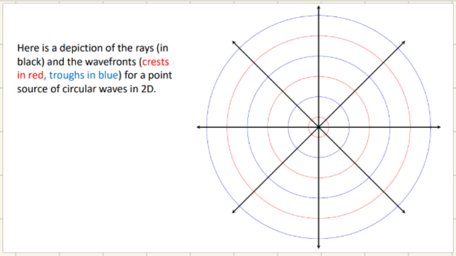
inference and diffraction

• rays

- ray are a method of depicting waves in 2d/3d



• here we have a wave the best way to imagine this is when you throw a rock in to the Pond this diagram is mapped out to show you the the movment of the wave

• reminder of what a crest and wave

• intensity

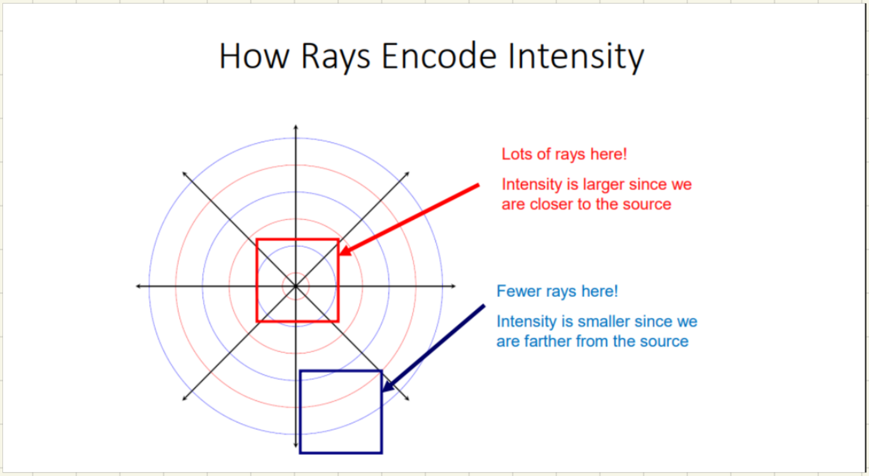
- is the Power Per unit area carried by a wave per unit are in the direction of Propagation (direction of Prop agation is the direction in which the wave energy and disturbance travel)

• the main idea

- there is the same amount of energy No matter where you stand

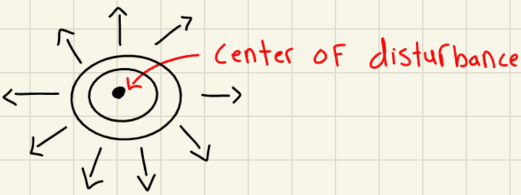
- but as you move further the energy has to cover a bigger area so intensity decreases

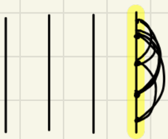
- if you move closer to the energy source the energy is concentrated over a small area so intensity increases



- Huygen Principle

- every point of an advancing wave front is a new center of disturbance from which they emanate waves from all direction

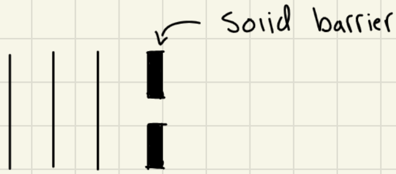




- every single point along that line is behaving like a new center of disturbance but you can keep adding more and more centers of disturbance and you will notice that notice that there is a specific spot all these waves are overlapping

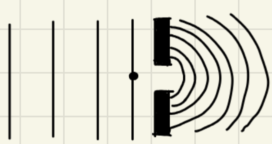
So thats going to be where the next wave front is going to be

- When waves of light pass through a small opening in a barrier and as a a result radiate out in all directions this is called diffraction



When that wave hits the solid barrier most of the wave is not able to move through the barrier

- the only point that will be able to get passed the solid barrier is the small point in the center



• inference

recall that:

$$\sin(\theta + 2n\pi) = \sin\theta$$

by adding an integer multiple of 2π to θ it will return you to the same position on the unit circle

- in other words its a even number this will return you to the same $\sin\theta$ position: 0, 2, 4, 6, 8 ....

