Empirical Methods

Topic 2d:

Panel Data

Panel Data

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Panel Data Intro I

- So far we have dealt with *cross-sectional data*:
  - Observations on economic agents (e.g. individuals, households, firms, etc.) collected at one point in time.
  - Model given by

$$y_i = x_i'\beta + \epsilon_i, \qquad i = 1, \ldots, N \tag{1}$$

    and variation is over individuals i and not over time t.
- *Panel data* includes multiple observations on agents *over time*:
  - Individuals indexed by i, $i = 1, \ldots, N$
  - Time indexed by t, $t = 1, \ldots, T$

Panel Data Intro II

- With both i's and t's, we can think about generalizing (1)
  - The most common generalization is to allow for separate effects for both individuals, i, and time periods, t:

$$y_{it} = x_{it}'\beta + \alpha_i + d_t + \epsilon_{it}$$

  - where...

Panel Data Notation I

$$y_{it} = x_{it}'\beta + \alpha_i + d_t + \epsilon_{it} \tag{2}$$

where

- x_{it}' is a $1 \times K$ row vector containing variables that vary
  - Across i only (e.g. gender, education) and/or
  - Across i and t (e.g. experience)
- α_i, $i = 2, \ldots, N$, discussed further in the coming slides
- d_t, $t = 2, \ldots, T$ is a vector of time intercepts
  - Excluding one to prevent multi-collinearity with the constant (inside x_{it})
  - (I'll show why this is necessary in a few slides)
- ϵ_{it} a time-specific deviation from α_i

Short Panels

- The typical focus of panel-data methods is *short panels*, i.e.
  - Large N but small T
  - Thus we rely on "cross-section-type" arguments for consistency and asymptotic normality
    - ★ e.g. T fixed, but $N \to \infty$
    - ★ (That's fine - these are what I've shown you so far)

Panel Data Notation II

- Notation gets more complicated once we introduce panel data
  - As there are now two dimensions of variation, *i and t*
  - (This very important to understanding panel data methods!)
- So far we have written our estimating equation with double subscripts:

$$y_{it} = x_{it}'\beta + \alpha_i + d_t + \epsilon_{it}$$

- Let's now put this into matrix notation

Panel Data Notation III

- Stack the T observations for each individual i into its own vector

$$y_i = X_i\beta + \alpha_i \iota_T + \epsilon_i$$

- where we've subsumed d_t into X_i, ι_T is a $T \times 1$ vector of ones, and

$$y_i = \underbrace{\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix}}_{T \times 1} \quad X_i = \underbrace{\begin{bmatrix} x_{i11} & x_{i21} & \cdots & x_{iK1} \\ x_{i12} & x_{i22} & \cdots & x_{iK2} \\ \vdots & & & \vdots \\ x_{i1T} & x_{i2T} & \cdots & x_{iKT} \end{bmatrix}}_{T \times K} \quad \alpha_i \iota_T = \underbrace{\begin{bmatrix} \alpha_i \\ \alpha_i \\ \vdots \\ \alpha_i \end{bmatrix}}_{T \times 1} \quad \epsilon_i = \underbrace{\begin{bmatrix} \epsilon_{i1} \\ \epsilon_{i2} \\ \vdots \\ \epsilon_{iT} \end{bmatrix}}_{T \times 1}$$

- Note:
  - If we had enough observations for each i, we could in principle run separate regressions for each person!
    - ★ (In which case we would drop d_t and α_i would be i's constant term)
  - This is rare except with Big Data applications - there it's common
    - ★ Assume this away in what follows...

Panel Data Notation IV

- And then stack each of the N individuals:

$$y = X\beta + \alpha + \epsilon$$

- where

$$y = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}}_{NT \times 1} \quad X = \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}}_{NT \times K} \quad \alpha = \underbrace{\begin{bmatrix} \alpha_1 \iota_T \\ \alpha_2 \iota_T \\ \vdots \\ \alpha_N \iota_T \end{bmatrix}}_{NT \times 1} \quad \epsilon = \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix}}_{NT \times 1}$$

- And the properties of ϵ, e.g. $E(\epsilon|X)$ and $V(\epsilon)$, will be described in detail in what follows.

Aside: Dummy Variable Multicollinearity I

- I said earlier that we only include
  - $N-1$ dummy variables for each i
  - $T-1$ dummy variables for each t
- Showing why this is also gives us practice with panel data notation...
- Suppose you had 3 individuals and 2 time periods
- Let
  - $x_{1i}=1$ be the constant term (as always)
  - d_t be a dummy variable for each time period
    - ★ For our example, we'd have d_1 and d_2

Aside: Dummy Variable Multicollinearity II

- Suppose we were to include a constant and *both* time dummies in our regression
- These three covariates for $i = 1, 2, 3$ and $t = 1, 2$ are given by:

$$\text{For } \begin{bmatrix} x_{it} \end{bmatrix} = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \\ x_{31} \\ x_{32} \end{bmatrix} \quad \text{constant} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad d_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad d_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

- Do you see why this would be problematic?*
  - _____

Aside: Dummy Variable Multicollinearity III

- More generally, whenever you add a set of dummy variables that span your data
  - e.g. men and women, all the individuals or years (in panel data)
- You have two choices:
  1. Drop the constant term, and both...
     - ★ Interpret each dummy variable as $E(y_i|x_i = 0, \text{that dummy} = 1)$
  2. Drop one of the dummy variables, and...
     - ★ Interpret the constant as $E(y_i|x_i = 0, \text{the excluded dummy} = 1)$
     - ★ Interpret each dummy variable as *the difference between* $E(y_i|x_i, \text{that dummy} = 1)$ and $E(y_i|x_i, \text{the excluded dummy} = 1)$
- Introduced for the first time here, but an important general principle!

Panel Data Examples I

- There are many examples of well-known and much-used panel datasets:
  - PSID: Panel Study of Income Dynamics (USA)
  - BHPS: British Household Panel Survey (UK)
  - ECHP: European Community Household Panel
  - GSOEP: German Socioeconomic Panel
  - SHP: Swiss Household Panel

Panel Data Examples II

- Looking at the determinants of a wide variety of outcomes:
  - Individuals' earnings
  - Households' expenditures
  - Firms' investments
  - Firms' productivities
  - Regions' migration patterns
  - Countries' income per capita

Unobserved Heterogeneity (α_i)

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Unobserved Heterogeneity (α_i)

$$y_{it} = x_{it}'\beta + \alpha_i + d_t + \epsilon_{it}$$

- α_i plays a very important role in panel data analysis
  - And in econometrics more generally
- Note:
  - α_i is *unobserved*
    - ★ It's measures tastes or costs that are (or might be) different for each individual in the sample....
    - ★ That yield different choices of y_{it}.
  - It is constant across time
    - ★ Note no t attached to it

α_i is called **Unobserved (individual-specific) Heterogeneity**

Example of Unobserved Heterogeneity and its Effects

- Unobserved heterogeneity can be very important
- For example, when analyzing the impact of aggregate (country-level) investment on income per capita
  - α_i could measure "good institutions":
    - ★ Countries with good institutions (e.g. infrastructure, rule of law) are likely to have high per-capita income
    - ★ This characteristic is likely to (often) be unchanging over time.
    - ★ It's perhaps (positively) correlated with firms' decisions to invest.
    - ★ $\Rightarrow$ ignoring it could yield an (upwardly) biased estimate of the effect of investment on income.
    - ★ (We will show this shortly...)

Unobserved Heterogeneity Bias I

How do you know there is bias from Unobserved Heterogeneity?

- If we ignore α_i in our regression, it effectively becomes part of the error term:

$$\begin{aligned} y_{it} &= x_{it}'\beta + \underbrace{d_t + \alpha_i}_{\text{Switched order!}} + \epsilon_{it} \\ &= x_{it}'\beta + d_t + \nu_{it} \end{aligned}$$

where $\nu_{it} = \alpha_i + \epsilon_{it}$ is the *composite error term* that includes α_i

Unobserved Heterogeneity Bias II

$$\begin{aligned} y_{it} &= x_{it}'\beta + d_t + \alpha_i + \epsilon_{it} \\ &= x_{it}'\beta + d_t + \nu_{it} \end{aligned}$$

- Since
  - α_i is part of the error term...
  - It may also correlated with one of our x's...
  - If so, we've violated our most important assumption: $E(\nu_i|x_i) = 0$
  - $\Rightarrow$ our estimate of β is biased!

Unobserved Heterogeneity Bias III

- Unobserved heterogeneity in this case is just an omitted variable
- And we know the formula for omitted variable bias, which here is:

$$\begin{aligned} \text{Truth:} \quad & y = X\beta + (\gamma)\alpha + \epsilon \\ \text{You estimate:} \quad & y = X\beta + \epsilon \\ \Rightarrow \quad E(\hat{\beta}) &= \beta + \gamma(X'X)^{-1}X'\alpha \\ &= \beta + \gamma\hat{\beta}_{\alpha\_on\_X} \end{aligned}$$

where

- $\gamma = \{-1, 1\}$ is the (implicit) sign of impact of α_i on y_i
- $\hat{\beta}_{\alpha\_on\_X}$ measures the correlation between α_i and x_i

Unobserved Heterogeneity Bias IV

$$E(\hat{\beta}) = \beta + \gamma\hat{\beta}_{\alpha\_on\_X}$$

- For our investment-income example:
  - We worry α_i measures "good institutions" and better institutions increase income ($\gamma > 0$)
  - We further worry that firms in countries with good institutions invest more ($\hat{\beta}_{\alpha\_on\_X} > 0$)
  - $E(\hat{\beta}_{inv}) = \beta_{inv} + (+)(+)$
    - ★ Meaning that we think their may be *positive* bias on our estimate of the impact of investment on income.

