Amperes law and Magnetism

to describe magnetism, we introduce a New Vector Field called the Magnetic Flux density $\vec{B}$

* the Magnetic Flux density is distinct From the Magnetic Field $\vec{H}$
  - $\vec{B}$ and $\vec{H}$ have Similar relation as $\vec{E}$ (electric Field) and $\vec{D}$ (electric Flux density)

Mono Pole: a hypothetical north or South Pole existing alone

net Magnetic Charges (MonoPole) Never exist because everytime you cut a Magnet you get a North and South Pole

if total magnetic monoPoles existed then

$$\oint_A \vec{B} \cdot d\vec{A} = \mu_0 Q_{m,\text{enclosed}}$$

which would tell us how much net Magnetic Charge is in that surface

but in real life $Q_{m,\text{enclosed}} = 0$ always the reasoning why this occurs is every magnetic Field line that leaves a surface also enters it again somewhere else So in simplier terms Magnetic Field lines go in and out of a surface in loops or circles and these Magnetic are caused by moving electric charges

So gauss's law For Magnetism is:

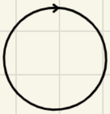
$$\oint_A \vec{B} \cdot d\vec{A} = 0$$

gauss's law For magnetism divergence Form

$$\nabla \cdot \vec{B} = 0$$

based on what we know about electricity electric field lines began on positive charges and ended on negitive charges well for magnetism, the magnetic field lines would began on positive magnetic charges and end on negitive magnetic charges but we have said we don't have positive magnetic charges and we don't have negitive magnetic charges that means they can't began or. end

hence magnetic flux density lines will form closed loops



- Magnetic mono poles
- While no magnetic monopoles have yet to be observed, their existence is possible
- Such monopole would source a $\vec{B}$

$$\vec{B} = \frac{Q_m}{4\pi r^2}\hat{r}$$

- monopole- monopole interaction would the resemble coulomb's law:

$$\vec{F}_m = \frac{Q_{m1}\, Q_{m2}}{4\pi \mu_0 r^2}\hat{r}$$

- ampere's law

$$\mu_0 = 4\pi \times 10^{-7}\ \frac{T \cdot m}{A} \qquad (T = \text{tesla})$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

it relates magnetic field along a closed loop to the current passing through that loop key idea: current (moving charges) are the source of magnetic fields

field lines form loops around currents

• notes on ampere's law

- amperes law only applies for static charges we would need to modify it for dynamic ones

- we also assumed vaccum

Static charges: steady currents that don't vary with time

• amperes law in differential form

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

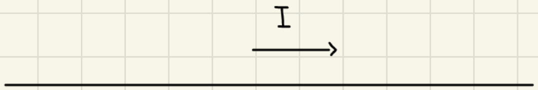
$\vec{J}$ is the current density

• Field of a long, straight wire

z = length of wire

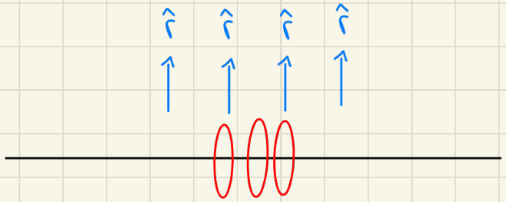
r = distance some length perpendicular to the wire

θ = angle around it



We cannot depend on z or θ because if we change z we just slide along the wire and if we change θ im just rotating which way i'm spinning the wire around but that dosen't look any different so the only thing B can depend on is r

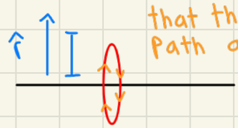
$$\vec{B} = \vec{B}(r)$$



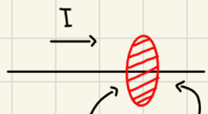
• the loops are the the magnetic field lines $\vec{B}$ notice how $\vec{B}$ points along $\hat{\theta}$ direction

You can also think of the reasoning on why $\vec{B}$ depends on θ instead of $\hat{r}$ like this: if $\vec{B}$ depends on $\hat{r}$ it would Not be able to Form a closed loop So For it to Form a closed loop it needs to depend on θ

but another area of confusion pops up if $\vec{B}$ depends on θ why do we have $B(r)$ well that r is how far the loop extends outward like the radius but the movement of $\vec{B}(r)$ is only depends on $\hat{\theta}$ by following the $\hat{\theta}$ $\vec{B}$ forms a loop/circle



that the movment of $\vec{B}$ going in a circular path along θ



Flux is equal on both side and opposite

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I \longrightarrow \oint B(r)\hat{\theta}\cdot d\vec{l} \rightarrow B(r)\hat{\theta}\oint d\vec{l} = 2\pi r B(r)$$

$B(r)$ is a constant

$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}\hat{\theta}}$$

this here is the magnetic flux density of a infinitly long wire

Summary of gauss's law for closed surface: any closed surface must have No net flux through it

• Biot-Savart law

the Biot-Savart law is more Fundmental it can be used For any Shape of wire even though there is No symmetry

amperes law is more Simplified Version that works when there is Symmetry (long straight wire, Solenoid, toroid)

its essentially a way to "add up" (integrate) the tiny contributions of the current carrying wire

$d\ell$: this Vector Points along the wire, in the direction of current Flow

example: if current is going upward $d\ell$ Points up

$\hat{r}$: this is a unit Vector Pointing From the current element (where the source is) toward the Point where your Calculating the Field

example: if your calculating B at a Point to the right of the wire $\hat{r}$ Points right

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{|\vec{r}|^2}$$

this result can also be written in terms of the density by using $\vec{J}\, dA\, d\ell = \vec{J}\, dV$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \hat{r}}{|\vec{r}|^2}\, dV$$

$$B = \frac{-\mu_0 I R^2}{2(r^2+R^2)^{3/2}} \hat{x}$$

this is the magnetic Field Produced by a circular loop, derived directely From the Biot- Savart law

Notice in the limit that $r \to \infty$

$$\lim_{r \to \infty} \vec{B} = -\frac{\mu_0 I R^2}{2r^3} \hat{x}$$

the Far Field limit means you are very far away From the Source of the magnetic Field that is $r >> R$ when $r >> R$, the loop looks like a single Point to you like how the sun and moon look like small disks even though they are huge, because they are so far away so back to how this relates to dipole at this distance the detailed shape of the loop does not matter the magnetic Field behaves like a dipole

now if $r >> R$, then $R^2 + r^2 \approx r^2$

- defining a magnetic dipole moment

- For a current loop, the magnitude dipole moment is:

$$m = I A$$

where:

I = current through the loop

A = area of the loop

For a circular loop, $A = \pi R^2$

So

$$m = -I \pi R^2 \hat{x}$$

thus, For a current loop in the far limit:

$$\vec{B} = \frac{\mu_0 \vec{m}}{2\pi r^3}$$

this is the magnetic dipole moment of current loop

$$M = AI\,\hat{r}$$

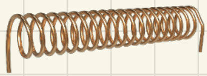
Where A is the area enclosed by the loop and $\hat{r}$ the direction given by the right hand rule

this Form is always correct the component Form like $(-\pi R^2 I\,\hat{x})$ is Just the same thing Just expressed For a Specific loop orientation

M tells you the strength and orientation of the magnetic Field Produced by the loop, especially Far away (the Far Field)

Solenoid

- a solenoid is essentially a coil of wire, usually in spiral shape, designed to produce a magnetic field when a electric current passes through it



When you are dealing with a short or finite solenoid you will use

$$B = \frac{\mu_0 I N L}{2\sqrt{(L/2)^2 + q^2}}$$

$N = \frac{N}{L}$ $N \longrightarrow$ number of loops, $L \longrightarrow$ length

μ_0 = permeability of free space
I = current
n = number of turns per unit length
L = length of solenoid or wire
q = distance from the center

When you are dealing with a long solenoid (length L much greater then its radius (

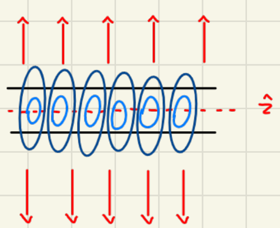
$$B \approx \mu_0 n I$$

$N = \frac{N}{L}$ $N \longrightarrow$ number of loops, $L \longrightarrow$ length

