Interpreting Regression Coefficients

Empirical Methods

Fall 2019*

1 Coefficient Interpretation

1.1 Non-binary variables

1.1.1 Level - Level Specification

Suppose the dependent variable and independent variable of interest are both in level form. For example, suppose we want to know the effect of an additional year of schooling on hourly wages and from available data we fit the following equation:

$$W_i = \alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \epsilon_i \tag{1}$$

where Ed_i are years of completed education and Exp_i are years of working experience. In this case, the coefficient on Ed_i can be interpreted as the **marginal effect**. The marginal effect is how the dependent variable changes when the independent variable changes by an additional unit holding all other variables in the equation constant (i.e. partial derivative) or:

*Written by Matteo R. Greco in 2018. Notes not for distribution, as they mostly rely on personal notes and ingenuity. A previous version of these notes existed under the title "Fantastic coefficients and how to interpret them".

$$\frac{\partial W_i}{\partial Ed_i} = \beta_1$$

Therefore, β_j can be interpreted as the change in wages from a one unit increase (or state change if dummy variable) of X_j holding all other independent variables *constant*.

Equivalent, using the definition of differential, we can write:

$$\frac{dW_i}{dEd_i} = \beta_1$$

Since $dEd_i \approx \Delta Ed_i$, which represents an increment in Ed_i (i.e. $\Delta Ed_i = (Ed_i+\delta)-Ed_i = \delta$), we can think of β_1 as the change in hourly wages induced by a unitary increment in education.

Example 1. Suppose the fitted equation for (1) is:

$$\hat{W}_i = 3.5 + 0.75Ed_i + 0.25Age_i + 0.30Exp_i$$

Based on the data used in this regression, an additional year of education corresponds to an increase in hourly wages of \$0.75. Similarly, an additional year of experience is associated with a \$0.30 per hour wage increase.

1.1.2 Log - Log Specification

Consider interpreting coefficients from a regression where the dependent and independent variable of interest are in log form (**from now on** $log = ln$; at least this is the common approach in economics). The coefficients can no longer be interpreted as marginal effects. Suppose economic theory suggests estimation of our wage equation with the dependent variable in log form and inclusion of community volunteer hours per week (Comm) also in log form. The equation of interest is now:

$$log(W_i) = \alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4 log(Comm_i) + \epsilon_i \tag{2}$$

We would like to interpret the coefficient on the community volunteer variable (β_4). To better understand the interpretation, consider taking the differential of (2) holding all independent variables constant except $log(Comm_i)$.

$$d[log(W_i)] = d[log(Comm_i)]\beta_4$$
$$\frac{1}{W_i} dW_i = \frac{1}{Comm_i} dComm_i \beta_4$$

since $d[log(X)] = \frac{1}{X} d(X)$. This final equation can be rearranged in the following way:

$$\frac{\frac{100 * dW_i}{W_i}}{\frac{100 * dComm_i}{Comm_i}} = \beta_4$$

where the left hand side is the (partial) elasticity of W with respect to $Comm$. Elasticity is the ratio of the percent change in one variable to the percent change in another variable. The coefficient in a regression is a partial elasticity since all other variables in the equation are held constant. Therefore, β_4 can be interpreted as the percent change in hourly wages from a one percent increase in community volunteer hours per week holding education, age and experience constant. We can use the following notation:

$$100 * \frac{dW_i}{W_i} = 100 * \frac{\Delta W_i}{W_i} = \% \Delta W_i$$

Example 2. Suppose that the fitted equation for (2) is:

$$\widehat{log(W_i)} = 3.26 + 0.24Ed_i + 0.08Age_i + 0.16Exp_i + 1.2log(Comm_i)$$

Based on these regression results, a one percent increase in community volunteer hours per week is associated with a 1.2% increase in hourly wages.