$$\sin(\theta + (2n+1)\pi) = -\sin\theta$$

this also take you to the same $\sin\theta$ position but in the negitives so for any odd number: 1, 3, 5, 7....

these are not equation they help explain and derive deconstructive and constructive equations I won't derive them

our real plane wave solution can be written as:

$$\Phi = kx + wt + \phi_0$$

now suppose we have two waves

$$\Phi_1 = kx_1 + wt + \phi_{01} \qquad \Phi_2 = kx_2 + wt + \phi_{02}$$

$$\Delta \Phi = (kx_1 + \cancel{\omega t} + \phi_{01}) - (kx_2 + \cancel{\omega t} + \phi_{02})$$

$$\Delta \Phi = k(x_1 - x_2) + (\phi_{01} - \phi_{02})$$

$$\Delta \Phi = \frac{2\pi}{\lambda} \Delta x + \Delta \phi_0$$

$$\Delta \Phi = k \Delta x + \Delta \phi_0$$

- this is the total Phase difference at the Point where the waves meet

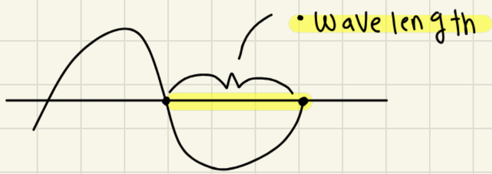
★ remember From waves:

$$V = \lambda f$$

- this is what it looked like:

$$V = \frac{\lambda}{T} \quad \text{as} \quad f = \frac{1}{T}$$

- λ (lamda) is the wavelength



For two Slits

- Constructive inter Ference

- When waves line up Peak to Peak, through to through they will add up to Make up a bigger

- on Problems you might get a hint Saying the light get brighter or the Sound is getting louder

- For constructive interference the Phase differnce (Φ) is even number = $0, 2\pi, 4\pi, 6\pi$

equation For constructive inFerence:

$$n\lambda = d\sin\theta$$

n = any number representing the order of the bright Fringe
λ = wavelength
d = distance between the two Slits

θ = angle directly in Front of the two Slits

For two Slits

- Destructive interference

- one wave Peak lines up with the others through to limit it or Cancel it

- on Problems you might get a hint saying the light gets dimmer or darker or the sound is getting smaller they might even tell you No Sound or No light

- For destructive interference the Phase Shift Should be odd : $\pi, 3\pi, 5\pi, 7\pi, \ldots\ldots$

equation for destructive interference

$$(n+1/2)\lambda = d\sin\theta$$

n = any integer representing the order of a bright Fringe

λ = wavelength of the light

d = distance between slits

θ = angle infront of Slits

For single slit

- destructive interference

- in single- slit diffraction the waves are canceling each other at certain angle

- on Problems you might get a hint saying its getting darker or forms dark spots or getting darker for sound the hint can be given saying it getting quiter or there is No Sound

For destructive interference (single slit) the Phase shift should be odd
$= \pi, 3\pi, 5\pi, 7\pi$

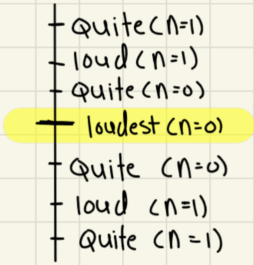
equation For destructive inference For Single Slit

$$n\lambda = d\sin\theta$$

the constructive equation for a single is a very complicated and beyond all of this

- <u>two Slits</u>

- when you two slits $n=0$ the central maximum is always the brighest or loudest on the screen

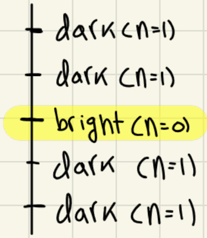


- the loud and Quite are all treated seperatly

☆ these are only For two Slits

- <u>Single Slit</u>

- the middle or central axis is always going to be the brightest or loudest Spot on the screen but the rest are all dark



