

1. Theory

Question 1

a) i)

Given by lecture:

$$\text{TSS} = \text{ESS} + \text{RSS}$$

$$\text{TSS} = \sum_{i=1}^n y_i^2$$

$$\text{ESS} = \sum_{i=1}^n \hat{y}_i^2$$

$$\text{RSS} = \sum_{i=1}^n e_i^2$$

Formal proof of $\text{TSS} = \text{ESS} + \text{RSS}$:

$$\begin{aligned} \text{TSS} &= \sum_{i=1}^n y_i^2 = \sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n (y_i - \bar{y} + \hat{y}_i - \hat{y}_i)^2 = \sum_{i=1}^n ((\hat{y}_i - \bar{y}) + (y_i - \hat{y}_i))^2 = \\ &\quad \sum_{i=1}^n ((\hat{y}_i - \bar{y}) + \hat{e}_i)^2 = \sum_{i=1}^n ((\hat{y}_i - \bar{y})^2 + 2\hat{e}_i(\hat{y}_i - \bar{y}) + \hat{e}_i^2) = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 + \\ &\quad 2\sum_{i=1}^n \hat{e}_i(\hat{y}_i - \bar{y}) + \sum_{i=1}^n \hat{e}_i^2 = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 + \sum_{i=1}^n \hat{e}_i^2 + 2\sum_{i=1}^n \hat{e}_i(\widehat{\beta_0} + \widehat{\beta_1 x_{i1}} + \dots) + \\ &\quad \widehat{\beta_k x_{ik}} - \bar{y}) = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 + \sum_{i=1}^n \hat{e}_i^2 + 2(\widehat{\beta_0} - \bar{y})\sum_{i=1}^n \hat{e}_i^2 + 2\widehat{\beta_1} \sum_{i=1}^n \hat{e}_i x_{i1} + \dots + \\ &\quad 2\widehat{\beta_k} \sum_{i=1}^n \hat{e}_i x_{i1} = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 + \sum_{i=1}^n \hat{e}_i^2 = \sum_{i=1}^n \hat{y}_i^2 + \sum_{i=1}^n \hat{e}_i^2 = \text{ESS} + \text{RSS} \end{aligned}$$

ii)

$$\text{Formal proof of } R^2 = 1 - \frac{\text{RSS}}{\text{TSS}} = 1 - \frac{e'e}{\tilde{y}'\tilde{y}}$$

$$\begin{aligned} R^2 &= \frac{\text{ESS}}{\text{TSS}} = \frac{\sum_{i=1}^n \hat{y}_i^2}{\sum_{i=1}^n y_i^2} = \frac{\text{ESS}}{\text{ESS} + \text{RSS}} = \frac{\sum_{i=1}^n \hat{y}_i^2}{\sum_{i=1}^n \hat{y}_i^2 + \sum_{i=1}^n e_i^2} = \frac{\sum_{i=1}^n \hat{y}_i^2 + \sum_{i=1}^n e_i^2 - \sum_{i=1}^n e_i^2}{\sum_{i=1}^n \hat{y}_i^2 + \sum_{i=1}^n e_i^2} \\ &= \frac{\sum_{i=1}^n y_i^2 - \sum_{i=1}^n e_i^2}{\sum_{i=1}^n y_i^2} = 1 - \frac{\sum_{i=1}^n e_i^2}{\sum_{i=1}^n y_i^2} = 1 - \frac{\text{RSS}}{\text{TSS}} = 1 - \frac{e'e}{\tilde{y}'\tilde{y}} \end{aligned}$$

b)

$$\text{Formal proof of } R^2 = \text{corr}^2(\mathbf{y}, \hat{\mathbf{y}}) = \rho_{\mathbf{y}, \hat{\mathbf{y}}}^2 = \frac{\text{ESS}}{\text{TSS}}$$

→ First take the square root of $\rho_{\mathbf{y}, \hat{\mathbf{y}}}^2$

$$\begin{aligned}
\rho_{y,\hat{y}} &= \frac{\sigma_{y,\hat{y}}^2}{\sqrt{\sigma_y^2 * \sigma_{\hat{y}}^2}} = \frac{\sum_{i=1}^n (y_i - \bar{y})(\hat{y}_i - \bar{y})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 * \sum_{i=1}^n (\hat{y}_i - \bar{y})^2}} = \frac{\sum_{i=1}^n (y_i + \hat{y}_i - \bar{y}_i - \bar{y})(\hat{y}_i - \bar{y})}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 * \sum_{i=1}^n (\hat{y}_i - \bar{y})^2}} \\
&= \frac{\sum_{i=1}^n (y_i \hat{y}_i + \hat{y}_i^2 - \hat{y}_i^2 - \bar{y} \hat{y}_i - \bar{y} y_i + \hat{y}_i \bar{y} - \bar{y} \hat{y}_i + \bar{y}^2)}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 * \sum_{i=1}^n (\hat{y}_i - \bar{y})^2}} \\
&= \frac{\sum_{i=1}^n ((y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) + (\hat{y}_i - \bar{y})^2)}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 * \sum_{i=1}^n (\hat{y}_i - \bar{y})^2}} \\
&= \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2 * \sum_{i=1}^n (\hat{y}_i - \bar{y})^2}} = \sqrt{\frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2}} = \sqrt{\frac{ESS}{TSS}}
\end{aligned}$$

→ And now square it: $\frac{ESS}{TSS} = \rho_{y,\hat{y}}^2 = corr^2(y, \hat{y}) = R^2$

One can interpret R^2 as the squared correlation coefficient between the true value y_i and the estimated value \hat{y}_i . In a regression model R^2 measures how good the estimated value explains the true value. In other words, it explains the variation in the estimated \hat{y}_i and its true value y_i . Thus, this can be translated to the correlation between these two variables as shown above. One can generally say: The higher R^2 or $\rho_{y,\hat{y}}^2$ the better the model can predict the true values where the R^2 is in a range between 0 and 1 and the correlation between -1 and 1.