When do we use log-transformations? Here there is a bunch of examples:

- Theory drives our decision. For instance suppose you think firms produce their output using a Cobb-Douglas production function, i.e. $Y = AK^{\alpha}L^{\gamma}$, and that you are interested in estimating α and γ. Then you can take logs of both the right-hand-side and left-hand-side to obtain

$$logY = logA + \alpha logK + \beta logL$$

  Which, if you have a sample of n observations and data on Y, K and L, can be estimated as the following model:

$$logY_i = \beta_0 + \beta_1 logK_i + \beta_2 logL_i + \epsilon_i$$

- To reduce variation in a variable. This is commonly used for variables like *Population*. This will automatically improve the fit of your model (i.e. increasing the R^2) regardless of whether it is the right choice to use the log-transformation (which in principle is not a good reason to use logs)

- Data drive our decision. After producing some explanatory plots, you may realize that it is more likely that the relationship between y and x is non-linear and you may want to transform your variables.

- One of the most important is probably related to the distribution of residuals. One of the OLS assumptions refers to Normality of the error term. If we run our regression and plot the residuals and find a very skewed uncentered distribution, we may want to log-transform the dependent variable. This would lead to a more Normal-like distribution of residuals.

- Log-transformations give less weight to outliers. In principle this is something good, because we do not want our results to be driven by outliers (i.e. very extreme and isolated values). However, a simpler (and more recommended) solution would be to cut outliers from your sample.

There are other more technical reasons for using logs, but for now it is sufficient for you to know the most commonly used ones (and why they are good or bad).

1.1.3 Log - Level Specification

In equation (2), education, age and experience are in level terms while the dependent variable (wage) is in log terms. We would like to interpret the coefficients on these variables. First, consider education. Take the differential holding all other independent variables constant.

$$
\begin{aligned}
dlog(W_i) &= dEd_i\beta_1 \\
\frac{dW_i}{W_i} &= dEd_i\beta_1
\end{aligned}
$$

Multiply both sides by 100 and rearrange:

$$
\begin{aligned}
\frac{100 * dW_i}{W_i} &= 100 * dEd_i\beta_1 \\
100 * \beta_1 = \frac{\frac{100*dW_i}{W_i}}{dEd_i} &= \frac{\%\Delta W_i}{unit\,\Delta Ed_i}
\end{aligned}
$$

Therefore, $100 * \beta_1$ can be interpreted as the percentage change in W_i for a unit increase in Ed_i, holding all other independent variables constant. In analogous way you can derive the interpretation for the coefficients on age and experience.

Example 3. Consider the fitted equation for (2):

$$\hat{log(W_i)} = 3.26 + 0.24Ed_i + 0.08Age_i + 0.16Exp_i + 1.2log(Comm_i)$$

Therefore, holding all other independent variables constant, an additional year of schooling is associated with a 24% increase in hourly wages. Similarly, an additional year of experience is associated with a 16% increase in hourly wages.

1.1.4 Level - Log Specification

Consider a regression where the dependent variable is in level terms and the independent variable of interest is in log terms. For example, consider the following equation:

$$W_i = \alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4 log(Comm_i) + \epsilon_i \tag{3}$$

Recall from the section on level-level regressions that the coefficients on education, age and experience can be interpreted as marginal effects. We would like to interpret the coefficient on community volunteer hours (β_4). Again, take the differential on both sides, holding all independent variables constant except community volunteer hours:

$$dW_i = dlog(Comm_i)\beta_4$$
$$dW_i = \frac{1}{Comm_i} dComm_i \beta_4$$

Divide both sides by 100 and rearrange:

$$\frac{\beta_4}{100} = \frac{dW_i}{\frac{100*dComm_i}{Comm_i}} = \frac{unit\,\Delta W_i}{\%\Delta Comm_i}$$

Therefore, $\frac{\beta_4}{100}$ can be interpreted as the increase in hourly wages from a one percent increase in community volunteer hours per week.

Example 4. Suppose that the fitted equation for (3) is:

$$\hat{W}_i = 3 + 0.67Ed_i + 0.28Age_i + 0.34Exp_i + 13.2log(Comm_i)$$

Therefore, holding education, age and experience constant, a one percent increase in community volunteer hours per week is associated with a \$0.132 increase in hourly wages.

