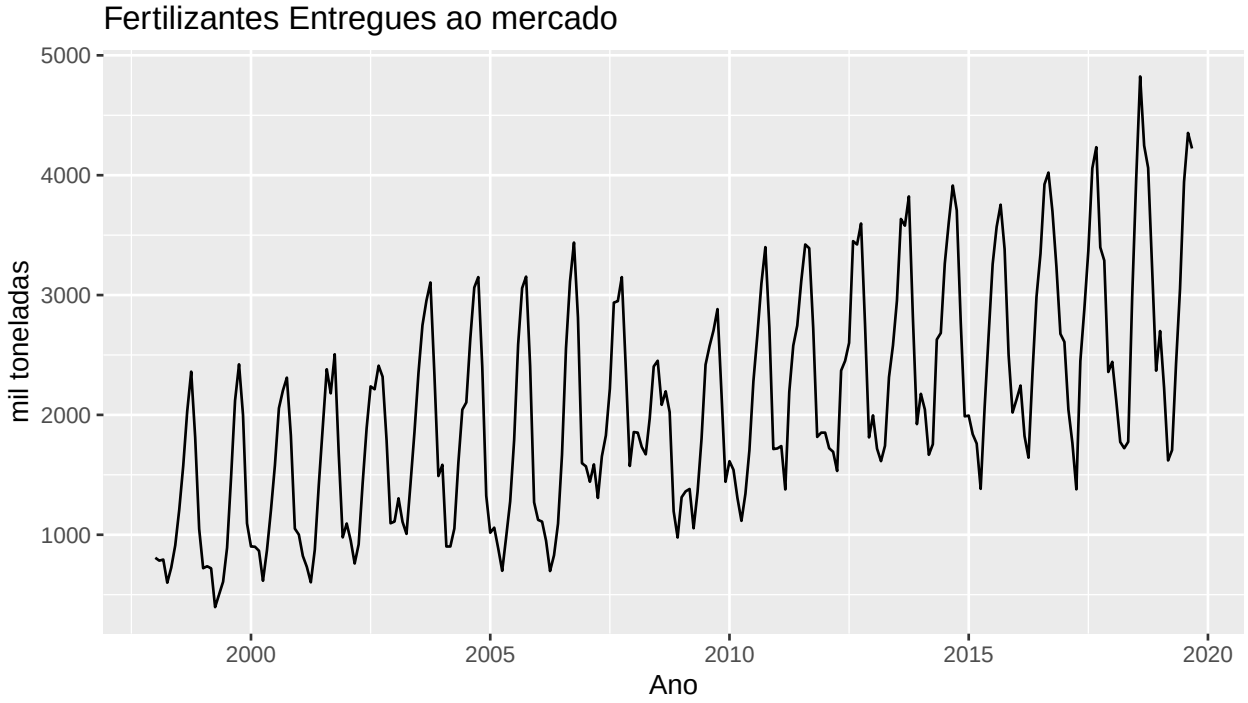
Fertilizantes

Nome: Matheus Amaral Moes
Disciplina: Análise de Séries Temporais
Professor: Alvaro Villarinho
Matrícula:

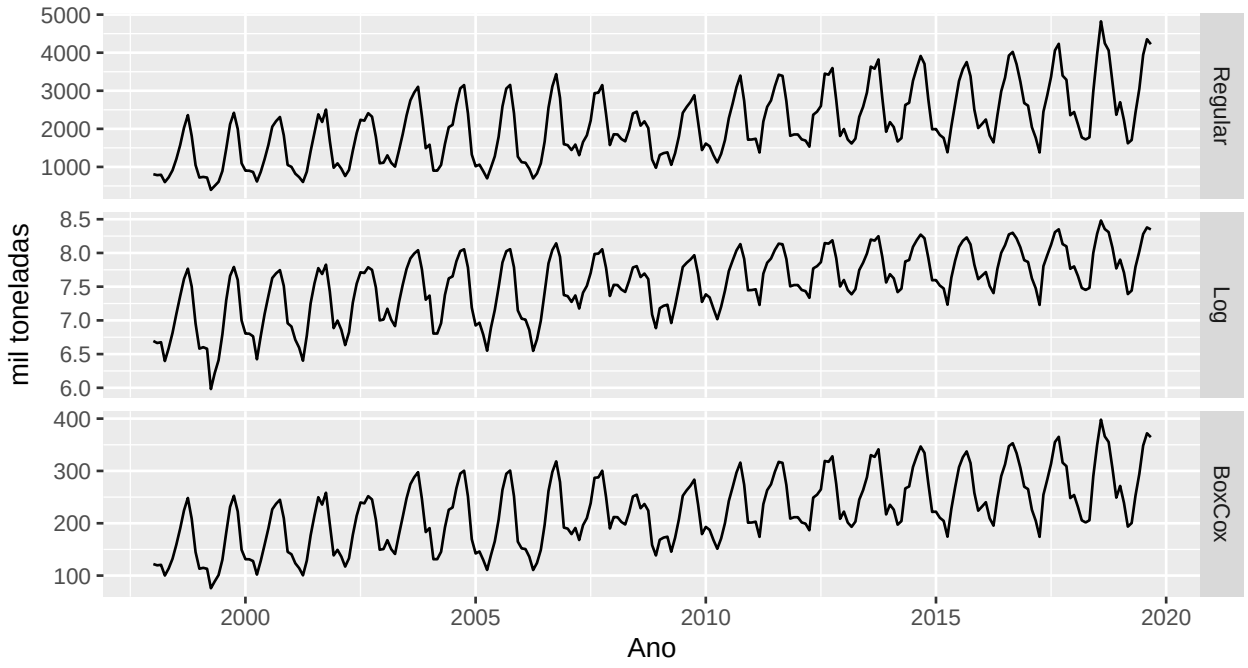
Análise exploratória

O conjunto de dados possui a quantidade de fertilizantes entregues mensalmente em uma série temporal de 1998 a Setembro de 2019. O volume de fertilizantes na série é sazonal com frequência anual, atingindo o pico todos os anos nos mêses de Setembro e Outubro, como é possível notar no gráfico a seguir. É de se notar que a série possui um aumento na amplitude da sazonalidade, indicando uma serie multiplicativa, e uma tendência de aumento no nível ao longo de toda sua duração dois fatos que podem ser melhor observados na decomposição da série posteriormente.