- c) By transforming our X variables linearly, one cannot gain more information out of our variables than before. For each datapoint the true value of y_i and the estimated value of \hat{y}_i do not change at all by linear transformation. It neither change the relation nor the underlying data. Since we have showed above that $R^2 = \rho_{y,\hat{y}}^2$, one can see that the correlation between y_i and \hat{y}_i is not affected either and the calculated R^2 remains the same.
- d) Intuitively the residual sum squared always decreases if one adds another regressor into the model (see formal proof 1.e) and so R^2 will increase. That is because with a new regressor one might gather more information from the data, since the estimated values may be predicted more precisely. Only in a special case, if the added regressor is perfectly correlated with an already existing one in the model, the RSS would stay the same, since one cannot gather more information by adding this new but perfectly correlated variable.

e)

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^n e_i^2}{\sum_{i=1}^n y_i^2} = 1 - \frac{\sum_{i=1}^n (y_i - x_i \hat{\beta} - \bar{e})^2}{\sum_{i=1}^n y_i^2}$$

If one adds an additional regressor x_i , the $\hat{\beta}$ will increase and therefore the RSS is getting smaller as a direct consequence of this. Furthermore, R^2 will increase. In other words, the precision of the prediction is increasing until the error term is hypothetically equal to zero.

- f) Working with R^2 could lead to multiple types of misinterpretation:
- The **type** of data implies different values of R^2 . While time-series data often have a high R^2 , the exact opposite is often true for cross-section data.
 - Different functional forms for y_i can **change** R^2 (for example: using $\log(y_i)$ often increases R^2)
 - Adding explanatory variables to a model always **increase** R^2
 - o This may lead one towards overfitting the model with too many variables. So, one can increase the R^2 even by just adding a large set of totally random predictors.

Furthermore, it is only an in-sample measurement: That means R^2 only measures the precision within the sample. A better way to measure the precision between samples is for example the measurement techniques of Cross-Validation.

2. Empirical Question

- a) Big school dummy
 - i) The coefficient on *classsize* is 0.134 and statistically significant. In this case, this means that the average marks in a grammar tests rises by 0.134 per additional student in class.
 - ii) The new coefficient on *classsize* shrinks to 0.102, the one of the new dummy *big school* is 1.246 (both are statistically significant). This means that, while an additional student only brings a rise in the average grammar test marks of 0.102 by controlling for school size, the bigger schools have higher averages.
- b) Natural log
 - i) The coefficient is -0.0603 on *classsize* (-0.0007 in the log-model) and -0.335 on *pct_dis* (-0.0048 in the log model). The effect of *classsize* is now even smaller than before and now slightly negative. The effect of the percentage of disadvantaged kids is also negative. With a rise of the amount of disadvantaged kids of 1%, the average mark decreases by -0.335 points. The model with the logged grammar scores shows the coefficients as an (approximated) percentage change with respect to the constant. The coefficients in the log model show percentage changes, meaning with one unit change in *classsize* or *pct_dis*, the grammar marks change by 0.07% and 0.5%.
 - ii) In the regression of grammar scores on *classsize* and *pct_dis*, the coefficient of the latter variable can be interpreted as follows: If *pct_dis* rises by one unit, the average grammar mark decreases by 0.335 points controlling for class size.
- c) Small size dummy
 - i) The coefficient of small size tells us that small classes score 2.56 points higher on grammar tests than big classes, controlling for the percentage of disadvantaged kids. In terms of economical significance, this coefficient seems rather large, comparing it with previous coefficients and with respect to the constant. In Stata, the two-sided test $\beta_2 = 0$ yields a p-value of 0.2016. Assuming that a small sizes class has a positive effect on grammar scores, we also use a one-sided test $\beta_2 \leq 0$, yielding a p-value of 0.1008 (which in this scenario is exactly 50% of the two-sided p-value). This means we cannot reject our hypotheses.

We recommend using the two-sided hypothesis test, because with *small_size* we are looking at an extreme attribute of class sizes and should not assume that the effect of *small_size* is positive (only 8 classes fall into the category small sized). This lack of

observations for small class sizes explains why the results are far from significant.

