AC circuts

Resonanace occurs when a circut naturally osicillates at a particular frequency

- resonance occurs when inductive reatance (x_L) and capacitive reactance (x_C) are equal to each other or another way to say this is when reactance is elimated:

$$x_L = x_C \overset{or}{\Rightarrow} x_L - x_C = 0$$

where x represents reactance

Resonance

- If $|z|$ has a local minimum, the system has a locally larger current response, and the system is a *resonator*.
- If $|z|$ has a local maximum, the system has a locally smaller current response, and acts as an *anti-resonator*.

an LRC circut obeys the same differential equation as a damped harmonic oscillator (which means it can show reasonace when driven by alternating current (AC) emf)

in AC circuts the imaginary part of the impedance (b) becomes zero when the inductor and capacitor cancel each other out

$$z = a + \underbrace{bi}_{\to 0}$$

when you have $z_{eq} = a$ that means the current and emf are in phase (they point in the same direction)

$$I = \frac{\mathcal{E} e^{i\omega t}}{R}$$

✅ WHEN this equation applies

This formula is **only valid for a *purely resistive* AC circuit**, meaning:

- **There is only a resistor**
- **No inductor** → $X_L = 0$
- **No capacitor** → $X_C = 0$
- Therefore **no reactance**, no imaginary part

So the impedance is simply:

$$Z = R$$

When asked to write equation in Polar Form using the two equations below

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \quad \text{and} \quad Z_{eq} = |Z|\, e^{i\theta}$$

you get the b and a from $z = a + bi$

magnitude "Pythagleom therom"

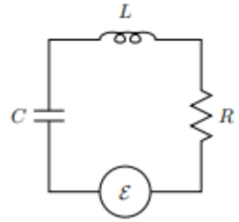
$$W_0 = \sqrt{\frac{1}{LC}}$$

✅ WHEN YOU CAN USE $\omega_0 = \sqrt{\frac{1}{LC}}$

This formula applies **ONLY** when you are finding the **resonant angular frequency** of a circuit that contains:

- an **inductor** L
- a **capacitor** C

Consider a series LRC series circuit driven by an AC voltage source with angular frequency ω like the one we examined last class, a schematic of which is shown below



1. In terms of C, L, R, and ω what is the total impedance of this LRC circuit?
2. Notice that the real part of Z is just the resistance. The imaginary part is called the *reactance*, denoted X. At high frequencies, is the reactance dominated by the *capacitive reactance* X_C or the *inductive reactance* X_L? What about at low frequencies?
3. Notice that the magnitude of the oscillating current $|I|$ will be at an extremum whenever the absolute value of the total impedance $|Z|$ is at the opposite extremum (that is, whenever $|Z|$ has a maximum, $|I|$ has a minimum, and vice versa). Using what you know about complex numbers, find $|Z|$, and then take a derivative with respect to ω to find the resonance frequency ω_r corresponding to the maximal current response.
4. Now rewrite Z for this series LRC circuit in polar form.
5. A way of depicting I and ΔV is to use a *phasor* (short for *phase vector*). To draw a phasor, we plot I or V in the complex plane and then draw a vector from the origin to that point. Try sketching a phasor for $\Delta V = \mathcal{E}e^{i\omega t}$ for a case where ωt is between π and $\frac{3\pi}{2}$.
6. Notice that the phasors for $I(t)$ and $\mathcal{E}(t)$ will, generically, point in slightly different directions. Under what circumstances will the phasor for $I(t)$ be ahead of that for $V(t)$? (Notice this would mean the sinusoidal peaks of the $I(t)$ graph would happen at a time ahead of those for the $V(t)$ graph.) Under what circumstances will the phasor for the $I(t)$ graph lag behind the $V(t)$ graph? Hint: remember from last class the impedance was

$$Z = \frac{\Delta V}{I} \tag{3.49}$$

7. Under what circumstances while the two phasors for $I(t)$ and $\mathcal{E}(t)$ point the same direction? How does this condition compare to the resonance peak you found in question three?

