

intro to electric circuits

• electric Charge

- electric Charge is what causes objects to feel force from electric and magnetic field

$$U = qV$$

- U : Potential electric energy
- q : the electric charge
- V : electric Potential

• the electric Potential

- the field " V " is termed the electric Potential it describes Potential energy per unit charge of all points space
- the absolute value of V has no physical meaning it may be set to zero this is called choice of gauge
- thus we talk primarily about ΔV , Not V

- Setting $V=0$ (or to ground "gnd") think of this as a change in positive (like Final minus initial) the reference point is mostly set to ground which is zero think of this as the change from ground to whatever point you are calculating

• Current

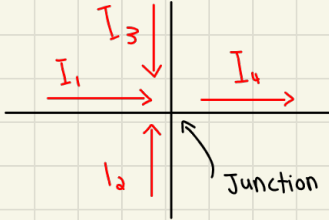
- an electric current through an area is defined to be the amount of charge flowing through that area per unit time

$$I = \frac{dq}{dt} \frac{(\text{change in charge})}{(\text{change in time})} \quad \text{units of Amps (A)}$$

- the Junction rule

$$\sum I_{in} = \sum I_{out}$$

- this is Kirchhoff First rule



- the Junction law is about current Conservation
- the sum of current into a Junction must equal the sum of current out the Junction

- Ohms law

- For most objects that carry a current (I) we can model the electric potential change (ΔV) by Ohms law

$$\Delta V = IR$$

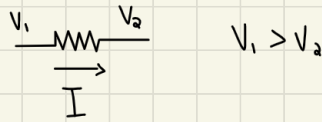
ΔV = represents difference in voltage also called potential difference

I = is the current flowing through the conductor

R = the resistor

the ΔV Voltage difference between two points, and one of those points can be ground

✱ resistance has units of ohms or Ω



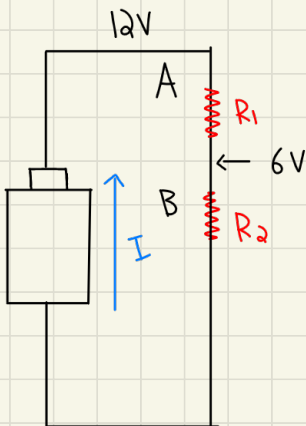
- Current always Flows From higher Potential to lower Potential looking at the drawing V_1 must be higher than V_2
- Current Flows in the direction Positive Charges move
 - For example: If you have a battery. The Current Flows From the Positive terminal (higher Potential) to negative terminal (lower Potential) of the battery
- Positive Charges move in the direction of decreasing Potential
 - Positive charges Naturally move From higher Potential to lower Potential
- ΔV is Negative in the direction of current Flow
 - ΔV refers to Change in Voltage between two Points
 - If current is Flowing in the direction that Positive Charges would move, that Means the Voltage is decreasing along the Path of current
 - In other words moving along the Path of current results in negative Change in Potential because you are moving From region of higher Potential to lower Potential
 - Voltage drop: a current Flows through Components like resistors, energy is lost

- the Positive terminal of the battery is at higher Potential (say $+V$)
- the Negative terminal of the battery is at lower Potential (say $0V$)
- Current Flows From the Positive terminal of the battery (higher Potential) to negative terminal (lower Potential) and this Movement of charge (current) corresponds to a decrease in Voltage

• electromotive Force (EMF)

~~✗~~ this is not an actual Force

- in order exist a steady state current, the Path of the current must Form a complete loop
- Sources of emf include batteries solar cells, and thermocouples
- inside the battery the emf moves electrons From the negative terminal to the Positive terminal
- outside the battery (current): the Current Flows From the Positive to the Negative terminal



- Current Flows From high Potential to low Potential
- think of it like this



- as a ball Flows From high Potential to low Potential

- as current flows from A to B its electric potential energy (ΔV) decreases, a resistor consumes energy as a current flows through the resistor ΔV losses energy
- a battery generates a EMF and as a result the EMF increases the electric potential energy



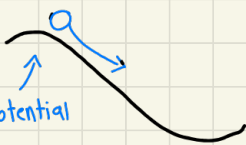
- you can think of it like this you can see the ball is on the ground to get it up to point D you would need to give it a push or something because that cannot happen by itself that's how you can imagine EMF