- the middle central axis is the brightest

☆ this is only For Single Slit

- diffraction grating

- this is a optical device that has a large number of equally spaced parallel slits or lines

equation for diffraction grating:

$$n\lambda = d \sin\theta$$

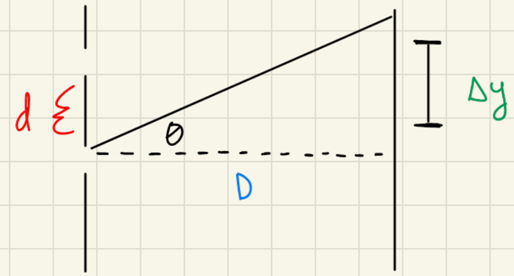
- grating spaces "d"
- if the grating has N lines per mm then:

$$d = \frac{1}{\text{number of slits per unit length}}$$

$$d = \frac{1}{N}$$

- Why don't we use destructive formula:

- When we are studying diffraction grating we only usally care about bright lines but destructive inference gives you dark gaps
- diffraction grating produces constructive interference given by the grating equation
- For grating we use the constructive interference formula because it gives the well defined bright lines we can actually measure
- the dark area exist but there isn't a simple equation for them



d: tells you how far the slits are if there is one slit they d is the slit width

D: the length from the laser to the screen

Δy: the distance on the screen between two point (n-values)

equation summary

$$\Delta \Phi = k \Delta x + \Delta \phi_0$$

$$(n + 1/2)\lambda = d \sin\theta$$

$$n\lambda = d \sin\theta$$

$$v = \lambda f$$

$$d = \frac{1}{N}$$

1.2 Class 2: Diffraction

1.2.1 An Interference Question

An astronaut travels to a distant planet where the speed of sound in the atmosphere is unknown. Standing exactly midway between two speakers playing 256 Hz sinusoidal waves, as shown in the picture below, he finds he hears destructive interference (i.e., no sound).

The astronaut then walks towards speaker 1 and find that the sound gets louder, first reaching maximum loudness after he has walked a distance of 32 cm.

1. What is the difference in phase constant $\phi_{02} - \phi_{01}$ for the two speakers?

   Notice in his initial location, $\Delta x = 0$ so $\Delta\Phi = \Delta\phi = \pi$ (or any odd multiple of π), since the interference is destructive.

2. What is the speed of sound in the atmosphere of this distant planet?

   Assume the astronaut is initially d cm from both speakers. Then after moving 32 cm toward speaker 1, he is $d-32$ from speaker 1 and $d+32$ from speaker 2. Hence

   $$\Delta x = x_2 - x_1 = d + 32 - (d - 32) = 64 \text{ cm}$$

   Notice that if $\Delta\Phi = \pi$ initially, then $\Delta\Phi = 2\pi$ now, with no ambiguity, as we want the first constructive interference point (4π would correspond to the second, 6π to the third, etc, while 0 would correspond to the first in the opposite direction). Hence

   $$\Delta\Phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0 \Rightarrow 2\pi = \frac{2\pi}{\lambda}(0.64 \text{ m}) + \pi \Rightarrow \lambda = (2(0.64 \text{ m})) \approx 1.28 \text{ m}$$

   so using the wave velocity equation

   $$v = \lambda f = (1.28 \text{ m})(256 \text{ Hz}) \approx 328 \text{ m/s}$$

   which is slower than that on earth's surface under normal conditions.

1.2.2 A Two Source Sound Experiment

Since Huygens's Principle is a property of all waves, not just light, a two-source interference pattern can be produced with sound waves, too, characterized by the equations. Constructive interference points - i.e., loud sound - is found whenever

$$n\lambda = d\sin\theta \tag{1.3}$$

for n any integer, where d is the source separation distance, λ the wavelength, and θ the angle made by the rays of sound relative to the horizontal. Meanwhile, the points of destructive interference - i.e., points of quiet or almost no sound - obey the equation

$$\left(n + \frac{1}{2}\right)\lambda = d\sin\theta \tag{1.4}$$

Consider two speakers, both playing the same 850 Hz sinusoidal tone with the same fixed phase constant ϕ_0, that are placed 2.5 meters apart from one another, as shown in the picture below (as viewed from above). Felicity, who starts an unknown distance D from the midway point between the two speakers in the direction perpendicular to the line separating the speakers, finds that if she walks 14.0 m in the direction parallel to the speakers, the 850 Hz sound gets quieter, then louder, then quieter again, so that, at the end of her walk, she is hearing almost no sound.