1) notice this circut is all in series so you can use this impedance formula:

$$Z_{eq} = Z_1 + Z_2$$

also notice that it has a inductor (L), capacitor (C), and resistor (R) so you can use these impedance equations $Z_R = R$, $Z_C = \frac{1}{i\omega C}$, and $Z_L = i\omega L$ Plug these into series impedance equation

$$Z_{eq} = R + \frac{1}{i\omega C} + i\omega L$$

notice how they are broken into two part real and imaginary you want to combine the two imaginary terms but keep both the real and imaginary seperate

$\frac{1}{i} = -i$ · you don't want to keep imaginary in denominator

$$Z = R - \frac{i}{\omega C} + i\omega L \implies Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

notice how these are in complex number form $z = a + bi$

2)

the imaginary part is called the reactance because it replesent energy that osscillates back and fourth between the source and reactive component (inductor or capacitor) R is the resistor which is the real part of impedance it represents true energy dissipation: energy converted into heat unlike reactance it does not store energy it consumes it

1. Complex impedance

For a series RLC circuit:

$$Z = R + j(X_L - X_C)$$

- **Real part** R → resistance (doesn't depend on frequency)
- **Imaginary part** $X = X_L - X_C$ → reactance (depends on frequency)

reactance is only caused by inductors and capacitors

capictors and inductors in Ac current shift the phase of our waves

- inductive reactance: $x_L = \omega L$ → increases with Frequency
- capacitive reactance: $x_c = 1/\omega c$ → decreases with Frequency

2. Reactance definitions

- Inductive reactance: $X_L = \omega L$ → **increases with frequency**
- Capacitive reactance: $X_C = \frac{1}{\omega C}$ → **decreases with frequency**

at high Frequencies (ω), ω is big so $\frac{1}{\omega c} \ll \omega L$ or in other words $x_L = \omega L$ is much larger then $x_c = 1/\omega c$ so the inductor dominates the reactance

3. High-frequency behavior

- At **high** ω (large frequency):

$$X_L = \omega L \text{ is large}, \quad X_C = \frac{1}{\omega C} \text{ is small}$$

- So the **inductor dominates** the reactance.

at low Frequencies (ω), ω is small so $\frac{1}{\omega c} \gg \omega L$ or in other words $x_L = xL$ is small and $x_c = 1/\omega c$ is much larger so the capacitor dominates the reactance

4. Low-frequency behavior

- At **low** ω (small frequency):

$$X_L = \omega L \text{ is small}, \quad X_C = \frac{1}{\omega C} \text{ is large}$$

- So the **capacitor dominates** the reactance.

✅ **Key takeaway**

- **Real part of Z** → always just R
- **Imaginary part of Z (reactance)** → changes with frequency:
  - **High frequency** → **inductor dominates**
  - **Low frequency** → **capacitor dominates**
- Resonance occurs when the inductor and capacitor **exactly cancel each other**, i.e., $X_L = X_C$.

3) We will Find the magnitude by apply "Pythagreom therom"

$$|z| = \sqrt{(R)^2 + \left(i\left(\omega L - \frac{1}{\omega C}\right)\right)^2}$$

$i^2 = -1$

$$|z| = \sqrt{(R)^2 - \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Now take the dervative with respect to ω

$$|z| = \left((R)^2 - \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}$$

$$\frac{1}{\cancel{2}}\left((R)^2 - \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{-1/2} \cdot \left(-\cancel{2}\left(\omega L - \frac{1}{\omega C}\right)\right)\left(L + \frac{1}{\omega^2 C}\right)$$

$$\boxed{|z| = \frac{-\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)}{\sqrt{(R)^2 - \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

the critical Points are where

$$\omega L - \frac{1}{\omega C} = 0 \implies \cancel{+}\frac{1}{\omega C} = \cancel{+}\,\omega L \implies \sqrt{\omega^2} = \sqrt{\frac{1}{LC}} \implies \boxed{\omega = \pm\sqrt{\frac{1}{LC}}}$$

$$L + \frac{1}{\omega^2 C} = 0 \implies \frac{1}{\omega^2} = -LC \implies \sqrt{\omega^2} = \sqrt{\frac{1}{-LC}} \implies \boxed{\omega = \pm\sqrt{\frac{1}{LC}}\, i}$$

4)

to write this in Polar Form you can use $\tan\theta = \frac{b}{a}$

$$Z = \underbrace{R}_{a} + i\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{b}$$

$$\tan\theta = \frac{\omega L}{R} - \frac{1}{\omega C} \longrightarrow \theta = \tan^{-1}\left(\frac{\omega L}{R} - \frac{1}{\omega C}\right)$$

in exponential Form, a complex is written as $Z = |Z| e^{i\theta}$

$$\boxed{Z = \left(\sqrt{(R)^2 + \left(i\left(\omega L - \frac{1}{\omega C}\right)\right)^2}\right) e^{i\tan^{-1}\left(\frac{\omega L}{R} - \frac{1}{\omega C}\right)}}$$