1.2 Dummy Variables

1.2.1 Level - Dummy

Consider a regression where the dependent variable is in level terms and the independent variable of interest is a binary variable. For example, consider the equation:

$$W_i = \alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4 Male_i + \epsilon_i \tag{4}$$

where:

$$Male_i = \begin{cases} 1 & if\ i\ is\ male \\ 0 & else \end{cases}$$

Clearly the approach we used in Section 1.1 cannot be applied in this context, as a binary variable cannot be differentiated. But do not worry, interpreting dummies is actually quite easy (at least when our dependent variable is expressed in levels). Consider the conditional expectation function (from now on CEF) under the two different scenarios:

$$E(W_i|Male_i) = \begin{cases} \alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i & Male_i = 0 \\ \alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4 & Male_i = 1 \end{cases}$$

The coefficient of the dummy variable can then simply be interpreted as $E(W_i|D_i = 1) - E(W_i|D_i = 0)$, namely the difference in average wages across the groups defined by the dummy variable, males versus females in this context.

Example 5. Suppose the fitted equation for (4) is:

$$\hat{W}_i = 3 + 0.67Ed_i + 0.28Age_i + 0.34Exp_i + 0.8Male_i$$

Therefore, holding education, age and experience constant, males on average earn an additional 0.8$ of hourly wage if compared to women.

1.2.2 Log - Dummy

Suppose the dependent variable is in log terms this time, while the independent variable of interest is again a dummy. For example consider the equation:

$$log(W_i) = \alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4 Male_i + \epsilon_i \tag{5}$$

Intuitively one may think β_4 can be interpreted as average percent difference, since now W_i is expressed in logs. This is only partially correct. Let us see when this is actually the case. First, using the same approach of the above section, we know that:

$$\beta_4 = E(logW_i|Male_i = 1) - E(logW_i|Male_i = 0)$$

To simplify notation let us call these two cases $logW_1$ and $logW_0$. Let's rewrite the formula for β_4:

$$\beta_4 = E(logW_1) - E(logW_0) = E(logW_1 - logW_0) = E(log\frac{W_1}{W_0})$$

If $E(log\frac{W_1}{W_0}) \approx 0$ (so if $\beta_4 \approx 0$) then the coefficient for the dummy ($\times 100$) would tell us how much more (or less) males earn with respect to women, in percentage terms. Where does this result come from? Let's start from the concept of first-order Taylor expansion for a differentiable function $f(x)$. So if the real valued function $f(x)$ is differentiable at x_0 one can approximate f as follows:

$$f(x) \approx f(x_o) + f'(x)|_{x=x_0}(x - x_0)$$

If we apply this formula to $f(x) = log(x)$ (recall we refer to the natural log ln) around $x_0 = 1$ we obtain:

$$ln(x) \approx ln(1) + \frac{1}{x}|_{x=1}(x - 1) = x - 1$$

where the last equality follows from $ln(1) = 0$ and $\frac{1}{x}|_{x=1} = \frac{1}{1} = 1$.

So we can go back to our previous formula and eventually obtain:

$$\beta_4 = E(log\frac{W_1}{W_0}) \approx E(\frac{W_1}{W_0} - 1) = E(\frac{W_1 - W_0}{W_0})$$

Interpretation then is straightforward. Suppose $\hat{\beta}_4 = 0.03$, this would mean that males on average earn 3% more than women. However, one should pay attention and use this interpretation for values of the estimated coefficient sufficiently close to 0. Indeed consider $\hat{\beta}_4 = 0.03$.

Then you can verify $\hat{\beta}_4 = 0.03 \approx E(ln(1.03)) = E(ln(\frac{W_1}{W_0})) \approx E(\frac{W_1}{W_0} - 1) = 0.03$. Conversely, suppose one obtains $\hat{\beta}_4 = 0.3 \approx E(ln(1.35)) = E(ln(\frac{W_1}{W_0})) \neq E(\frac{W_1}{W_0} - 1) = 0.35$. As you see, getting away from 0 can lead to very incorrect interpretations of the coefficient.

