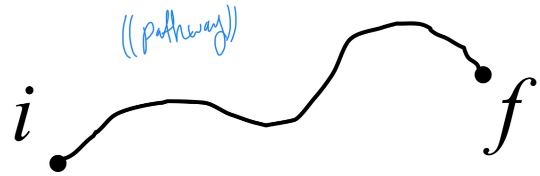
$$\boxed{W_{net} = F_{net} \times d}$$



Since force can vary along pathway, then it values changes along the way. Therefore, we would need to use integral to find the work done:

$$\boxed{W_{net} = \int F_{net} \cdot dx}$$

⇒ However, this method becomes harder to deal with as we consider 2D/3D path... for us to be able to solve this we would need a good understanding of ((dot product))... Nevertheless, we would also be dealing with multiple forces...

Solution:

$$dW_{net} = \vec{F}_{net} \cdot \vec{dx}$$

$$dW_{net} = m\,\vec{a} \cdot \vec{dx}$$

$$dW_{net} = m\,\frac{d\vec{v}}{dt} \cdot \vec{dx}$$

$$dW_{net} = m\,\vec{v}\,\frac{d\vec{v}}{dt} \cdot dt$$

$$dW_{net} = \frac{1}{2}m\,\frac{dv^2}{dt} \cdot dt$$

$$\int_i^f dW_{net} = \int_i^f \frac{1}{2} m\, dv^2$$

$$W_{net} = \frac{1}{2} m\,(v_f^2 - v_i^2)$$

$$* \quad \vec{F}_{net} = m\vec{a}$$

$$* \quad \vec{a} = \frac{dv}{dt}$$

$$* \quad d\vec{x} = \frac{d\vec{x}}{dt} \cdot dt = \vec{v} \cdot dt$$

$$* \quad \frac{dv^2}{dt} = \frac{d(\vec{v} \cdot \vec{v})}{dt} = \frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt}$$

$$\frac{dv^2}{dt} = 2\,\vec{v}\,\frac{d\vec{v}}{dt}$$

$$\frac{1}{2}\,\frac{dv^2}{dt} = \vec{v}\,\frac{d\vec{v}}{dt}$$

$$\boxed{W_{net} = \frac{1}{2} m\,(\Delta v)^2 = \Delta K}$$

⇒ Kinetic energy is path independent meaning ΔK will be the same regardless of path taken as long as it has the same initial & final position...

However, this only applies if energy is conserved, which means the object is in a Conservative force field as ((gravity)), electrostatics, & Springs... ((functions of position))

$$dW_{net} = dW_g = \vec{F}_g \cdot dx = -mg \cdot dy$$

$$W_{net} = W_g = \int_i^f -mg \cdot dy = -mg(y_f - y_i)$$

$$\boxed{W_{net} = W_g = -mg\Delta y = -\Delta U_g}$$

$$\boxed{F = -\frac{dU}{dx}}$$

$$\boxed{\Delta E = \Delta K + \Delta U = 0}$$ ((Energy conserved)) ((no change of energy))

$$\boxed{W_{net} = W_g = -\Delta U = \Delta K}$$ * Unit: $Kg \cdot m^2/s^2$ = Joule

$$\boxed{E_{total} = K + U = \text{constant}}$$ * K = always positive (velocity squared)

If Non-conservative forces are present as ((friction)), drag, & normal ((non-position functions))

$$\Delta E = \Delta K + \Delta U = W_{nc}$$

$W_{nc} < 0$ ... negative, meaning that energy was released from the system in form of thermal energy (heat) ...