Aside: Bias in Cross-Section Settings I

- You may be thinking...
  - **Wait a minute!**
  - **Isn't Heterogeneity Bias relevant for Cross-Section data???**
  - (Why are we hearing about it only now???)

Aside: Bias in Cross-Section Settings II

- Heterogeneity Bias *is* relevant for cross-section analysis - it's an *i*-specific correlated unobservable.
  - Q: Why do you think we don't teach it before now?
  - A: _____?
    - ★
    - ★

Aside: Bias in Cross-Section Settings III

- The good news: Panel Data *is* rich enough to let us handle problems of unobserved heterogeneity.
  - It is for this reason that it is our second tool (after IV) to help resolve endogeneity issues in econometrics
  - With...
    - ★ Panel data, and
    - ★ A time-constant correlated unobservable
  - ...we can use panel data methods (especially Fixed Effects and/or First Differences) to resolve the endogeneity problem
- The coming slides show you how this works.

Panel Data Assumptions and Alternative Estimators

Panel Data Assumptions

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Panel Data Assumptions Intro

- Before showing you how it works, I need to describe a few assumptions needed by these methods
- These include assumptions about:
  1. (Seen before:) Correlation between ϵ_{it} and ϵ_{js}
     - ⋆ i.e. Panel-data versions of the CLRM Assumptions 3 and 4 (Homoskedasticity and No Correlation)
  2. (Seen before:) Correlation between x_{is} and ϵ_{it}
     - ⋆ i.e. Panel-data versions of the CLRM Assumption 2 (Mean-zero error)
  3. (New:) Correlation between x_{it} and α_i
     - ⋆ New with panel data because α_i is new
- There is a lot here, so I will try to simplify as much as possible!

Panel Data Assumptions (1): $Cov(\epsilon_{is}, \epsilon_{it})$ I

- And so: do the easiest first
- Our baseline assumptions on heteroscedasticity and autocorrelation in ϵ_{it} across i and t are the same as for the CLRM:

$$\begin{aligned} (A3, \text{Homoskedasticity}): \quad Var(\epsilon_{it}) &= \sigma_\epsilon^2 \quad t = 1, \ldots, T \\ (A4, \text{No Correlation}): \quad Cov(\epsilon_{is}, \epsilon_{it}) &= 0 \quad s \neq t \end{aligned}$$

  - (Also that $Cov(\epsilon_{js}, \epsilon_{it}) = 0$, i.e. no correlation *across individuals*)
  - (That's usually not controversial)

Panel Data Assumptions (1): $Cov(\epsilon_{is}, \epsilon_{it})$ II

- That being said,
  - The standard estimation methods presented below are consistent (for fixed T and $N \to \infty$) even if ϵ_{it} has arbitrary heteroscedasticity and/or serial correlation (that isn't too strong)
    - ★ (Serial correlation in ϵ_{it} appears to be particularly common)
  - Though they will of course impact the efficiency of an estimator
    - ★ (i.e. standard error calculations)

The other two panel data assumptions

- The other two panel data assumptions are the key ones:
  - (2) Between x_{is} and ϵ_{it}
  - (3) Between x_{it} and α_i
- Some of the most common panel data methods rely differentially on these two assumptions (e.g. Pooled OLS v Fixed Effects):
  - Strong on one and weak on the other
  - (A common pattern in econometrics)
- We will discuss this tradeoff once we introduce the two simplest versions (weak and strong) of each assumption

Panel Data Assumptions (2): $Cov(x_{is}, \epsilon_{it})$ I

- The two most common assumptions about x_{is} and ϵ_{it}:
  1. Contemporaneous Exogeneity:

$$Cov(x_{it}, \epsilon_{it}) = 0 \qquad t = 1, \ldots, T$$

     - ⋆ This is just the panel-data version of our normal CLRM assumption (A2, Mean-zero error)
     - ⋆ (And the weaker of our two assumptions re: $Cov(x_{is}, \epsilon_{it})$)

Panel Data Assumptions (2): $Cov(x_{is}, \epsilon_{it})$ II

- Two assumptions about x_{is} and ϵ_{it}, cont:
  2. Strict Exogeneity:

$$Cov(x_{is}, \epsilon_{it}) = 0 \qquad s, t = 1, \ldots, T$$

  - ★ The covariates at any time s are uncorrelated with the idiosyncratic errors at any time t
  - ★ This is a strong assumption...
  - ★ ...that is used by *almost all* of the standard panel data estimation methods

Panel Data Assumptions (2): $Cov(x_{is}, \epsilon_{it})$ III

- Implications of Strict Exogeneity include
  1. One must correctly specify the dynamic structure of the model
     - ⋆ (e.g., the *exactly* correct lag structure of covariates)
  2. One cannot have lagged dependent variables
     - ⋆ (e.g., if $s = t + 1$, then including $y_{i,t-1}$ as a covariate implies including $x_{is} = y_{it}$, which is necessarily correlated with ϵ_{it})
  3. Shocks today cannot affect future values of the covariates
     - ⋆ (e.g. A good outcome today cannot change my investment tomorrow)
- $\Rightarrow$ Must evaluate how reasonable is this assumption in your application!
  - (Tho note often you're stuck with it)
  - (In which case must evaluate how much you believe your results under this assumption)

Panel Data Assumptions (3): $Cov(x_{it}, \alpha_i)$

- There are two common assumptions invoked about the relationship between x_{it} and α_i:
  1. Arbitrary Effects:
     - ★ No restrictions are placed on the relationship between x_{it} and α_i
     - ★ Not really an assumption; more like "no assumption".
     - ★ This is Very Good.
  2. Uncorrelated Effects:

$$Cov(x_{it}, \alpha_i) = 0 \qquad t = 1, \ldots, T$$

     - ★ This is Very Strong.
     - ★ (The whole point of worrying about unobserved heterogeneity is because you think this assumption is violated!)
- We will discuss each of these in more detail when introducing the estimators that rely on them

Key Panel Data Assumptions Overview

- To summarize:

Table: Strength of two key Panel Data Assumptions

	Weak(er)	Strong(er)
(2) $Cov(x_{is}, \epsilon_{it})$	Contemporaneous Exogeneity	**Strict Exogeneity**
(3) $Cov(x_{it}, \alpha_i)$	Arbitrary Effects	Uncorrelated Effects

- The colors in the table are indicative:
  - Arbitrary Effects is no assumption at all
  - Contemporaneous Exogeneity is our regular Assumption 2
    - ★ (Strong, but we have tools to address it if violated)
  - **Strict Exogeneity** is a stronger version of our Assumption 2
    - ★ (Stronger, but unfortunately not much we can do if violated)
  - Uncorrelated Effects
    - ★ (Strongest - tho remember always part of cross-section analysis)

Four Estimation Methods

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Panel Data Estimation Methods Intro

- There are four common ways to estimate panel data models:
  1. Pooled OLS
     - ⋆ (The thing you already know... applied to panel data)
  2. Fixed Effects
     - ⋆ (The really useful estimator that can resolve some kinds of endogeneity)
  3. Random Effects
     - ⋆ (Most efficient estimator - but relying on the strongest assumptions)
  4. First Differences
     - ⋆ (An estimator very similar to Fixed Effects)
- We'll cover each in turn

Pooled OLS

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Pooled OLS I

- Our estimating equation for Pooled OLS is

$$\begin{aligned} y_{it} &= x_{it}'\beta + \alpha_i + \epsilon_{it} \\ &= x_{it}'\beta + \nu_{it} \end{aligned}$$

  - where we've subsumed d_t into x_{it} and $\nu_{it} = \alpha_i + \epsilon_{it}$ is, as before, a composite error term.

- As you can see, we are ignoring the unobserved heterogeneity, α_i:
  - It is just part of the composite error, $\nu_{it} = \alpha_i + \epsilon_{it}$.

