total momentum of the basketball and tennis ball system before the collision? Which direction does this total momentum point?

   (b) Using that both kinetic energy and momentum are conserved in the collision, set up two equations for the unknowns $v_{f,B}$ and $v_{f,T}$, the velocities after the collision for the basketball and tennis ball, respectively.

   (c) Now solve these two equations in two variables using algebra to find $v_{f,T}$, the final velocity of the tennis ball after the collision. Remember the trick from lecture: the quadratic equation you get for $v_{f,T}$ should be factorable, with one of the factors being the initial velocity of the tennis ball.

4. Using kinematics or conservation of energy, calculate the maximum height attained by the tennis ball when it rebounds from the collision with the velocity you found in 3(c). How does this height compare to the initial height of 10 cm at which the tennis ball was dropped?

5. Now go ask your teacher for the Galilean cannon demonstration, so you can qualitatively confirm your calculations!

$\boxed{2.5.2}$ $m_1 = 0.75\,kg$, $m_2 = 1.5\,kg$ $V_{01} = 4\,m/s$ $V_{02} = 0\,m/s$

$\boxed{1}$ $(m_1+m_2)V_{f1}^2 - 2m_1 V_{01} V_{f1} + (m_1 - m_2)V_{01}^2 = 0$

$2.25\,V_{f1}^2 - 12\,V_{f1} - 12 = 0$

$$V_{f1} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{+12 \mp \sqrt{144-108}}{4.50} \Rightarrow V_{f1} = 4$$

$$\boxed{V_{f1} = 1.33\,m/s}$$

$\boxed{2}$ $E_{total} = U + KE$

$= 0 + \frac{1}{2} m_2 V_{f2}^2$??

$= \frac{1}{2} \cdot 1.5 \cdot 1.33^2$

$= \boxed{1.33\,J}$

$$V_{f2} = \frac{m_1(V_{01} - V_{f1})}{m_2}$$

$$= \frac{0.75(4-1.33)}{1.5}$$

$$= \boxed{1.33\,m/s}$$

$E_{total} = U_{max} + KE$

$1.33 = m_2 \cdot g \cdot h + 0$

$1.33 = 1.5 \cdot 9.8 \cdot h$

$$\boxed{h = 8.71\,m}$$

$\boxed{2.5.3}$ $m_1 = 0.6\ kg$ $m_2 = 0.06\ kg$ $h = 0.10\ m$

$\boxed{1}$ $2ad = V_f^2 - V_i^2$

$+2 \cdot 9.8 \cdot 0.10 = V_f^2 - 0$

$\boxed{V_f = 1.4\ m/s}$

$\boxed{2}$ a) Earth (m_2) is bigger in mass than the basketball (m_1)

b) $(m_1 \cdot m_2) V_{f_1}^2$ & $(m_1 \cdot m_2) V_{i_1}^2$

c) $m_1 m_2 V_{f_1}^2 - m_1 m_2 V_{i_1}^2 = 0$

$V_{f_1}^2 = V_{i_1}^2$

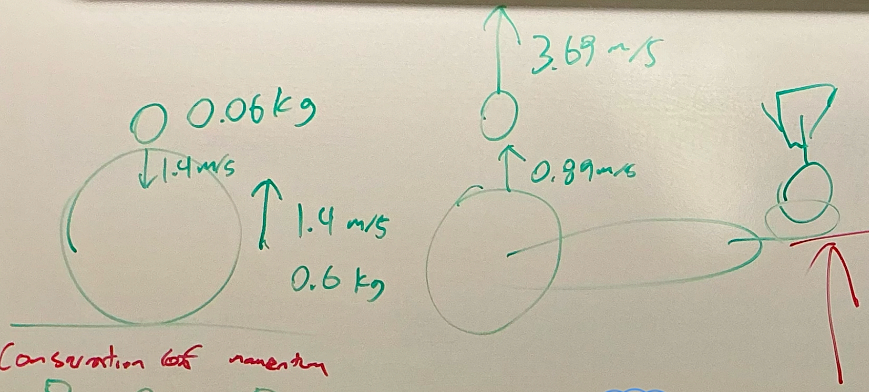
$V_{f_1}^2 = (1.4)^2$

$V_f = 1.4\ m/s$ or $\boxed{V_f = -1.4\ m/s}$

$\boxed{3}$ $m_1 = 0.6\ kg$ $m_2 = 0.06\ kg$ $V_1 = -1.4\ m/s$ $V_2 = 1.4\ m/s$

a) $P_{total} = m_1 V_1 + m_2 V_1 = 0.6 \cdot -1.4 + 0.06 \cdot 1.4$

$P_{total} = -0.716\ kg \cdot m/s$ upward



Conservation of momentum

$$P_{i1} + P_{i2} = P_{f1} + P_{f2}$$

3(b)

$$(0.06)(-1.4) + (0.6)(1.4) = 0.06V_{f1} + 0.6V_{f2}$$

$$\cancel{\tfrac{1}{2}}(0.06)(1.4)^2 + \tfrac{1}{2}(0.6)(1.4)^2 = \cancel{\tfrac{1}{2}}(0.06)V_{f1}^2 + 0.6V_{f2}^2$$

3(c)

$$V_{f1} = 3.69 \text{ m/s}$$

$$V_{f2} = 0.89 \text{ m/s}$$

4

$$K_i + \cancel{U_i} = \cancel{K_f} + U_f$$

$$\frac{1}{2}\cancel{m}(3.69)^2 = \cancel{m}(9.8)h$$

$$h = \frac{(3.69)^2}{2(9.8)}$$