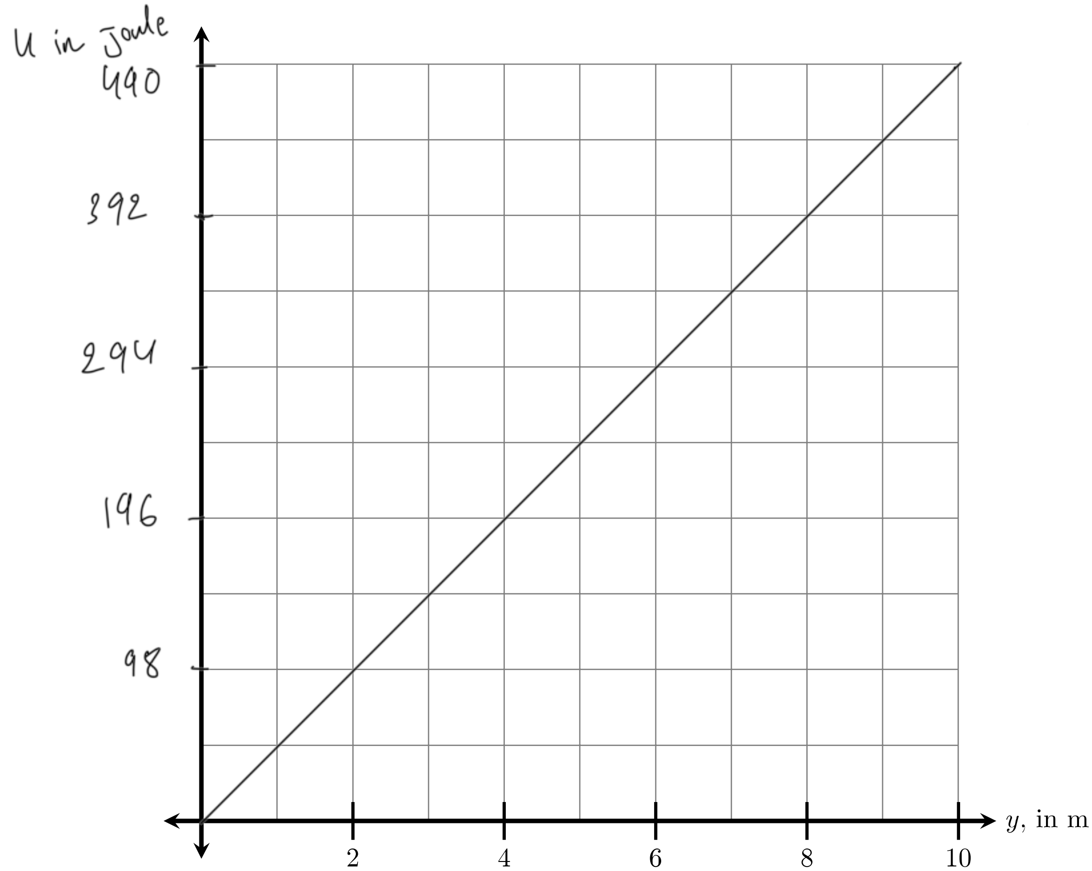
2.1 Potential and Kinetic Energy

2.1.1 The Gravitational Potential Energy

In lecture we learned that the potential energy of an object of mass m due to gravity is given by

$$U = mgy \tag{2.1}$$

1. On the graph below, sketch a U vs. y graph for the potential energy of a 5 kg object. Be sure to label this graph as U. I have labeled the y axis for you, but you will need to label the energy axis.



2. Consider now a situation where the mass has a total energy of 196 J. On your graph above, sketch this total energy as a function of position. Label this as E_{tot}.

3. Assuming there are no nonconservative forces doing work on the object, sketch a graph of the kinetic energy, K, as a function of position on your graph above for the case where total energy is 196 J. Label this curve K. What is the allowed domain for K? (Hint: can K be negative? Why or why not?)

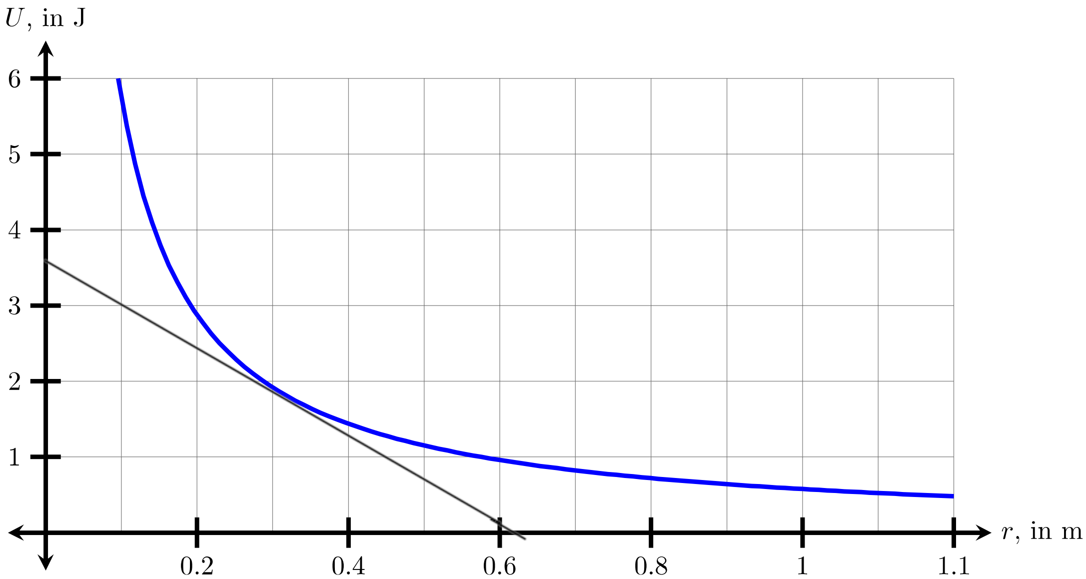
4. From the graph you drew on the previous page, estimate the maximum height the 5 kg object attains with this initial kinetic energy. Does this answer match the answer you get if you solve the problem using kinematics?