The alternative approach (still an approximation), in case β_4 was not sufficiently close to 0 would be to take exponentials on both sides of the equation and then evaluate it when $Male_i = 0$ or $Male_i = 1$. The difference between these two values, divided by the expression for W_i based on the starting value of $Male_i$ will provide the correct interpretation.[1]

$$W_i = \begin{cases} exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \epsilon_i\} & Male_i = 0 \\ exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4 + \epsilon_i\} & Male_i = 1 \end{cases}$$

Thus if we want to compute the percentage change in wages induced by a change from 0 to 1 in the $Male_i$ dummy, i.e. if we want to know on average how much more (or less) males earn in percentage terms, we compute:

$$\frac{W_i|Male_i = 1 - W_i|Male_i = 0}{W_i|Male_i = 0} * 100 =$$

$$\frac{exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4\} - exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i\}}{exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i\}} * 100 =$$

$$(exp\{\beta_4\} - 1) * 100$$

Conversely, if we want to know how much less (or more) females earn in percentage terms if compared to males, we would need to compute:

[1]A proper quasi-unbiased estimator for β_4 has been derived almost 40 years ago by Kennedy (1981) under a proposed set of assumptions, and more have been derived later on.

$$\frac{W_i|Male_i = 0 - W_i|Male_i = 1}{W_i|Male_i = 1} * 100 =$$

$$\frac{exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i\} - exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4\}}{exp\{\alpha + \beta_1 Ed_i + \beta_2 Age_i + \beta_3 Exp_i + \beta_4\}} * 100 =$$

$$(exp\{-\beta_4\} - 1) * 100$$

Example 6. Suppose then that the fitted equation for (5) is:

$$log(\hat{W_i}) = 2.4 + 1.5Ed_i + 3.2Age_i + 2.2Exp_i + 0.2Male_i$$

Therefore, holding education, age and experience constant, males on average have an hourly salary 22.1% higher than females, or females have an hourly salary 18.1% lower than males. Neither of these numbers matches closely the coefficient.

1.3 Interactions between variables

1.3.1 Interactions between binary and non-binary variables

Suppose you want to estimate the following model:

$$W_i = \alpha + \beta_1 M_i + \beta_2 Ed_i + \beta_3 M_i Ed_i + \epsilon_i \tag{6}$$

where W_i are our usual wages, Ed_i is education while:[2]

$$M_i = \begin{cases} 1 & if\ i\ is\ male \\ 0 & else \end{cases}$$

Would it be sufficient to say that β_2 captures the marginal effect of Ed_i on W_i? In this case the answer is most likely to be negative because of the inclusion of an interaction term in the equation. More precisely, our marginal effect of Ed_i would depend on the specific values of M_i:

$$\frac{\partial W_i}{\partial Ed_i} = \beta_2 + \beta_3 M_i$$

As before, consider the CEF cases where $M_i = 0$ and $M_i = 1$ separately:

$$E(W_i|Ed_i, M_i) = \begin{cases} \alpha + \beta_2 Ed_i & M_i = 0 \\ \alpha + \beta_1 + \beta_2 Ed_i + \beta_3 Ed_i & M_i = 1 \end{cases}$$

So how do we read the coefficients? We can use the formulas first, and then look at them graphically.

- $\alpha = E(W_i|M_i = 0, Ed_i = 0)$; you immediately see this from the first line in the CEF formula above. If you plug $Ed_i = 0$, then α can be read as the average salary for female individuals whose have completed zero years of schooling (the intercept of the female curve in the graph below).

- $\beta_1 = E(W_i|M_i = 1, Ed_i = 0) - E(W_i|M_i = 0, Ed_i = 0)$; this is just the difference between the lower and the upper row of the above CEF when $Ed_i = 0$. So β_1 tells us