So as current goes down the hill as shown here it reaches the bottom and stops you do not want that so you use EMF (battery) to get it back to where it was at a higher potential and you keep doing this until the battery dies

Current



• high Potential



• low Potential

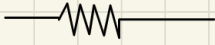
- as soon it drops to floor EMF get it back to high potential



• circuit diagram



- ideal source of EMF
battery (the long side is positive terminal)



- resistor



- ideal wire ($r=0$)

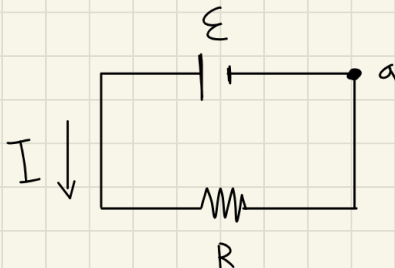
• Kirchhoff's loop rule

- the total in potential around any closed loop for steady state current must be zero

- mathematically

$$\Delta V_{a \rightarrow a} = 0$$

- as long as you move around a completely closed circuit (starting and ending at the same point) the total sum of voltage change must be zero



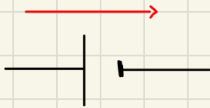
$$\Delta V_{a \rightarrow a} = 0 \Rightarrow \mathcal{E} - IR$$

$$= I = \frac{\mathcal{E}}{R}$$



- electron going From Negative to Positive

$$\Delta V = \mathcal{E}$$



- electrons going From Positive to Negative

$$\Delta V = -\mathcal{E}$$



- If you are moving in the direction of current Flow, the Potential drops So the voltage drop will be Negative

$$\Delta V = -IR$$



- in this case the voltage drop becomes Positive because you are opposing the Flow of current which means you're going against the natural drop in Potential

$$\Delta V = IR$$

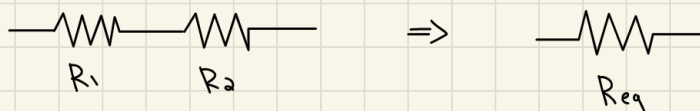
Voltage rise: occurs when you move from negative to the positive terminal of the battery

Voltage drop: occurs when you move with the direction of current through resistors

- Kirchhoff loop rule says the sum of all voltages in a closed loop is zero written as:

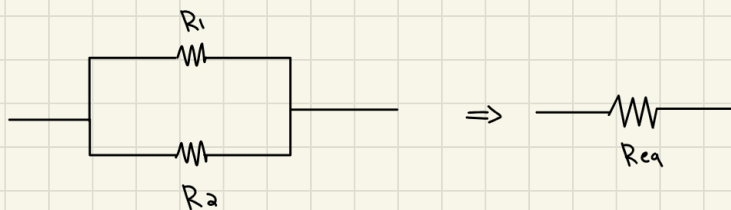
$$0 = \sum \mathcal{E} - \sum IR$$

• Resistors in Series



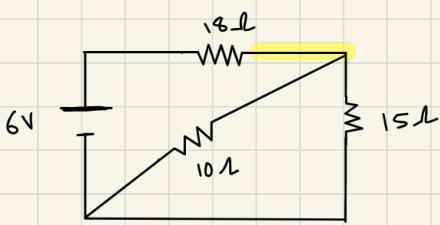
$$R_1 + R_2 = R_{eq}$$

• Resistors in Parallel



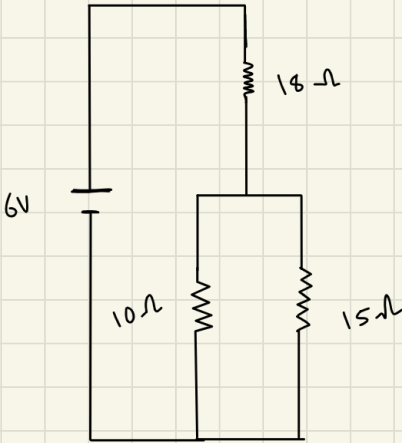
$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}}$$

★ remember to flip the answer you get



- Notice that this is not in Series with 15Ω the electrons Flow to both 10Ω and 15Ω