Practice/explinations

Consider the case of an infinitely long thick wire of radius R. Inside the wire there is a uniform current density $\vec{J} = J_0 \hat{z}$ while outside the wire the current density is zero

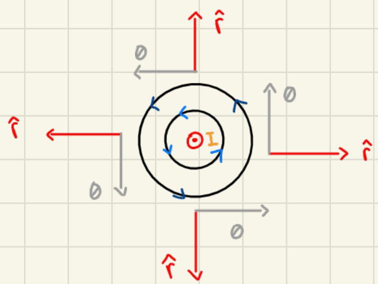
Side view



- $\vec{B}$ field inside wire
- $\vec{B}$ field outside wire
- $\hat{r}$ field

---- = J/I current density

Front View



- z Pointing out
- I current
- $\vec{B}$ Field inside wire
- $\vec{B}$ Field outside wire

$$\nabla \times B = J_0 \hat{z}$$

looking at the vector calc cheatsheet

$$\nabla \times \vec{B} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{z}$$

✱ we don't need all them because the current is only traveling in the $\hat{z}$ direction so you can get rid of the rest

$$\mu_0 \vec{J}_0 \cancel{\hat{z}} = \frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \cancel{\frac{\partial A_r}{\partial \theta}}\right)\cancel{\hat{z}}$$

✱ $\vec{B}$ is going in the θ direction if it wasn't then we violate our loop

$$\mu_0 \vec{J}_0 = \frac{1}{r}\frac{\partial (r B_\theta)}{\partial r} \Rightarrow \int \mu_0 J_0 r \, dr = \int \frac{\partial (r B_\theta)}{\partial r}$$

$$\frac{\mu_0 J r^{\cancel{2}}}{2} = \cancel{r} B_\theta \Rightarrow B_\theta = \frac{\mu_0 J_0 r}{2} + C \Rightarrow \boxed{B_\theta = \frac{\mu_0 J_0 r}{2}}$$

C gets set to zero because the magnetic field must remain finite and symmetric at the center of the wire

Find the $\vec{B}$ outside we were told that the current density outside is zero

$$\nabla \times B = 0$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r} = 0$$

$$\frac{\partial(rB_\theta)}{\partial r} = 0$$

$$\int \frac{\partial(rB_\theta)}{\partial r} = \int 0$$

$$rB_\theta = c$$

$$B_\theta = \frac{c}{r}$$

you can set the boundaries equal and solve for c

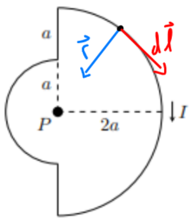
$r = R$

$$\frac{\mu_0 J_0 R}{2} = \frac{c}{R}$$

$$c = \frac{\mu_0 J_0 R^2}{2}$$

$$\boxed{B_\theta = \frac{\mu_0 J_0 R^2}{2r}}$$

where $\vec{r}$ points from the location of the differential length of current carrying wire to the point in space where $\vec{B}$ is being found. (Note here that $\hat{r}$ is a unit vector pointing in the direction of $\vec{r}$, not a standard coordinate basis vector!) Consider the wire loop below, which carries a current of I in the direction pictured. The wire loop consists of two half circles, the first of radius a and the second of radius $2a$, connect by two radial wire segments of length a, as shown.



$d\vec{l} \times \vec{r} \approx d\vec{l}$

this can only happen if $d\vec{l}$ is perpendicular to $\vec{r}$

dl points where ever the current is going

$\hat{r}$ point where ever your point P is

Where is the $\vec{B}$ Pointing?

$d\vec{l}$ = index, $\hat{r}$ = middle Finger and apply the right hand rule
you should be getting inward

another thing you can Notice here is the cross Product vanishes because the angel between the two vectors is 90°

$$A \times B = |A||B|\sin\theta \ @ \ 90° = |A||B|$$

What is the contribution to $\vec{B}$ From the semi circle of radius a?

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{|r|^2} \Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}}{a^2}$$

- you would need to convert to Polar Coordinates because it is a circle

$d\vec{l} = a\,d\theta$

$$\int d\vec{B} = \int_0^{\pi} \frac{\mu_0 I}{4\pi a^2}\,a\,d\theta = \left[\frac{\mu_0 I}{4\pi a}\theta\right]_0^{\pi}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4a}}$$

What is the contributions of $\vec{B}$ From the the semi circle of radius 2a?

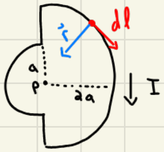
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{|\vec{r}|^2} \Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}}{(2a)^2}$$

- you need to convert to Polar coordinates because it is a circle

$dl = 2a\,d\theta$
the radius Changed to 2a

$$\int d\vec{B} = \int_0^{\pi} \frac{\mu_0 I}{4\pi (2a)^2}\,2a\,d\theta \Rightarrow \left[\frac{\mu_0 I 2\cancel{a}\theta}{16a^{\cancel{2}}\pi}\right]_0^{\pi}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{8a}}$$



$d\vec{l}$ and $\vec{r}$ are perpendicular

$\sin(90) = 1$

$d\vec{l} \times \vec{r} = |d\vec{l}||\vec{r}|$

if you convert r to the position vector in polar coordinites:

$\vec{r} = R\,\hat{r}$

radius unit vector

$$d\vec{l} \times R\hat{r} \rightarrow R\,(d\vec{l} \times 1)$$

- we replaced $\vec{r}$ with $R\hat{r}$ and the unit vector is equal to one so you get:

$R\,d\vec{l}$

$R\,d\theta$

- where R can be any radius you have $(a, 2a)$ and $d\vec{l}$ would be $d\theta$ because we convert to polar coordinates

☆ For this to work $d\vec{l}$ and $\vec{r}$ need to be perpendicular

What is the magnetic dipole moment $\vec{m}$ associated with this wire loop?

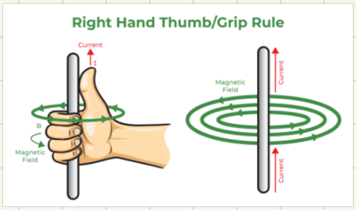
$$m = A I \hat{r}$$

$$\text{Area} = \pi r^2$$

$$m = -\left(\frac{\pi (a)^2}{2} + \frac{\pi (2a)^2}{2}\right) I \hat{r}$$

• dividing by two because its only half the circle

★ to Find what direction $\vec{m}$ is going you will use right hand curl in direction of current



• to Find $\vec{B}$ you aim thumb to where every current is Pointing and curl Fingers

the current is going down so you need to use a Negitive sign

More explanations on Q_m enclosed $= 0$

For gauss's law for magnetism we write

$$\oint B \cdot dA = 0$$

this says the net magnetic flux through any closed surface is zero

in electric fields this equation looks like

$$\oint E \cdot dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

So $Q_{enclosed}$ is total charge inside that means electric field lines start and end on charges

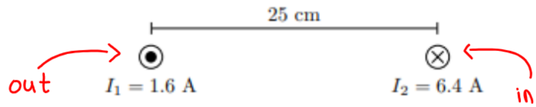
For magnetism the right hand side is zero, which means:

There are no isolated magnetic charges (no magnetic monopoles) inside the surface

So the number of magnetic field lines entering a closed surface always equals amount leaving

So the field lines go in one side and out the other, but they can't start or stop inside the surface

1. Two infinitely long straight wires separated by a distance of 25 cm are observed to pass perpendicularly through the plane of the page, as shown in the picture below. The wire to the left carries a current of 1.6 A in the direction out of the plane of the page, while the wire to the right carries a current of 6.4 A and points into the plane of the page.



(a) What is the magnetic flux density field $\vec{B}$ at the point midway between the two wires? Please be sure to specify both the magnitude and direction of the field $\vec{B}$?