Calculation of the hypothesis test that $\beta_2 = 0$ by hand:

$$1: \quad H_0: \beta_2 = 0, H_A: \beta_2 \neq 0$$

$$2: \quad \text{Degrees of freedom: } N - K = 1967 - 2 = 1965$$

$$s^2 = \frac{1}{N-K} \sum_{i=1}^n e_i^2$$

$$\text{t-value: } \frac{2.560}{2.004} = 1.277$$

$$3: \quad \text{At the 5% level: } P(t(1965) \leq \overline{t_{0.975}}) = 0.975 \rightarrow \overline{t_{0.975}} = 1.96$$

$$|1.277| > 1.96 \rightarrow H_0 \text{ cannot be rejected}$$

- ii) First, we need to regress the grammar scores only on *pct_dis* and then take the residuals from this model (see table below: 1). Then we regress *small_size* on *pct_dis* too and also take the residuals (see table below: 2). Now we can regress the residuals from the first model one the residuals on the second model. The resulting coefficient is the coefficient of grammar score on small sized classes (and is exactly the same as in i).

Y-VARIABLE: VARIABLES	(mrkgrm) 1	(small_size) 2	(residuals1) 3
pct_dis	-0.327*** (0.00977)	9.57e-06 (0.000110)	
residuals2			2.560 (2.003)
Constant	77.11*** (0.183)	0.00394* (0.00206)	-6.64e-09 (0.128)
Observations	1,967	1,967	1,967
R-squared	0.363	0.000	0.001

- iii) To show that $\hat{\beta}_1 = \bar{y} - \bar{X}_{-1}\hat{\beta}_{-1}$, we need the means of *mrkgrm*, *small_size* and *pct_dis*. Now by deducting the means of *small_size* and *pct_dis* multiplied with their respective coefficient from the mean of *mrkgrm*, we get $\hat{\beta}_1$:

$$\overline{mrkgrm} - \overline{small_size} \times 2.5597 - \overline{pct_dis} \times (-0.32677) = 77.099$$

- iv) The correct interpretation is that small classes have an average grammar score that is 3.65% higher than in bigger classes.
- d) Many disadvantaged dummy
- i) The joint hypothesis that $\beta_3 = 0$ and $\beta_4 = 0$ has an F-value of 401.85 (p-value = 0). Calculating this by hand shows the same results (small difference due to rounding):

$$\frac{(R_U^2 - R_R^2)/q}{(1 - R_U^2)/(N - K)} = \frac{(0.3005 - 0.0142)/2}{(1 - 0.3005)/(1966-3)} = 401.72$$

In conclusion, we reject the joint hypothesis that $\beta_3 = 0$ and $\beta_4 = 0$. Many disadvantaged kids (alone and combined with class size) affects grammar scores.

- ii) The effect of having 10 additional students in a class with less than 10% disadvantaged kids is -1.1 (since *many_dis* is a dummy with value 0, both β_3 and β_4 are not relevant in this specific case here)
- e) Separated regressions

The table below shows the results separating classes with high and low percentages of disadvantaged kids as well as the results from d). The coefficient on *classsize* in (2) is exactly the same as in (3), which makes sense because both only consider class size for classes with less than 10% disadvantaged kids. For classes with more than 10% disadvantaged kids, one can calculate that the separate regressions (1) and (2) and the combined regression (3) also yield the same results:

$$\begin{aligned} & 63.81 + \text{classsize} \times 0.159 \\ &= 79.52 + \text{classsize} \times (-0.110) + (-15.71) + (\text{classsize} \times \text{many_dis}) \times 0.269 \end{aligned}$$

In conclusion, model (3) is a combination of models (1) and (2)

VARIABLES	(1) high dis	(2) low dis	(3) d)
classsize	0.159*** (0.0373)	-0.110*** (0.0255)	-0.110*** (0.0291)
many_dis			-15.71*** (1.346)
classsize \times many_dis			0.269*** (0.0438)
Constant	63.81*** (1.103)	79.52*** (0.821)	79.52*** (0.937)
Observations	858	1,109	1,967
R-squared	0.021	0.017	0.301

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

- f) Region dummies

The region dummies cannot all be included in the same model because of multicollinearity (dummy variable trap). If one wants to include region dummies, one needs to be omitted.

- g) The table below shows the results for regression separated by region. While the coefficients of *pct_dis* are all negative and in the same range, the coefficients on *classize* are not.

VARIABLES	(1) Reg1	(2) Reg2	(3) Reg3	(4) Reg4	(5) Reg5	(6) Reg6
classize	-0.0901** (0.0451)	-0.0550 (0.0823)	0.168** (0.0669)	0.00741 (0.0459)	0.0212 (0.0429)	-0.0758 (0.0541)
pct_dis	-0.249*** (0.0218)	-0.252*** (0.0309)	-0.213*** (0.0275)	-0.490*** (0.0389)	-0.319*** (0.0195)	-0.404*** (0.0232)
Constant	81.01*** (1.311)	77.17*** (2.511)	69.65*** (2.226)	80.18*** (1.479)	75.24*** (1.557)	79.62*** (1.883)
Observations	255	195	267	276	574	400
R-squared	0.344	0.266	0.257	0.382	0.373	0.460

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

We propose a model that includes all the regions. To do so, we use the region dummies and omit region 5 (the region with most observations):

VARIABLES	mrkgrm
classize	0.00654 (0.0218)
pct_dis	-0.309*** (0.0103)
Reg1	3.501*** (0.431)
Reg2	0.969** (0.467)
Reg3	0.399 (0.410)
Reg4	3.258*** (0.419)
Reg6	0.249 (0.360)
Constant	75.55*** (0.801)
Observations	1,967
R-squared	0.400

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

This model shows positive coefficients for all regions, suggesting a positive effect of class size on grammar scores. The effect for region 5 is shown in the coefficient of *classize*, for the other regions one needs to add the effect of the respective region dummy.