Pooled OLS II

- To estimate, we run a simple OLS regression on all the data

$$\begin{aligned} \hat{\beta}_{POLS} &= (X'X)^{-1}X'y \\ V(\hat{\beta}_{POLS}) &= \sigma^2(X'X)^{-1} \end{aligned}$$

  - For X and Y defined on Slide 9

Pooled OLS III

- We know for OLS that the key thing for consistency and asymptotic normality is that we satisfy Assumption 2 (Mean-zero error)
  - $E(\nu_{it}|x_{it}) = 0$
- Because $\nu_{it} = \alpha_i + \epsilon_{it}$, to do so requires...
  - *Contemporaneous Exogeneity*...

$$Cov(x_{it}, \epsilon_{it}) = 0 \qquad t = 1, \ldots, T$$

  - ... and *Uncorrelated Effects*:

$$Cov(x_{it}, \alpha_i) = 0 \qquad t = 1, \ldots, T$$

    - ★ (We'll discuss later how reasonable are these assumptions)
    - ★ (As well as how to test the latter)

Pooled OLS IV*

- We can allow arbitrary heteroskedasticity and serial correlation in ν_{it}
  - We just must be sure to use variance formulas that accommodate that
- For example, let $\hat{v}_{it} = y_{it} - x_{it}'\hat{\beta}_{POLS}$
  - Then we should calculate standard errors with the formula:

$$\hat{V}(\hat{\beta}_{POLS}) = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}x_{it}'\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} \hat{v}_{is}\hat{v}_{it}x_{is}x_{it}'\right)\left(\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}x_{it}'\right)^{-1}$$

  - (The more econometrics you do the more you get comfortable with formulas like this!)

Pooled OLS: Intuition

- After introducing each estimator, I'll provide some intuition for it.
  - And after introducing them all, I'll provide some intuition about the tradeoffs between them
- The intuition for Pooled OLS is the easiest: it's the same intuition as for "regular old OLS"...
  - ...when applied to panel data
- The key addition relative to regular old OLS? Covered 2 slides ago:
  - We still have our "regular" Assumption 2 (Contemporaneous Exogeneity)
  - As well as the new addition of Uncorrelated Effects
    - ★ (Which was also there with OLS - we just didn't talk about it!)

Fixed Effects

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Fixed Effects I

- Fixed Effect estimation starts by *de-meaning* the data
- Let

$$\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}, \quad \bar{x}_i = \frac{1}{T}\sum_{t=1}^{T} x_{it}, \text{ and } \bar{\epsilon}_i = \frac{1}{T}\sum_{t=1}^{T} \epsilon_{it}$$

- Then

$$\begin{aligned} y_{it} &= x_{it}'\beta + \alpha_i + \epsilon_{it} \\ \Rightarrow \bar{y}_i &= \bar{x}_i'\beta + \alpha_i + \bar{\epsilon}_i \end{aligned}$$

where $\frac{1}{T}\sum_{t=1}^{T} \alpha_i = \frac{1}{T}(T\alpha_i) = \alpha_i$ as α_i doesn't vary with t

Fixed Effects II

$$y_{it} = x_{it}'\beta + \alpha_i + \epsilon_{it}$$
$$\bar{y}_i = \bar{x}_i'\beta + \alpha_i + \bar{\epsilon}_i$$

- Subtracting the second equation from the first yields

$$y_{it} - \bar{y}_i = (x_{it} - \bar{x}_i)'\beta + \epsilon_{it} - \bar{\epsilon}_i$$

- Which is normally written as

$$\boxed{\ddot{y}_{it} = \ddot{x}_{it}'\beta + \ddot{\epsilon}_{it}}$$

where

  - $\ddot{y}_{it} \equiv y_{it} - \bar{y}_i$ (similarly for $\ddot{x}_{it}$ and $\ddot{\epsilon}_{it}$)...
  - ...and α_i has dropped out as it is constant across time

Fixed Effects III

$$\boxed{\ddot{y}_{it} = \ddot{x}'_{it}\beta + \ddot{\epsilon}_{it}}$$

- To estimate, we simply run OLS on the (transformed) equation above
- Let
  - $\ddot{y}$ be the $NT \times 1$ vector with typical element $\ddot{y}_{it}$
  - $\ddot{X}$ be the $NT \times K$ matrix with typical element $\ddot{x}_{ikt}$
  - $\ddot{\epsilon}$ be the $NT \times 1$ vector with typical element $\ddot{\epsilon}_{it}$
- Then

$$\boxed{\hat{\beta}_{FE} = (\ddot{X}'\ddot{X})^{-1}\ddot{X}'\ddot{y}}$$

Fixed Effects IV

$$\boxed{\ddot{y}_{it} = \ddot{x}'_{it}\beta + \ddot{\epsilon}_{it}}$$

- Looking at $\ddot{x}_{it}$ and $\ddot{\epsilon}_{it}$, it's clear why we need strict exogeneity for consistency
- As always for consistency, we need

$$Cov(\ddot{x}_{it}, \ddot{\epsilon}_{it}) = 0$$

- But

$$\ddot{x}_{it} \equiv x_{it} - \frac{1}{T}(x_{i1} + \ldots + x_{IT}) \quad \text{and} \quad \ddot{\epsilon}_{it} \equiv \epsilon_{it} - \frac{1}{T}(\epsilon_{i1} + \ldots + \epsilon_{IT})$$

- Thus $Cov(\ddot{x}_{it}, \ddot{\epsilon}_{it}) = 0 \Rightarrow$
  - We need $Cov(x_{is}, \epsilon_{it}) = 0$ for *each pair* of x_{is} and ϵ_{it}
  - And that's **Strict Exogeneity**

Fixed Effects V

$$\boxed{\ddot{y}_{it} = \ddot{x}'_{it}\beta + \ddot{\epsilon}_{it}}$$

- Note we also have *Arbitrary Effects*,
  - i.e. The relationship between α_i and x_{it} is completely unrestricted
    - ★ (Because α_i doesn't enter anywhere into the estimating equation)
    - ★ (As we've differenced it out of our estimating equation)
    - ★ (This is really good)

Fixed Effects VI

- Under homoskedasticity and no serial correlation...

$$\begin{aligned} V(\epsilon_i|x_i,\alpha_i) &= \sigma_\epsilon^2 I_T \\ \Rightarrow V(\hat{\beta}_{FE}) &= \sigma_\epsilon^2(\ddot{X}'\ddot{X})^{-1} \end{aligned}$$

  - ...we can estimate the asymptotic variance of $\hat{\beta}_{FE}$ as

$$\hat{V}(\hat{\beta}_{FE}) = \hat{\sigma}_\epsilon^2 \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \ddot{x}_{it} \ddot{x}_{it}' \right)^{-1}$$

  - where $\hat{\ddot{\epsilon}}_{it} = \ddot{y}_{it} - \ddot{x}_{it}'\hat{\beta}_{FE}$ and

$$\boxed{\hat{\sigma}_\epsilon^2 = \frac{1}{N(T-1)-K} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\ddot{\epsilon}}_{it}^2}$$

    - ★ (This is an important input into something we'll need later)

Least Squares Dummy Variable Estimator I

$$y_{it} = x'_{it}\beta + \alpha_i + \epsilon_{it}$$
$$\ddot{y}_{it} = \ddot{x}'_{it}\beta + \ddot{\epsilon}_{it}$$

- An equivalent way to estimate a model with fixed effects...
  - (tho it takes longer - and sometimes *much longer* - in Stata or R)
- ...is to include dummy variables, a_i, for each of the α_i

$$y_{it} = x'_{it}\beta + a_i + \epsilon_{it}$$

  - This means estimating as many extra parameters as you have i
    - ★ (Could be a lot!)
- Aka the *Least Squares Dummy Variable* (LSDV) estimator

Least Squares Dummy Variable Estimator II

$$y_{it} = x_{it}'\beta + a_i + \epsilon_{it}$$

- There is an important limitation of the LSDV estimator with short panels
  - i.e. large N and small T
- Can you guess what it is?*
- Despite this problem, $\hat{\beta}_{FE}$ *is* consistent for β

Least Squares Dummy Variable Estimator III

$$y_{it} = x_{it}'\beta + a_i + \epsilon_{it}$$

- The a_i in the LSDV estimator are sometimes called *nuisance parameters* or *incidental parameters*
  - Nuisance parameters $\equiv$
    - ★ Parameters in the model we're not inherently interested in
  - Incidental parameters $\equiv$
    - ★ Nuisance parameters that grow with the sample size
- The lack of consistent estimation of α_i in panel data models is often called the "*incidental parameters problem*"

Fixed Effects VII

- Regardless of the representation of the FE model,

$$\begin{aligned} \ddot{y}_{it} &= \ddot{x}'_{it}\beta + \ddot{\epsilon}_{it} \qquad \text{or} \\ y_{it} &= x'_{it}\beta + a_i + \epsilon_{it} \end{aligned}$$

- Because we've included a separate effect for each i,
  - We only rely on variation *within individuals over time* to identify β
- For this reason, de-meaning is called the *within transformation*
  - And Fixed Effects is called the *Within Estimator*

The Between Estimator I

- The Fixed Effects estimator relies on de-meaning the data
- We could, however, also run a regression on the "meaned data":

$$\begin{aligned} \bar{y}_i &= \bar{x}_i'\beta + \alpha_i + \bar{\epsilon}_i \\ \bar{y}_i &= \bar{x}_i'\beta + \bar{\nu}_i \end{aligned}$$

where $\bar{\nu}_i = \alpha_i + \bar{\epsilon}_i$

- This is called the *Between Estimator* for panel data
  - Because we are only relying on variation *between individuals*
    - ★ (Note this is a single cross-section...
    - ★ ...as we've averaged across time for each individual...
    - ★ ...thus only i and no t subscripts)

The Between Estimator II

- The Between Estimator isn't often used (as we still have α_i)
  - And if we're going to ignore unobserved heterogeneity, we might as well use the Random Effects estimator introduced next.
- It is still useful, however, for a few things:
  - Understanding the sources of variation in the data
  - The relationship between the FE and RE estimators
    - ★ (Which I won't teach but you sometimes see in textbooks)
  - And...