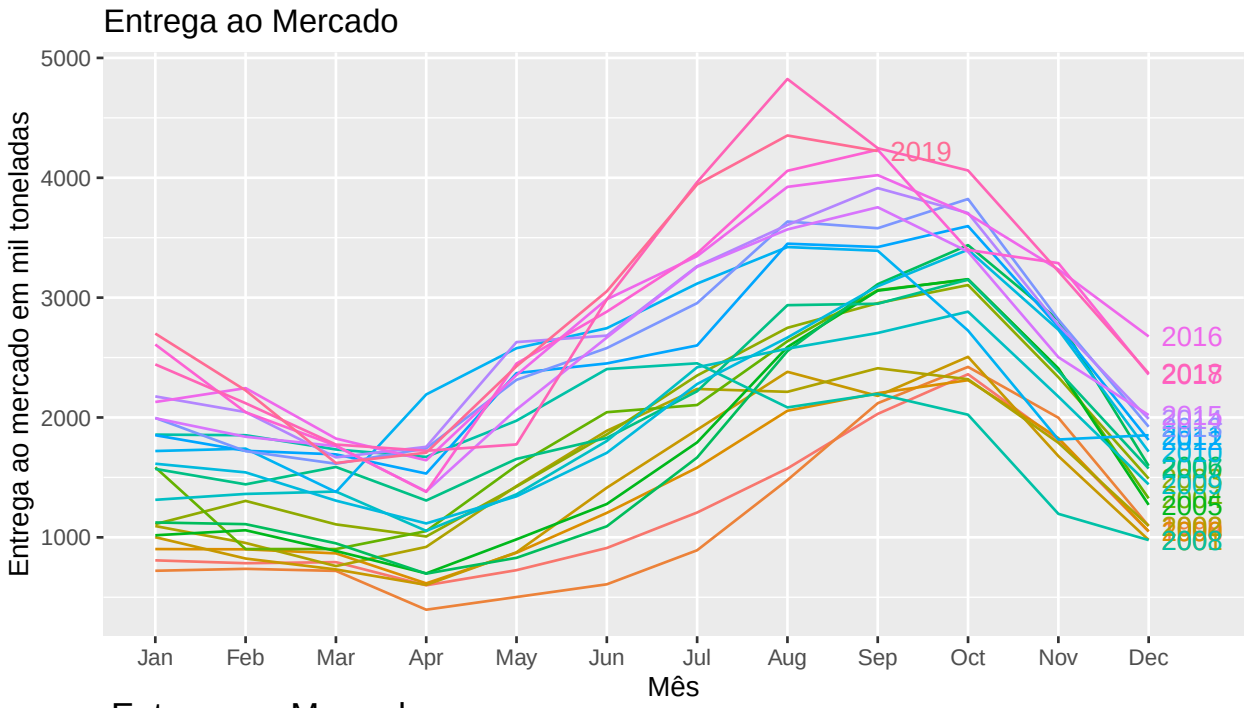
Transofrmações

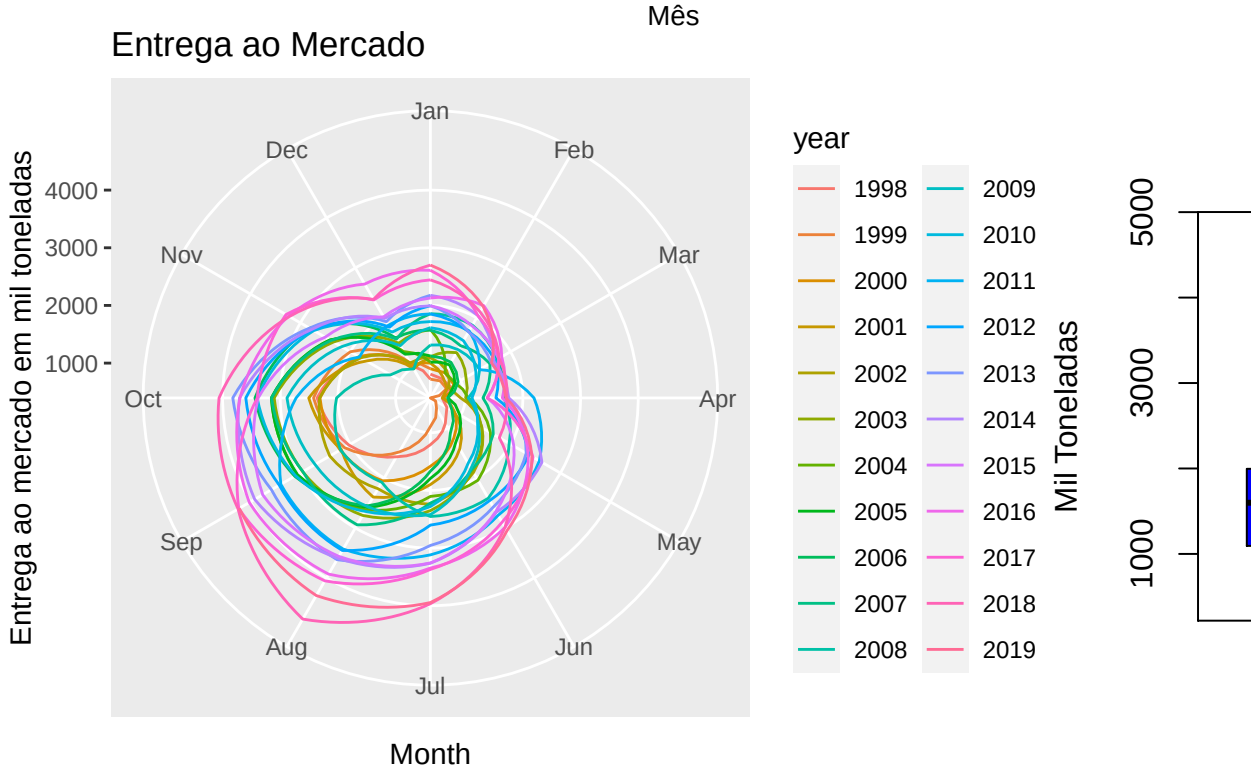
Realizando transformacoes BoxCox e Logarítimica utilizando a base natural dois resultados são observados. Na transformação Logarítimica a amplitude da serie diminui ao longo do tempo e na transformacao BoxCox a amplitude se manteve estavel ao longo do periodo indicando que a transformação mais adequada para a séria é a BoxCox.