- h) Subsample with only one class
 - i) The coefficient of *sc_boys* is -0.302 and statistically significant, meaning that with one additional boy in the school, the average grammar scores decrease by 0.302 points. The coefficient of *classize* is 0.0961, suggesting that the average grammar scores increase by this much with one additional student in class. However, this effect is not statistically significant anymore.
 - ii) The coefficient of *sc_boys* is -0.206. This means, controlling for the number of girls per school, an additional boy in the school decreases the average grammar scores by 0.206 points, which is slightly less than before.
 - iii) From the estimation in h-ii) one cannot say anything about the exact effect of one pupil in general, because the effects differ with respect to gender. However, we can expect the variance in the number of girls and boys to be roughly the same:

$$\text{Cov}(\text{sc}_\text{boys}, \text{sc}_\text{girls}) \approx \text{Var}(\text{sc}_\text{boys}) = \text{Var}(\text{sc}_\text{girls})$$

The standard deviations in model h-ii) suggest this expectation to be true. Also, the correlation between *sc_boys* and *sc_girls* is with 0.7918 close to 1.

Now we can compare the two models:

$$\begin{aligned}\hat{y} &= \hat{\beta}_0 + \hat{\beta}_1 \times \text{sc}_\text{girls} + \hat{\beta}_2 \times \text{sc}_\text{boys} + \epsilon \\ \hat{y} &= \hat{\gamma}_0 + \hat{\gamma}_1 \times (\text{sc}_\text{girls} + \text{sc}_\text{boys}) + \mu\end{aligned}$$

Solving the second model for $\hat{\gamma}_1$, we get:

$$\begin{aligned}\hat{\gamma}_1 &= \frac{\text{Cov}(\hat{y}, \text{sc}_\text{girls})}{\text{Var}(\text{sc}_\text{girls})} \frac{\text{Var}(\text{sc}_\text{girls})}{\text{Var}(\text{sc}_\text{girls} + \text{sc}_\text{boys})} \\ &\quad + \frac{\text{Cov}(\hat{y}, \text{sc}_\text{boys})}{\text{Var}(\text{sc}_\text{boys})} \frac{\text{Var}(\text{sc}_\text{boys})}{\text{Var}(\text{sc}_\text{girls} + \text{sc}_\text{boys})}\end{aligned}$$

Combining this equation with auxiliary models only including *sc_boys* or *sc_girls* solved for their respective coefficients, we get the following formula:

$$\begin{aligned}\hat{\gamma}_1 &= \frac{\text{Var}(\text{sc}_\text{girls}) + \text{Cov}(\text{sc}_\text{girls}, \text{sc}_\text{boys})}{\text{Var}(\text{sc}_\text{girls} + \text{sc}_\text{boys})} \hat{\beta}_1 \\ &\quad + \frac{\text{Var}(\text{sc}_\text{boys}) + \text{Cov}(\text{sc}_\text{boys}, \text{sc}_\text{girls})}{\text{Var}(\text{sc}_\text{girls} + \text{sc}_\text{boys})} \hat{\beta}_2\end{aligned}$$

Using the expectation with the similar variances and the correlations stated at the beginning of h-iii), we get the following approximation:

$$\hat{\gamma}_1 \approx \frac{1}{2}(\hat{\beta}_1 + \hat{\beta}_2) = \frac{1}{2}(0.096 - 0.206) = -0.055$$

Compared to the coefficient-value of the regression of *mrkgrm* on *classize* (-.0475), it seems that one can say something about the effect of increasing the class size by one pupil, even though the effect is different for boys and girls.

- i) It is very unlikely that assumption 2 holds. There are definitely omitted variables that influence the grammar scores. Possible examples are cultural background, family income, school types (public or private) or the degree of preparation the schools provide for this standardized grammar tests.

3. Log-file

See attachment



```
(863 real changes made)
  name: <unnamed>
  log: C:\Users\ramon\Desktop\UZH\Empirical Methods\Problem Sets\Problem Set 2\Stata\log_gm
log type: smcl
opened on: 11 Nov 2019, 17:42:25

1 .
2 . use "C:\Users\ramon\Desktop\UZH\Empirical Methods\Problem Sets\Problem Set 2\Stata\class_size_p
3 .
4 . *Empirical Question
5 .
6 . **a)
7 .
8 . gen big_school = 0
9 . replace big_school = 1 if n_classes > 2
(863 real changes made)

10 .
11 . ***a-i)
12 .
13 . reg mrkgrm classize

      Source |       SS           df          MS      Number of obs   =
             | 1396.59756        1  1396.59756   F(1, 1965)   =
             | 97259.1452       1,965  49.4957482 Prob > F     =
                                         R-squared   =
                                         Adj R-squared   =
                                         Root MSE    =
             | 98655.7428       1,966  50.1809475

      mrkgrm |      Coef.    Std. Err.      t     P>|t| [95% Conf. Interval]
             | .1341112  .0252472      5.31  0.000  .0845971  .1836254
             | 68.6283  .7839904     87.54  0.000  67.09076  70.16584

14 .
15 . outreg2 using "PS2_regressiona.doc", replace ctitle(a-i)
PS2_regressiona.doc
dir : seeout

16 .
17 . ***a-ii)
18 .
19 . reg mrkgrm classize big_school

      Source |       SS           df          MS      Number of obs   =
             | 2068.58052        2  1034.29026   F(2, 1964)   =
             | 96587.1622       1,964  49.1787995 Prob > F     =
                                         R-squared   =
                                         Adj R-squared   =
                                         Root MSE    =
             | 98655.7428       1,966  50.1809475

      mrkgrm |      Coef.    Std. Err.      t     P>|t| [95% Conf. Interval]
             | .1019125  .0266311      3.83  0.000  .0496844  .1541407
             | 1.246412  .3371876      3.70  0.000  .5851293  1.907695
             | 69.06062  .7901793     87.40  0.000  67.51094  70.6103
```

```

20 .
21 . outreg2 using "PS2_regressiona.doc", append ctitle(a-ii)
  PS2 regressiona.doc
  dir : seeout

22 .
23 . **b)
24 .
25 . drop big_school

26 .
27 . ***b-i)
28 .
29 . gen ln_mrkgrm = log(mrkgrm)

30 . reg mrkgrm classize pct_dis