The Between Estimator III

$$\bar{y}_i = \bar{x}_i'\beta + \bar{\nu}_i$$

- We can use the Between Estimator to help estimate σ_α^2
  - Which we're going to need momentarily
- To get there, note

$$\begin{aligned} \bar{\nu}_i &= \alpha_i + \bar{\epsilon}_i \\ &= \alpha_i + \frac{1}{T}\sum_i \epsilon_{it} \\ \Rightarrow \sigma_{\bar{\nu}}^2 &= \sigma_\alpha^2 + \frac{1}{T^2}\sum_i \sigma_\epsilon^2 \\ &= \sigma_\alpha^2 + \frac{1}{T}\sigma_\epsilon^2 \end{aligned}$$

The Between Estimator IV

$$\bar{y}_i = \bar{x}_i'\beta + \bar{\nu}_i$$

$$\sigma^2_{\bar{\nu}} = \sigma^2_\alpha + \frac{1}{T}\sigma^2_\epsilon$$

- We then use this to estimate σ^2_α by:
  - Estimating the first line of equation above (by OLS), $\Rightarrow \hat{\beta}_{Between}$
  - $\Rightarrow \hat{\bar{\nu}}_i = \bar{y}_i - \bar{x}_i'\hat{\beta}_{Between}$
  - $\Rightarrow \hat{\sigma}^2_{\bar{\nu}} = \frac{1}{N-K}\sum_i \hat{\bar{\nu}}^2_i$
  - And we can use our estimate of $\hat{\sigma}^2_\epsilon$ from Slide 49 to calculate

$$\boxed{\hat{\sigma}^2_\alpha = \hat{\sigma}^2_{\bar{\nu}} - \frac{1}{T}\hat{\sigma}^2_\epsilon}$$

Fixed Effects: Intuition I

- The fixed effects estimator is a *very important estimator*
- It's primary advantage is that it resolves - *completely* - concerns about (time-constant) unobserved heterogeneity
  - (What again is the intuition for how?)
  - _____
- The consequence of this very attractive feature is that estimation of β must therefore come only from the time-series variation in the data
  - In essence, all of the cross-sectional variation is "used up" to identify the fixed effects
- This is both Good *and* Bad
  - The Good is described above

Fixed Effects: Intuition II

- The Bad comes in two flavors
- First, including FEs means we can't estimate any covariate that doesn't vary across time. *At All!*
  - E.g., suppose we wanted to estimate the effects of education on wages
    - ★ If education is constant across time for each i...
    - ★ ...then the FE estimator *cannot* estimate $\beta_{Education}$!
- Second, adding FEs may wipe out *much* of the variation in the data
  - If there isn't much time-series variation within each i, you'll have very imprecise estimates of β
  - But such is life! This is a common tradeoff between the econometric goals of consistency and efficiency

Random Effects

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Random Effects I

- The assumptions for the Random Effects estimator are the same as Pooled OLS,
  - But it is more efficient
- The reasons is that α_i induces *serial correlation* in the composite error term, $\nu_{it} = \alpha_i + \epsilon_{it}$
  - Even if we assume there is no serial correlation in ϵ_{it}, i.e. $E(\epsilon_{is}\epsilon_{it}) = 0$
  - (...along with $E(\epsilon_{it}, \alpha_i) = 0$, an often-weak assumption)

$$\begin{aligned} Cov(\nu_{is}, \nu_{it}) &= Cov(\alpha_i + \epsilon_{is}, \alpha_i + \epsilon_{it}) \\ &= Cov(\alpha_i, \alpha_i) \\ &= Var(\alpha_i) \\ &= \sigma_\alpha^2 \end{aligned}$$

Random Effects II

- With this panel-induced serial correlation, we can always use the general formula for $\hat{V}(\hat{\beta}_{POLS})$ that accommodates it
  - (As we showed on Slide 42)
- But we can also do better
  - And get a more efficient estimator by modeling the serial correlation
- This is just Generalized Least Squares (GLS)
  - (Q: I didn't teach you GLS this semester, so what is it?)
  - (A: A way to get a more efficient estimator than OLS when there is heteroskedasticity and/or serial correlation and/or clustering in the error term)
  - (Here we have serial correlation across time within i induced by α_i)

Random Effects III

- The Random Effects estimator is a particular version of (F)GLS
- We assume, for $\nu_{it} = \alpha_i + \epsilon_{it}$, that

$$\begin{aligned} V(\epsilon_{it}) &= \sigma^2_\epsilon \\ Cov(\epsilon_{is}, \epsilon_{it}) &= 0 \\ Cov(\alpha_i, \epsilon_{it}) &= 0 \end{aligned}$$

- *And*
  - **Strict Exogeneity** (as for FE, **pretty strong**):

$$Cov(x_{is}, \epsilon_{it}) = 0 \qquad s, t = 1, \ldots, T$$

  - Uncorrelated Effects (as for Pooled OLS, very strong):

$$Cov(x_{it}, \alpha_i) = 0 \qquad t = 1, \ldots, T$$

  - (These are strong assumptions; hope the efficiency gains are worth it!)

Random Effects IV

- Given these assumptions, ν_i is a $T \times 1$ vector of composite errors w/

$$E(\nu_i \nu_i') = \Omega_i = \begin{bmatrix} \sigma_\alpha^2 + \sigma_\epsilon^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \sigma_\alpha^2 + \sigma_\epsilon^2 & \cdots & \sigma_\alpha^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 + \sigma_\epsilon^2 \end{bmatrix}$$
$$= \sigma_\alpha^2 \iota_T \iota_T' + \sigma_\epsilon^2 I_T$$

where ι_T is a $T \times 1$ vector of ones and I_T a $T \times T$ identity matrix.

Random Effects V

- And the overall variance-covariance matrix for ν, a $NT \times 1$ vector is

$$E(\nu\nu') = \Omega = \begin{bmatrix} \Omega_i & 0 & \cdots & 0 \\ 0 & \Omega_i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Omega_i \end{bmatrix}$$

Random Effects VI

- The GLS estimator in this setting is given by:

$$\boxed{\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y} \quad \boxed{V(\hat{\beta}_{GLS}) = (X'\Omega^{-1}X)^{-1}}$$

- And the Feasible GLS estimator by:

$$\boxed{\hat{\beta}_{FGLS} = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}y} \quad \boxed{V(\hat{\beta}_{FGLS}) = (X'\hat{\Omega}^{-1}X)^{-1}}$$

  - Where the only difference between the two is that we estimate Ω with $\hat{\Omega}$ in the latter

- To implement, we only need an estimate of Ω

Random Effects VII

- Estimating $\Omega_i = \sigma_\alpha^2 \iota_T \iota_T' + \sigma_\epsilon^2 I_T$ requires estimates of σ_α^2 and σ_ϵ^2
- We have an estimate of σ_ϵ^2 from our Fixed Effects (Within Estimator):

$$\boxed{\hat{\sigma}_\epsilon^2 = \frac{1}{N(T-1)-K} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\ddot{\epsilon}}_{it}^2}$$

- And we have an estimate of σ_α^2 from our Between Estimator:

$$\boxed{\hat{\sigma}_\alpha^2 = \hat{\sigma}_{\bar{\nu}}^2 - \frac{1}{T}\hat{\sigma}_\epsilon^2}$$

- Therefore it's quite easy to estimate $\hat{\Omega}$

Random Effects VIII*

- It turns out that there is a way to transform (or weight) observations to "correct" for the structure of the error variance-covariance matrix
  - That yields the Random Effects estimator
- To do so, let

$$\begin{aligned} \lambda &= 1 - \sqrt{\frac{\sigma_\alpha^2}{\sigma_\alpha^2 + T\sigma_\epsilon^2}} \\ \Rightarrow \hat{\lambda} &= 1 - \sqrt{\frac{\hat{\sigma}_\alpha^2}{\hat{\sigma}_\alpha^2 + T\hat{\sigma}_\epsilon^2}} \end{aligned}$$

  - (Note we can estimate λ using our estimates on the previous slide)
  - Intuition?
    - ★ λ measures the relative importance of ν_{it}'s variation due to ϵ_{it} v α_i

Random Effects IX*

- Given $\hat{\lambda}$ we first transform the data

$$\begin{aligned} \tilde{y}_{it} &= y_{it} - \hat{\lambda}\bar{y}_i \\ \tilde{x}_{it} &= x_{it} - \hat{\lambda}\bar{x}_i \\ \tilde{\nu}_{it} &= \nu_{it} - \hat{\lambda}\bar{\nu}_i \end{aligned}$$

  - (This transformation ensures that $\tilde{\nu}_{it}$ has nice properties)
    - ★ (Given our assumptions about α_i and ϵ_{it})

- Then the Random Effects estimating equation can be written as

$$\tilde{y}_{it} = \tilde{x}'_{it}\beta + \tilde{\nu}_{it}$$

  - OLS estimation of which yields the random effects estimator, $\hat{\beta}_{RE}$