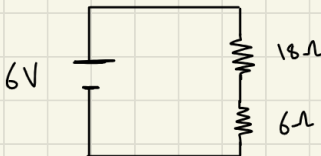
- You can redraw it



- those two circuits are the same, 18Ω is still Flowing to 10Ω and 15Ω

$$\frac{1}{10} + \frac{1}{15} = \frac{1}{6} = 6\Omega$$

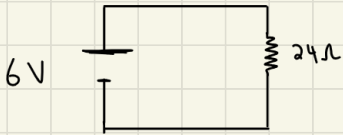
- redraw it again



$$R_1 + R_2 = R_{eq}$$

$$18 + 6 = 24\Omega$$

- redraw it again

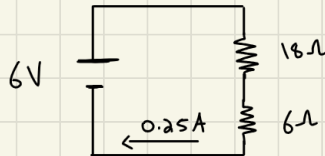


apply loop rule

$$\Delta V_{a \rightarrow a} = 0 \quad \Sigma - I R = 0$$

$$I = \frac{\Sigma}{R} = \frac{6}{24} = 0.25 \text{ A} \quad (\text{whole circuit})$$

- We must undo our combinations, since your last combination was in series the current is preserved meaning the current at all points that were in series are preserved



- thus the voltage drop across 18Ω is equivalent by ohms law

$$\Delta V = IR \Rightarrow (0.25)(18) = 4.5 \text{ V}$$

- we thus conclude that the 18Ω has a voltage drop of 4.5 V and a current of 0.25 A
- the voltage drop by the 6Ω resistor can be done by ohms law:

$$\Delta V = IR = (0.25)(6) = 1.5 \text{ V}$$

or by loop law

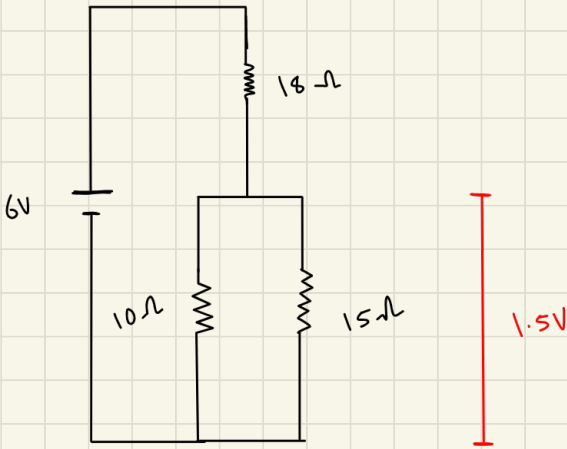
$$\Sigma \mathcal{E} - \Sigma I R = 0$$

$$\mathcal{E} - (\Delta V_{18\Omega} + \Delta V_{6\Omega}) = 0$$

$$6 - 4.5 = \Delta V_{6\Omega}$$

$$\Delta V_{6\Omega} = 1.5 \text{ V}$$

- Since our Next Was in Parallel this 1.5V must be across our 15Ω and 10Ω as well



- You can undo the combinations of the 6Ω resistor which will give you a parallel combo consisting of a 10Ω and a 15Ω resistor

When resistors are parallel to one another the Voltage drops (ΔV) are equivalent while in series the current is equivalent

- in simple words

Parallel = ΔV is the same

Series = I is the same

Power in Circuits

- Power in electric circuits

$$P = \Delta V I$$

- Where I is the current through the resistor and ΔV the voltage across the resistor

- Substituting $\Delta V = IR$

We can rewrite our previous result in two additional equivalent way by using algebra and substitution

$$P = \Delta V I \Rightarrow (IR) I$$

- You can use any equation depending on what is given to you

\Rightarrow

$$P = I^2 R$$

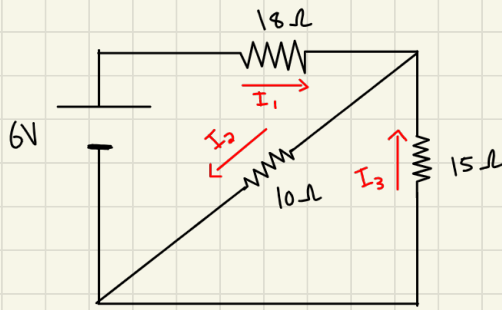
$$P = \Delta V I \Rightarrow \Delta V \left(\frac{\Delta V}{R} \right)$$

$$P = \frac{(\Delta V)^2}{R}$$

Method #1: Use equivalent resistance to systematically simplify the resistor network