```

Source	SS	df	MS	Number of obs	=	1,967
Model	36025.757	2	18012.8785	F(2, 1964)	=	564.86
Residual	62629.9858	1,964	31.8889948	Prob > F	=	0.0000
Total	98655.7428	1,966	50.1809475	R-squared	=	0.3652
				Adj R-squared	=	0.3645
				Root MSE	=	5.647

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.0602863	.0211063	-2.86	0.004	-.1016794 -.0188931
pct_dis	-.3348571	.0101615	-32.95	0.000	-.3547856 -.3149286
_cons	79.05196	.7043112	112.24	0.000	77.67068 80.43323

```

31 . outreg2 using "PS2_regressionbi.doc", replace ctitle(normal)
  PS2 regressionbi.doc
  dir : seeout

```

```
32 . reg ln_mrkgrm classize pct_dis
```

Source	SS	df	MS	Number of obs	=	1,967
Model	7.53443573	2	3.76721786	F(2, 1964)	=	573.18
Residual	12.9082879	1,964	.006572448	Prob > F	=	0.0000
Total	20.4427236	1,966	.01039813	R-squared	=	0.3686
				Adj R-squared	=	0.3679
				Root MSE	=	.08107

ln_mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.0007001	.000303	-2.31	0.021	-.0012944 -.0001059
pct_dis	-.0048256	.0001459	-33.08	0.000	-.0051117 -.0045395
_cons	4.367718	.0101113	431.96	0.000	4.347888 4.387548

```

33 . outreg2 using "PS2_regressionbi.doc", append ctitle(log)
PS2 regressionbi.doc
dir : seeout

34 .
35 . ***b-ii)
36 .
37 . **c)
38 .
39 . gen small_size = 0

40 . replace small_size = 1 if classize <= 10
(8 real changes made)

41 .
42 . reg mrkgrm small_size pct_dis

```

Source	SS	df	MS	Number of obs	=	1,967
Model	35817.7919	2	17908.8959	F(2, 1964)	=	559.74
Residual	62837.9509	1,964	31.9948833	Prob > F	=	0.0000
Total	98655.7428	1,966	50.1809475	R-squared	=	0.3631
				Adj R-squared	=	0.3624
				Root MSE	=	5.6564

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
small_size	2.559678	2.003923	1.28	0.202	-1.370361 6.489718
pct_dis	-.3267693	.0097728	-33.44	0.000	-.3459354 -.3076032
_cons	77.09925	.1834885	420.19	0.000	76.73939 77.4591

```

43 . outreg2 using "PS2_regressionnc.doc", replace ctitle(c)
PS2 regressionnc.doc
dir : seeout

44 . test _b[small_size]=0
( 1) small_size = 0

      F( 1, 1964) = 1.63
      Prob > F = 0.2016

45 . local sign_ss = sign(_b[small_size])

46 . display "Ho: coef <= 0 p-value = " ttail(r(df_r),`sign_ss'*sqrt(r(F)))
Ho: coef <= 0 p-value = .10081773

47 .
48 . ***c-i)
49 .

```

```

50 . ****Hand- and Stata-Testing!!!
51 .
52 . ***c-ii)
53 .
54 . reg mrkgrm pct_dis

```

Source	SS	df	MS	Number of obs	=	1,967
Model	35765.5896	1	35765.5896	F(1, 1965)	=	1117.49
Residual	62890.1531	1,965	32.005167	Prob > F	=	0.0000
Total	98655.7428	1,966	50.1809475	R-squared	=	0.3625
				Adj R-squared	=	0.3622
				Root MSE	=	5.6573

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
pct_dis	-.3267448	.0097743	-33.43	0.000	-.3459139	-.3075757
_cons	77.10933	.1833481	420.56	0.000	76.74975	77.4689

```

55 . outreg2 using "PS2_regressionci.doc", replace ctitle(1)
PS2 regressionci.doc
dir : seeout

```

```
56 . predict residuals1, residuals
```

```

57 .
58 . reg small_size pct_dis