2.1.2 The Electrostatic Potential Energy

As you will explore more fully in physics 232, the electrostatic potential energy of a test charge q placed a distance r away from another charge Q is given by

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r} \tag{2.2}$$

where $\epsilon_0 \approx 8.85 \times 10^{-12}$ $C^2/J{\cdot}m$ is a constant called the *vacuum permittivity*. The potential energy as a function of separation distance r for two 8 μC charges is plotted on the next page. (Note: $1\mu C = 1 \times 10^{-6}$ C.)



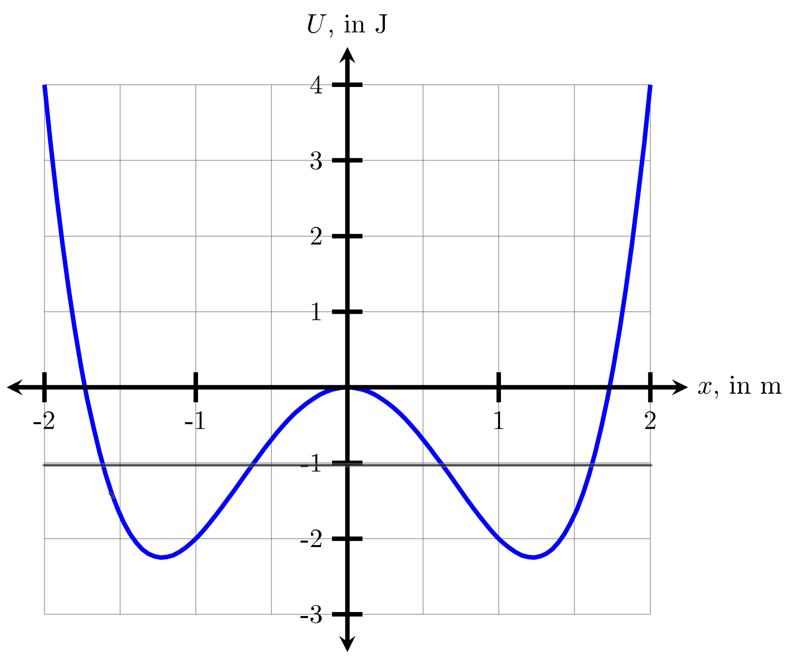
1. Approximately how much work must be done to bring one of the 8 μC charges from very far away to a point 0.3 m away from the other charge?
2. From the graph, estimate the approximate electrostatic force on one of the 8 μC charges experiences at separation distance $r = 0.3$ m by drawing in the tangent line and estimating its slope.
3. Now, using the functional form of the electrostatic potential energy given in equation (1) above, as well as the relationship $F = -\frac{dU}{dr}$, calculate the actual force experienced by the 8 μC charge. Do your answers come close to agreeing? (Hint: take the derivative first, before plugging in numbers!)
4. If one of the 8 μC charges starts from rest at a separation distance of 0.2 m, approximately how fast is the charge traveling at a separation distance of 0.6 m, if it is observed to have a mass of 3.5×10^{-6} kg? Assume the other 8 μC charge remains stationary.

