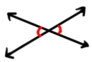
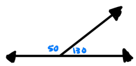
Cicular Motion

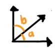
geometry/angle review



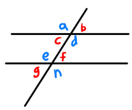
• Vertical angle are directly opposite from each other also they are equal to each other



• Supplementry angles are angles that add to 180° (180° is a straight line) they can be more then one meaning you can have two diffrient line as long as they add to 180° they are supplementry

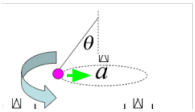


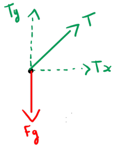
• Complementary angles add up to 90°



the ones that match colors are all equal to one and another

this will be helpful when we derive equations later





$T_y = F_g = mg$

$\cos\theta = \frac{T_y}{T} \Rightarrow T\cos\theta = T_y \approx T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta}$

$\sin\theta = \frac{T_x}{T} \Rightarrow T\sin\theta = T_x$

$F_{net} = T_x$

$\left(\frac{mg}{\cos\theta}\right) \cdot \sin\theta = F_{net}$

$\downarrow$

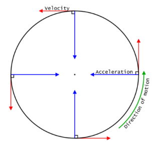
$mg \cdot \tan\theta = F_{net}$

(this equation will help when we derive another one for velocity)

- a pendulum was used because it has circular arc and it essentially undergoes circular motion which needs centripetal force we know because the acceleration is pointing to the center

- notice that T_y and F_g will cancel each other out leaving us with only T_x so that means $F_{net} = T_x$

Centripetal Force



Centripetal Force is the net force this means that the centripetal force is not a seperate force but it comes from whatever forces are acting and adding up towards the center of an object moving in a circular path. the force is always directed towards the center of the circle thats why its called the "centripetal" which means center-seeking

- $\vec{F} = m\vec{a}$ the force and acceleration will point in the same direction if the acceleration is pointed to the middle then the force also points to the middle if you take a rope you will a bit of force/pressure when you spin it

- centripetal acceleration

$\boxed{a_c = \frac{v^2}{r}}$ $\Rightarrow$ to get the centripetal force we can substitue the centripetal acceleration into $F = ma$

$\boxed{F_c = m \cdot \frac{v^2}{r}}$

$\downarrow$

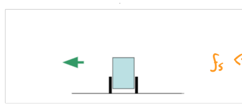
This is the net force

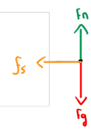
$F_{net} = F_c$

turns

$\boxed{V = \sqrt{\mu_s g r}}$

- this equation tells you the maximum speed at which at which a vechicle can go around a flat curve without slipping purely due to friction





☆ notice that Fn and Fg cancel each other out leaving us only with f_s so $F_{net} = f_s$

$f_s \leq \mu_s F_n$
$F_n = mg$

$F_{net} = f_s$

$m \cdot \frac{v^2}{r} \leq \mu_s F_n$

$\cancel{m} \cdot \frac{v^2}{r} \leq \mu_s \cdot \cancel{m} g \Rightarrow \boxed{V = \sqrt{\mu_s g r}}$

Velocity on banked curve (No Friction)

- imagine a car moving on a curve thats tited (banked) at an angle and the road is frictionless to stay to stay on the curve (without sliding) the car the car has has to move at exactly the right speed that where the formula comes in it tells you it tells you the speed needed to stay in circular motion on a banked curve without friction

$F_{net} = F_c = mg\tan\theta$

$\cancel{m} \cdot \frac{v^2}{r} = \cancel{m} g \tan\theta \Rightarrow \boxed{V = \sqrt{gr\tan\theta}}$

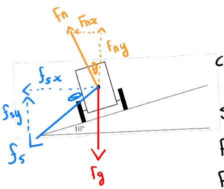
$\boxed{V = \frac{2\pi r}{T}}$: this equation is used in circular motion and tells you the speed of an object moving in circular path with radius r, Period T

Banked turns (Fast)

- at high speed

if a car is going too fast the required centripetal force will be too large for the horizontal component the car will want to slide outward from the center of the curve