```

Source	SS	df	MS	Number of obs	=	1,967
Model	.00003067	1	.00003067	F(1, 1965)	=	0.01
Residual	7.96743247	1,965	.004054673	Prob > F	=	0.9307
Total	7.96746314	1,966	.004052626	R-squared	=	0.0000
				Adj R-squared	=	-0.0005
				Root MSE	=	.06368

small_size	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
pct_dis	9.57e-06	.00011	0.09	0.931	-.0002062	.0002253
_cons	.0039382	.0020637	1.91	0.056	-.0001091	.0079854

```

59 . outreg2 using "PS2_regressionci.doc", append ctitle(2)
PS2 regressionci.doc
dir : seeout

```

```
60 . predict residuals2, residuals
```

```
61 .
```

```
62 . reg residuals1 residuals2
```

Source	SS	df	MS	Number of obs	=	1,967
Model	52.2022502	1	52.2022502	F(1, 1965)	=	1.63
Residual	62837.9512	1,965	31.9786011	Prob > F	=	0.2015
Total	62890.1534	1,966	31.9888878	R-squared	=	0.0008
				Adj R-squared	=	0.0003
				Root MSE	=	5.655

residuals1	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
residuals2	2.559679	2.003413	1.28	0.202	-1.369359	6.488716
_cons	-6.64e-09	.1275051	-0.00	1.000	-.2500594	.2500594

```

63 . outreg2 using "PS2_regressionci.doc", append ctitle(3)
PS2 regressionci.doc
dir : seeout

64 .
65 . ***c-iii)
66 .
67 . egen mean_mrkgrm = mean(mrkgrm)

68 . egen mean_small_size = mean(small_size)

69 . egen mean_pct_dis = mean(pct_dis)

70 .
71 . display mean_mrkgrm - mean_small_size*2.559768 - mean_pct_dis*-.3267693
77.099243

72 .
73 . ***c-iv)
74 .
75 . reg ln_mrkgrm small_size pct_dis

```

Source	SS	df	MS	Number of obs	=	1,967
Model	7.5099808	2	3.7549904	F(2, 1964)	=	570.24
Residual	12.9327428	1,964	.0065849	Prob > F	=	0.0000
Total	20.4427236	1,966	.01039813	R-squared	=	0.3674
				Adj R-squared	=	0.3667
				Root MSE	=	.08115

ln_mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
small_size	.0365346	.0287485	1.27	0.204	-.0198462	.0929154
pct_dis	-.0047317	.0001402	-33.75	0.000	-.0050067	-.0044568
_cons	4.345013	.0026323	1650.62	0.000	4.339851	4.350176

```

76 . outreg2 using "PS2_regressionciv.doc", replace ctitle(c-iv)
PS2 regressionciv.doc
dir : seeout

77 .
78 .
79 . **d)

```

```

80 .
81 . gen many_dis = 0
82 . replace many_dis = 1 if pct_dis > 10
(858 real changes made)
83 . gen many_disXclassize = many_dis*classize
84 . reg mrkgrm classize many_dis many_disXclassize

```

Source	SS	df	MS	Number of obs	=	1,967
Model	29649.3319	3	9883.11065	F(3, 1963)	=	281.14
Residual	69006.4108	1,963	35.153546	Prob > F	=	0.0000
Total	98655.7428	1,966	50.1809475	R-squared	=	0.3005
				Adj R-squared	=	0.2995
				Root MSE	=	5.929

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.1101571	.0291463	-3.78	0.000	-.1673179 -.0529962
many_dis	-15.71298	1.346291	-11.67	0.000	-18.35329 -13.07267
many_disXclassize	.2686753	.043792	6.14	0.000	.1827916 .354559
_cons	79.52074	.9369263	84.87	0.000	77.68326 81.35821

```

85 . outreg2 using "PS2_regressionond.doc", replace ctitle(d)
PS2 regressionond.doc
dir : seeout

86 .
87 . ***d-i)
88 .
89 . test _b[many_dis]=0
( 1) many_dis = 0
      F( 1, 1963) = 136.22
      Prob > F = 0.0000

90 . test _b[many_disXclassize]=0, accumulate
( 1) many_dis = 0
( 2) many_disXclassize = 0
      F( 2, 1963) = 401.85
      Prob > F = 0.0000

91 .
92 .
93 . ***Ru^2=0.3005

```

94 . reg mrkgrm classize

Source	SS	df	MS	Number of obs	=	1,967
Model	1396.59756	1	1396.59756	F(1, 1965)	=	28.22
Residual	97259.1452	1,965	49.4957482	Prob > F	=	0.0000
Total	98655.7428	1,966	50.1809475	R-squared	=	0.0142
				Adj R-squared	=	0.0137
				Root MSE	=	7.0353

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	.1341112	.0252472	5.31	0.000	.0845971 .1836254
_cons	68.6283	.7839904	87.54	0.000	67.09076 70.16584

95 . ****Rr^2=0.0142
 96 . ****q=2, N-K=df=1, 963
 97 . display ((0.3005-0.0142)/2)/((1-0.3005)/1963)
401.72044

98 .
 99 .
 100 . ***d-ii)
 101 .
 102 . **e)
 103 .
 104 . reg mrkgrm classize if many_dis == 1

Source	SS	df	MS	Number of obs	=	858
Model	826.915211	1	826.915211	F(1, 856)	=	18.08
Residual	39159.72	856	45.7473364	Prob > F	=	0.0000
Total	39986.6352	857	46.6588509	R-squared	=	0.0207