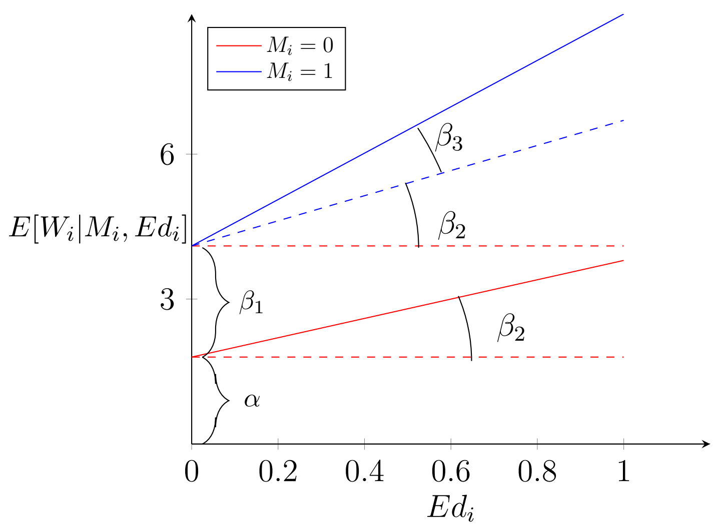
[2] I switch to M_i as it is less time consuming than writing $Male_i$ every time.

the average salary gap between males and females whose level of education is zero (the difference in intercepts in the graph below).

- $\beta_2 = \frac{\partial W_i}{\partial Ed_i}|_{M_i=0}$; so this is the slope coefficient for the CEF of females in our example (see the graph below). This tells us what the marginal effect of an additional year of education on wages is for females only.

- $\beta_3 = \frac{\partial W_i}{\partial Ed_i}|_{M_i=1} - \frac{\partial W_i}{\partial Ed_i}|_{M_i=0}$; so this coefficients represents the difference in the marginal effect of an additional year of education on wages for males relative to females (see the graph below.)

Suppose all coefficients are positive. The figure below provides a graphical intuition of how coefficients should be read, in line with the formulas in the bullets.



Example 7. Suppose the fitted equation for (6) is:

$$\hat{W}_i = 2.3 + 1.3M_i + 0.7Ed_i + 0.2M_iEd_i$$

Therefore, we can conclude that the marginal effect of an additional year of education on income for females is given by \$0.7, whereas for males the marginal effect is higher by a factor of \$0.2. Suppose we are interested in the total marginal effect of education for males, then we would have to sum the coefficients: an additional year of education increases males' wages by $\$0.7 + \$0.2 = \$0.9$.

1.3.2 Interactions between binary variables

Suppose you want to estimate the following model:

$$W_i = \alpha + \beta_1 S_i + \beta_2 M_i + \beta_3 M_i S_i + \epsilon_i \tag{7}$$

where:

$$S_i = \begin{cases} 1 & if\ i\ is\ Swiss \\ 0 & else \end{cases}$$

What would the three coefficients capture in this case? Let us again consider the CEF ($E(W_i|S_i, M_i)$) in all the possible subcases:

	$M_i = 0$	$M_i = 1$
$S_i = 0$	α	$\alpha + \beta_2$
$S_i = 1$	$\alpha + \beta_1$	$\alpha + \beta_1 + \beta_2 + \beta_3$

Then in order to get a better understanding of how to interpret the β coefficients we can add a few cells to the table and also use a graphical representation.

	$M_i = 0$	$M_i = 1$	$M_i = 1 - M_i = 0$
$S_i = 0$	α	$\alpha + \beta_2$	β_2
$S_i = 1$	$\alpha + \beta_1$	$\alpha + \beta_1 + \beta_2 + \beta_3$	$\beta_2 + \beta_3$
$S_i = 1 - S_i = 0$	β_1	$\beta_1 + \beta_3$	β_3

So now suppose you are asked to comment on all coefficients. What is each of them?

- $\alpha = E(W_i|M_i = 0, S_i = 0)$; so this coefficient in this specification just reports the average salary of non-Swiss females.

- $\beta_1 = E(W_i|M_i = 0, S_i = 1) - E(W_i|M_i = 0, S_i = 0)$; as indicated by the red coefficient in the table, this represents the average difference in salaries between Swiss females and non-Swiss ones.

- $\beta_2 = E(W_i|M_i = 1, S_i = 0) - E(W_i|M_i = 0, S_i = 0)$; this blue coefficient represents instead the average difference in salaries between non-Swiss males and non-Swiss females.