6)

a Phasor is a way to represent a sinusoidal signal (like Ac voltage or current) as a rotating vector

in Polar Form $\Delta V = \mathcal{E} e^{i\omega t}$ and $Z = |Z| e^{i\theta}$

$$I = \frac{\Delta V}{Z} = \frac{\mathcal{E} e^{i\omega t}}{|Z| e^{i\theta}} \Rightarrow$$

$V = IR$

$\mathcal{E} = IZ$

$I = \frac{\mathcal{E}}{Z}$

$i\omega t - i\theta$

$i(\omega t - \theta)$

$$\Rightarrow \boxed{|I| = e^{i(\omega t - \theta)}}$$

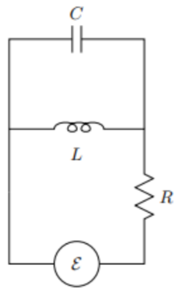
7)

the two Phasors will be in Phase when z is real which happens when $\theta = 0$, $x_L = x_C$ and:

$$\omega L = \frac{1}{\omega C} \implies \boxed{\omega = \sqrt{\frac{1}{LC}}}$$

3.9.2 A Second LRC Circuit

Consider now the case of an AC voltage input of angular frequency ω to the circuit depicted below, in which the capacitor and inductor are in parallel.



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1. What is the impedance Z of this LRC circuit?
2. Find an expression for $|Z|$, and then open up a computer and plot $|Z|$ as a function of ω in Desmos. Is there a value for ω at which $|Z|$ has an extremum? Is this circuit therefore an example of a resonator or an antiresonator?

1) notice that the inductor and capacitor are in parallel so we will use this equation:

$$Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

the impedance for a capacitor is $Z_C = 1/i\omega C$, for inductor its $Z_L = i\omega L$

now plug these into the equation

$$Z = i\omega C + \frac{1}{i\omega L} \implies \frac{1 - \omega^2 LC}{i\omega L}$$

\# flip these around

now you will combine this in series with the left over resistor which has impedance $Z_R = R$ using this equation:

$$Z_{eq} = Z_1 + Z_2$$

now you can combine them in series

$$\boxed{Z_{eq} = R + \frac{i\omega L}{1-\omega^2 LC}}$$

2) we will now Find the magnitude For $|Z|$ For that we will do "Pythagreom therom"

$$|Z| = \sqrt{(R)^2 + \left(\frac{i\omega L}{1-\omega^2 LC}\right)^2}$$

now take the derivative with respect to angular Frequency (ω)

$$|Z| = \left((R)^2 + \left(\frac{i\omega L}{1-\omega^2 LC}\right)^2\right)^{1/2} \Longrightarrow \frac{1}{\cancel{2}}\left((R)^2 + \left(\frac{i\omega L}{1-\omega^2 LC}\right)^2\right)^{-1/2} \cancel{2}\left(\frac{i\omega L}{1-\omega^2 LC}\right)\ldots\ldots$$

$$\frac{\frac{df}{dx}\cdot g - f\cdot\frac{dg}{dx}}{g^2}$$

$F = i\omega L$ $\quad \frac{df}{d\omega} = iL$

$g = 1-\omega^2 LC$ $\quad \frac{dg}{d\omega} = -2\omega LC$

$$\frac{(iL)(1-\omega^2 LC) - (i\omega L)(-2\omega LC)}{(1-\omega^2 LC)^2}$$

$$= \frac{iL - \omega^2 L^2 C i - 2\omega^2 L^2 i C}{(1-\omega^2 LC)^2} \longrightarrow \frac{\left(\frac{\omega L.}{1-\omega^2 LC}\right)\left(\frac{L(1-\omega^2 LC) + 2\omega^2 L^2 C}{(1-\omega^2 LC)^2}\right)}{\sqrt{R^2 + \left(\frac{\omega L}{1-\omega^2 L}\right)^2}}$$

$$\longrightarrow \frac{\omega L^2(1+\omega^2 LC)}{(1-\omega^2 LC)^3\sqrt{R^2 + \left(\frac{\omega L}{1-\omega^2 LC}\right)^2}}$$