Method #2: Use Kirchhoff's laws directly to find a system of n linearly independent

- advantage: will always work to reduce problems to algebra
- disadvantage: the resulting algebra might be tedious (without matrices)



- We will resolve this by using Kirchhoff law we can begin by labeling these I_1 , I_2 , and I_3 , it does not matter what you label each resistor and what direction it's going
- if you walk the current direction \rightarrow it's a drop ($-IR$)
- if you walk against the current direction \rightarrow it's a rise ($+IR$)
- We can begin by applying the Junction rule or loop rule but I will use the Junction rule

$$I_1 + I_3 = I_2$$

$$6V - I_1(18\Omega) - I_2(10\Omega) = 0$$

★ We are using $IR + IR$ but remember $IR = \Delta V$ so you can replace IR with ΔV

$$-I_2(10\Omega) - I_3(15\Omega) = 0$$

- We have made around 3 equations but we can create as many as we want

• if they told you $I_1 = 10\text{ A}$ and solve for I_3

$$I_1 + I_3 = I_2 \Rightarrow 10 + I_3 = I_2$$

$$-(10 + I_3)(10\ \Omega) - I_3(15\ \Omega) = 0$$

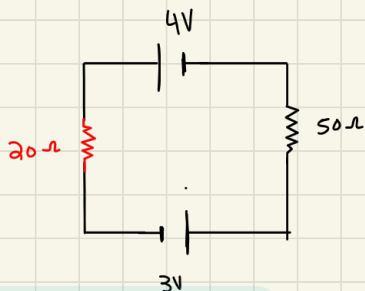
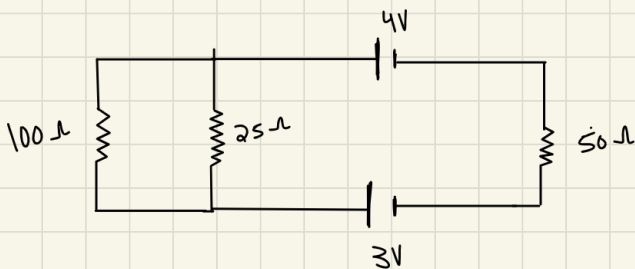
$$-100 - 10I_3 - 15I_3 = 0$$

$$-100 = 25I_3$$

$$I_3 = 4\text{ A}$$

• we only used two equations out of the three

• Circuits with 2 EMF's

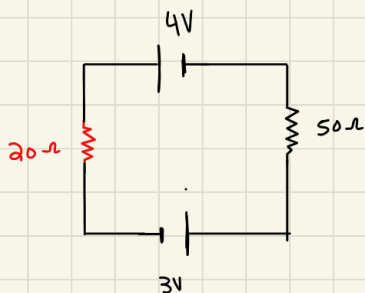


• Notice the $100\ \Omega$ and the $25\ \Omega$ are parallel

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}}$$

$$\frac{1}{25} + \frac{1}{100} = \frac{1}{20}$$

$$R_{eq} = 20$$



$$4V - I_1(20\Omega) + 3V - I_1(50\Omega) = 0$$

- Notice there are no junctions so the current is the same everywhere

✦ if there are no junctions in a circuit (meaning the circuit is a simple series loop) the current is going to be the same in that loop