(b) Is there a point in the plane pictured, not infinitely far away from the wires, where the magnetic flux density field $\vec{B}$ vanishes? If so, find the location of this point.

a) $$B = \frac{\mu_0 I}{2\pi r} \Rightarrow \frac{(4\pi \times 10^{-7})(1.6)}{2\pi(0.125)} = 2.55 \times 10^{-6}$$

$$\Rightarrow \frac{(4\pi \times 10^{-7})(6.4)}{2\pi(0.125)} = 1.02 \times 10^{-5}$$

$$2.55 \times 10^{-6} + 1.02 \times 10^{-5} = \boxed{1.28 \times 10^{-5}\ \text{T}}$$

an infinetely long cylindricaly symmetric wire of radius $2a$ has the following current density $\vec{J}$ as a function of radial distance r from center of wire

$$\vec{J} = \begin{cases} J_0 \hat{z} & \text{if } r \leq a \\ -J_0 \hat{z} & \text{if } a \leq r \leq 2a \end{cases}$$

a) What is the magnetic flux density field $\vec{B}$ in region $r \leq a$

$$\nabla \times B = \mu_0 J \qquad J = J_0 \hat{z}$$

$$\frac{1}{r}\left(\frac{\partial(r A_\theta)}{\partial r} - \cancel{\frac{\partial(A_r)}{\partial \theta}}\right)\cancel{\hat{z}} = \mu_0 J_0 \cancel{\hat{z}}$$

★ $\vec{B}$ cannot depend on r because if it did it could Not move around in a loop

$$\frac{1}{r}\left(\frac{\partial(rB_\theta)}{\partial r}\right) = J\mu_0$$

$$\int \frac{\partial(rB_\theta)}{\partial r}\,dr = \int \mu_0 J r\,dr$$

$$rB_\theta = \frac{Jr^2}{2} + c$$

$$B_\theta = \frac{\mu_0 J r}{2} + \cancel{\frac{c}{r}}^{\,0}$$

$$\boxed{B_\theta = \frac{\mu_0 J r}{2}\hat{\theta}}$$

c=0 the reasoning behind why is as you approach the center there is a force pulling it from all direction so that means its balanced from every direction so the net magnetic force there would be zero

b) what is the magnetic flux density field in the region $a < r \leq 2a$?

$$\nabla \times B = \mu_0 J \qquad J = -J\hat{z}$$

$$\frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \cancel{\frac{\partial A_r}{\partial \theta}}^{\,0}\right)\cancel{\hat{z}} = -\mu_0 J\cancel{\hat{z}}$$

★ $\vec{B}$ cannot depend on r if it did it cannot produce a loop

$$\int \frac{\partial(rB_\theta)}{\partial r}\,dr = \int -\mu_0 J r\,dr$$

$$B_\theta = \frac{-\mu_0 J r}{2} + \frac{c}{r}$$

You now need to set r=a and solve for c because we are moving from center

$$\frac{\mu_0 J a}{2} = -\frac{\mu_0 J a}{2} + \frac{C}{a}$$

$$\mu_0 J a^2 = C$$

• Plug back into C

$$\boxed{B_\theta = -\frac{\mu_0 J r}{2} + \frac{\mu_0 J a^2}{r}\,\hat{\theta}}$$

c) What is the magnetic Flux density Field in the region $2a < r$ assuming there is no current on the outside of the wire?

$$\nabla \times \vec{B} = 0$$

$$\frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \cancelto{0}{\frac{\partial (r A_r)}{\partial \theta}}\right) = 0$$

$$\int \frac{\partial (r B_\theta)}{\partial r}\, dr = \int 0 \, dr$$

$$r B_\theta = C$$

$$B_\theta = \frac{C}{r}$$

★ set $r = 2a$ and solve for the constant C

$$r = 2a$$

$$-\mu_0 J a + \frac{\mu_0 J a}{2} = \frac{C}{2a}$$

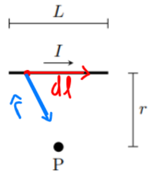
$$-2\mu_0 J a^2 + \mu_0 J a^2 = C$$

• Plug back into C

$$C = -\mu_0 J a^2$$

$$\boxed{B_\theta = -\frac{\mu_0 J a^2}{r}\,\hat{\theta}}$$

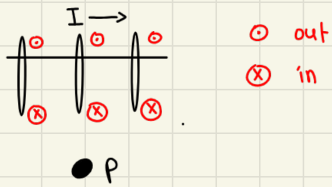
3. Consider a segment of thin wire of length L carrying a current I to the right, as shown in the picture below. The point labeled P is located a distance r away from the center of the wire segment in the same plane as the segment, as shown.



(a) What direction does $\vec{B}$ point at P?

(b) Using the generalized Biot-Savart law, set up an equation for $\left|d\vec{B}\right|$, the differential magnitude of the magnetic flux density, due to the differential length $d\vec{l}$ of the current carrying wire. Hint: use the cross product magnitude formula and then trigonometry to solve for $\sin\theta$.

a) We know dl Points directly where the current is going and $\hat{r}$ Point directly where Point P is by using our right hand rule dl = index Finger $\hat{r}$ = middle Finger We can see its going into the Page We can also use the curl by Pointing our right thumb where the current is going and curling our Fingers where we see its going inward

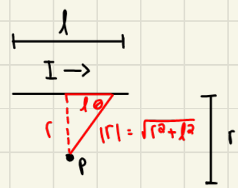


b)

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{|\vec{r}|^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I |dl| \sin\theta}{(r^2 + l^2)}$$

• the square root get squared so it cancels out

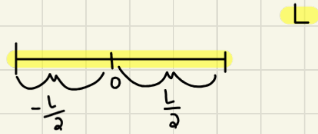


$$\sin\theta = \frac{r}{\sqrt{r^2 + l^2}}$$

• Plug this in For $\sin\theta$

$$d\vec{B} = \frac{\mu_0 I |dl|}{4\pi (r^2 + l^2)} \cdot \left(\frac{r}{\sqrt{r^2 + l^2}} \right)$$

$$d\vec{B} = \frac{\mu_0 I \, dl \, r}{4\pi (r^2 + l)^{3/2}}$$



We were already told that the length is L and this is a Finite length So the Statment $-L/2$ to $L/2$ is under Standable by Placing zero at the center but if we Place zero on the Far left and try to integrate From zero to L we would Need the actual length but We want the answer to work for all length L we give it we want to know the differential magnitude of magnetic flux density due to dl

$$\vec{B} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\mu_0 I \, dl \, r}{4\pi (r^2 + l)^{3/2}} \Rightarrow B = \frac{\mu_0 I r}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{(r^2 + l^2)^{3/2}} \, dl$$

$$\frac{\mu_0 I}{4\pi r} \left(\frac{l}{\sqrt{r^2 + l^2}} \right) \Bigg|_{-\frac{L}{2}}^{\frac{L}{2}} = \boxed{\frac{\mu_0 I L}{4\pi r \sqrt{\frac{L^2}{4} + r^2}}}$$

Summary

$$\oint_A \vec{B} \cdot d\vec{A} = 0$$

$$\nabla \cdot B = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

$$\nabla \times B = \mu_0 J$$

$$B(r) = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$

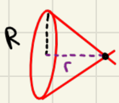
$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I \, d\vec{l} \times \vec{r}}{|r|^2}$$

$d\vec{l} \times \vec{r}$

$d\vec{l}$ = Points in the direction of the current

$\vec{r}$ = Points toward Point P

$$B = \frac{-\mu_0 I R^2}{2(r^2+R^2)^{3/2}}$$



$$M = I A$$

$$B = \frac{\mu_0 I n L}{2\sqrt{(L/2)^2 + q^2}}$$

$$B \approx \mu_0 N L$$

$N = \frac{N}{L}$ ← number of loops, ← length

• Magnetic Forces

- lorenz- Force law

a Charge q traveling with Velocity $\vec{v}$ in local magnetic Flux density field $\vec{B}$ Feels a Force given by:

$$\vec{F} = q\vec{v} \times \vec{B} = qVB\sin\theta$$

Where θ is between the $\vec{v}$ and $\vec{B}$ you can also use your right hand rule by the Very Bad Finger Mnemonice Very (Velocity) = thumb, Bad (magnetic Field) = index Finger, Finger (Force) = middle Finger

✱ if you are dealing with a Negitive charge you can use your left hand but this only works for a Negitive charge

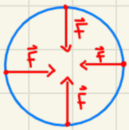
• the Full Lorentz Force law Which combines electric and magnetic forces on a moving charge