Seasonal Plot

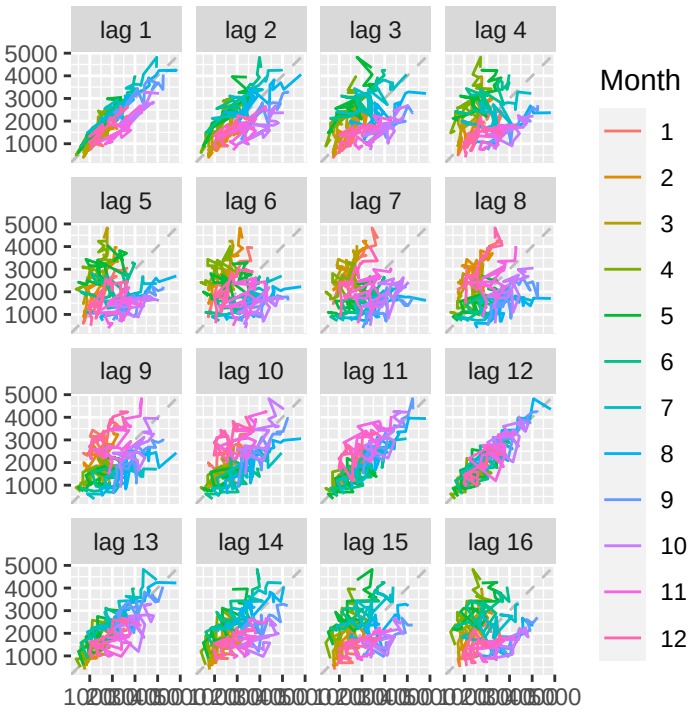
Com o Seasonal Plot é possivel ver com maior claridade a sazonalidade da serie e a forma como o pico se concentra entre os meses de Agosto e Outubro e o momento de maior baixa entre os meses de Fevereiro e Abril.