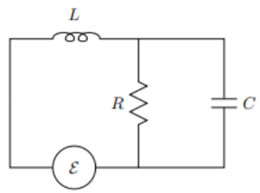
$1-\omega^2 LC = \quad \omega = \sqrt{\frac{1}{LC}}$

✱ we need to understand when ω (angular Frequency) goes to zero to know when that happens we set ω to zero

You can also plug this into ω (angular frequency) and you can see that it will take the z and see that it will take $z \to \infty$

3.9.3 Another Definition of Resonance

In the two previous circuits, note that the resonance frequency corresponded to the point where the total reactance was zero and therefore the total impedance was purely real, meaning that the phasors for $\mathcal{E}$ and the response current I pointed in the same direction. While it is not always the case that the point of vanishing X corresponds to an extremum of $|Z|$, the point of vanishing X is one definition of the resonance frequency. For the circuit depicted below, determine the overall impedance Z and then find the frequency ω at which the total reactance vanishes.



What you can do here is instead of taking the derivative you can get them into complex form $(a+bi)$ and just set the B equal to zero this method works for all AC circuts

Notice the resistor and capacitor are in parallel so you can use this equation

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

the impedance for a resistor is $Z_R = R$, impedance for a capacitor is $Z_C = 1/i\omega C$

$$\frac{1}{Z_{eq}} = \frac{1}{R} + i\omega C \longrightarrow \frac{1 + i\omega CR}{R} \longrightarrow Z_{eq} = \frac{R}{1 + i\omega CR}$$

now we add on the inductor which is in series: $Z_{eq} = Z_1 + Z_2$ the impedance for a inductor is $Z_L = i\omega L$

$$Z_{eq} = i\omega L + \frac{R}{1 + i\omega CR} \cdot \frac{(1 - i\omega CR)}{(1 - i\omega CR)}$$

• multiply by the conjucate

$$Z_{eq} = i\omega L + \frac{R - i\omega CR^2}{1 - \cancel{i\omega CR^2} + \cancel{i\omega CR^2} + \omega^2 C^2 R^2} \longrightarrow Z = i\omega L + \frac{R - i\omega CR^2}{1 + \omega^2 C^2 R^2}$$

$$Z = i\omega L + \frac{R - i\omega C R^2}{1 + \omega^2 C^2 R^2} \longrightarrow Z = i\omega L + \frac{R}{1+\omega^2 C^2 R^2} - \frac{i\omega C R^2}{1+\omega^2 C^2 R^2}$$

now get the terms with i togther

$$Z = \frac{R}{1+\omega^2 C^2 R^2} + i\left(\omega L - \frac{\omega C R^2}{1+\omega^2 C^2 R^2}\right)$$

notice now this is in $Z = a + bi$ Format

You will take b and set it to Zero

$$\omega L - \frac{\omega C R^2}{1+\omega^2 C^2 R^2} = 0 \longrightarrow \frac{\omega L}{1} = \frac{\omega C R^2}{1+\omega^2 C^2 R^2} \longrightarrow \cancel{\omega} C R^2 = \cancel{\omega} L (1 + \omega^2 C^2 R^2)$$

$$C R^2 = L + \omega^2 L C^2 R^2 \longrightarrow \frac{CR^2 - L}{L^2 C^2 R^2} = \omega^2 \longrightarrow \boxed{\omega = \sqrt{\frac{CR^2 - L}{L^2 C^2 R^2}}}$$

3.9 AC LRC Circuits - Class Period #25

1. A real inductor of inductance L and internal resistance R_L is wired in series with a real leaky capacitor of internal resistance R_C and capacitance C and an AC EMF source of angular frequency ω.
   (a) What is the total impedance Z of this circuit?
   (b) In the DC limit (i.e. $\omega \to 0$) what is the total impedance of this circuit?
   (c) At what angular frequency ω is the current response of this circuit in phase with the input EMF?