$$4V - I_1(20\Omega) + 3V - I_1(50\Omega) = 0$$

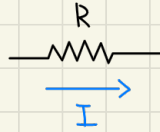
$$7 - 70 I_1 = 0$$

$$-70 I_1 = -7$$

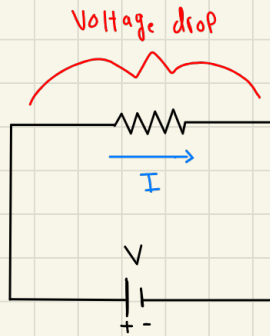
$$I_1 = 0.1 A$$

• summary

- if we have a resistor and current flows through it in this *direction*

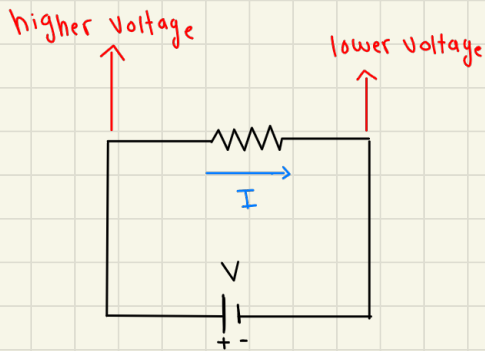


- the resistor acts like a voltage eater it takes away from the energy from the battery



• the voltage drops in the direction of the current

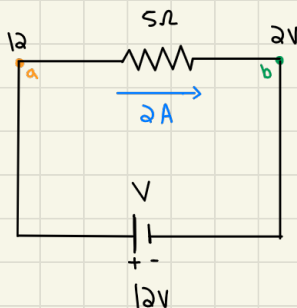
- if the current goes from left to right the left side of the resistor is at a higher voltage and the right side is at a lower voltage



• this is given or explained by $V = IR$ (ohm's law)

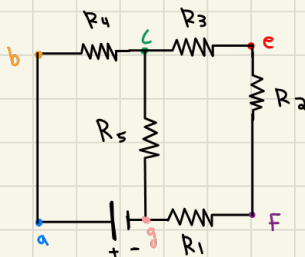
ex: if a 2 A current flows through a $5\ \Omega$ resistor the resistor will eat 10 Volts $V = (5\ \Omega)(2\text{ A})$

this means if the left side (a) has 12V the right side (b) will have 2V as $12 - 10 = 2V$

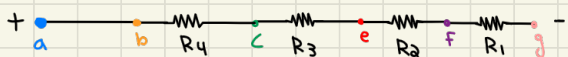


Practice Problem

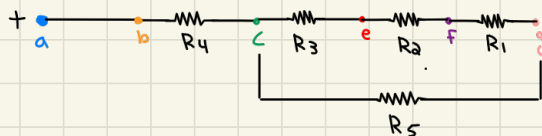
Consider the circuit depicted below. Each resistor has resistance R (all resistance is the same) what Power is being put into the circuit by the ideal battery?



• began by labeling a, b, c this will help you break down and see which one is Parallel and which ones are Series



✗ Notice how we went through the whole circuit and its resistors except R_5 because it goes from c to g

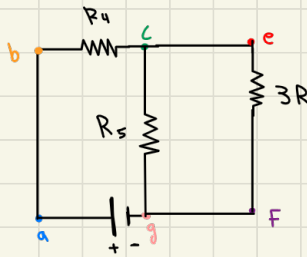


✗ R_5 is parallel to R_3, R_2 and, R_1

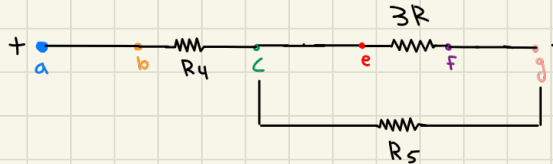
• We can use series equation on R_3, R_2 and, R_1 also they have the same resistance

$$(R + R + R) = 3R$$

• redraw

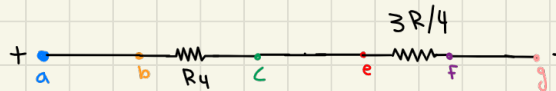
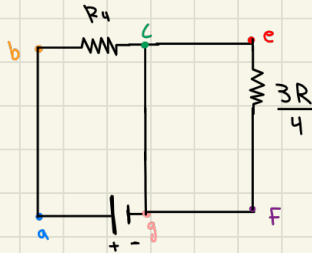