Random Effects X

$$\tilde{y}_{it} = \tilde{x}'_{it}\beta + \tilde{v}_{it}$$
$$\equiv (y_{it} - \hat{\lambda}\bar{y}_i) = (x_{it} - \hat{\lambda}\bar{x}_i)'\beta + \tilde{\nu}_{it}$$
$$\lambda = 1 - \sqrt{\frac{\sigma_\alpha^2}{\sigma_\alpha^2 + T\sigma_\epsilon^2}}$$

Note:

- As $\sigma_\epsilon^2 \to \infty$ (or $T \to \infty$), the square root term $\to 0$ and $\lambda \to 1$, and
  - $\hat{\beta}_{RE} \to \hat{\beta}_{FE}$
- As $\sigma_\epsilon^2 \to 0$, the square root term $\to 1$ and $\lambda \to 0$, and
  - $\hat{\beta}_{RE} \to \hat{\beta}_{POLS}$
- $\Rightarrow$ (Loosely:) RE is a weighted combination of the FE and POLS estimators

Random Effects XI*

- The Random Effects estimator can fail to be the true GLS estimator for (at least) two reasons:
  1. $V(\nu_i)$ may not have the specified form
     - ★ In particular, there could be further unmodelled serial correlation
  2. Maybe $V(\nu_i) \neq V(\nu_i|x_i)$
- These aren't horrible
  - As long as strict exogeneity holds, the RE estimator is consistent
    - ★ And is likely to be more efficient than Pooled OLS
  - Tho we should be sure to still use robust variance estimation
    - ★ Even though in principle we're estimating a GLS model!

Random Effects: Intuition I

- What's the intuition for the Random Effects estimator?
  - Basically it's a more efficient version of Pooled OLS
  - Where the efficiency gain comes from there being a common element, α_i, in the variance-covariance matrix of the econometric error
    - ★ (Which is $\nu_{it} = \alpha_i + \epsilon_{it}$)
- Accounting for the serial correlation in ν_{it} induced by the presence of the constant-across-time α_i means RE makes "better use" of the information contained in the data compared to Pooled OLS
  - Shows up as smaller standard errors
  - (Tho note it *also* changes the coefficient estimates, β)

Random Effects: Intuition II

- Efficiency is good, right? Right!
  - So it must come with some strings attached...
  - And so it does
- The key downside of the RE estimator is that it relies on the strongest assumptions
  - And if these assumptions are wrong, then it is inconsistent
  - (And that's Very Bad)

First Differences

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First Differences I

- Our final estimator can be handled quickly
- Like the Fixed Effects estimator, it eliminates α_i
  - But instead of doing it by de-meaning the data...
  - ...it does so by taking differences in adjacent observations, i.e.

$$\begin{aligned} y_{it} &= x_{it}'\beta + \alpha_i + \epsilon_{it} \qquad t = 1, \ldots, T \\ y_{i,t-1} &= x_{i,t-1}'\beta + \alpha_i + \epsilon_{i,t-1} \qquad t = 2, \ldots, T \\ \Rightarrow \Delta y_{it} &= \Delta x_{it}'\beta + \Delta \epsilon_{it} \qquad t = 2, \ldots, T \end{aligned}$$

where $\Delta y_{it} \equiv y_{it} - y_{i,t-1}$ (and similarly for Δx_{it} and $\Delta \epsilon_{it}$)

- Note we've lost the first observation in our dataset
- (Tho researchers often cheat and assume that we had a y_{i0})

First Differences II

- The assumptions for consistency are similar to those for the FE estimator
- The least restrictive condition is:

$$Cov(\Delta x_{it} \Delta \epsilon_{it}) = 0 \qquad t = 2, \ldots, T$$

  - Which itself holds if

$$\begin{aligned} E(x_{it}\epsilon_{it}) &= 0 \\ E(x_{i,t-1}\epsilon_{it}) &= 0, \quad \text{and} \\ E(x_{i,t+1}\epsilon_{it}) &= 0 \end{aligned}$$

    - ★ (Note this is slightly-weaker-version of *Strict Exogeneity*)
    - ★ (This fact underlies much of dynamic panel data estimation)

- As for the FE estimator, we again have *Arbitrary Effects*,
  - i.e. The relationship between α_i and x_{it} is completely unrestricted

First Differences III

$$\Delta y_{it} = \Delta x_{it}'\beta + \Delta \epsilon_{it} \qquad t = 2, \ldots, T$$

- We estimate the FD estimator by OLS on the equation above
- Since we have a differenced error term,

$$\begin{aligned} V(\Delta \epsilon_{it}) &= V(\epsilon_{it} - \epsilon_{i,t-1}) \\ &= 2\sigma_\epsilon^2 \\ Cov(\Delta \epsilon_{i,t-1}, \Delta \epsilon_{it}) &= E(\epsilon_{i,t-1} - \epsilon_{i,t-2})(\epsilon_{it} - \epsilon_{i,t-1}) \\ &= -\sigma_\epsilon^2 \\ \Rightarrow Corr(\Delta \epsilon_{i,t-1}, \Delta \epsilon_{it}) &= -0.5 \end{aligned}$$

  - ... and we should allow for heteroskedasticity and (especially) serial correlation when calculating standard errors

First Differences: Intuition

- The intuition for First Differences is the same as for Fixed Effects
  - You solve unobserved heterogeneity by differencing the data
  - For FE, you subtract the mean
  - For FD, you subtract the previous observation
    - ★ (Six of one, half-a-dozen of the other...)
    - ★ (i.e., this isn't a big conceptual differences)
- You'll be safe if you simply think of FD as a version of FE

Mapping Estimators to their Assumptions

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Mapping Estimators to Assumptions I

- Recall our key panel data assumptions:

	Weak	Strong
(2) $Cov(x_{is}, \epsilon_{it})$	Contemporaneous Exogeneity	**Strict Exogeneity**
(3) $Cov(x_{it}, \alpha_i)$	Arbitrary Effects	Uncorrelated Effects

- How do each of our four panel data estimators rely on each of these?

	Assumption on $Cov(x_{is}, \epsilon_{it})$	Assumption on $Cov(x_{it}, \alpha_i)$
Pooled OLS	Contemporaneous Exogeneity	Uncorrelated Effects
Fixed Effects	**Strict Exogeneity**	Arbitrary Effects
Random Effects	**Strict Exogeneity**	Uncorrelated Effects
First Differences	**(slightly weaker) Strict Exogeneity**	Arbitrary Effects

Mapping Estimators to Assumptions II

- Let's re-order our list, ranking estimators by the strength of the assumptions on which they rely
  - Where it's always better to have _____ assumptions!
- And list also their efficiency properties ...
  - (And combine FE with FD as they are so close conceptually)

Asumptions	Estimator	Assumption on $Cov(x_{is}, \epsilon_{it})$	Assumption on $Cov(x_{it}, \alpha_i)$	Efficiency
Stronger	Random Effects	**Strict Exogeneity**	Uncorrelated Effects	Most efficient
↓	Pooled OLS	Contemporaneous Exogeneity	Uncorrelated Effects	↑
Weaker	Fixed Effects / First Differences	**Strict Exogeneity**	Arbitrary Effects	Least efficient

Mapping Estimators to Assumptions III

Asumptions	Estimator	Assumption on $Cov(x_{is}, \epsilon_{it})$	Assumption on $Cov(x_{it}, \alpha_i)$	Efficiency
Stronger	Random Effects	**Strict Exogeneity**	Uncorrelated Effects	Most efficient
↓	Pooled OLS	Contemporaneous Exogeneity	Uncorrelated Effects	↑
Weaker	Fixed Effects / First Differences	**Strict Exogeneity**	Arbitrary Effects	Least efficient

- This table highlights a common lesson in econometrics:
  - There is a tradeoff between the strength of your assumptions and the efficiency of your estimator!
- And so...
  - Which panel data estimator should you use???
  - It turns out there is a test that can help you make the choice

Testing Uncorrelated Effects

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Which Estimator to Use? I

- The previous table suggests that you should use one of only two (classes of) estimators
  - Fixed Effects / First Differences
    - ⋆ (If you're worried about the bias from unobserved heterogeneity)
    - ⋆ (As α_i is differenced out/estimated)
    - ⋆ (And is thus not in the error term)
  - Random Effects
    - ⋆ (If you're not worried about U.H. bias)
    - ⋆ (As it's most efficient when not)
- (Usually don't bother with Pooled OLS...
  - ... unless very worried about Strict Exogeneity of x_{is} and ϵ_{it})

Which Estimator to Use? II

- How shall we choose between them?
  - Using straightforward logic...
- We often care most about two different properties of estimators
  1. Unbiasedness/Consistency
  2. Efficiency
     - ★ In *that* order!
- So the goal is the most efficient consistent estimator.
  - Which is that between FE and RE?
    - ★ *

Testing Uncorrelated Effects I

- To determine the best estimator, we must therefore test whether or not uncorrelated effects is satisfied
  - i.e. whether $Cov(x_{it}, \alpha_i) = 0$
- We test it by comparing the (time-varying) coefficients in the FE and RE models
  - Called *The Hausman Test*