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \xrightarrow{\text{rewrite}} F = q\vec{E} + q(\vec{v} \times \vec{B})$$

• the cyclotron radius

✱ the magnetic Force always acts like a centripetal Force

What is a Centripetal Force: a centripetal Force always acts inward



also the Force is Perpendicular to our Velocity Which Means the Velocity Vector of the charged Particle always Points along the circumference of the circular Path or to simplify everything we can say the velocity is tangential at every Point

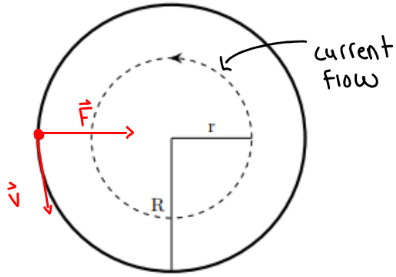
$$r = \frac{mv}{qB}$$

the current and $\vec{B}$ don't depend on r (radius) in a Solenoid

gives the radius of the circular Path that a charged particle traces when moving perpendicular to a uniform magnetic charge

- When you are describing where the $\vec{B}$ Field or any Field is going don't Say left or right use clockwise or counterclockwise

An N-turn solenoid has a total length L and radius R (such that $R << L$). Inside the solenoid, an electron follows a helical path of radius $r < R$, as shown in the picture below, such that it circles in the counterclockwise direction as viewed from the front of the solenoid, as shown. The electron completes one full helix in time T.



1. What is the direction of the magnetic flux density field $\vec{B}$ experienced by the electron? (Hint: pick a point on the trajectory. What is the direction of the force on the electron at this point?)
2. In what direction does the current flow around the solenoid?
3. In terms of the variables given as well as e, the fundamental charge, and m_e, the mass of the electron, what is the current, I, carried by the solenoid? (Hint: how does the time T relate to the speed v of the electron?)

1) we can use our right hand rule (Very, bad, Finger) also do keep in mind that we are dealing with an electron so it is negitive and instead of using our right hand we would need to use our left hand so the $\vec{B}$ is going out the Page $\vec{B}$ is spinning around the current

2) We already know where $\vec{B}$ is going we can find where I is going because $\vec{B}$ is spinning around I so we know $\vec{B}$ is going out the page so by curling our right hand in that direction we can see the current is going in the "left" direction but we say counter clockwise

⊙ in
⊗ out

- this is how the $\vec{B}$ field look its not pointing (↑) out its spinning it just happens to point outward in that direction

3)

$$I = \frac{2\pi m_e L}{\mu_0 N e T}$$

m_e = mass of electron
L = length of solenoid
N = total number of turns in solenoid
e = charge of electron (1.6×10^{-19})
T = period of electrons circular motion

* this formula assumes $L >> R$

• lorentz forces on currents

whenever we have a closed loop wire placed near an ambient $\vec{B}$ field ($\vec{B}$ is external), it will produce a torque

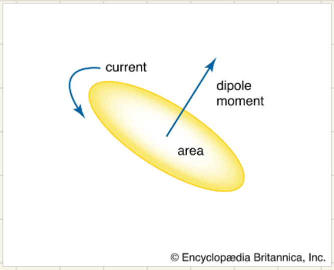
We get a magnetic dipole:

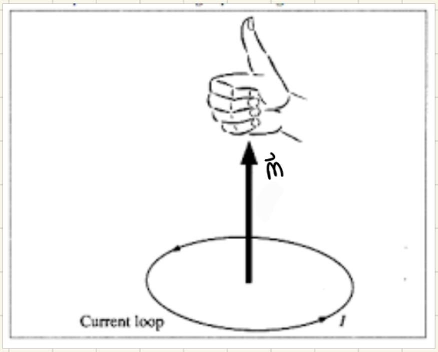
$$\tau = \vec{m} \times \vec{B} \Rightarrow \tau = |\vec{m}||\vec{B}| \sin\theta$$

(works for any magnetic dipole)

m = magnetic dipole moment ($A \cdot m^2$)
$\vec{B}$ = magnetic field (T)
θ = angel between m and B

the magnetic dipole moment measures how strong the magnetic field is
you can find the magnetic dipole by: curling your right hand fingers around the current and thumb will point in the direction of the magnetic dipole





magnetic dipole moment arises from current that circulates in a loop if there is no loop then there is no magnetic dipole moment so if you have a line of current it produces a magnetic field but no magnetic dipole moment

So for a straight wire you cannot define m pointing outwards or anywhere its simply not defined

Made with Goodnotes

- we can also define a dipole Potential energy such that:

$$U = -\vec{m}\cdot\vec{B} \longrightarrow U = -|\vec{m}||\vec{B}|\cos\theta$$

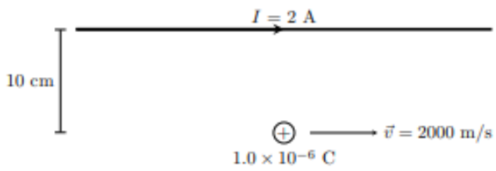
this works for any magnetic dipole

It is observed experimentally that the magnetic force $\vec{F}$ experienced by a charge q moving with velocity $\vec{v}$ in the presence of a magnetic flux density field $\vec{B}$ is given by

$$\vec{F} = q\vec{v}\times\vec{B} \tag{3.4}$$

1. As we saw last class, the SI unit of $\vec{B}$ is the *Tesla*, denoted T. Based on the equation given above, what is a Tesla in terms of other SI units?

2. Consider the long straight wire pictured below, which is carrying a current of 2 A. A distance 10 cm away from the wire is a positive 1.0×10^{-6} C charge moving at a speed of 2000 m/s in the direction indicated.



(a) What is the magnitude and direction of the magnetic flux density field $\vec{B}$ experienced by the charge?

(b) To find the magnitude of the force, recall from introductory mechanics that the magnitude of the vector cross product was given by

$$\left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta \tag{3.5}$$

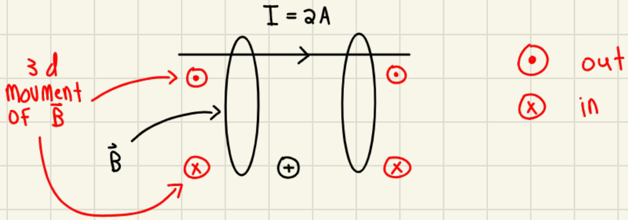
where θ is the angle in between the two vectors. Use this result to find $\left|\vec{F}\right|$.

(c) To find the direction of $\vec{F}$, we can use mnemonic "very bad finger". Point your right thumb in the direction of $\vec{v}$ (for very) and your right index finger in the direction of $\vec{B}$ (for bad). (This might require you to rotate your wrist and/or the paper on which the figure is drawn, however your fingers should be held naturally.) The middle finger on your right hand then gives the direction of $\vec{F}$, the force, a direction that must be perpendicular to both $\vec{B}$ and $\vec{v}$ by the nature of the vector cross product. Use this mnemonic to find the direction of $\vec{F}$ on the charge pictured.

a) this is a straight long wire the magnetic flux density for a long straight wire is:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7})(2)}{2\pi(0.1)} = \boxed{4.0\times10^{-6}\ \text{T}}$$

to find the direction we need to apply the curl we already know the current is going clockwise so if we stick our right hand thumb in the direction of clockwise and curl our fing we can see at I its going out ward but at the charge it will be going inward because $\vec{B}$ goes in a loop around the current

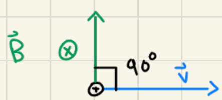


So $\vec{B}$ is going inward at the charge

b) $F = q\vec{V} \times \vec{B} \implies q|V||B| \sin\theta$

$q = 4.0 \times 10^{-6}$
$V = 2000 \text{ m/s}$
$B = 4.0 \times 10^{-6}\ T$
$\theta = 90°$

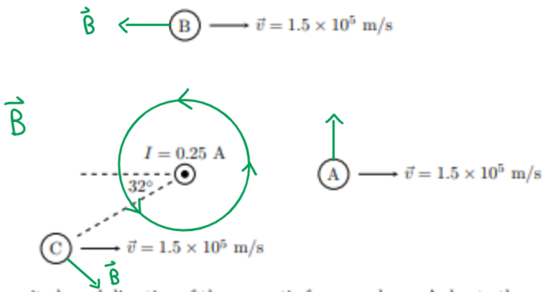
• We are told where $\vec{V}$ is going
We also know were $\vec{B}$ is going



$\vec{F} = (1.0 \times 10^{-6})(2000)(4.0 \times 10^{-6}) \sin(90°) = \boxed{8 \times 10^{-9}\ N}$

c) $\vec{F}$ is going upward

3. In the picture below, three negative charges (labeled A, B, and C) are all 20 cm from a long straight wire carrying a current of 0.25 A directed out of the page, as pictured. All three charges are traveling to the right at a speed $v = 1.5 \times 10^5$ m/s, as shown. Charge A is -1.5×10^{-6} C, charge B is -2.5×10^{-6} C, and charge C is -1.0×10^{-6} C.