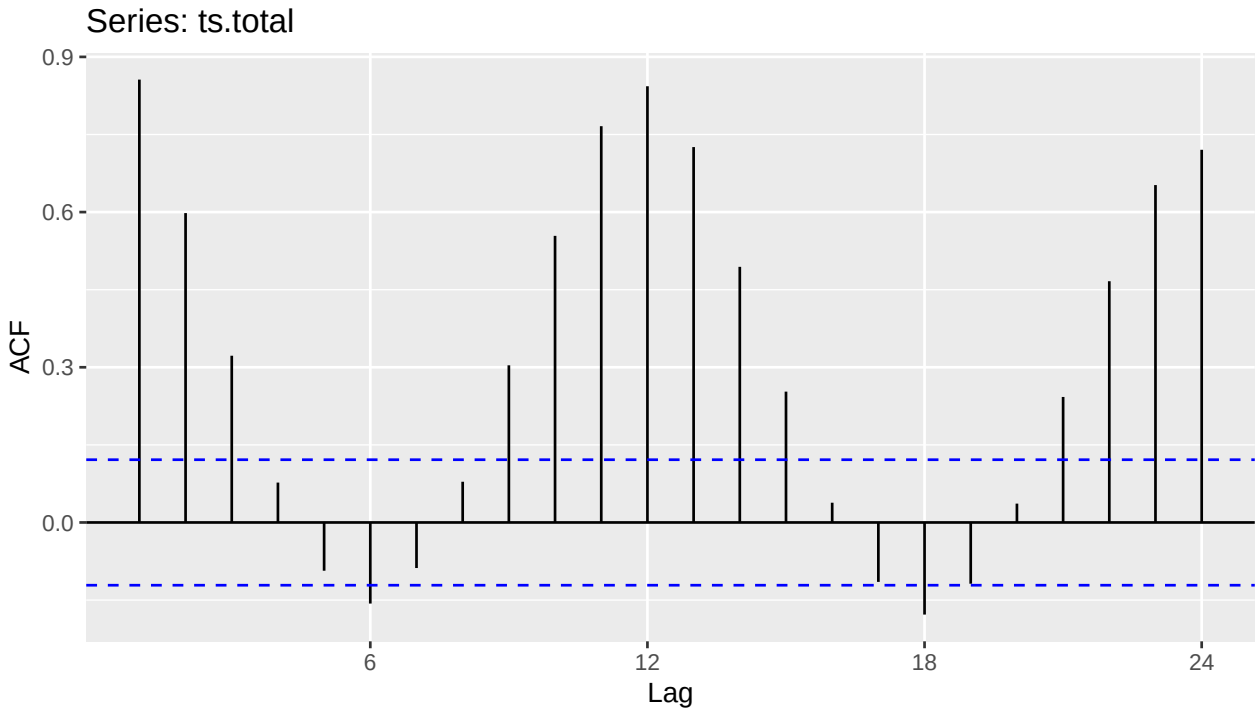
Lag Plot

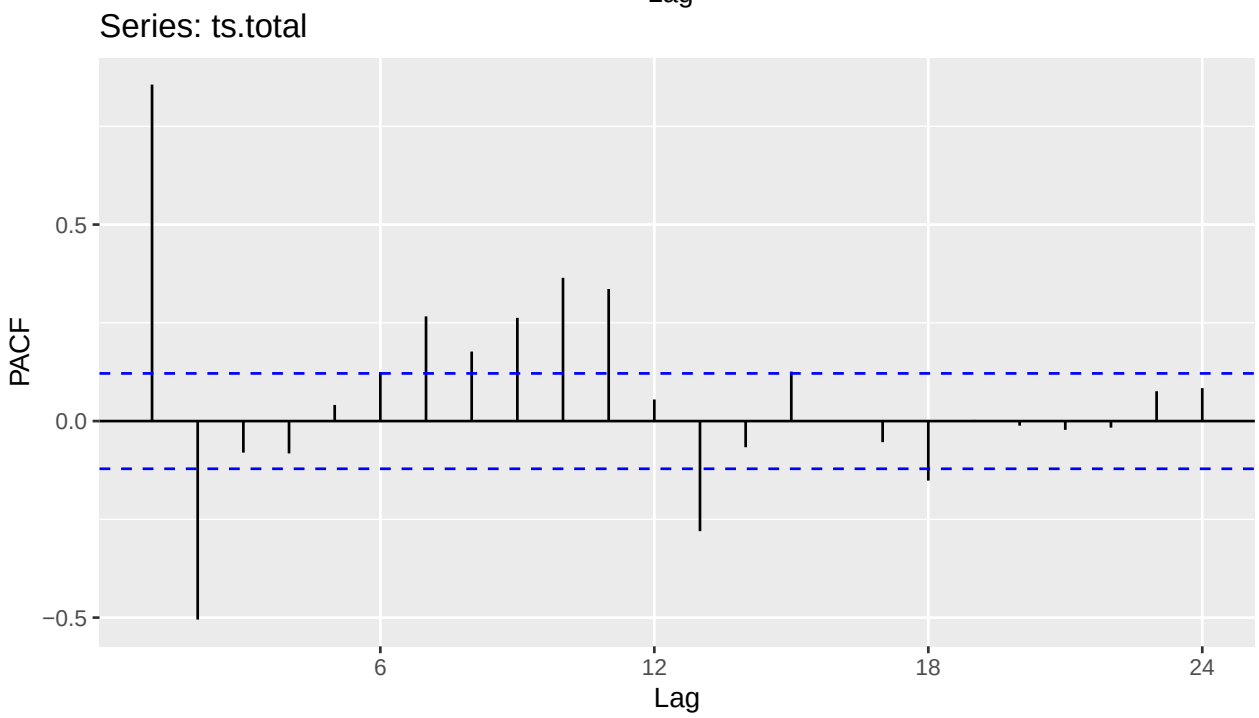
A visao do Lag Plot da serie corrabora com a visão inicial de que a serie possui uma sazonalidade de 12 meses, uma vez que o valor de lag para 12 meses apresenta os menores valores no lag plot.



Auto Correlation Function (AFC) e Partial Auto Correlation Function (PACF)

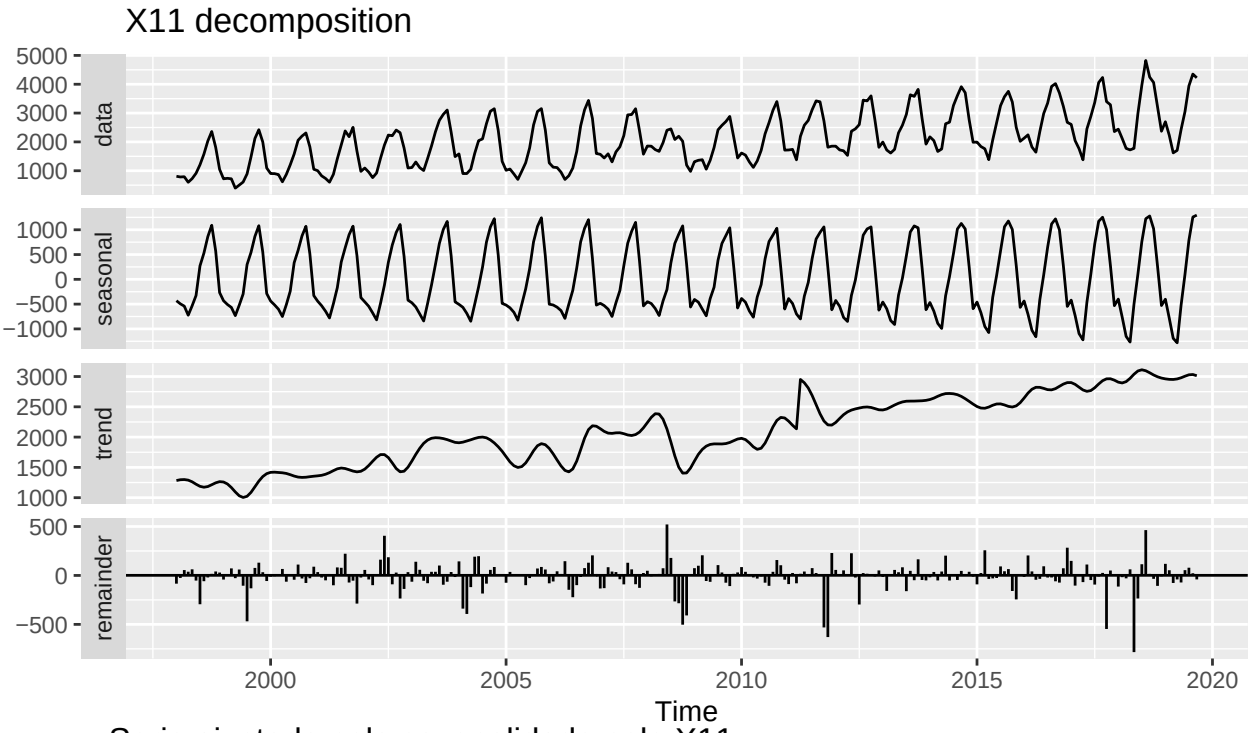
Observando a função de autocorrelação da série podemos observar que o maior valor de correlação ocorre para um lag de 12 meses, reforcando a visão de que a serie possui uma sazonalidade de 12 meses.