1. Assuming the speed of sound in the air is about 340 m/s, what is the wavelength, λ, of the sound being by the two speakers?

   Notice that the speed of the wave must be such that

   $$v = \frac{\lambda}{T} \Rightarrow \lambda = vT = \frac{v}{f} = \frac{340 \text{ m/s}}{850Hz} \approx 0.4 \text{ m}$$

2. Which equation (1.3 or 1.4) is the correct one to use in this problem?

   Notice this problem can be treated as a two-slit interference question for which $d = 2.5$ m, $\Delta y = 1$ $\lambda = 0.4$ m, and where Felicity ends on a point of destructive interference. Hence equation 1.4 must be em

3. What is the value of n corresponding to the end point of Felicity's walk?

   To find the correct n, notice that Felicity starts at the $n = 0$ constructive point, then walks past tl destructive point (the first quiet spot) and the $n = 1$ constructive point (the second loud spot) before end the $n = 1$ destructive point. Hence $n = 1$.

4. What is the unknown distance D? Hint: Find θ first, and then use trigonometry!

   Plugging everything in, we have:

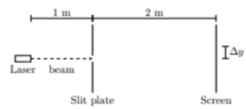
   $$\left(n + \frac{1}{2}\right)\lambda = d\sin\theta \to \theta = \sin^{-1}\left(\frac{(n+\frac{1}{2})\lambda}{d}\right) \approx \sin^{-1}\left(\frac{(1+\frac{1}{2})(0.4 \text{ m})}{(2.5 \text{ m})}\right) \approx 13.89^\circ$$

   Using trigonometry, we then get

   $$\tan\theta = \frac{\Delta y}{D} \Rightarrow D = \frac{\Delta y}{\tan\theta} \approx \frac{14 \text{ m}}{\tan(13.89^\circ)} \approx 56.6 \text{ m}$$

1.2.3 A Single Slit Example

For single slit diffraction, the points of destructive interference (i.e., dark spots in the pattern) obey equation 1 with the case $n = 0$ omitted (and corresponding to the constructive interference point at the center of the pat d corresponding to the width of the slit. Consider the theoretical example of a 650 nm red laser, whose beam incident on a narrow slit of width 15 μm spaced 1 m from the laser, as shown in the picture below. Two mete on is a screen upon which the interference pattern will be projected. (Note: the figure is not drawn to scale.)



Applying the single slit diffraction equation, we have (for $n = 1$)

$$n\lambda = d\sin\theta \Rightarrow \sin\theta = \frac{n\lambda}{d} = \frac{1(650 \text{ nm})}{15\ \mu m} = \frac{650 \times 10^{-9} \text{ m}}{15 \times 10^{-6} \text{ m}} \Rightarrow \theta \approx \sin^{-1}\left(\frac{650 \times 10^{-9} \text{ m}}{15 \times 10^{-6} \text{ m}}\right) \approx 2.48^\circ$$

while for $n = 3$

$$\sin\theta = \frac{n\lambda}{d} = \frac{1(650 \text{ nm})}{15\ \mu m} = \frac{1950 \times 10^{-9} \text{ m}}{15 \times 10^{-6} \text{ m}} \Rightarrow \theta \approx \sin^{-1}\left(\frac{1950 \times 10^{-9} \text{ m}}{15 \times 10^{-6} \text{ m}}\right) \approx 7.47^\circ$$

2. Notice that this angle, while easy to calculate, is much more difficult to measure experimentally. Instead, in a real experiment, we would typically measure the distance on the screen from the center of the pattern to the dark spot in question (Δy in the figure above). For this example, what is Δy for the $n = 1$ dark spot? What about Δy for the $n = 3$ dark spot? Hint: use trigonometry.