(a) What is the magnitude and direction of the magnetic force on charge A due to the current in the wire? (Hint: note that A is negative. How would that affect the direction of the force?)

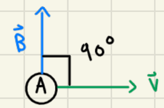
(b) What is the magnetic and direction of the magnetic force on charge B due to the current in the wire?

(c) What is the magnitude and direction of the magnetic force on charge C due to the current in the wire?

✯ when you are given a quetion like this began by Finding where $\vec{B}$ Field is you can see that you are already given the I (current) and its direction which is comming directly at you by using curl stiking out your thumb (right hand) directly at your Self and curl your Finger the $\vec{B}$ Field goes counter clockwise

a)

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow \frac{(4\pi\times10^{-7})(0.25)}{2\pi\ (0.2)} = 2.49\times10^{-7}\ T$$



$$\vec{F} = q\,\vec{v}\times\vec{B} = q\,|V|\,|B|\sin\theta$$

$$\vec{F} = (-1.5\times10^{-6})(1.5\times10^{5})(2.49\times10^{-7})\sin(90) = \boxed{-5.60\times10^{-8}\ N}$$

✯ Charge A is Negitive that Means instead of using your right hand For the right hand rule you will use your left (this only works if you are dealing with a negitive charge)

So $\vec{F}$ goes into the Page

b) $F = q\vec{v}\times\vec{B} \Rightarrow \vec{F} = q\,|V|\,|B|\sin\theta$

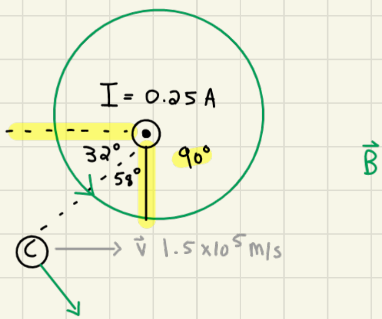
$$\vec{F} = (-2.5\times10^{-6})(1.5\times10^{5})(2.49\times10^{-7})\sin(180°)$$

$$\boxed{F = 0N}$$

the $\vec{B}$ is going Counter clockwise

c)

$F = q\vec{v}\times\vec{B} \Rightarrow \vec{F} = q\,|\vec{V}|\,|\vec{B}|\sin\theta$



$q = -1.0\times10^{-6}\ C$

$V = 1.5\times10^{5}\ m/s$

$\vec{B} = 2.49\times10^{-7}\ T$

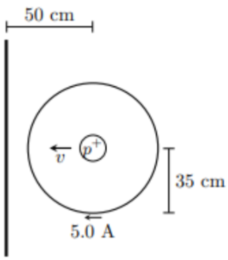
$\theta = 58°$

$\vec{F}$ Points out at us

you want to use angel 58 instead 32 because the angel needs to be between $\vec{v}$ and $\vec{B}$

$$F = (-1.0\times10^{-6})(1.5\times10^{5})(2.49\times10^{-7})\sin(58) = \boxed{-3.16\times10^{-8}\ N}$$

1. A proton traveling to the left at a speed of $v = 1.4 \times 10^5$ m/s lies in the center of a circular loop of radius 35 cm that carries a 5.0 A current in the clockwise direction, as shown in the picture below. Additionally, there is an infinitely long straight wire positioned to be 50 cm from the proton, as illustrated., but the current carried by this wire is unknown.



(a) (*Draws on Class #17*) What is the magnitude and direction of the magnetic force on the proton due to just the current in the circular loop?

(b) If the net magnetic force on the proton is observed to be zero, how large and in what direction is the current in the infinitely long straight wire?

Set $r = 0$

a) $$B = \frac{-\mu_0 I R^2}{2(r^2 + R^2)^{3/2}} \Rightarrow B = \frac{-\mu I R^2}{2(R^2)^{3/2}} \Rightarrow B = \frac{-\mu_0 I \cancel{R^2}}{2R^{\cancel{3}}} \Rightarrow B = \frac{-\mu_0 I}{2R}$$

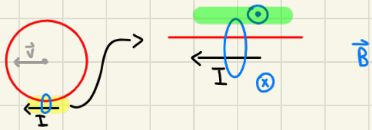
$$B = \frac{-(4\pi \times 10^{-7})(5.0)}{2\,(0.35)} = -9.0 \times 10^{-6}\ \text{T}$$

$$\vec{F} = q\,\vec{v} \times \vec{B} \Rightarrow \vec{F} = q\,|\vec{v}|\,|\vec{B}| \sin\theta$$

$$(1.6 \times 10^{-19})(1.4 \times 10^{5})(9.0 \times 10^{-6}) \sin(90)$$

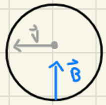
$$\boxed{F = 2.01 \times 10^{-9}\ \text{N}}$$

Force is going inward or into the Page



at this moment $\vec{B}$ going inward toward $\vec{v}$ we will use our right hand rule with $\vec{B}$ going inward

looking From top



* angel that it makes between $\vec{v}$ and $\vec{B}$ is $90°$

b) For the Force to be zero they need to be equal and opposite of each other

$$\frac{-\cancel{\mu_0} I_s}{\cancel{2}R} = \frac{\cancel{\mu_0} I_w}{\cancel{2}\pi r}$$

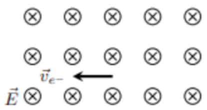
$$\frac{\pi r I_s}{R} = I_w$$

$$\frac{(\pi)(0.5)(5.0)}{(0.35)} = I_w$$

$$\boxed{I_w = 22.4\,A}$$

* you cannot just plug the answer you got for a into the equation for a sphere to get the answer for B

2. A beam of electrons traveling to the left at a speed of 4.5×10^5 m/s enter a region of space with a uniform *electric* field of magnitude 350 N/C directed into the plane of the page, as shown.



(a) (*Draws on Class #9*) What is the magnitude and direction of the constant $\vec{B}$ field perpendicular to the electron beam the beam must experience in order for the net electromagnetic force on the electrons in the beam to be zero?

(b) Assuming the constant $\vec{B}$ field you found in part a is present, what would be the direction of the net force on an electron traveling slower than the 4.5×10^5 m/s speed of the other electrons in the beam? Please explain briefly.

(c) Assuming once again that the $\vec{B}$ field you found in part a is present, what would be the direction of the net force on an alpha particle traveling to the left at the same 4.5×10^5 m/s speed? (Note: an alpha particle, being a helium nucleus, has a electric charge of $+2e$.) Once again, please explain your answer briefly.