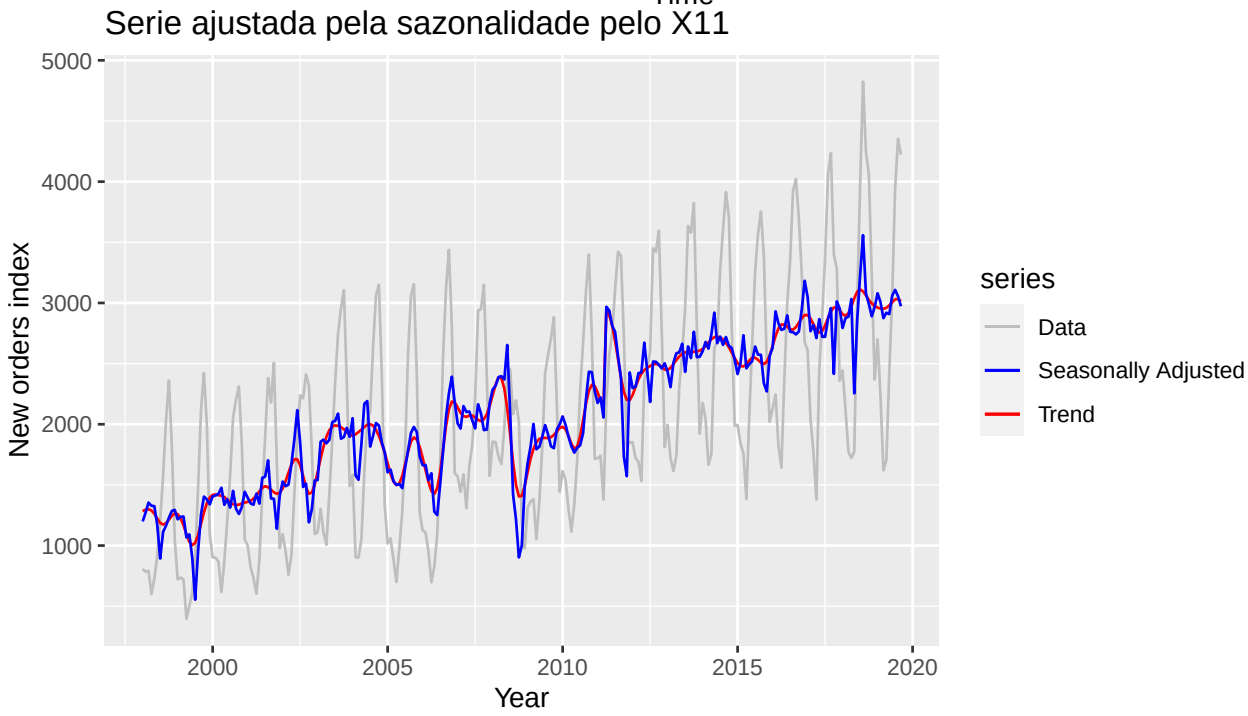




Decomposição

Após a decomposicao da série em sazonalidade, tendencia e residuos alguns pontos observados anteriormente ficam mais claros. O primeiro deles é o aumento de amplitude na série ao longo do tempo, o segundo é o aumento no nível da série ao longo do tempo, partindo de um patamar inferior a 1.500 e chegando a um patamar de 3.000 ao fim da série.





Teste Unitário

Realizando o teste unitário da série temporal 2,6 no teste de hipótese indicando que a série é não estacionária.

```
########################
# KPSS Unit Root Test #
########################

Test is of type: mu with 5 lags.
```

```
Value of test-statistic is: 2.6176

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

Utilizando o algoritmo fornecido pelo programa R para determinar o número de diferenciações necessárias para atingir a estacionariedade chega-se a conclusão de que é necessária uma diferenciação para atingir a estacionariedade da série. Realizando o teste unitário após uma diferenciação é obtido um p-value de 0,011 indicando que de fato ocorre a estacionariedade após uma diferenciação, essa característica indica que em um modelo ARIMA provavlemnte será necessário um modelo com uma diferenciação.

```
[1] "Diferenciações para estacionariedade:"

[1] 1


#######################
# KPSS Unit Root Test #
#######################

Test is of type: mu with 5 lags.

Value of test-statistic is: 0.0111

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

Modelo

Tanto de acordo com as métricas de erro simple como erro médio quadrado (RMSE) e com o critério de Akaike (AICc) o modelo mais adequado foi o modelo ARIMA com sazonalidade e por isso esse foi o modelo adotado. Também tiveram resultados muito positivos os modelos de regressão linear utilizando Dummies e o modelo de regressão utilizando a transformada de Fourier. Outro ponto a ser destacado é a alta correlação observada nos modelos que naão consideram sazonalidade como Suavização Exponencial, Holt e ARIMA não sazonal, indicando que esses modelos não foram capazes de captar adequadamente as variações na série temporal.

```
Series: ts.train
ARIMA(1,0,1)(0,1,2)[12] with drift

Coefficients:
         ar1     ma1     sma1     sma2   drift
      0.6276  0.1693  -0.6984  -0.1545  0.5351
s.e.  0.0713  0.0916   0.0796   0.0781  0.0669

sigma^2 estimated as 273.9:  log likelihood=-968.35
AIC=1948.69   AICc=1949.07   BIC=1969.27

Training set error measures:
                     ME     RMSE     MAE        MPE     MAPE      MASE
Training set 0.08641124 15.95277 11.6103 -0.3792579 5.795372 0.5357892
                    ACF1
Training set 0.008179276
```

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Naive	35,0	74,4	59,8	7,5	19,6	2,8	0,8
Naïve Sazonal	9,0	22,5	16,4	2,6	6,0	0,8	−0,2
Drift	29,2	71,0	57,9	5,4	19,4	2,7	0,8
Suavização Exponencial	35,0	74,4	59,8	7,5	19,6	2,8	0,8
Holt	290,3	319,8	290,3	99,9	99,9	13,4	0,8
Holt Winther Aditivo	1,0	25,1	19,1	-1,4	7,2	0,9	0,5
Holt Winther Multiplicativo	-25,2	32,3	26,9	-9,7	10,1	1,2	0,2
Dummy	-1,2	22,4	17,2	-2,0	6,6	0,5	0,6
Fourier	-1,2	22,4	17,2	-2,0	6,6	0,5	0,6
ARIMA	65,3	92,6	73,2	18,8	22,7	3,4	0,8
Seasonal ARIMA	-0,7	22,0	15,8	-1,8	6,1	0,7	0,4