2.1.3 The Higgs Potential

A potential energy commonly encountered in high energy physics has the functional form

$$U(x) = -Ax^2 + Bx^4 \tag{2.3}$$

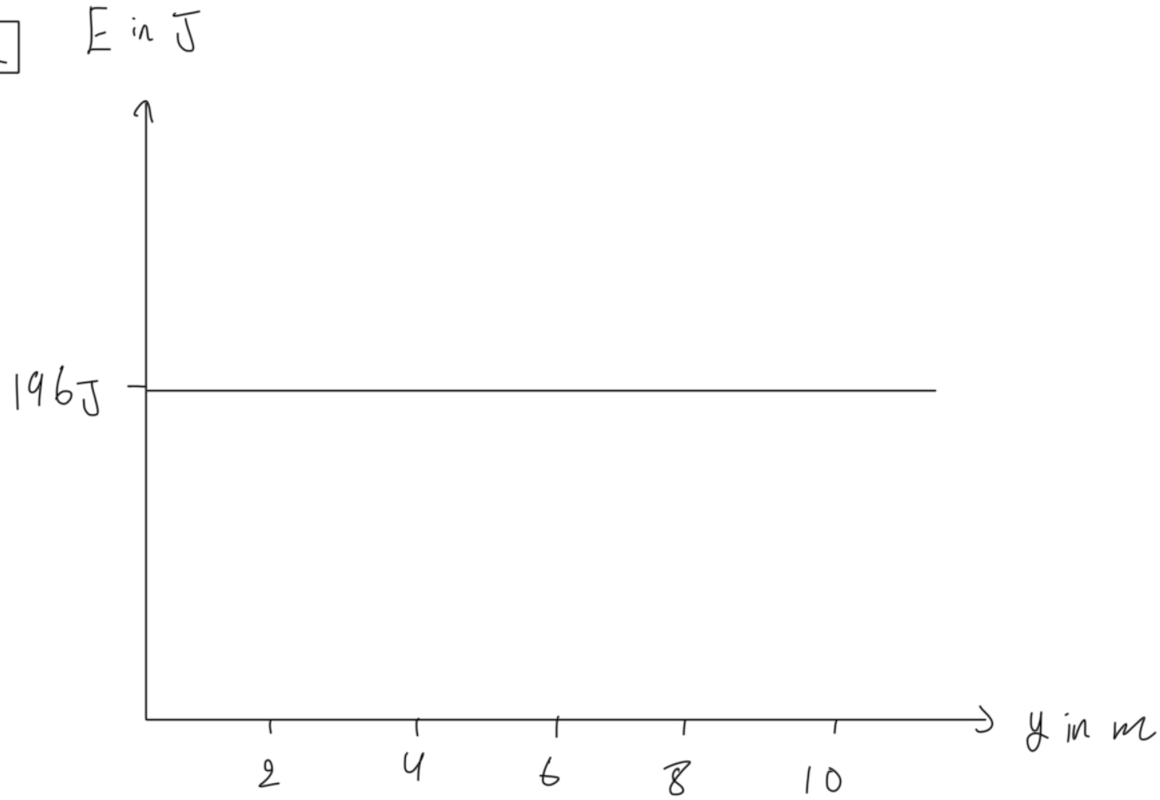
where A and B are both positive parameters and x is the position of the particle experiencing the potential. In the plot below, I have graphed the situation in which $A = 3$ J/m^2 and $B = 1$ J/m^4.



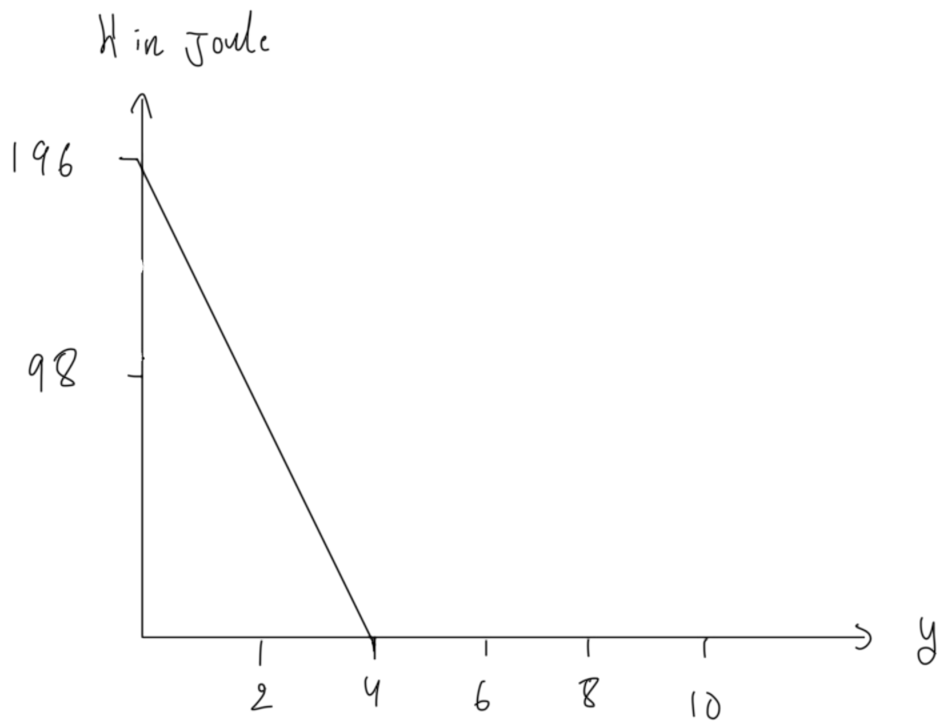
1. Assume at first the particle has a total energy of -1 J and is at position $x = 1.5$ meter.
   (a) Sketch a graph of the total energy E_{tot} as a function of the position.
   (b) Use your E_{tot} graph above to estimate between what two positions x_{min} and x_{max} the particle is confined.
   (c) Now sketch an approximate graph of K, the kinetic energy, as a function of the position, on the relevant domain.
2. The particle is now placed at position $x = -2$ m such that its initial velocity is zero. Will the particle be able to get to position $x = 0$ m? If so, what kinetic energy will the particle have at this point?
3. The particle is now placed at position $x = 1$ m such that its initial velocity is zero. Will the particle be able to get to position $x = 0$ m? If so, what kinetic energy will the particle have at this point?
4. Using the functional form of $U(x)$, find the magnitude of the force experienced by the particle when it is at position $x = -1$ m. In what direction (i.e., towards $+x$ or towards $-x$) does this force point?