* every electric field has a force if you are dealing with a electron the field will point opposing the electric field if you have a proton the force will go toward the electric field

a) $F = qE + q(\vec{v} \times \vec{B})$

$0 = q(E + \vec{v} \times \vec{B})$

$E = -\vec{v} \times \vec{B} \Rightarrow E = -|\vec{v}||\vec{B}|\sin\theta$

you are dealing with a electron so the force will point opposing the electric field we are also told that the force need to be zero so that would mean the magnigtic force is opposing the force or going toward the electric field

Magnetic Force
velocity
Force (electric)
electric field going into page

the two forces would cancel out

because we are dealing with a electron we need to use our left instead of right for the right hand rule so $\vec{B}$ is going downward

Magnetic Force
Velocity
Force (electric)
magnitic Field
electric Field going into Page

the angle velocity and magnitic Field make is 90°

$$E = -|V||B| \sin\theta$$

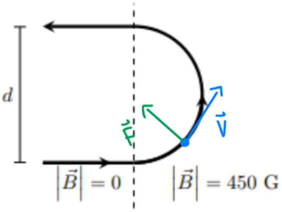
$$\frac{E}{-|V| \sin\theta} = B$$

$$|B| = \frac{(350)}{(4.5\times10^{5}) \sin(90)} = \boxed{7.77\times10^{-4}\ T}$$

b) if the electron was traveling slower the magnitude would Not change but the Net Force would Not cancel out the Magnetic Force will go in the same direction as the Force or opposing the electric Field the net Force is No longer zero

c) the experinced net Force on the alpha Particle is zero because its traveling at a velocity of 4.5×10^{5} m/s anything traveling with that speed will experince No net Force regardless of charge

3. A beam of antiprotons (which have the same mass $m_p = 1.67 \times 10^{-27}$ kg as regular protons, but the opposite sign for their electric charges) traveling at a speed of 1.56×10^5 m/s passes from a region of space where $\vec{B} = 0$ into one with a constant $\vec{B}$ field of magnitude $|\vec{B}| = 450$ G. (Note: the *Gauss*, denoted G, is a unit of magnetic flux density defined such that 1 T = 10000 G.) As a result, the antiproton beam follows the semi-circular trajectory pictured.



(a) In what direction does the 450 G $\vec{B}$ field point?
(b) What is the distance d, depicted above?

a) We have a Circular movement we know the Force in this circular movement will point tangential and the Force will go to the Center as it will act like a Centripital Force we will use our left hand to find the direction $\vec{B}$ is pointing in

So the magnetic Field is going upward or out the Page

b) $r = \frac{MV}{qB}$ $\longrightarrow$ $(1.6 \times 10^{-27})\,\text{Kg} \times \frac{1000\,g}{1\,\text{Kg}} = 1.6 \times 10^{-24}\,g$ $\quad 450\,G \times \frac{1\,T}{10000} = 0.045\,T$

$$\frac{(1.6 \times 10^{-24})(1.56 \times 10^{5})}{(-1.6 \times 10^{-19})(0.045)} = -34.6\,m \qquad -34.6 \times 2 = \boxed{-69.3\,M}$$

Summary

$\vec{m}$ (magnetic dipole moment) = $A \cdot m^2$

$$F = q(\vec{v} \times \vec{B}) \rightarrow q|\vec{v}||\vec{B}|\sin\theta$$

$$F = q(\vec{E} + \vec{v} \times \vec{B}) \rightarrow F = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$r = \frac{mv}{qB}$$

$$I = \frac{2\pi M_e L}{\mu_0 N e t}$$

$$\tau = \vec{m} \times \vec{B} \rightarrow \tau = |\vec{m}||\vec{B}|\sin\theta \quad (n \cdot m)$$

$$U = -\vec{m} \cdot \vec{B} \rightarrow U = -|\vec{m}||\vec{B}|\cos\theta \quad (J)$$

- Magnetic Materials

materials with oservable macroscopic magnetic Field are termed Paramagnetic alighnment in Paramagnets is usally achieved via external Fields this is linear

Materials where the atomic scale magnetic Moments cause mesoscale orientations of magnetic domains are termed Ferromagnetic Non-linear

all material will exhibit Some magnetic responses to changing electric Fields materials to which this is only measurable response are termed diamagnetic linear

the magnetization M of a material is the magnetic dipole moment per unit volume:

$$\vec{M} = n\vec{m}$$

Where n is the number of (aligned) dipoles per unit volume

this equation is conceptually always true by definition but in Practice it applies under certain conditions

it becomes straight forward for simple materials like: Paramagnetic and diamagnetic or in linear material

but it becomes much more complicated when we are talking about non-linear cases or in Ferromagnetic

Magnetization contributes an effective current density given by:

$$\vec{J}_m = \nabla \times \vec{m}$$

bound current

curl of magnetization

☆ Magnetization only "sees" or accounts for bound current in magnetic material amperes law becomes:

total current → $J_{total} = \underset{\text{Free current}}{J_f} + \underset{\text{bound current}}{J_m}$

$$\nabla \times B = \mu_0 (J_f + J_m)$$

$$\boxed{\nabla \times B = \mu_0 (J_f + \nabla \times m)}$$

$\vec{B}$ can see free and bound current

We can also define a new field $\vec{H}$ which corresponds to applied magnetic field the field you control (wires, or coils) it can only see free current

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{m}$$

this is alway true even for non-linear

this field $\vec{H}$ is called the magnetic field by definition

$$\vec{J}_f = \nabla \times \vec{H}$$

Notice that $\vec{H}$ is to $\vec{B}$ what $\vec{D}$ is to $\vec{E}$ for electrostatics

Field	Name / Symbol	Sees / Depends on	Physical Meaning / Role
B	Magnetic flux density / magnetic field	**Both free ($\mathbf{J}_f$) and bound currents ($\mathbf{J}_m$)**	Total, measurable magnetic field. The actual field you would detect with a sensor.
H	Magnetic field intensity / auxiliary field	**Free currents only ($\mathbf{J}_f$)**	Applied field produced by external sources (coils, wires). Bound currents are absorbed into $\mathbf{M}$. Used for solving problems in materials.
M	Magnetization	**Bound currents only ($\mathbf{J}_m$)**	Material's response to $\mathbf{H}$. Represents net magnetic dipole moment per unit volume.
Jf	Free current density	N/A	Actual moving charges (e.g., in wires). Drives $\mathbf{H}$.
Jm	Bound (magnetization) current density	N/A	Effective current density from aligned atomic/molecular dipoles. Contributes to $\mathbf{B}$. Calculated as $\mathbf{J}_m = \nabla \times \mathbf{M}$.

$\vec{B}$ (magnetic field) and H (magnetic feild intensity) point in the same direction

For linear media, the Magnetization is directly proportional to the magnetic Field

$$\vec{M} = (\mu_r - 1)\vec{H}$$

In such media (linear), the relationship between $\vec{B}$ and $\vec{H}$ becomes:

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

☆ only if material is linear you can use this

- Permeability
- the quantity μ_r is termed the relative permeability unlike the dielectric constant it can be less than one For some materials

$$\mu_0 \mu_r = \mu$$

- **Diamagnetic:** $\mu < \mu_0$ → oppose magnetic field slightly.
- **Paramagnetic:** $\mu > \mu_0$ → weak attraction to field.
- **Ferromagnetic:** $\mu \gg \mu_0$ → very strong magnetic response.
- **Vacuum / Air:** $\mu = \mu_0$ → baseline reference.

diamagnet $\mu_r < 1$
Paramagnet $\mu_r > 1$
} expressed different way's

Permeability

- In normal media the quantity $\mu_r - 1$ is typically small. For diamagnetic materials, it is typically ~ 10^{-5} (and negative!). For paramagnetic materials, it is typically no larger than ~ 10^{-3}.
- For ferromagnetic materials, μ_r is usually both large and variable; ferromagnetic materials are almost always highly non-linear.

- hysteresis
- Means lag or remember in magnetic material, hysteresis describes how magnetization of a material does Not immediately Follow the applied Magnetic field instead it lags behind due to Material Magnetic history so it depends on Past Magnetic states

☆ this occurs <u>only</u> in FerroMagnetic material it does <u>Not</u> occur in Paramagnetic or diamagnetic material

- Forces on Permanet Magnets
- if two maynets are aligned North to South, then the angel between one dipole and the $\vec{B}$ Field Produced by the other is zero hence:

$$U = -\vec{m} \cdot \vec{B} = -|\vec{m}||\vec{B}|$$

- notice that bringing the dipoles closer togther increases the strength of $\vec{B}$

- Summary

1. **You** create a magnetic field by running current in a coil.
   → That field is called $\mathbf{H}$ — it's the "applied" or "external" field.
2. **The material responds** — the atoms inside act like tiny magnets that try to align or oppose the external field.
   → That internal response is called $\mathbf{M}$ — the "magnetization."
3. The **total field inside the material** (what you could measure with a magnetometer) is the combination of the two effects.
   → That's $\mathbf{B}$ — the actual magnetic field that exists in space.