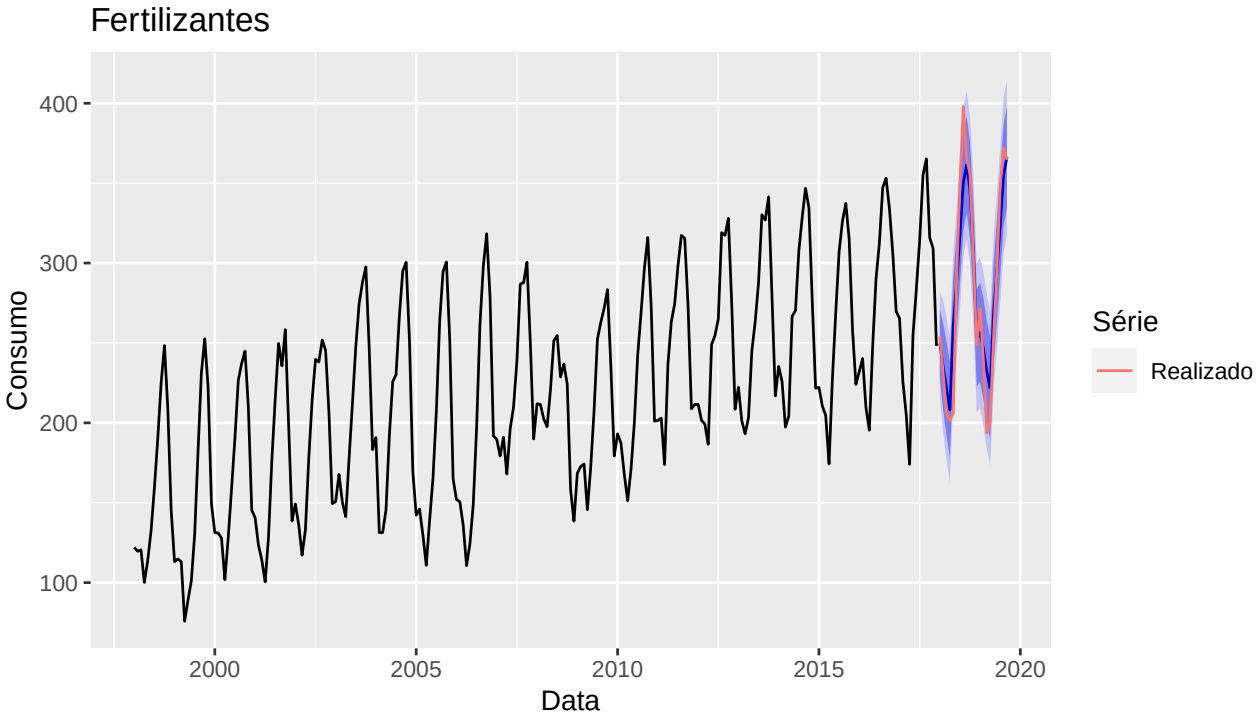
Figure 1: resultados

	AIC	AICc	BIC
Suavização Exponencial	3.005,39	3.005,49	3.015,83
Holt	3.003,28	3.003,38	3.013,72
Holt Winther Aditivo	2.714,29	2.714,29	2.773,46
Holt Winther Multiplicativo	2.757,70	2.760,46	2.816,87
Dummy	2.399,14		
Fourier	2.399,14		
ARIMA	2.311,60	2.311,65	2.318,56
Seasonal ARIMA	1.948,69	1.949,07	1.969,27

Figure 2: AIC

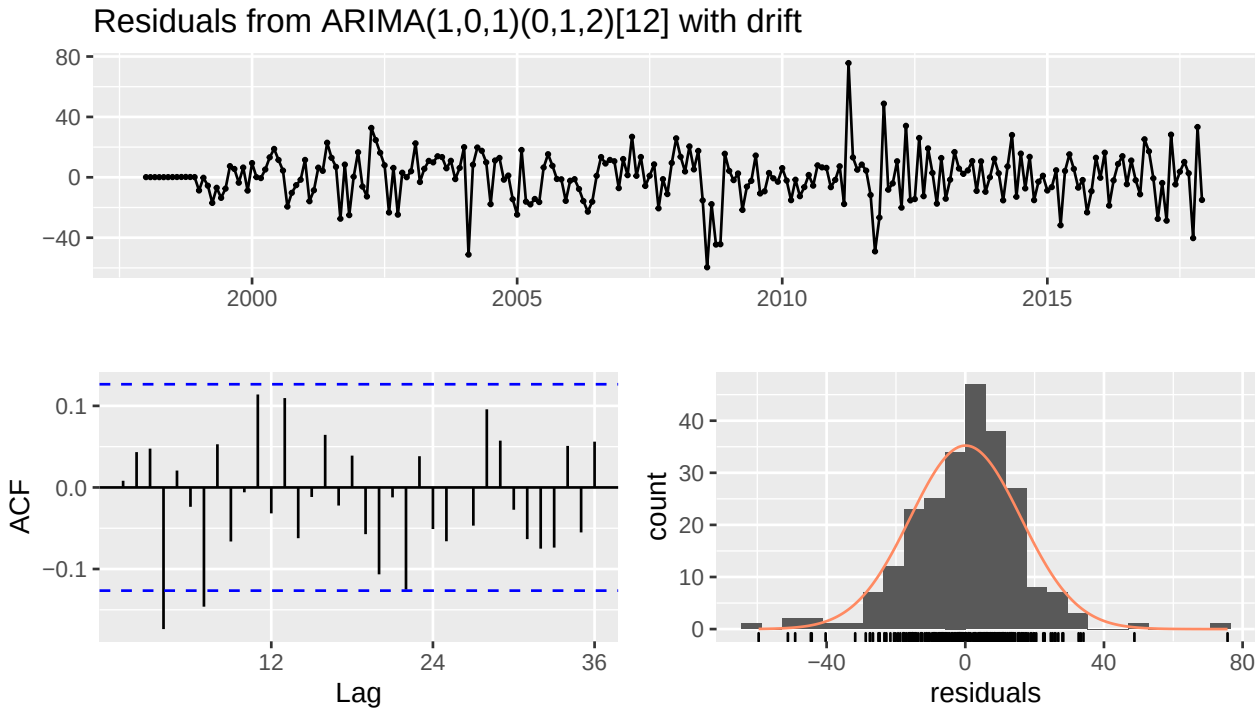
Previsões

Alta aderência das previsões com baixo erro médio quadrado e pode ser visto visualmente também pela proximidade dos dados projetados e os dados realizados.



