The Kepler Problem

When a question says relative to smaller mass "m" this means from the perspective of smaller m this would mean that smaller "m" has $x = 0$ (position = 0) because you are looking from smaller "m" perspective it does not have to be small "m" it can be any position

the equation $l = \mu r^2 \omega$ is vaild for both circular and elliptical orbits

- circular orbits the angular momentum is easy to compute because both r and ω are constant
- elliptical orbits r and ω change but the total angular momentum of the system remains constant

reduced mass

$$\mu = \frac{m \cdot M}{m + M}$$

- this equation would be used when you have two body system where two masses are interating with each other like Planets that are moving togther in someway such as orbiting each other you can apply this equation to help reduce the mass and calculations

ex:

you have two Planets earth and Cookie these two Planets are orbiting orbiting the sun which has a much bigger Mass

in this Scenario you would apply the reduced mass in cases if you wanted to model relative Motion between earth and Cookie such if youre interested in the gravational interaction between them

- but applying it to any Planet and the sun would Just give you the sun's mass because how massive the sun is compared to each Planet

angular momentum

$$l = \mu r^2 \omega$$

the equation describes the angular Momentum of a two-body system if there are No external Force (like torques) acting on the system the angular momentum of the two-body system is conseved this means:

$$l_{inital} = l_{Final} \Rightarrow \mu r^2 \omega = \mu r^2 \omega$$

this means you can calculate the angular momentum l at any Point in the orbit and get the correct answer in a elliptical orbit, while r and ω change the total angular will remain constant

$$l = \mu r^2 \omega$$

$v = r\omega$

$\Rightarrow \omega = \frac{v}{r}$

areal Velocity

$dA = \frac{1}{2} r^2 d\theta$

- You are working with motion over time
- you want to Find the Small area swept out by a Small angle

think "I want to find the area swept out per angle"

$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$

- your dealing with motion over time
- your relating the orbit angular "ω"

think "I care about how fast the sweep is, I need time"

how these tie to angular momentum

$L = I\omega \approx> L = MR^2 \cdot \omega$

- implying that your considering a Point mass $I = MR^2$

$\omega = \frac{d\theta}{dt}$

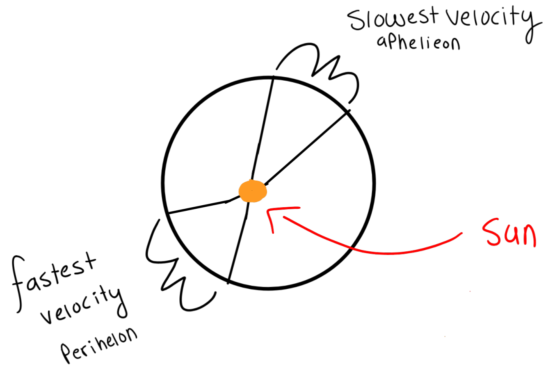
$L = mr^2 \frac{d\theta}{dt} \approx> \frac{L}{m} = r^2 \frac{d\theta}{dt}$

Plug these into the velocity Formula

$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} \approx> \boxed{\frac{dA}{dt} = \frac{1}{2} \cdot \frac{L}{m}}$

this equation gives you the same answer as $\frac{dA}{dt} = \frac{1}{2} r^2 \cdot \frac{d\theta}{dt}$ the only difference is that it does it by using angular momentum at the Perihelion, closet to the sun the radius is the smallest while the velocity is the largest

at the aphelion Farthest From the sun the radius is the largest while the velocity is the smallest



- the Position closest to the sun is called Perihelion that is also where velocity is fastest the Furthest one From the Sun is called the aphelion that is also where Velocity is the Slowest

$$r_1 V_1 = r_2 V_2$$

this can be written as $r_1 V_1 = l$ Where l is a constant r and V are not constant because the radius is changing between Perihelon and aphelion

- Simplifed Form of Conservation of angular momentum
- r_1 = Perhelion
- r_2 = aphelion

Center of mass

$$x_{com} = \sum \frac{m \cdot x}{M_{total}}$$

- this equation is used to calculate the center of mass along the x-axis the goal is to Find one Single Point that represents the average Position of all the mass in the system

or you can think of it like this "if you could treat all these objects as one big object this is where the mass would be balanced"

M (mass on the numerator) is the mass of whatever object you are dealing with the bottom mass is the total mass when you get to "x" you can set the orgin anywhere also remember if it say relative to some mass you are looking at the "world" from its point of view meaning the "x" is going to be zero

ex:
lets say you have a three body system consisting of the earth, sun, and planet cookie

when choosing a reference point pick one that is simple and not going to overcomplicate the calculation a great and common choice is the sun this will give the sun a a position of $x=0$ now you can describe the position of earth and cookie relative to the sun

Summary of equations

$$x_{com} = \sum \frac{m \cdot x}{M_{total}}$$

$$r_1 V_1 = r_2 V_2$$

$$dA = \frac{1}{2} r^2 \, d\theta$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

$$\mu r^2 \omega = \mu r^2 \omega$$

$$\mu = \frac{m \cdot M}{m + M}$$