---

the magnetic Flux density is made up of applied Field H and material response M

H = applied Field
M = material response
B = total magnetic Field

$\vec{B}$ can see Free and bound current

diamagnets (linear)

- they weakly repel magnetic fields

- magnetization is very small, opposite the applied field
- $\vec{M}$ is opposing $\vec{B}$ and $\vec{H}$

$\mu_r < 1$

Key Fact: No permanent magnetization. only responds while external field is applied

Paramagnet (linear)

- they weekly attract magnetic fields

- Magnetization is weak, and along the applied
- $\vec{M}$ goes along with $\vec{B}$ and $\vec{m}$

$\mu_r > 1$

Key Fact: only magnetized when the external field is present; thermal motion tends to randomize the dipoles, so effect is weak

Ferromagnets (non-linear)

- they strongly attract and can retain magnetization

M can be very large and constant in a domain

- they often have permanent magnetization

Consider the case of an infinitely long cylindrical wire of radius R made from a diamagnetic material of permeability μ. Inside the wire, the free current density $\vec{J}_f$ as a function of the radial coordinate r is given as

$$\vec{J}_f = \frac{A}{r}\hat{z} \text{ for } r \leq R \tag{3.6}$$

where A is a positive constant. Outside the wire ($r > R$) is a vacuum that contains no free currents.

1. What is the magnetic field $\vec{H}$ inside the wire?
2. What is the magnetic flux density field $\vec{B}$ inside the wire?
3. What is the magnetization $\vec{M}$ inside the wire? Which way does it point? Sketch a picture showing the $\vec{H}$ field lines and the orientations of the magnetic dipoles that give rise to $\vec{M}$ in the region inside the wire.
4. What is the magnetic field $\vec{H}$ outside the wire?
5. What is the magnetic flux density field $\vec{B}$ outside the wire?

1)

$$\nabla \times H = J_F$$

$$\left(\cancel{\frac{1}{r}\frac{\partial A_z}{\partial \theta}} - \frac{\partial A_\theta}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \cancel{\frac{\partial A_z}{\partial r}}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{z} = \frac{A}{r}\hat{z}$$

remember that H also goes in a loop it also depends on θ

$$\frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \cancel{\frac{\partial A_r}{\partial \theta}}\right)\cancel{\hat{z}} = \frac{A}{r}\cancel{\hat{z}}$$

$$\frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r}\right) = \frac{A}{r}$$

$$\int \frac{\partial (r H_\theta)}{\partial r}\, dr = \int A \, dr$$

$$r H_\theta = Ar + c$$

$$\boxed{H_\theta = A\hat{\theta}}$$

* inside the cylinderical wire c goes to zero as r goes to 0

2) $\vec{B} = \mu_0 \mu_r H$

$\mu_0 \mu_r = \mu$

$H = A\hat{\theta}$

$\boxed{B = \mu A \hat{\theta}}$

3) $\vec{M} = (\mu_r - 1)\vec{H}$

$\mu_0 \mu_r = \mu$

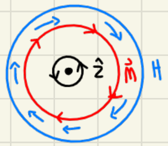
$\mu_r = \dfrac{\mu}{\mu_0}$

$H = A\hat{\theta}$

$\boxed{M = \left(\frac{\mu}{\mu_0} - 1\right) A \hat{\theta}}$

When you are dealing with diamagnetic material $\mu < \mu_0$ if that happens $\left(\frac{\mu}{\mu_0} - 1\right)$ = negitive For diamagnetic material $\mu < 1$ it Points in the negitive $\hat{\theta}$

Sketch:



4) $\nabla \times H = 0$

$$\frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r}\right) = 0$$

$$\int \frac{\partial (r H_\theta)}{\partial r} dr = \int 0 \, dr$$

$$r H_\theta = C$$

$$H_\theta = \frac{C}{r}$$

$$A = \frac{C}{R}$$

$$C = AR$$

$$\boxed{H_\theta = \frac{AR}{r}\hat{\theta}}$$

☆ Set the inside and outside equal and solve for C

5) $B = \mu_0 \mu_r H$

$$\boxed{B = \mu_0 \frac{AR}{r} \hat{\theta}}$$

✱ we are told to assume vaccum outside so $\mu_r = 1$

An infinitely long conducting cylindrical wire of radius R is made from an Ohmic ferromagnetic material of conductivity σ that has been uniformly magnetized in the $\hat{z}$ direction such that

uniformly magnitized = constant

$$\vec{M} = M\hat{z} \tag{3.7}$$

for some positive constant M. The wire carries a free current density $\vec{J}_f$ given by (in the region $r < R$)

$$J_f = -Ar^2\hat{z} \tag{3.8}$$

where A is another positive constant and r the radial distance from the center of the wire.

1. What is the magnetic field $\vec{H}$ in the region inside the wire?
2. What is the effective magnetization current density $\vec{J}_m$ inside the wire?
3. What is the magnetic flux density field $\vec{B}$ in the region inside the wire? (Hint: do not use Ampère's law!)
4. Notice that if we solve for $\vec{B}$ in part c using Ampère's law and our usual procedure, we get the wrong answer. Can you think about why this might be? (Hint: does specifying the curl of a vector field uniquely define that vector field?)

1) $\nabla \times H = J_f$

H does not depend on r

$$\frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \cancel{\frac{\partial A_r}{\partial \theta}}\right) \cancel{\hat{z}} = -Ar^2 \cancel{\hat{z}}$$

$$\int \frac{\partial (r H_\theta)}{\partial r} = \int -Ar^3 \, dr$$

$$r H_\theta = -\frac{Ar^4}{4} + C$$

✱ you can set $C=0$ because as r gets closer to the center $C=0$

$$\boxed{H_\theta = -\frac{Ar^3}{4} \hat{\theta}}$$

2)

$$\nabla \times M = J_m$$

We are told that its a constant so:

$$J_m = 0$$

3)

✱ you cannot use $B = \mu_0 \mu_r H$ because Ferromagnets are not linear Ferromagnets are non-linear you need to use $B = \mu_0 (H + M)$

$$H = \frac{\vec{B}}{\mu_0} - M \implies B = \mu_0 (\vec{H} + \vec{M})$$

$$\boxed{B = \mu_0 \left(-\frac{Ar^3}{4} + M\hat{z}\right)}$$

1. For her birthday, Maggie receives an infinitely long, 0.1 m radius wire made of platinum, a paramagnetic conductor with a relative permeability of 1.00265. Maggie celebrates this gift by running a constant 3.1415 A total current through the wire. (Feel free to assume that the current density in Maggie's wire is uniform.)

   (a) What is the magnetic field, $\vec{H}$, that is produced inside Maggie's wire a distance of 0.05 m from the center (that is, halfway between the center and the surface of the wire)?

   (b) What is the magnetic flux density field, $\vec{B}$, that is produced in Maggie's wire a distance of 0.05 m from the center (that is, halfway between the center and the surface of the wire)?