• We combined R_1, R_2 and R_3
Now we have $3R$ Parallel
to R_5

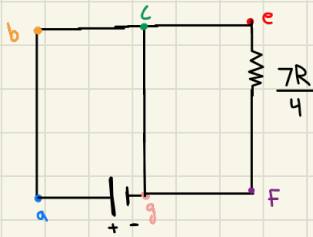


$$\frac{1}{R_5} + \frac{1}{3R} = \frac{3+1}{3R} = \frac{4}{3R} = \frac{3R}{4} \quad \bullet \text{ Flip this around}$$

• redraw



$$\frac{3R}{4} + R_4 = \frac{3R + 4R}{4} = \frac{7R}{4}$$



• Find Current

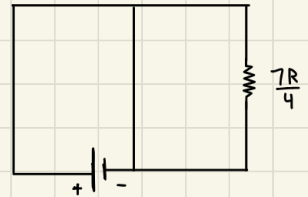
$$P = \frac{(\Delta V)^2}{R} \Rightarrow \frac{(\mathcal{E})^2}{R} \Rightarrow \frac{\mathcal{E}^2}{\left(\frac{7R}{4}\right)} \Rightarrow \boxed{\frac{4\mathcal{E}^2}{7R}}$$

Find the Power dissipated by resistor R_3

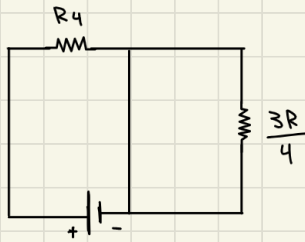
$$R_{\text{total}} = \frac{7R}{4}$$

• We can use this to Find the total current

$$I = \frac{\mathcal{E}}{R_{\text{tot}}} = \frac{\mathcal{E}}{\left(\frac{7R}{4}\right)} = \frac{4\mathcal{E}}{7R}$$



• if we go back we will have a circuit in series which means that both resistors have the same current flowing through them

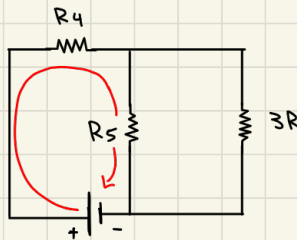


• these two resistors both have the same current flowing through them, again this is happening because they are in series

• we can find the drop in voltage in R_4

$$\Delta V_{R_4} = I R \Rightarrow \left(\frac{4\mathcal{E}}{7R} \right) (R_4) = \frac{4\mathcal{E}}{7}$$

• if we take a step back further



• we can now use loop law to figure what R_5 is

$$\sum \mathcal{E} - \sum I R = 0$$

$$\sum \mathcal{E} - \sum \Delta V = 0$$

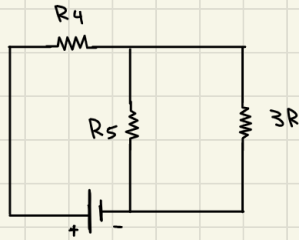
$$\mathcal{E} - (\Delta V_{R_4} + \Delta V_{R_5}) = 0$$

$$\mathcal{E} - \Delta V_{R_4} = \Delta V_{R_5}$$

$$\mathcal{E} - \frac{4\mathcal{E}}{7} = \Delta V_{R_5}$$

$$\Delta V_{R_5} = \frac{3\mathcal{E}}{R}$$

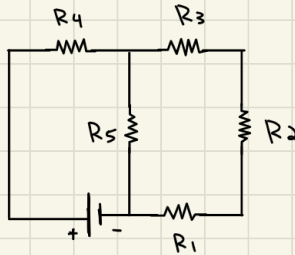
- We got ΔV_{R5} now R_5 and $3R$ are parallel to one another which means they have the same ΔV so the ΔV for $3R$ is $\frac{3\varepsilon}{7}$



We will find the current for $3R$

$$\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow \frac{\left(\frac{3\varepsilon}{7}\right)}{3R}$$

$$I = \left(\frac{3\varepsilon}{7}\right) \frac{1}{3R} = \frac{\varepsilon}{7R}$$



- after breaking down $3R$ even further we get three resistors all in series which means they all have the same current

$$P = I^2 R \Rightarrow \left(\frac{\varepsilon}{7R}\right)^2 \cdot (R_2)$$

$$\left(\frac{\varepsilon^2}{49R^2}\right) (R_2)$$

$$= \frac{\varepsilon^2}{49R}$$

~~★~~ all resistance is the same