Parallel to series

a) Whenever the word "leaky" is used almost always means Parallel resistor so the inductance and resistance are in series and the capacitor is Parallel to them

$$Z_{eq} = Z_1 + Z_2 \implies Z_R = R \text{ and } Z_L = i\omega L \implies Z_L = R_L + i\omega L$$

now the "leaky" capacitor is parallel so:

$$\frac{1}{Z_C} = \frac{1}{R_C} = i\omega C \implies \frac{R_C}{1 + i\omega R_C C}$$

b) what they mean by DC limit is set angular Frequency (ω) to zero

$$Z = Z_R + Z_L \Rightarrow R_L + i\omega L + \frac{R_c}{1 + i\omega R_c C}$$

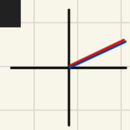
$$\boxed{Z = R_L + R_c}$$

c) what this question is asking Find the Frequency where the imaginary Part of impedance is zero

1️⃣ Phase angle

- In AC circuits, voltage and current are **phasors** (vectors in the complex plane).
- The **phase angle** ϕ is the angle between the **voltage vector** V and the **current vector** I.
- If the vectors are aligned → $\phi = 0$ → **current in phase with voltage**.

Vectors are alighned meaing they look something like this:



· the angle between them is zero

$$Z = R_L + i\omega L + \frac{R_c}{1 + i\omega R_c C} \cdot \frac{(1 - i\omega R_c C)}{(1 - i\omega R_c C)} \Rightarrow Z = R_L + i\omega L + \frac{R_c - i\omega R_c^2 C}{1 - \cancel{i\omega R_c C} + \cancel{i\omega R_c C} + \omega^2 R_c^2 C^2}$$

$$Z = R_L + i\omega L + \frac{R_c - i\omega R_c^2 C}{1 + \omega^2 R_c^2 C^2} \Rightarrow Z = R_L + i\omega L + \frac{R_c}{1 + \omega^2 R_c^2 C^2} - \frac{i\omega R_c^2 C}{1 + \omega^2 R_c^2 C^2}$$

- The **resistive part** R is always **in phase** with the current.
- The **reactive part** X (from L and C) is always **90° out of phase**.
- To make the **total voltage vector align with the current** (phase angle = 0), the **out-of-phase reactive component must be zero**.

So yes: the **only way** for current and EMF to be in phase is to **eliminate the reactive (imaginary) part**.

* the resistive Part is always in Phase with the current but the reactive Part (L and C) is alway 90° out of Phase or lagging

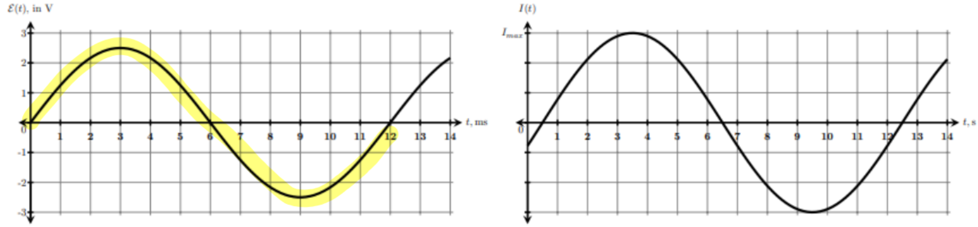
with all that said to get the current response of this circut in Phase with EMF we need to eliminate the lag by setting x (or the Part with imaginary to zero)

$$\omega L - \frac{\omega R_c^2 C}{1+\omega^2 R_c^2 C^2} = 0 \Rightarrow \frac{\omega L}{1} = \frac{\omega R_c^2 C}{1+\omega^2 R_c^2 C^2} \Rightarrow \cancel{\omega} L(1+\omega^2 R_c^2 C^2) = \cancel{\omega} R_c^2 C$$

$$L + L\omega^2 R_c^2 C^2 = R_c^2 C \Rightarrow L\omega^2 R_c^2 C^2 = R_c^2 C - L \Rightarrow \omega^2 = \frac{R_c^2 C - L}{L R_c^2 C^2}$$

$$\boxed{\omega = \sqrt{\frac{R_c^2 C - L}{L R_c^2 C^2}}}$$

2. The graph below on the left shows the EMF $\mathcal{E}$ input into a series LRC circuit consisting of a 1.5 mH inductor, a 1.2Ω resistor, and an unknown ideal capacitor. The graph on the right then shows the current in the circuit, though the maximum current I_{max} is not known.



(a) (*Draws on Class #1*) What is ω, the angular frequency of this AC EMF?

(b) What is the phase difference $\Delta\theta$ between the EMF and the current? Then use this value to calculate the capacitance C of the unknown capacitor.