1) $J = \frac{I}{A}$ • this expression relates current density (J) to current in a conducter

the area A we used is πr^2 because $\vec{B}$ and H move in a loop so that moves a circle

$$J = \frac{3.1415}{\pi (0.1)^2} = 100 \text{ A/m}^2$$

$$\nabla \times H = J_f$$

$$\frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r}\right) = 100$$

$$\int \frac{\partial (r H_\theta)}{\partial r} = \int 100 r \, dr$$

$$r H_\theta = 50 r^2$$

$$H_\theta = 50 r$$

$$r = 0.05 \text{ m}$$

$$H_\theta = (50)(0.05)$$

$$\boxed{H_\theta = 2.5 \hat{\theta} \text{ A/m}}$$

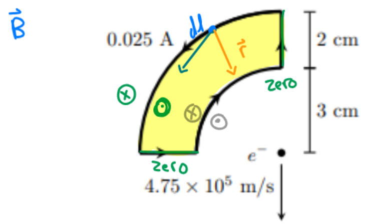
2)

$$B = \mu_0 \mu_r H$$

$$B = \mu_0 \mu_r H$$

$$B = (4\pi \times 10^{-7})(1.00265)(2.5) = \boxed{3.14 \times 10^{-6}\,T}$$

2. A wire loop made from two concentric quarter circles connected by radial segments carries a current of 0.025 A directed in the counterclockwise direction, as shown in the picture below. An electron traveling downward with speed $v = 4.75 \times 10^5$ m/s is located in the same plane as the wire loop, at the central point of the two concentric quarter circles.



thats the area we are accounting For

a). (*Draws on Class #17*) What is the magnitude and direction of the magnetic flux density field $\vec{B}$ experienced by the electron?

b). What is the magnitude and direction of the magnetic field $\vec{H}$ experienced by the electron?

c). (*Draws on Class #18*) What is the magnitude and direction of the net magnetic force on the electron?

d). If this wire loop were to be placed in a constant ambient $\vec{B}$ field of magnitude $B = 0.048$ T, directed downward (i.e., in the same direction as the electron's velocity vector), then what is the magnitude and direction of the net torque τ on the wire loop?

a)

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\, d\vec{l} \times \vec{r}}{|\vec{r}|^2}$$

Notice that $d\vec{l}$ and $\vec{r}$ are Perpendicular

$$d\vec{l} \times \hat{r} \approx d\vec{l}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl}{|r|^2}$$

$$\vec{B}_{out} = \int_l \frac{\mu_0}{4\pi} \frac{I}{r^2}\, dl \Rightarrow \left[\frac{\mu_0 I l}{4\pi r^2}\right]$$

• the $2\pi r$ is what we got From l which is arc length but we need to divide by Four because thats one quarter of the circumference of the Full circle

$$\frac{(4\pi \times 10^{-7})(0.025)}{4\pi (0.05)^2}\left(\frac{2\pi r}{4}\right) \Rightarrow \frac{(4\pi \times 10^{-7})(0.025)}{4\pi (0.05)^2}\left(\frac{2\pi (0.05)}{4}\right)$$

$$= \boxed{7.85 \times 10^{-8}}$$

$$B_{in} = \int_l \frac{\mu_0}{4\pi} \cdot \frac{I}{|r|^2} \, dl \longrightarrow \left[\frac{\mu_0 \; I \; l}{4\pi \; r^2}\right] \qquad l = \frac{2\pi r}{4}$$

$$\longrightarrow \left[\frac{\mu_0 I}{4\pi r^2}\right]\left(\frac{2\pi r}{4}\right) \longrightarrow \left[\frac{(4\pi \times 10^{-7})\,(0.025)}{4\pi \;\; (0.03)^2}\right]\left(\frac{2\pi\,(0.03)}{4}\right)$$

$$= \boxed{1.30 \times 10^{-7}\,T}$$

to get B_{total} we need to subtract B_{in} from B_{out}

$$|B_{tot}| = |B_{in}| - |B_{out}| = \boxed{5.25 \times 10^{-8}\,T}$$

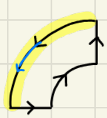
b) $B = \mu_0 \mu_r H$ ✱ We will assume we are in a vaccum because it does not tell us $\mu_r = 1$

$B = \mu_0 H$

$$H = \frac{B}{\mu_0} = \frac{5.25 \times 10^{-8}}{4\pi \times 10^{-7}} = \boxed{0.0418\ A/M}$$

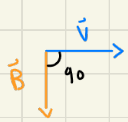
c) $F = q(\vec{v} \times \vec{B}) \Rightarrow q\,|\vec{V}||\vec{B}|\sin\theta = (-1.6 \times 10^{-19})(4.75 \times 10^{5})(5.25 \times 10^{-8})\sin(90) =$

$$= \boxed{-3.98 \times 10^{-21}\,N}$$



the $\vec{B}$ goes into the page

Velocity

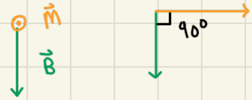


d)

$$\tau = \vec{m} \times \vec{B} \implies |\vec{m}||\vec{B}| \sin\theta$$

by curling our right hand Fingers around the current the magnetic dipole moment Points directly at us

Side View



$$m = AI = \left(\frac{\pi r^2}{4} - \pi r^2\right) I$$

$$= \left(\frac{\pi (0.05)^2}{4} - \pi (0.03)^2\right)(0.025) \approx \pi \times 10^{-5} \, \text{Am}^2$$

$$\tau = (\pi \times 10^{-5})(0.048)\sin(90°) = \boxed{1.51 \times 10^{-6} \, \text{N} \cdot \text{m}}$$

3. An infinitely long cylindrical wire of radius R, made from a linear paramagnetic material of relative permeability μ_r is observed to have the following magnetic field $\vec{H}$ as a function of the radial coordinate r in the region inside the wire (i.e., for $r < R$).

$$\vec{H}(r) = -\sqrt{\aleph r}\hat{\theta} \tag{3.3}$$

where $\aleph$ is a positive constant. Outside the wire is empty vacuum.

(a) In SI, what are the units of $\aleph$?

(b) What is $\vec{B}$ in the region inside the wire?

(c) What is $\vec{J}$, the total current density (not just $\vec{J}_f$, the free current density), as a function of r in the region inside the wire (i.e., for $r < R$)?

(d) What is $\vec{H}$ in the region outside the wire?

a)

$H = A/m \qquad r = m$

$$\frac{A}{m} = -\sqrt{\aleph}\, m$$

$$\aleph = \frac{A}{m^2}$$

b)

$B = \mu_0 \mu_r H \Rightarrow \boxed{B = -\mu_0 \mu_r \sqrt{nr}\,\hat{\theta}}$

c)

$\nabla \times B = \mu_0 J$

$\frac{1}{r}\left(\frac{\partial (r B_\theta)}{\partial r}\right) = \mu_0 J$

$\frac{1}{r} \frac{\partial(-\mu_0 \mu_r (r\sqrt{N r}\,\hat{\theta})}{\partial r} = \mu_0 J$

$(r)^{1/2} = \frac{1}{2} r$

$\frac{-\mu_0 \mu_r \sqrt{n}}{r} \frac{\partial (r^{3/2})}{\partial r} = \mu_0 J$

$\frac{-\cancel{\mu_0} \mu_r \sqrt{n}}{r} \cdot \left(\frac{3}{2}\sqrt{r}\right) = \cancel{\mu_0} J$

$1/2 - 1 = -1/2$

$-\frac{3}{2} \frac{\mu_r \sqrt{n}\ r^{1/2}}{r^1} = J$

$\boxed{-\frac{3}{2}\mu_0 \sqrt{\frac{n}{r}}\,\hat{z} = J}$

• current density is in the $\hat{z}$ direction

d)

$\nabla \times H = J_f$

$\frac{1}{r}\left(\frac{\partial (r H_\theta)}{\partial r}\right) = 0$

$\int \frac{\partial (r H_\theta)}{\partial r} dr = \int 0\, dr$

$r H_\theta = C$

$H_\theta = \frac{C}{r}$

you can set the inside and out equal at $r = R$ and solve for c

$$H_{in} = H_{out}$$

$$-\sqrt{NR} = \frac{C}{R}$$

$$-R\sqrt{NR} = C$$

• Plug this back into C

$$\boxed{H_\theta = \frac{-R\sqrt{NR}}{r}}$$

Summary

$$\nabla \times \vec{m} = \vec{J}_m$$

$$\boxed{H = A/m}$$

$$\nabla \times B = \mu_0 (J_f + \nabla \times \vec{m})$$

$$\boxed{B = T}$$

$$H = \frac{\vec{B}}{\mu_0} - \vec{m} \xrightarrow{\text{you can also write it as}} \vec{B} = \mu_0 (\vec{H} + \vec{m})$$

$$\boxed{M = A/m}$$

$$\vec{\nabla}_f = \nabla \times \vec{H}$$

$$\boxed{J = \frac{A}{m^2}}$$

$$M = (\mu_r - 1)\vec{H}$$

$$B = \mu_0 \mu_r \vec{H}$$

$$\mu_0 \mu_r = \mu$$

$$U = -\vec{m} \cdot \vec{B} = -|\vec{m}||\vec{B}|$$

$$J = \frac{I}{A}$$