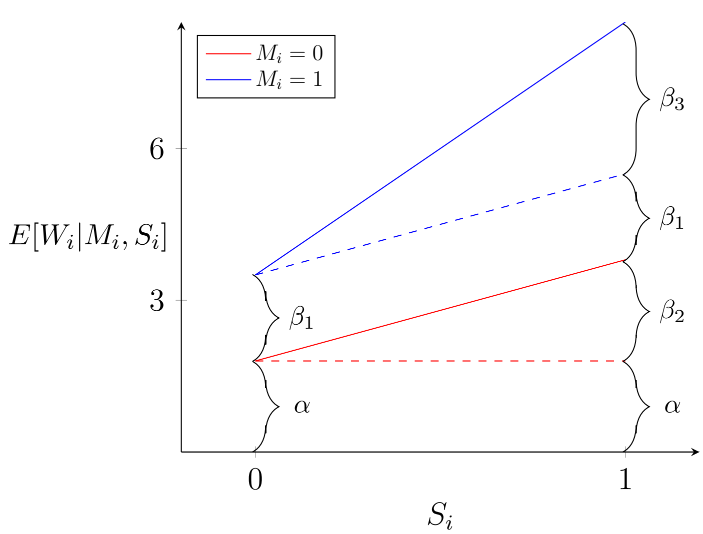
- $\beta_3 = ...$; well there are actually two ways of looking at it:

  - $\beta_3 = [E(W_i|M_i = 1, S_i = 1) - E(W_i|M_i = 0, S_i = 1)] - [E(W_i|M_i = 1, S_i = 0) - E(W_i|M_i = 0, S_i = 0)]$; in this case the coefficient can be read as the **additional** average difference in wages that exists in Switzerland between males and females, compared to the gender gap in other countries. Or...

  - $\beta_3 = [E(W_i|M_i = 1, S_i = 1) - E(W_i|M_i = 1, S_i = 0)] - [E(W_i|M_i = 0, S_i = 1) - E(W_i|M_i = 0, S_i = 0)]$; in this case the coefficient can be read as the **additional** average difference in wages that exists between Swiss males and non-Swiss males as compared to the existing salary gap between Swiss females and non-Swiss females.

  In experimental settings, this coefficient is commonly referred to as "Difference-in-Differences" estimator (as you can see, it is a difference of differences...).

The figure below provides a graphical intuition of how to read the coefficients.



Example 8. Suppose the fitted equation for (7) is:

$$\hat{W}_i = 1.3 + 0.2S_i + 0.4M_i + 0.3M_iS_i$$

For example we can read that average hourly salaries are \$0.2 higher for Swiss females than non-Swiss females ($\hat{\beta}_1$), while this gap is even higher for males, as Swiss males earn on average an additional \$0.3 ($\hat{\beta}_3$) per hour compared to their non-Swiss counterparts (so the total gap for males across countries amounts to \$0.5). The remaining ways of interpreting these numbers are left to the Empirical Methods amazing students.

1.3.3 Interactions between non-binary variables

Suppose you want to estimate the following model:

$$W_i = \alpha + \beta_1 Ed_i + \beta_2 FI_i + \beta_3 Ed_i FI_i + \epsilon_i \tag{8}$$

where FI_i is family income. If we are interested in the usual marginal impact of an additional year of education, we follow the same approach as before:

$$\frac{\partial W_i}{\partial Ed_i} = \beta_1 + \beta_3 FI_i$$

As you can see also this time the marginal effect of education depends on the values taken by another variable. There are two main take-aways when you have to comment the estimated coefficients from such a specification:

- We usually compute the marginal effect of our variable of interest for the average level of the other. So, after you run the regression you use the estimated parameters to say that the marginal effect of an additional year of education for an individual with average family income is given by $\hat{\beta}_1 + \hat{\beta}_3 \bar{FI}$

- β_1 in such a specification is rarely informative, as it would represent the marginal effect of an additional year of education on hourly wages for individual with no family income. In most cases (unless you have individual data from developing countries) you will not find a single observation with zero family income. So what would be the point of interpreting it?