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	.1585182	.0372848	4.25	0.000	.0853379 .2316986
_cons	63.80775	1.102878	57.86	0.000	61.64309 65.97241

105 . outreg2 using "PS2_regressione.doc", replace ctitle(high dis)
PS2 regressione.doc
dir : seeout

106 . reg mrkgrm classize if many_dis == 0

Source	SS	df	MS	Number of obs	=	1,109
Model	502.144157	1	502.144157	F(1, 1107)	=	18.62
Residual	29846.6908	1,107	26.9617803	Prob > F	=	0.0000
Total	30348.835	1,108	27.3906453	R-squared	=	0.0165

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.1101571	.0255254	-4.32	0.000	-.1602407 -.0600735
_cons	79.52074	.8205313	96.91	0.000	77.91076 81.13071

107 . outreg2 using "PS2_regressione.doc", append ctitle(low dis)
PS2 regressione.doc
dir : seeout

108 . reg mrkgrm classize many_dis many_disXclassize

Source	SS	df	MS	Number of obs	=	1,967
Model	29649.3319	3	9883.11065	F(3, 1963)	=	281.14
Residual	69006.4108	1,963	35.153546	Prob > F	=	0.0000
Total	98655.7428	1,966	50.1809475	R-squared	=	0.3005
				Adj R-squared	=	0.2995
				Root MSE	=	5.929

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.1101571	.0291463	-3.78	0.000	-.1673179 -.0529962
many_dis	-15.71298	1.346291	-11.67	0.000	-18.35329 -13.07267
many_disXclassize	.2686753	.043792	6.14	0.000	.1827916 .354559
_cons	79.52074	.9369263	84.87	0.000	77.68326 81.35821

109 . outreg2 using "PS2_regressione.doc", append ctitle(d)
PS2 regressione.doc
dir : seeout

110 .
111 . **f)
112 .
113 . foreach regioncode in Reg1 Reg2 Reg3 Reg4 Reg5 Reg6{
2. gen `regioncode' = 0
3. replace `regioncode' = 1 if regioncode == "`regioncode'"
4. }
(255 real changes made)
(195 real changes made)
(267 real changes made)
(276 real changes made)
(574 real changes made)
(400 real changes made)

114 .
115 . **g)
116 .
117 . reg mrkgrm classize pct_dis if regioncode == "Reg1"

Source	SS	df	MS	Number of obs	=	255
Model	2865.43406	2	1432.71703	F(2, 252)	=	66.06
Residual	5465.35025	252	21.6878978	Prob > F	=	0.0000
Total	8330.78431	254	32.7983634	R-squared	=	0.3440
				Adj R-squared	=	0.3388
				Root MSE	=	4.657

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.0900784	.0450526	-2.00	0.047	-.1788059 -.0013508
pct_dis	-.2490626	.0218086	-11.42	0.000	-.2920129 -.2061122
_cons	81.00851	1.311482	61.77	0.000	78.42564 83.59137

```
118 . outreg2 using "PS2_regressionong.doc", replace ctitle(Reg1)
PS2 regressionong.doc
dir : seeout

119 . foreach regioncode in Reg2 Reg3 Reg4 Reg5 Reg6{
2.          reg mrkgm classize pct_dis if regioncode ==`regioncode'
3.          outreg2 using "PS2_regressionong.doc", append ctitle(`regioncode')
4. }
```

Source	SS	df	MS	Number of obs	=	195
				F(2, 192)	=	34.84
Model	3017.96063	2	1508.98031	Prob > F	=	0.0000
Residual	8314.72655	192	43.3058675	R-squared	=	0.2663
Total	11332.6872	194	58.4159133	Adj R-squared	=	0.2587
				Root MSE	=	6.5807

mrkgm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.054951	.0822927	-0.67	0.505	-.2172648 .1073629
pct_dis	-.2521636	.0309288	-8.15	0.000	-.3131675 -.1911597
_cons	77.17122	2.510945	30.73	0.000	72.21864 82.1238

PS2 regressionong.doc
dir : seeout

Source	SS	df	MS	Number of obs	=	267
				F(2, 264)	=	45.65
Model	2924.30927	2	1462.15463	Prob > F	=	0.0000
Residual	8456.43979	264	32.0319689	R-squared	=	0.2570
Total	11380.7491	266	42.7847709	Adj R-squared	=	0.2513
				Root MSE	=	5.6597

mrkgm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	.1678957	.0669434	2.51	0.013	.0360848 .2997067
pct_dis	-.2127071	.027496	-7.74	0.000	-.2668463 -.1585678
_cons	69.64526	2.225889	31.29	0.000	65.2625 74.02801

PS2 regressionong.doc
dir : seeout

Source	SS	df	MS	Number of obs	=	276
				F(2, 273)	=	84.22
Model	4600.18127	2	2300.09063	Prob > F	=	0.0000
Residual	7455.58685	273	27.3098419	R-squared	=	0.3816
Total	12055.7681	275	43.8391568	Adj R-squared	=	0.3770
				Root MSE	=	5.2259

mrkgm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	.0074092	.0458776	0.16	0.872	-.0829096 .0977281
pct_dis	-.4898067	.0388518	-12.61	0.000	-.5662939 -.4133195
_cons	80.17505	1.479157	54.20	0.000	77.26304 83.08705