$\boxed{2.1.1}$

$\boxed{2}$



$\boxed{3}$



$K + U = 196$

$$\boxed{K = 196 - U}$$

$\boxed{4}$ the maximum height is (4), it is when $\boxed{K = 0}$

$\boxed{2.1.2}$

$\boxed{1}$ 1.9 J

$\boxed{2}$ $F = \frac{\Delta U}{\Delta x} = \frac{0 - 3.6}{0.63} = \boxed{-5.7 N}$

$\boxed{3}$ $U_e = \frac{1}{4\pi \varepsilon_0} \cdot \frac{qQ}{r} \Rightarrow \boxed{\frac{dU_e}{dr} = \frac{-1}{4\pi \varepsilon_0} \frac{qQ}{r^2}}$

$$F = -\frac{dU_e}{dr} = +\frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2}$$

$$= \frac{64 \times \cancel{10}^{-12}}{4\pi \times 8.85 \times \cancel{10}^{-12} \times (0.3)^2}$$

$$= \boxed{6.39 N}$$

$\boxed{4}$ $\Delta K = -\Delta U$

$$\frac{1}{2} m (v_f^2 - \underbrace{v_i^2}_{0}) = -(U_f - U_i)$$

$$\frac{1}{2} m v_f^2 = -(1 - 2.85)$$

$$v = \sqrt{\frac{1.85 \cdot 2}{3.5 \times 10^{-6}}} = \boxed{1028.17 \ m/s}$$

$\boxed{2.1.3}$

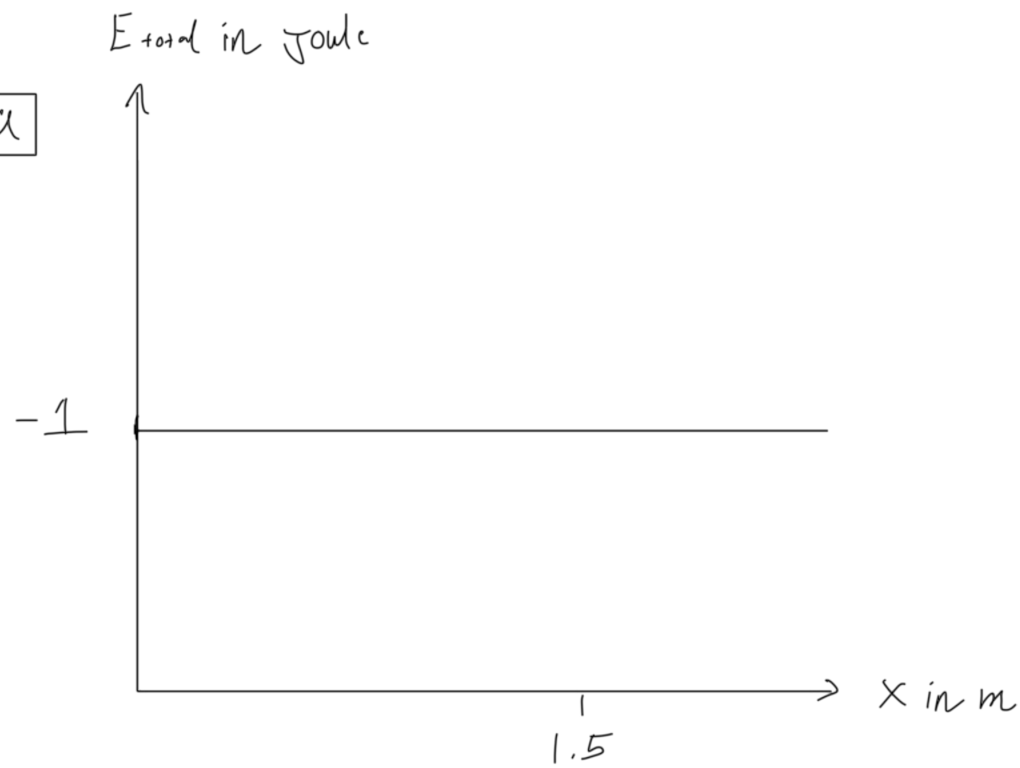
$U(x) = -Ax^2 + Bx^4$

$$\boxed{U(x) = -3x^2 + 1x^4}$$