Resíduos

O teste de Portmanteau indica que os resíduos da série não possuem autocorreleação, fato que também é reforçado pelos baixos valores encontrados na função de autocorrelação.



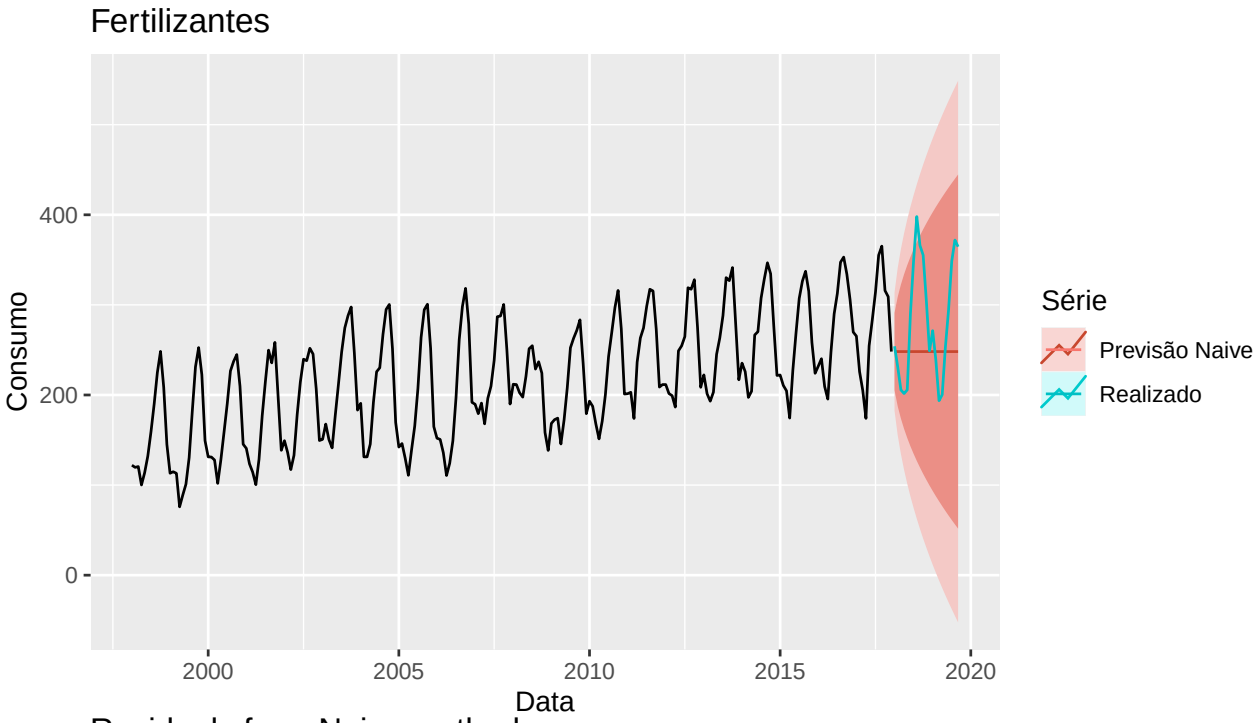

```
    Ljung-Box test

data:  Residuals from ARIMA(1,0,1)(0,1,2)[12] with drift
Q* = 34.256, df = 19, p-value = 0.01714

Model df: 5.   Total lags used: 24
```

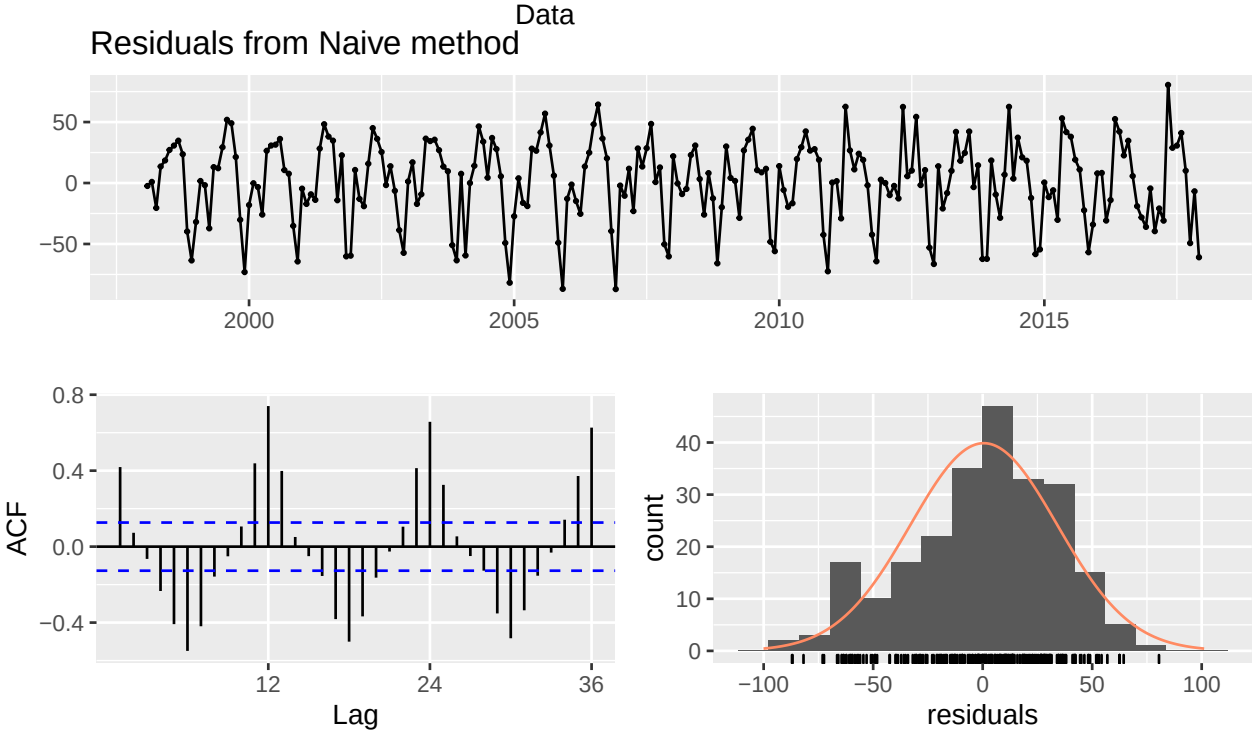
Conclusão

Os métricas calculadas mostram que o modelo de ARIMA com sazonalida é adequado para realização de previsões na série de tempora utilizada. Além do modelo ARIMA outros modelos também se mostraram adequados para a série em questão, principalmente os modelos de regressão utilizando Dummies e o modelo de regressão utilizando a transformada de Fourier. Foi possível notar também que modelos que não levam a sazonalidade em concideração não foram adequados para modelar o problme dado a sazonalidade da série temporal utilizada.