PS2 regressionong.doc
dir : seeout

Source	SS	df	MS	Number of obs	=	574
Model	10101.2705	2	5050.63523	F(2, 571)	=	169.65
Residual	16999.5884	571	29.7716085	Prob > F	=	0.0000
Total	27100.8589	573	47.2964378	R-squared	=	0.3727
				Adj R-squared	=	0.3705
				Root MSE	=	5.4563

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	.0212184	.0429338	0.49	0.621	-.0631092 .1055459
pct_dis	-.3191605	.0195322	-16.34	0.000	-.3575243 -.2807967
_cons	75.24485	1.556768	48.33	0.000	72.18715 78.30254

PS2 regressiong.docdir : seeout

Source	SS	df	MS	Number of obs	=	400
Model	8889.58149	2	4444.79075	F(2, 397)	=	168.84
Residual	10451.356	397	26.3258338	Prob > F	=	0.0000
Total	19340.9375	399	48.4735276	R-squared	=	0.4596
				Adj R-squared	=	0.4569
				Root MSE	=	5.1309

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.0758319	.0540577	-1.40	0.161	-.1821071 .0304432
pct_dis	-.4038452	.0231615	-17.44	0.000	-.4493797 -.3583107
_cons	79.62101	1.882575	42.29	0.000	75.91994 83.32207

PS2 regressiong.docdir : seeout

120 .
121 . ****Model alternative: Dummy with omitting Reg5:
122 . reg mrkgrm classize pct_dis Reg1 Reg2 Reg3 Reg4 Reg6

Source	SS	df	MS	Number of obs	=	1,967
Model	39479.1959	7	5639.88513	F(7, 1959)	=	186.70
Residual	59176.5468	1,959	30.2075277	Prob > F	=	0.0000
Total	98655.7428	1,966	50.1809475	R-squared	=	0.4002
				Adj R-squared	=	0.3980
				Root MSE	=	5.4961

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	.006539	.0218313	0.30	0.765	-.036276 .049354
pct_dis	-.3085222	.0102683	-30.05	0.000	-.3286603 -.2883842
Reg1	3.501301	.4311002	8.12	0.000	2.655837 4.346764
Reg2	.9691248	.4671522	2.07	0.038	.0529574 1.885292
Reg3	.3992164	.4098365	0.97	0.330	-.4045451 1.202978
Reg4	3.257748	.4188072	7.78	0.000	2.436394 4.079103
Reg6	.2489347	.3595311	0.69	0.489	-.456169 .9540384
_cons	75.55303	.8013656	94.28	0.000	73.98141 77.12465

```
123 . outreg2 using "PS2_regressiongalt.doc", replace ctitle(alt)
PS2 regressiongalt.doc
dir : seeout
```

```
124 .
125 . **h)
126 .
127 . ***h-i)
128 .
129 . reg mrkgrm classize sc_boys if n_classes == 1
```

Source	SS	df	MS	Number of obs	=	240
Model	378.269997	2	189.134999	F(2, 237)	=	2.12
Residual	21139.3133	237	89.1954149	Prob > F	=	0.1222
Total	21517.5833	239	90.0317294	R-squared	=	0.0176
				Adj R-squared	=	0.0093
				Root MSE	=	9.4443

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	.0964143	.1099979	0.88	0.382	-.1202843 .3131129
sc_boys	-.3024193	.1529586	-1.98	0.049	-.6037514 -.0010871
_cons	72.92232	2.177034	33.50	0.000	68.63352 77.21113

```
130 . outreg2 using "PS2_regressionhi.doc", replace ctitle(1)
PS2 regressionhi.doc
dir : seeout
```

```
131 .
132 . ***h-ii)
133 .
134 . reg mrkgrm sc_girls sc_boys if n_classes == 1
```

Source	SS	df	MS	Number of obs	=	240
Model	378.269997	2	189.134999	F(2, 237)	=	2.12
Residual	21139.3133	237	89.1954149	Prob > F	=	0.1222
Total	21517.5833	239	90.0317294	R-squared	=	0.0176
				Adj R-squared	=	0.0093
				Root MSE	=	9.4443

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
sc_girls	.0964143	.1099979	0.88	0.382	-.1202843 .3131129
sc_boys	-.206005	.1150097	-1.79	0.075	-.432577 .020567
_cons	72.92232	2.177034	33.50	0.000	68.63352 77.21113

```
135 . outreg2 using "PS2_regressionhii.doc", replace ctitle(1)
PS2 regressionhii.doc
dir : seeout
```

```
136 .
137 . correlate sc_boys sc_girls
      (obs=1,967)
```

	sc_boys	sc_girls
sc_boys	1.0000	
sc_girls	0.7918	1.0000

```
138 . reg mrkgrm classize if n_classes == 1
```

Source	SS	df	MS	Number of obs	=	240
Model	29.6008824	1	29.6008824	F(1, 238)	=	0.33
Residual	21487.9825	238	90.2856406	Prob > F	=	0.5675
Total	21517.5833	239	90.0317294	R-squared	=	0.0014
				Adj R-squared	=	-0.0028
				Root MSE	=	9.5019

mrkgrm	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
classize	-.0475071	.0829689	-0.57	0.567	-.2109544
_cons	72.65987	2.186222	33.24	0.000	68.35305
					76.96668

```
139 .
140 .
end of do-file
```