$\boxed{1}$ $E_{total} = -1\,J$

when $x = 1.5\,m$

$\boxed{a}$



$\boxed{b}$ since the particle started at $x = 1.5\,m$, then:

$$\boxed{\begin{aligned} \max(x) &= 1.7\,m \\ \min(x) &= 0.7\,m \end{aligned}}$$

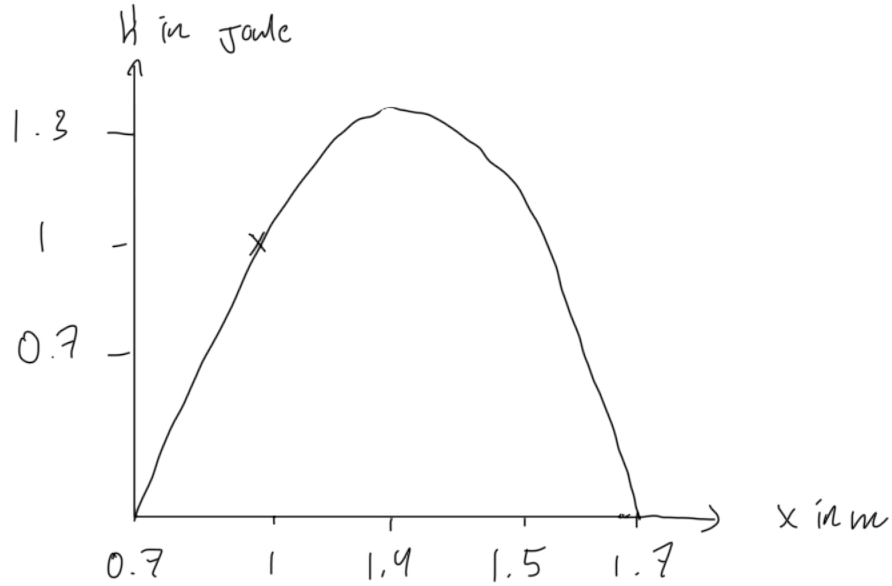
the other 2 points is excluded since $U > K$ ... for example:

when $U(0) = 0$ & $K + U = -1$, then:

$K + 0 = -1$

$$\boxed{K \neq -1}$$

C



$K + U = -1$

$\boxed{K = -1 - U}$

$\boxed{K = 1.3}$ $x = 1.4$

$\boxed{K = 1}$ $x = 1$

$\boxed{K = 0.7}$ $x = 1.5$

2 $x = -2$

$U_i = 0$

$K + U = 4$

$K + 0 = 4$

$\boxed{K = 4 \text{ Joule}}$

Note:

$\Delta K + \Delta U = 0$

$K + U = E_{total}$

3 $x = 1$

$U_i = 0$

$K + U = -2$

$K(0) + U(0) = -2$

$K(0) + 0 = -2$

$\boxed{K(0) \neq -2}$

$\boxed{DNE}$

4 Since $F = \frac{-\Delta U}{\Delta x}$, then $F = -U'(x) \Rightarrow U(x) = -3x^2 + 1x^4$

$U'(x) = -6x + 4x^3$

$F(-1) = U'(-1) = -6+4 = \boxed{-2}$ (negative x) $\quad -U'(x) = 6x - 4x^3$