   Notice this distance is such that

   $$\tan\theta = \frac{\Delta y}{D}$$

   where D is the distance between the slit plate and the screen (here, 2 meters). Hence, for the $n = 1$ case

   $$\Delta y = D\tan\theta = (2 \text{ m})\tan(2.48^\circ) \approx 0.0867 \text{ m} \approx 8.67 \text{ cm}$$

   while for the $n = 3$ case

   $$\Delta y = D\tan\theta = (2 \text{ m})\tan(7.47^\circ) \approx 0.262 \text{ m} \approx 26.2 \text{ cm}$$

3. The calculation you did in question 2 can be simplified by using a "small angle approximation". Recall that the Taylor expansions of $\sin\theta$ and $\tan\theta$ are both θ to order θ^3. Hence we can make the small angle approximation

   $$\sin\theta \approx \theta \approx \tan\theta \tag{1.5}$$

   Redo your calculations from question 2 using this approximation. Are your values for Δy noticeably different?

   Returning to first principles, we have, under this approximation

   $$n\lambda = d\sin\theta \approx d\tan\theta = d\frac{\Delta y}{D} \Rightarrow \Delta y = \frac{n\lambda D}{d}$$

   Hence, for the $n = 1$ case

   $$\Delta y = \frac{(1(650 \times 10^{-9} \text{ m})(2 \text{ m})}{15 \times 10^{-6} \text{ m}} \approx 0.0867 \text{ m} = 8.67 \text{ cm}$$

   which is the same as in question 2, while for the $n = 3$ case.

   $$\Delta y = \frac{(3(650 \times 10^{-9} \text{ m})(2 \text{ m})}{15 \times 10^{-6} \text{ m}} \approx 0.26 \text{ m} = 26.0 \text{ cm}$$

   which is off by 0.2 mm, not too shabby!

4. Does the distance between the laser and the slit plate matter for the purposes of this calculation?

   Nope! (Notice we never used the 1 m in any of the foregoing calculations!)

1.2.4 Quantum Mechanics?

The wave-particle debate surrounding light would eventually be resolved with the dawn of quantum mechanics, where it was realized that light is both a wave and a particle, depending on the sort of experiment you are running - that is to say, that the very idea of "waves" and "particles" as distinct concepts is not actually correct. One interesting result of this

$$\lambda = \frac{h}{p} \tag{1.6}$$

where p is the momentum and h Planck's constant; in SI units,

$$h \approx 6.626 \times 10^{-34} \text{ J}\cdot\text{s} \tag{1.7}$$

1. Consider the case of a baseball (of regulation mass 145 g) traveling at a speed of 45 m/s (or about 100 mph). What is the de Broglie wavelength of this baseball? In order to get a deflection angle of 0.5° for the $n = 1$ maximum for a double slit experiment involving a beam of these baseballs, what must the separation distance d between the two slits be? Would it be possible to create such a diffraction experiment involving baseballs?

   Notice the de Broglie wavelength is

   $$\lambda = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{(0.145 \text{ kg})(45 \text{ m/s})} \approx 1.02 \times 10^{-34} \text{ m}$$

   Hence

   $$n\lambda = d\sin\theta \Rightarrow d = \frac{n\lambda}{\sin\theta} = \frac{(1)(1.02 \times 10^{-34} \text{ m})}{\sin(0.5^\circ)} \approx 1.16 \times 10^{-32} \text{ m}$$

   which, considering the size of a baseball, is not realistic. Hence, we wouldn't expect to observe diffraction patterns involving baseballs.

2. Consider the case of an electron (of mass 9.11×10^{-31} kg) traveling at a speed of 10000 m/s. What is the de Broglie wavelength of this electron? In order to get a deflection angle of 0.5° for the $n = 1$ maximum for a double slit experiment involving a beam of these electrons, what must the separation distance d between the two slits be? Is this sort of diffraction experiment possible?

   Notice the de Broglie wavelength is

   $$\lambda = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(10000 \text{ m/s})} \approx 7.27 \times 10^{-8} \text{ m}$$

   Hence

   $$n\lambda = d\sin\theta \Rightarrow d = \frac{n\lambda}{\sin\theta} = \frac{(1)(7.27 \times 10^{-8} \text{ m})}{\sin(0.5^\circ)} \approx 8.33 \times 10^{-6} \text{ m}$$

   which is very reasonable, as it is on the micrometer scale (and, indeed, larger than the slit separation of the diffraction grating we will use in Lab 2). Hence this sort of experiment is indeed quite possible to run (and indeed electron diffraction is quite easy to observe).