Testing Uncorrelated Effects: Intuition

- Maybe that's not obvious. What's the *intuition* of the Hausman Test?
- Simple! We know that (under its weaker assumptions) the FE estimator is always consistent
  - But the RE estimator is only consistent if the Uncorrelated Effects assumption is true
- Thus compare the two estimates!
  - If Uncorrelated Effects is valid, the two sets of estimates shouldn't be "too different"
  - That's what the Hausman Test does

Testing Uncorrelated Effects II

- The "classic" Hausman Test computes a quadratic form in the difference in FE and RE coefficients:

$$H = (\hat{\beta}_{FE} - \hat{\beta}_{RE})'(\hat{V}_{FE} - \hat{V}_{RE})^{-}(\hat{\beta}_{FE} - \hat{\beta}_{RE})$$

  where $(\cdot)^{-}$ is the generalized matrix inverse
  - This is distributed as a χ^2_K under the standard Random Effects assumptions
  - (where recall β is a $K \times 1$ column vector)

Testing Uncorrelated Effects III

- If we're only interested in one of the elements of β,
  - E.g., one of the x_{it} is a key policy variable while others are simply control variables
  - Then we can write the test as a t-test focusing on that variable:

$$H = \frac{\hat{\beta}_{k,FE} - \hat{\beta}_{k,RE}}{\{se(\hat{\beta}_{k,FE})^2 - se(\hat{\beta}_{k,RE})^2\}^{1/2}}$$

Testing Uncorrelated Effects IV

- This "classic" version of the test has fallen out of favor a bit,
  - In part because it requires homoskedasticity and no serial correlation in the errors
  - These assumptions gives the simplified form of $(\hat{V}_{FE} - \hat{V}_{RE})$ without worrying about covariance terms
    - ★ (But they are often violated in typical datasets)

Testing Uncorrelated Effects V

- The current best method to run the Hausman Test is to use a regression approach...
  - ...that allows the nesting of both the FE and RE estimators
- Suppose we are interested in a subset of the elements in x_{it}'
  - Call these w_{it}', a $1 \times M$ row vector
  - (Cannot include the time dummies)
    - ★ (Which is fine - these usually aren't parameters of interest)

Testing Uncorrelated Effects VI

- Wooldridge (2012, Section 10.7.3) shows how to implement the Hausman test as an F-test
- The Restricted model is the Random Effects specification:

$$\tilde{y}_{it} = \tilde{x}'_{it}\beta + \tilde{\nu}_{it}$$

where

- $\tilde{y}_{it} \equiv (y_{it} - \hat{\lambda}\bar{y}_i)$,
- $\hat{\lambda} = 1 - \sqrt{\frac{\hat{\sigma}^2_\alpha}{\hat{\sigma}^2_\alpha + T\hat{\sigma}^2_\epsilon}}$, and
- $\hat{\sigma}^2_\alpha$ and $\hat{\sigma}^2_\epsilon$ were defined on Slide 68

Testing Uncorrelated Effects VII

- The Unrestricted model is given by

$$y_{it} = x_{it}'\beta + \bar{w}_i'\xi + \nu_{it}^*$$

where

  - $\bar{w}_i$ are the across-time mean values of w_{it} and
  - ν_{it}^* is an error term

- The clever algebraic trick is that the estimate of β when including $\bar{w}_i$ *is* the Fixed Effect estimator
  - (Not obvious! - this is why econometricians get the big bucks!)

Testing Uncorrelated Effects VIII

- The nice thing is that we can test $\xi = 0$ allowing for arbitrary heteroskedasticity and serial correlation
- Simply run a t- or F-test of $H_0 : \xi = 0$ in the unrestricted model
  - Using either Pooled OLS or RE...
    - ★ (Each yields identical coefficients when $\bar{w}_i$ included)
  - ...with standard errors calculated appropriately
    - ★ (i.e. with robust and/or clustered standard errors)

Panel Data Instrumental Variables*

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Panel Instrumental Variables Intro I*

- One of the major undertaking in econometrics is to worry about sources of bias and inconsistency in our econometric analyses
  - (usually induced by $Cov(x_{it}, \nu_{it}) \neq 0$)
  - And - of course - to develop methods to deal with them!
- We covered Instrumental Variables (IV) estimation in our last big set of slides
  - It is one of the primary tools we use to address correlation between our x's and the error

Panel Instrumental Variables Intro II*

- Panel data methods provide *another* potential solution to issues of bias due to correlation between x_{it} and ν_{it}:
  - *If* the correlation is driven *only* by unobserved heterogeneity
  - i.e., if we think
    - ★ $Cov(x_{it}, \alpha_i) \neq 0$, but
    - ★ $Cov(x_{it}, \epsilon_{it} | \alpha_i) = 0$
  - Then we can used FE or FD:
    - ★ FE/FD estimation controls for α_i, leaving variation in x_{it} within i across t to identify β
    - ★ Problem solved! No instruments needed!

Panel Instrumental Variables Intro III*

- Can we imagine cases like this? Perhaps yes!
- There is considerable analysis of supermarket scanner data:
  - Goal is to estimate demand curves for products
    - ★ e.g. varieties of butter, to analyze a "fat tax"
  - Panel dataset of market shares and prices for grocery stores (i) across weeks (t)
  - Lots of price variation driven by products on sale
    - ★ With sales determined months in advance in negotiations between manufacturers and retailers
    - ★ (i.e. plausibly unrelated to cost or demand shocks in a given week, ϵ_{it})
    - ★ (Right???)

Panel Instrumental Variables Intro IV*

- Reason to believe prices related to where a store is located, *i*
  - Due to cost differences of serving different locations
  - Or income differences among the people living in different locations
- Fixed effects soak of the location differences
  - Leaving sales to exogenously trace out demand curves for products

Reasonable???

Panel Instrumental Variables Intro V*

- But what if it's *not* true???
  - Q: Is it reasonable that the *only* source of correlation between ν_{it} and x_{it} is due to a time-constant effect across individuals???
  - A: _____
    - ★ _____

Panel Instrumental Variables Intro VI*

- Q: So what do we do if it's *not* true? i.e.,
  - What if we have *both* (1) unobserved heterogeneity...
    - ★ (Causing cross-sectional endogeneity)
  - ...*and* (2) other sources of bias???
    - ★ (Causing time-series endogeneity)
- A: _____

Panel Instrumental Variables I*

- Start with our basic panel data model:

$$y_{it} = x_{it}'\beta + \alpha_i + \epsilon_{it}$$

  - (where we've subsumed α_t into x_{it})
  - But allow for $Cov(x_{it}, \epsilon_{it}) \neq 0$
- Suppose one has a $1 \times L$ $(L > K)$ vector of possible instruments, z_{it}'

Panel Instrumental Variables II*

- There are analogous IV estimators to each of the panel data estimators we introduced earlier:
  1. Pooled OLS
  2. Fixed Effects
  3. Random Effects
  4. First Differences
- The assumptions underlying them are basically the IV analogs to the assumptions we introduced last time
  - (Will skip (4) First Differences - analogous to what we do for FE)

(1) Pooled IV*

- Pooling the data and estimating by 2SLS will be consistent if

$$Cov(z_{it}, \alpha_i) = 0$$
$$Cov(z_{it}, \epsilon_{it}) = 0$$

  - While theoretically possible...
    - ★ It may be hard to find an IV that is uncorrelated with both α_i and ϵ_{it}

(2) Fixed Effects IV I*

- Fixed Effects required **Strict Exogeneity** but allowed Arbitrary Effects
- FE IV requires an analogous strict exogeneity of the instruments:

$$Cov(z_{is}, \epsilon_{it}) = 0 \qquad s, t = 1, \ldots, T$$

- But allows arbitrary correlation with the unobserved heterogeneity:

$$Cov(z_{it}, \alpha_i) \neq 0 \qquad t = 1, \ldots, T$$

(2) Fixed Effects IV II*

- To estimate, run IV on the demeaned equation

$$\ddot{y}_{it} = \ddot{x}'_{it}\beta + \ddot{\epsilon}_{it}$$

using $\ddot{z}_{it} \equiv z_{it} - \bar{z}_i$ as IVs

  - Be sure to check the first stage(s) by regressing, for each potential RHS endogenous variables, x_{itk}

$$\ddot{x}_{itk} = \ddot{x}'_{it,-k}\pi + \ddot{z}'_{it}\rho + \omega_{it}$$

where $\ddot{x}_{it,-k}$ are the elements in x_{it} other than the k^{th}, x_{itk}

    - ★ In the first stage, test for instrument relevance with $H_0 : \rho = 0$
    - ★ (All standard steps, just with the de-meaned data)

- Time-constant covariates *and instruments* drop out of the analysis
- As always, use robust and/or clustered standard errors

(3) Random Effects IV*

- Random Effects required **Strict Exogeneity** and Uncorrelated Effects
- RE IV requires analogous conditions of the instruments:

$$Cov(z_{is}, \epsilon_{it}) = 0 \qquad s, t = 1, \ldots, T$$
$$Cov(z_{it}, \alpha_i) = 0 \qquad t = 1, \ldots, T$$