- the static friction will act downwards (towards the center of the center) to help provide the nessary centripetal force and prevent the car from sliding outwards



x-direction

SOH-CAH-TOA

$\cos\theta = \frac{f_{sx}}{f_s} \Rightarrow f_s \cos\theta = f_{sx}$

$\sin\theta = \frac{F_{nx}}{F_n} \Rightarrow F_n \sin\theta = F_{nx}$

$F_{net} = F_n \sin\theta + f_s \cos\theta$

$F_{net} = m \cdot \frac{v^2}{r}$ $\quad f_s \leq \mu_s F_n$

$$\boxed{m \cdot \frac{v^2}{r} = F_n \sin\theta + \mu_s F_n \cos\theta}$$

y-direction

$\cos\theta = \frac{F_{ny}}{F_n} = F_n \cos\theta = F_{ny}$

$\sin\theta = \frac{f_{sy}}{f_s} = f_s \sin\theta = f_{sy}$

$F_{net} = -mg + F_n \cos\theta - f_s \sin\theta$

F_{net} is zero in the y-direction because the car only travels horizontally not vertically

$f_s \leq \mu_s F_n$

$$\boxed{0 = -mg + F_n \cos\theta - \mu_s F_n \sin\theta}$$

we want to find the maximum speed we can go on the curve

$F_n(\cos\theta - \mu_s \sin\theta) = mg$

$F_n = \frac{mg}{\cos\theta - \mu_s \sin\theta}$

$m \cdot \frac{v^2}{r} = F_n(\sin\theta + \mu_s \cos\theta)$

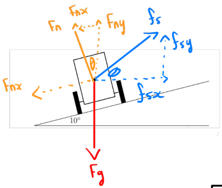
$\cancel{m} \cdot \frac{v^2}{r} = \left(\frac{\cancel{m}g}{\cos\theta - \mu_s \sin\theta}\right) \cdot (\sin\theta + \mu_s \cos\theta)$

$$\boxed{V_{max} = \sqrt{\frac{gr(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta}}}$$

banked turns (slow)

- if the car is moving too slow the required centripetal force will be too small the car won't be able to maintain its circular path without help

- in this case static friction will act up the blank (away from the center) to help provide the centripetal force needed to keep the car on its circular path



x-direction

$\sin\theta = \frac{F_{nx}}{F_n} = F_n \sin\theta = F_{nx}$

$\cos\theta = \frac{f_{sx}}{f_s} = f_s \cos\theta = f_{sx}$

$F_{netx} = F_n \sin\theta - f_s \cos\theta$

$F_{net} = m \cdot \frac{v^2}{r}$ $\qquad f_s \leq \mu_s \cdot F_n$

$$\boxed{m \cdot \frac{v^2}{r} = F_n \sin\theta - \mu_s F_n \cos\theta}$$

y-direction

$\cos\theta = \frac{F_{ny}}{F_n} = F_n \cos\theta = F_{ny}$

$\sin\theta = \frac{f_{sy}}{f_s} = f_s \sin\theta = f_{sy}$

$F_{nety} = -mg + F_n \cos\theta + f_s \sin\theta$

Fnet is zero because the car is traveling on the horizontal not on the vertical

$$\boxed{0 = -mg + F_n \cos\theta + \mu_s F_n \sin\theta}$$

we want to find the minimum velocity we can go on the curve

$F_n(\cos\theta + \mu_s \sin\theta) = mg$

$F_n = \frac{mg}{\cos\theta + \mu_s \sin\theta}$

$m \cdot \frac{v^2}{r} = F_n(\sin\theta - \mu_s \cos\theta)$

$\cancel{m} \cdot \frac{v^2}{r} = \left(\frac{\cancel{m}g}{\cos\theta + \mu_s \sin\theta}\right)(\sin\theta - \mu_s \cos\theta)$

$$\boxed{V_{min} = \sqrt{\frac{gr(\sin\theta - \mu_s \cos\theta)}{\cos\theta + \mu_s \sin\theta}}}$$

Summary of equations

banked slow

$$\boxed{V_{min} = \sqrt{\frac{gr(\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta}}}$$

$$\boxed{m \cdot \frac{v^2}{r} = F_n \sin\theta - \mu_s F_n \cos\theta}$$

$$\boxed{0 = -mg + F_n \cos\theta + \mu_s F_n \sin\theta}$$

banked fast

$$\boxed{V_{max} = \sqrt{\frac{gr(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta}}}$$

$$\boxed{0 = -mg + F_n \cos\theta - \mu_s F_n \sin\theta}$$

$$\boxed{m \cdot \frac{v^2}{r} = F_n \sin\theta + \mu_s F_n \cos\theta}$$

$$\boxed{V = \sqrt{\mu_s g r}}$$

$$\boxed{V = \frac{2\pi r}{T}}$$

$$\boxed{V = \sqrt{gr \tan\theta}}$$

$$\boxed{F_c = m \cdot \frac{v^2}{r}}$$

↓ This is the net Force

$F_{net} = F_c$

$$\boxed{a_c = \frac{v^2}{r}}$$