Given the discussion in the second bullet, we may think of using an alternative approach and estimate the following model:

$$W_i = \alpha + \beta_1 Ed_i + \beta_2 FI_i^* + \beta_3 Ed_i FI_i^* + \epsilon_i \tag{9}$$

where $FI_i^* = FI_i - \bar{FI}$. The marginal effect of an additional year of education would then be given by:

$$\frac{\partial W_i}{\partial Ed_i} = \beta_1 + \beta_3 FI_i^*$$

This time we can give a straightforward interpretation of β_1! This is the marginal effect of an additional year of education on wages for an individual with average family income (as $FI_i^* = 0 \iff FI_i = \bar{FI}$). In addition, we can compute the marginal effect of education when moving away from the mean by constant amounts. For example, suppose SD_{FI} is the standard deviation of family income in the sample. Then, the marginal effect of an additional year of education on wages for individuals whose family income is 1 SD above the average is given by: $\hat{\beta}_1 + \hat{\beta}_3 SD_{FI}$.

1.4 Alternative ways of reading coefficients

1.4.1 Percent or percentage points?

Many students confuse "percent" with "percentage points" (or p.p.). What is the difference? How to detect it? Let's see it with an example. In 2010 unemployment in Italy amounted

to 14% of the total labor force. In 2017 it amounted to 18%. So it raised by **4 percentage points**, which corresponds a **22% increase** in unemployment rate.
Let's see it in a regression framework. Suppose you want to estimate:

$$T_c = \alpha + \beta_1 Size_c + \beta_2 IU_c + \beta_3 log(Tr)_c + \epsilon_c \tag{10}$$

where T_c is voters' turnout (i.e. share of individuals eligible for voting and who actually voted) in canton c, $Size_c$ is the size of the canton (in thousands of people), IU_c is the share of internet users, and $log(TR)_c$ is the log of federal transfers received by canton c. Shares are usually coded as numbers between 0 and 1 in the datasets. the unemployment rate from the example above is also a share (as all rates...). So how do we read these coefficients?

- β_1: a unitary increase in $Size_c$ (i.e. an additional 1000 citizens) induces a change in T_c by β_1 **p.p.**.
- β_2: a unitary increase in UI_c (i.e. the share of internet users moving from 0% to 100%) is associated with a change in T_c by β_2 **p.p.**.
- β_3: (as we learnt above) a **1 percent** increase in federal transfers is associated with a change in T_c by $\frac{\beta_3}{100}$ **p.p.**.

1.4.2 Unitary increases or standard deviations?

Consider equation (10) just above. Does it make sense to talk about the change in turnout induced by an increase in the population size by 1000 individuals? Not much actually. What about an increase by a million? Nah... If you do not know what the average size of a canton is, neither of these methods is very informative. Econometricians (or someone else...) came up with a very elegant solution for cases like this one, when talking about unitary increases does not make much sense: use **standard deviation** increases. The standard deviation would put the sensitivity in terms of a "typical" deviation from the current value (is that why it is called "standard"?!), and it has the advantage of accounting for the average value of the variable of interest. Consider our example. To provide a nicer interpretation of β_1

you can say that **1 standard deviation (SD) increase in $Size_c$ is associated with a change in T_c by $\beta_1 \times SD_{Size_c}$ p.p.**.
Summing up: multiply the estimated coefficient by the standard deviation of the covariate of interest.

1.4.3 Now you know how to read a coefficient. Is that it?

Suppose that from equation (1) you learn that an additional year of completed education increases hourly wages by \$0.64, and that this coefficient displayes the beloved 3 stars (i.e. it is statistically significantly different from zero). Is that a large **economic** effect (as the three stars may suggest)? The main take away of this section is: don't just stop at the interpretation of the coefficient! What you want to do is to understand the **magnitude** of the estimated marginal effect. How do you do it? Simple, just compare the effect to the sample average value of the **dependent** variable.
Let's go back to our example, and suppose that the average hourly wage in the sample is \$6.9 per hour. Given our estimates ($\hat{\beta}_1 = 0.64$) we can say that the marginal increase in salary induced by an additional year of education for an individual with average salary would amount to $\frac{0.64}{6.9} * 100 = 9.27\%$, which is a very large economic effect!

Everything else you can learn about reading coefficients will come with time and practice. Have fun!