- Calculate $\hat{\sigma}^2_\epsilon$ and $\hat{\sigma}^2_\alpha$ per usual and estimate

$$\tilde{y}_{it} = \tilde{x}'_{it}\beta + \tilde{v}_{it}$$
$$\equiv (y_{it} - \hat{\lambda}\bar{y}_i) = (x_{it} - \hat{\lambda}\bar{x}_i)'\beta + \tilde{\nu}_{it}$$
$$\hat{\lambda} = 1 - \sqrt{\frac{\hat{\sigma}^2_\alpha}{\hat{\sigma}^2_\alpha + T\hat{\sigma}^2_\epsilon}}$$

by IV using $\tilde{z}_{it} \equiv z_{it} - \hat{\lambda}\bar{z}_i$ as instruments

Hausman Test for IV Estimates*

- There is an analogous augmented regression for comparing FEIV and REIV estimates

$$y_{it} = x_{it}'\beta + \bar{z}_i'\xi + \nu_{it}^*$$

- Test for $Cov(x_{it}, \alpha_i) = 0$ by
  1. Estimating the above by 2SLS or IV using $(1, z_{it}', \bar{z}_i')'$ as IVs
     - ★ $\hat{\beta}$ is the FE estimator
  2. Testing $H_0 : \xi = 0$
     - ★ Using a robust test

Panel Data Examples

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Panel Data Examples

- I think the best way to learn econometrics is to "do" it
- I therefore have slides for three panel data examples (*if time*)
  1. The student's thesis from the IV notes
     - ★ Analyzing the impact of CBW on votes "against foreigners"
     - ★ (as she also had panel data)
  2. A working paper of mine analyzing the impact of Twitter on movie demand
  3. Panel data of US airfares

Student Thesis Redux I

$$v_{it} = \beta_1 + \beta_2 CBW_{it} + \tilde{x}_i' \tilde{\beta} + \epsilon_{it}$$

Recall where we left off with the IV results of this thesis

- We thought there could be either negative or positive bias on β_2:
  - Negative from reverse causality and/or a correlated unobservable measuring "local openness" that wasn't adequately captured by dummies for city/suburbs/rural areas
  - Positive from a correlated unobservable measuring local (i.e. Swiss-population) unemployment rates

Student Thesis Redux II

- We tried a number of specfications and found a consistent story:
  - Positive/significant effects of CBW with OLS
  - Negative/insignificant effects of CBS with IV
    - ★ Using distance to the border as an IV
- Let's now try using panel data methods - what do we get?
  - See Stata results in class
  - Fixed effects: no effect of CBW share (like IV)
  - Random effects: pos and sig effects (like Pooled OLS)
- The Hausman test? Rejected!

Student Thesis Redux III

- And so the $64,000 Question: which results to "believe"?
  - Easy! All are consistent!
- Can never be sure, but based on the balance of evidence:
  1. There are no causal effects of cross-border workers on "anti-foreign votes" in Ticino municipalities, and
  2. Looking at OLS results is likely to yield the false conclusion that there is a positive effect of cross-border workers on anti-foreign votes

Twitter on Movie demand

- See slides to come

Online Word of Mouth Matters

- Consumers now spend more than 135 mins per day on social media
  - Social media sites contain a treasure-trove of decision-relevant information
  - Twitter is the main platform for opinion exchange about brands and products
- Online WoM particularly important for new products
- Chief Marketing Officers think online WoM matters
  - ... Rationalized by consumer's trust in online info from peers (Nielsen, 2013)

Hollywood & News Media Think Twitter Matters

What This Paper is About

Quantifying the impact of online WoM on new product performance

- Estimate demand elasticities w.r.t.:
  1. Volume of Tweets
  2. Tweet Sentiment
- How:
  - Structural model of consumer demand
  - Controlling for endogenous advertising and offline WoM
- Application:
  - Movie Industry
  - Twitter

Tweet Volume Measures Awareness and Buzz

- Awareness
  - Introduces new consumers to a movie
  - Reminds consumers about movie
  - Reinforce traditional advertising
- Buzz: expressions of anticipation
  - Increase in anticipation $\rightarrow$ increase in volume of posts
    - ★ By consumers who want to act as opinion leaders, and reflect their interests, excitement, and expectations
    - ★ Generally "neutral" in sentiment

Sentiment Measures Movie Quality

- Sentiment expressed tweet's text provides means to measure quality
- Tweet Sentiment impacts sales via social learning
  - Quality revealed through interactions with their peers
  - Relevant if consumers use these reviews to decide what movie to attend
- Important to control for other ways consumers learn about movie quality

Data Sources: Box Office

US Movie Industry 2014 & 2015:

- Wide release movies - opened to at least 600 cinemas
- Released on a Friday
- 222 movies out of approx. 300 wide releases

Box office data & movie characteristics from Box Office Mojo:

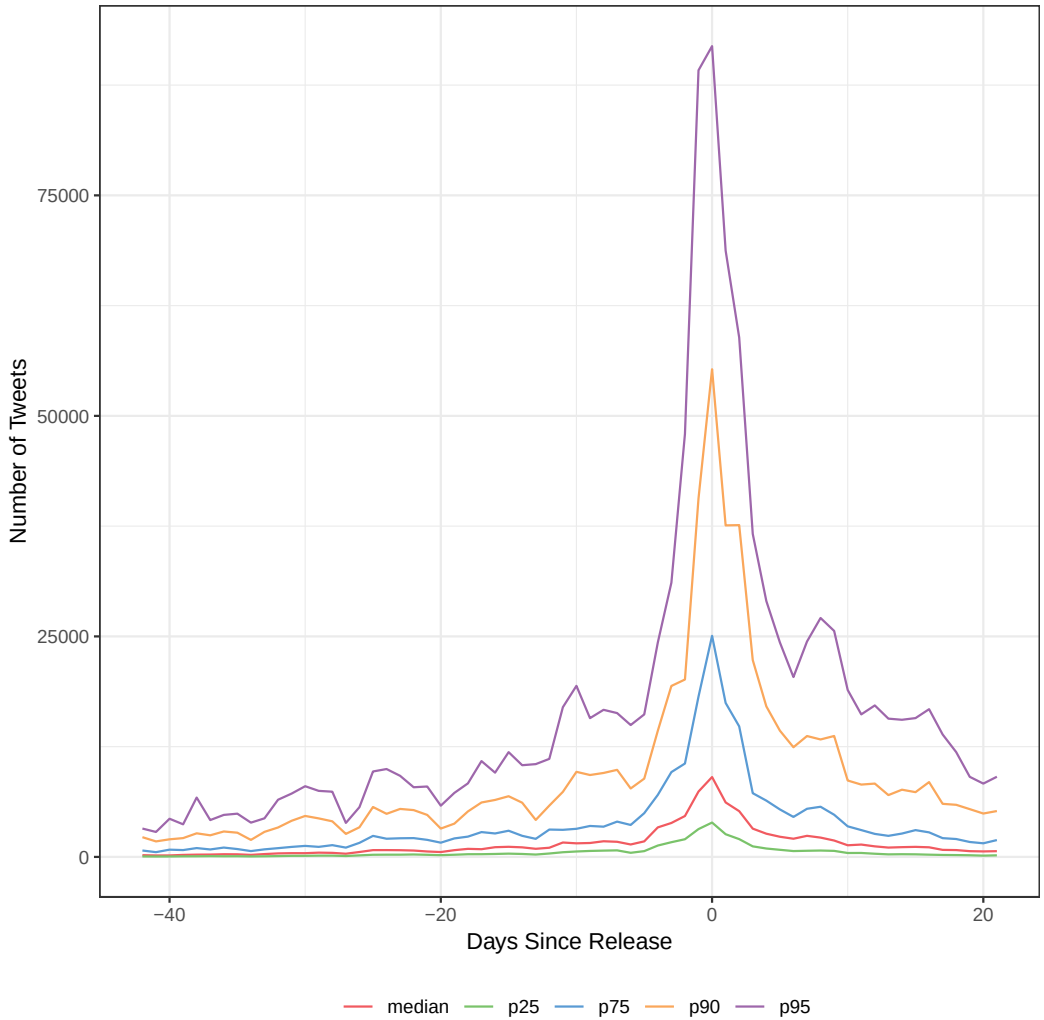
- Data from opening 3 weekends
  - Opening 3 weeks - 86 percent of US Box Office
  - 80 percent of opening 3 weeks is earned on weekends

Twitter

Tweets about each movie from Twitter's Historical Powertrack

- Search for tweets about each movie using
  1. Movie name
  2. Relevant hashtags
  3. Movie franchise + sequel indicators
- 60 days pre-release until end of the third release weekend
  - 48 million movie relevant tweets

Large Heterogeneity in Tweet Volume



Data Sources: Movie Industry

- Offline WoM and Advertising:
  - Advertising Spending
  - CinemaScore Grades
  - Expected Box Office Performance
- Additional Data:
  - Movie Characteristics
  - Critic Reviews