(c) What is the unknown maximal current I_{max}?

1) remember angular Frequency is $\omega = \frac{2\pi}{T}$ or $2\pi f$ the Period (T) is how long it takes to complete one ossillation which takes 12ms= 0.012s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.012} = \frac{500\pi}{3} = \boxed{523.6\ s^{-1}}$$

2) the $\Delta\theta$ is asking for the Phase difference between the voltage and the current the equation for $\Delta\theta$ is $\Delta\theta = \omega \Delta t$

$$\Delta\theta = \omega \Delta t$$

angular Frequency

the time delay between voltage wave and the current wave

in other words what the Δt is asking for is how long will the EMF wave take to reach the I wave this is a estimate

$$\Delta\theta = \frac{2\pi}{(12ms)}(0.5ms) = \boxed{\frac{\pi}{12}}$$

to Find the Capacitor we will use equation that relates Phase angle between the voltage and the current in a LRC circut:

$$\tan\Delta\theta = \frac{\omega L}{R} - \frac{1}{\omega RC} \Rightarrow \tan(\Delta\theta) = \frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right) \Rightarrow R\tan(\Delta\theta) = \omega L - \frac{1}{\omega C}$$

$$R\tan(\Delta\theta) - \omega L = -\frac{1}{\omega C} \Rightarrow \frac{\omega R\tan(\Delta\theta) - \omega^2 L}{1} = -\frac{1}{C}$$

$$C\left(\omega R\tan(\Delta\theta) - \omega^2 L\right) = -1 \Rightarrow C = \frac{-1}{\left(\omega R\tan(\Delta\theta) - \omega^2 L\right)}$$

$$C = \frac{-1}{(523.6)(1.2)\tan(\frac{\pi}{12}) - (523.6)^2(0.0015)} = \boxed{3.7\times10^{-3}\,F}$$

c) to Find the maximum current We will use $I = \mathcal{E}/|Z|$ which can also be used even when you arent Finding the current

$$I = \frac{\mathcal{E}}{|Z|} \rightsquigarrow Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \Rightarrow I = \frac{\mathcal{E}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

We use this because we know the maximum EmF which can be read from the graph

$$I = \frac{2.5}{\sqrt{(1.2)^2 + \left((523.6)(1.5\times10^{-3}) - \frac{1}{(523.6)(4.12\times10^{-3})}\right)^2}} = \boxed{2.01\ A}$$

3. An inductor of inductance L, capacitor of capacitance C, and resistor of resistance R are combined in parallel with one another and then driven by an AC voltage source whose EMF varies sinusoidally with angular frequency ω.

(a) What is the overall impedance Z of this parallel LRC circuit?

(b) In the DC limit, what is the overall impedance Z of this circuit? In this limit, which circuit element (i.e., the inductor, capacitor, or resistor) is the direct current passing through?

(c) At what value of ω, if any, does the total reactance of this circuit vanish?

a) because they are all combined in Parallel we will use this Parallel equation

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \Rightarrow \frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C \Rightarrow \frac{i\omega L + R + \omega^2 RLC}{i\omega L R}$$

* Flip these around:

$$\boxed{Z_{eq} = \frac{i\omega RL}{R - \omega^2 RLC + i\omega L}}$$

2) remember when it says DC limit you take $\omega \to 0$

$$Z_{eq} = \frac{i\omega RL}{R - \omega^2 RLC + i\omega L} \Rightarrow \frac{0}{R} = 0$$

3) remember you need to get this into complex form ($z = a + bi$) and set the imaginary to zero

$$z = \frac{i\omega RL}{(R - \omega^2 LRC + i\omega L)} \cdot \frac{(R - \omega^2 RLC - i\omega L)}{(R - \omega^2 RLC - i\omega L)} = \frac{i\omega R^2 L - i\omega^3 R^2 L^2 C + \omega^2 RL^2}{(R - \omega^2 RLC)^2 + \omega^2 L^2}$$

set these to zero as they have imaginary numbers in them

$$\omega R^2 L - \omega^3 R^2 L^2 C = 0 \Rightarrow \omega R^2 L (1 - \omega^2 LC) = 0$$

$$\omega R^2 L = 0 \qquad 1 - \omega^2 LC = 0$$

$$\boxed{\omega = 0} \qquad \boxed{\omega = \pm\sqrt{\frac{1}{LC}}}$$