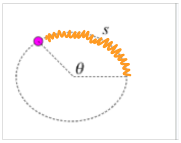
Rotational Motion



$s = r\theta$

s = arc length

θ = angular distance

r = radius

r is a constant it does not change

we can use this formula to get angular velocity and angular acceleration

angular velocity (rad/s)

$v = \frac{ds}{dt} \Rightarrow \frac{d}{dt}(r\theta) \Rightarrow r\frac{d\theta}{dt}$ r is a constant

$\boxed{\omega = \frac{d\theta}{dt}}$

where ω is the angular velocity

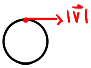
$\boxed{\omega = 2\pi f}$: this can also represent angular velocity / it will give you angular velocity aswell

$\boxed{f = \frac{\text{revolution}}{\text{time}}}$ Frequency is how frequent some thing is showing up

- linear velocity (tangential velocity) (m/s)

$\boxed{|\vec{v}| = r\omega}$

this will point tangentially to the curve along which it point



$v = r\omega$

Futhermore:

$a_c = \frac{v^2}{r} \Rightarrow \frac{(r\omega)^2}{r} \Rightarrow \frac{r^2\omega^2}{\cancel{r}} = r\omega$

$F_c = m \cdot \frac{v^2}{r} \Rightarrow F_c = mr\omega^2$

$\boxed{a_c = \omega^2 r}$ and $\boxed{F_c = mr\omega^2}$

both the centripetal formulas describe the same thing but they are written differently but a question can ask for centripetal acceleration giving you ω and r

angular acceleration (rad/s²)

if we take another derivative

$a_t = \frac{d}{dt}(r\omega) \Rightarrow r\frac{d\omega}{dt} = r\alpha$

$\boxed{a_t = r\alpha}$ tangential acceleration

$\boxed{\alpha = \frac{d\omega}{dt}}$ angular acceleration

- the tangential acceleration is the acceleration along a tanget line to a circular path

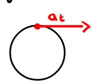
- angular acceleration
think: "is the object spinning faster or slower"

- tangential acceleration
think: "how fast is the object speeding up/down the circle edge"

difference between tangential acceleration and centripetal acceleration

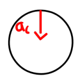
tangential acceleration $a_t = r\alpha$

the tangential acceleration changes the speed of the object along the tanget line along the edge of a circular path



centripetal acceleration $a_c = \frac{v^2}{r}$ or $a_c = \omega^2 r$

centripetal acceleration is the acceleration towards the center of the circular path it will point inward



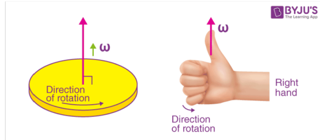
total acceleration

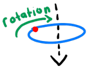
- to find the total acceleration we can use the Pythagreom theorm and combine a_c and a_t

$$\boxed{a_{total} = \sqrt{a_c^2 + a_t^2}}$$

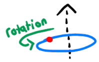
Sign Convention For ω (right hand rule)

- curl your Fingers on your right hand (it will not work on your left hand) direction of the rotation then ω Points in the direction of your right thumb





the direction of this rotion is going inward so using the right hand rule for ω your hand should look like a thumbs down your thumb is Pointing down so ω is down



- the direction of this rotation is going outward so using your right hand rule for ω your hand should look like a thumbs up

rotation dynamics

- For translational motion we have:

$$\vec{F} = m\vec{a}$$

- For rotational motion this becomes

$$\boxed{\vec{\tau} = I\vec{\alpha}}$$

angular acceleration

moment of interia

torque (N·m)

- measures how much Force will cause an object to rotate
- r is a vector that Points From the axis of rotation to the Point where the Force is applied

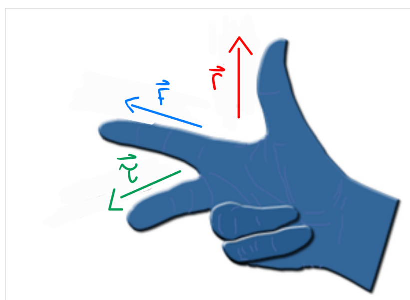
$$\boxed{\vec{\tau} = |\vec{r}|\sin\theta|\vec{F}|}$$

right hand rule for torque

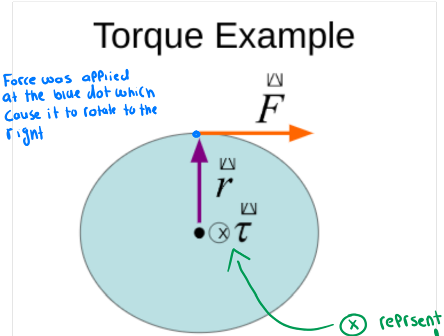
⊗ : represents inward movement (into Page)
⊙ : represents outward movement (out of Page)

- this only works for right hand
- Point out right thumb in the direction of r
- Point out right index Finger in direction of F
- as soon as you have both of these done keep your right hand in the same Position don't move anything open your middle Finger thats where $\vec{\tau}$ will be Pointing
- use right rule to find the torque, angular velocity but when you want to find the direction of angular acceleration it will Point in the same direction as torque because of the relationship they both have

$$\vec{\tau} = I\vec{\alpha}$$



Force was applied at the blue dot which couse it to rotate to the right



angular acceleration will follow the same direction as torque so it will go ⊗ inward

we can also find the angular velocity by using the right hand rule for angular velocity we know it rotates to the right by curling our right hand finger we can see it also goes ⊗ inward

⊗ repersent going inward

- notice r goes from the axis of rotation to where the force was applied

From	To	Conversion Formula
Radians	Revolutions	$\frac{\text{radians}}{2\pi}$
Revolutions	Radians	$\text{revolutions} \times 2\pi$
Radians	Degrees	$\text{radians} \times \frac{180^\circ}{\pi}$
Degrees	Radians	$\text{degrees} \times \frac{\pi}{180^\circ}$

- this table will help in converting from revolution to radian

Summary of equations

- $\omega = \frac{d\theta}{dt}$
- $\omega = 2\pi f$
- $f = \frac{\text{revolutions}}{\text{time}}$
- $|\vec{v}| = r\omega$
- $a_c = \omega^2 r$ or $a_c = \frac{v^2}{r}$

- $\alpha = \frac{d\omega}{dt}$
- $a_t = r\alpha$
- $a_{tot} = \sqrt{a_c^2 + a_t^2}$
- $\vec{\tau} = I\vec{a}$
- $\vec{\tau} = |\vec{r}| \sin\theta |\vec{F}|$

$$\omega_f = \alpha \Delta t + \omega_i$$

$$\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 + \omega_i \Delta t$$

$$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{\omega_f + \omega_i}{2}\Delta t$$

- these rotational kinematics where not included in these notes they are kinematics there really isn't any to write notes about