angular momentum

refresher: angular means "rotation" things with angular motion are moving in a circular path but the object does not need to be round

- linear momentum

$\vec{P} = m\vec{v}$

- angular momentum

$\vec{L} = I\vec{\omega}$

I = moment of interia

ω = angular velocity

rotational second law

$\tau = I\alpha + \frac{dI}{dt}\cdot\omega$

- this is a more accurate version of $\tau = I\alpha$

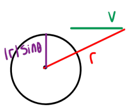
$\text{final}_I - \text{inital}_I$

- moment of interia "I": in the first half depends on the shape and how the mass is distrbuted
- rate of change of interia "$\frac{dI}{dt}$": is the final - intial of moment of interia of that shape you might need to use parallel axis therom

Point mass angular momentum

$\vec{L} = \vec{r} \times \vec{P} \Rightarrow |\vec{L}| = |\vec{r}|\sin\theta|\vec{P}|$

this works even for a straight-line motion as long as your looking at the angular momentum relative to a point or axis which means you looking at it from a point thats not changing



- r: (radius) from the pivot to velocity
- v: (velocity) will mostly be an object that has mass meaning it can be treated as momentum due to it having velocity and mass

r = (radius) thumb

P = (momentum) index · (looking at picture this will be velocity, it has velocity/mass it won't be labeled as P)

L = (angular momentum) middle finger

torque (τ)

torque is the change in angular momentum

$\tau = \frac{d\vec{L}}{dt}$

angular momentum conservation

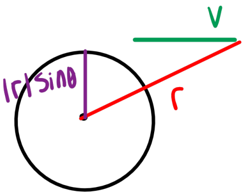
angular momentum is conserved if the net torque is zero

$\tau_{net} = 0 \Rightarrow \frac{dL}{dt} = 0$

- Just like linear momentum the statement of conservation of momentum can be though of as if energy comes in it either stays or goes out equaly like an action/reaction (newton 3rd law)

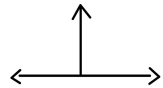
$\Rightarrow L_{final} - L_{inital}$

only for circular
cannot be applied to straight lines



$|\vec{r}| \sin\theta$ = represents perpendicular distance from the axis of rotation to the line of motion of the object

perpendicular lines



Statics

- a system is in static equalibrium if is neither moving or changing how its moving
  - this means that

    $\vec{F} = 0$

    $\vec{\tau} = 0$

- For the system to be zero it has to be torque going in and a torque going out

$$\tau_{in} = \tau_{out}$$

- r is measured from the axis of rotation (which is often the pivot) to where the force is applied

- when you see uniform density it is the center of mass which is the gravity location so if the object has uniform density then the center of mass is the location where gravity acts

Summary of equations

$\vec{L} = I\vec{\omega}$

$\tau = \frac{d\vec{L}}{dt}$

$\tau = I\alpha + \frac{dI}{dt} \cdot \omega$

$\vec{L} = \vec{r} \times \vec{p} \Rightarrow |\vec{L}| = |\vec{r}| \sin\theta |\vec{p}|$

$\tau_{net} = 0 \Rightarrow \frac{dL}{dt} = 0 \Rightarrow L_{final} - L_{inital}$