Tweet Volume

- Separate pre-release and post-release phase
  - Different information content in each phase
- Pre-release volume of tweets for movie j:

$$prevol_j = \sum_{\tau=-60}^{-1} \text{tweets}_{j\tau}$$

- Post-release volume on day t:

$$postvol_{jt} = \sum_{\tau=0}^{t-1} \text{tweets}_{j\tau}$$

Tweet Sentiment

- VADER Sentiment Lexicon (Hutto & Gilbert 2014)
  - Classify tweets as positive / neutral / negative
  - Extends 'standard' lexicons for online language

- **Movie Sentiment = Positive-Negative Ratio**:

$$sentiment_{jt} = \frac{\sum_{t=\tau}^{t-1} \#\text{ positive tweets}}{\sum_{t=\tau}^{t-1} \#\text{ negative tweets}}$$

Separate measures for pre- and post-release

Key Product Characteristics

Pre-release
Post-release

x Tweet Volume
x Tweet Sentiment
Advertising Expenditure

x β_{open}
$\beta_{post\text{-}open}$

Utility Specification - Movies

$$u_{ijt} = x_{jt}\beta_{t-r_j} + w_j^{(1)}\gamma + w_j^{(2)}\lambda_{t-r_j} + \sum_{s=1}^{S} d_{js}\theta_s + \xi_{jt} + \bar{\varepsilon}_{ijt}$$

Utility Specification - Movies

$$u_{ijt} = x_{jt}\beta_{t-r_j} + w_j^{(1)}\gamma + w_j^{(2)}\lambda_{t-r_j} + \sum_{s=1}^{S} d_{js}\theta_s + \xi_{jt} + \bar{\varepsilon}_{ijt}$$

- x_{jt}: Pre- and post-release measures of:
  - Twitter Volume
  - Twitter Sentiment
  - Advertising Expenditure
- β_{t-r_j}: Separate parameters for
  - Opening Weekend
  - Post-opening Weekends

Utility Specification - Movies

$$u_{ijt} = x_{jt}\beta_{t-r_j} + w_j^{(1)}\gamma + w_j^{(2)}\lambda_{t-r_j} + \sum_{s=1}^{S} d_{js}\theta_s + \xi_{jt} + \bar{\varepsilon}_{ijt}$$

- $w_j^{(1)}$: Movie Characteristics
  - Actor Starpower, critic review and production budget
  - **CinemaScore grades**
    - ★ Controls for offline WoM

Utility Specification - Movies

$$u_{ijt} = x_{jt}\beta_{t-r_j} + w_j^{(1)}\gamma + w_j^{(2)}\lambda_{t-r_j} + \sum_{s=1}^{S} d_{js}\theta_s + \xi_{jt} + \bar{\varepsilon}_{ijt}$$

- $w_j^{(2)}\lambda_{t-r_j}$: Market Share Decline
  - Day of release fixed effects for:
    - ★ Genre
    - ★ **Franchises**

Utility Specification - Movies

$$u_{ijt} = x_{jt}\beta_{t-r_j} + w_j^{(1)}\gamma + w_j^{(2)}\lambda_{t-r_j} + \sum_{s=1}^{S} d_{js}\theta_s + \xi_{jt} + \bar{\varepsilon}_{ijt}$$

- θ_s: **Expected Performance Tier fixed effects**
  - Absorbs potential endogeneity due to:
    - ★ Ad spending
    - ★ Awareness caused by offline WoM

Utility Specification - Movies

$$u_{ijt} = x_{jt}\beta_{t-r_j} + w_j^{(1)}\gamma + w_j^{(2)}\lambda_{t-r_j} + \sum_{s=1}^{S} d_{js}\theta_s + \xi_{jt} + \bar{\varepsilon}_{ijt}$$

- Unmodelled Product Characteristics, ξ_{jt}
- Nested logit individual specific valuations, $\bar{\varepsilon}_{ijt}$:

$$\bar{\varepsilon}_{ijt} = \sum_{g=0}^{G} d_{jg}\zeta_{igt} + (1-\rho)\varepsilon_{ijt}$$

  - ρ preference similarity between movies in same genre g

Utility Specification - Staying Home

$$u_{i0t} = -\tau_t + \bar{\varepsilon}_{i0t}$$

- τ_t: Seasonality in demand
  - Calendar Week FE
  - Public Holidays FE

Results

- OK, let's see what kind of results we get!
  - OLS (sort of - IV for within-group share)
    - ★ Interpret result magnitudes
  - IV for tweet volume
    - ★ Better?
  - FE
    - ★ Better?
  - Rich Controls
  - Which results do you believe?

OLS Elasticities

	Own Demand Elasticities	
	Opening Weekend	Post Opening
Volume_{pre}	0.18***	0.03
Volume_{post}	-	0.11
Sentiment_{pre}	0.06	-0.13
Sentiment_{post}	-	0.25
Ad Spend_{pre}	0.44**	0.48*
Ad Spend_{post}	-	0.16

- What do you think of the magnitudes here? Reasonable?
- If not, what kind of bias? What direction?

IV Elasticities

- Let's instrument for Tweet volume, both pre- and post-release
- What might make a good instrument? Ideas???
  - Is it relevant? Is it exogenous?
- We settled on "news pressure" - what do you think? - and got the following results:

	Own Demand Elasticities	
	Opening Weekend	Post Opening
Volume_{pre}	1.16	-.26
Volume_{post}	-	0.48
Sentiment_{pre}	0.22	-0.14
Sentiment_{post}	-	0.25
Ad Spend_{pre}	0.50	0.36
Ad Spend_{post}	-	0.24

- Nothing significant - and crazy magnitudes. Our Tweet volume IVs just don't have power.

FE Elasticities

- What of fixed effects? Is this likely to help?

	Own Demand Elasticities	
	Opening Weekend	Post Opening
Volume_{pre}	-	-0.06**
Volume_{post}	-	0.05**
Sentiment_{pre}	-	-0.10***
Sentiment_{post}	-	0.18***
Ad Spend_{pre}	-	0.06
Ad Spend_{post}	-	0.10***

- Wow - big differences! More reasonable?
- But why don't we see estimates for pre-release variables?
- Is there anything else we can try???

Rich Controls to Minimize Endogeneity Concerns

Fixed Effects for:

- Expected Performance Tier ("Hollywood Stock Exchange")
- Genre $\times$ Franchise $\times$ Day-of-release
- CinemaScore (day-of-release measure of movie quality)

$\Longrightarrow$ identification from 'within category variation'

- The 'right' variation for understanding marketing interventions
  - Example: (Expected to be) Large action movies, 2nd Saturday of release, CinemaScore rating of A

Rich Controls Elasticities

	Own Demand Elasticities	
	Opening Weekend	Post Opening
Volume_{pre}	0.06*	-0.04
Volume_{post}	-	0.08
Sentiment_{pre}	-0.02	-0.17
Sentiment_{post}	-	0.27
Ad Spend_{pre}	0.04	0.18
Ad Spend_{post}	-	0.12

- Interesting - what do you think?

Which results do you believe???

		OLS	Rich	FE
Opening Weekend	Volume_{pre}	0.18***	0.06*	
	Sentiment_{pre}	0.06	-0.02	
	Ad Spend_{pre}	0.44**	0.04	
Post-Opening Weekend	Volume_{pre}	0.03	-0.04	-0.06**
	Volume_{post}	0.11*	0.08*	0.05**
	Sentiment_{pre}	-0.13	-0.17	-0.10***
	Sentiment_{post}	0.25	0.27*	0.18***
	Ad Spend_{pre}	0.48*	0.18	0.06
	Ad Spend_{post}	0.16	0.12	0.10***

- We like the Rich Controls specification...
- Do you agree?

Airfares Example I

- Let's do one last example (if time)
- It's a panel dataset of airfares in the U.S. market for airplane flights
  - $i = 1, \ldots, 1,149$ U.S. air routes
    - ★ 94 origin cities, 97 destination cities
  - $t = 1997, \ldots, 2000$
  - Dependent variable: $y_{it} = \log(fare_{it})$
    - ★ Measured as the average one-way fare on route i in period t
  - Key independent variable: $x_{it} = concentration_{it}$
    - ★ Measured as the passenger (market) share of the largest airline on i in t
    - ★ (Other measures are possible, indeed more common)
  - Other control variable: $distance_{it}$
    - ★ Measuring both cost and demand factors
    - ★ Include as a quadratic

Airfares Example II

- Before we dig into the data, let's do some higher-level thinking:
  1. What does economic theory suggest about how prices should respond to increases in concentration?*
  2. Why might we be worried about unobserved heterogeneity, i.e....
     - ★ Why might we think air routes differ in important ways (α_i) that might be correlated with concentration (x_{it})?*
     - ★ What would be the sign of any bias in this case?*

Airfares Example III

- OK, let's see whether the data confirm our prior beliefs

See Stata Output in Class*

  - (Basic Summary Statistics)
  - (Pooled OLS - interpret $\hat{\beta}_{conc}$?)
  - (Fixed Effects)
    - ★ (What happened to $\hat{\beta}_{dist}$?)
    - ★ (What happened to $\hat{\beta}_{conc}$?)
  - (Random Effects)
    - ★ (What happened to $\hat{\beta}_{conc}$?)
    - ★ (Compare POLS to RE - does GLS 'help'?)
    - ★ (Run the Augmented Regression Hausman Test)
  - Which results do you believe???

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