Anexo

Naive

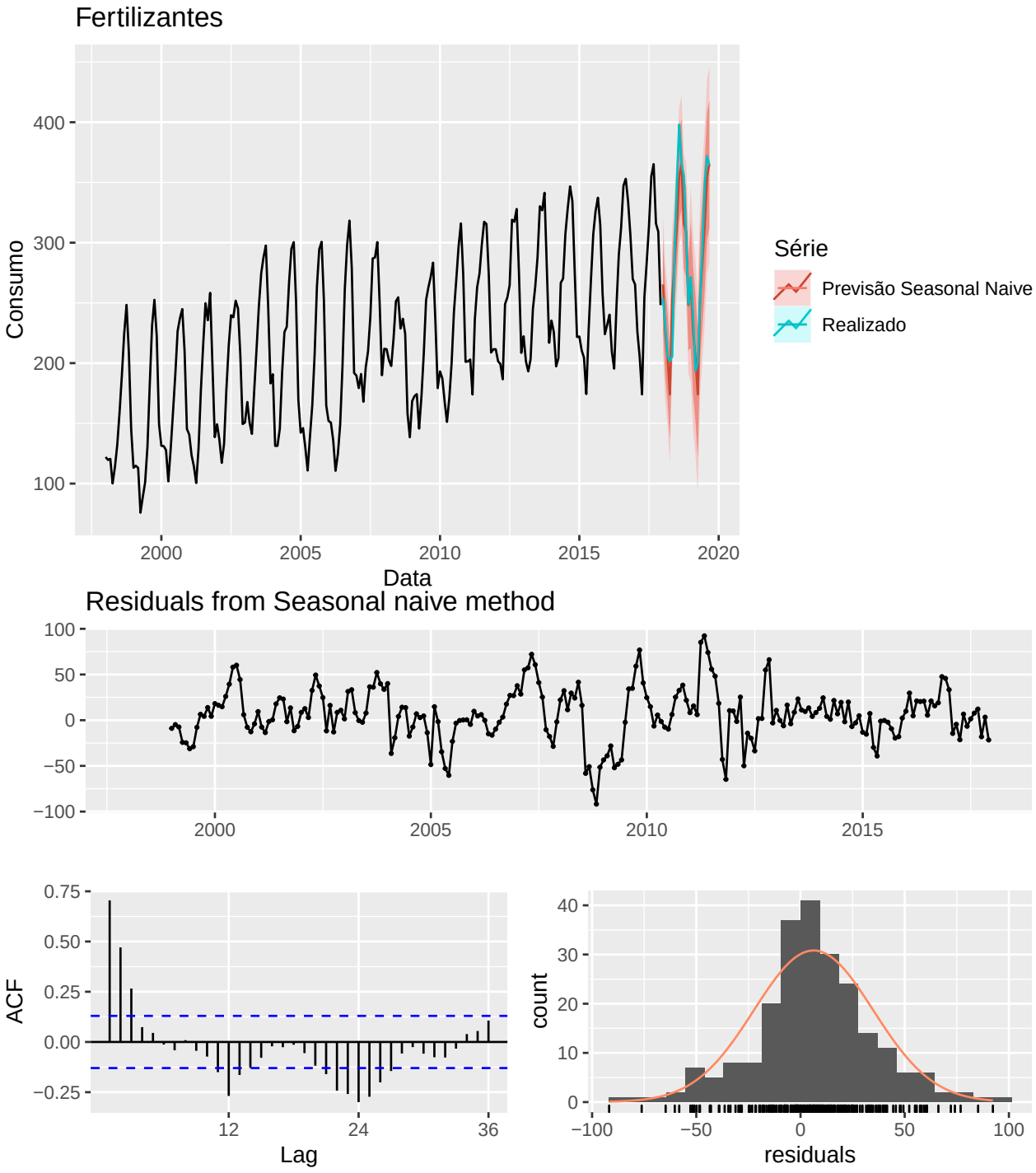





```
    Ljung-Box test

data:  Residuals from Naive method
Q* = 772.42, df = 24, p-value < 2.2e-16
```

```
Model df: 0.   Total lags used: 24

                    ME     RMSE      MAE       MPE     MAPE     MASE      ACF1
Training set  0.5276758 33.46313 26.91195 -1.070538 13.30554 1.241926 0.4189777
Test set     35.0437972 74.37272 59.82303  7.480002 19.64148 2.760699 0.7777121
             Theil's U
Training set        NA
Test set      1.525694


Forecast method: Naive method

Model Information:
Call: naive(y = ts.train, h = 21)

Residual sd: 33.5292

Error measures:
                   ME     RMSE      MAE       MPE     MAPE     MASE      ACF1
Training set 0.5276758 33.46313 26.91195 -1.070538 13.30554 1.241926 0.4189777

Forecasts:
         Point Forecast     Lo 80    Hi 80      Lo 95    Hi 95
Jan 2018       248.1736 205.28888 291.0583 182.587078 313.7601
Feb 2018       248.1736 187.52545 308.8218 155.420245 340.9270
Mar 2018       248.1736 173.89508 322.4521 134.574401 361.7728
Apr 2018       248.1736 162.40415 333.9431 117.000542 379.3467
May 2018       248.1736 152.28044 344.0668 101.517661 394.8296
Jun 2018       248.1736 143.12791 353.2193  87.520067 408.8272
Jul 2018       248.1736 134.71128 361.6359  74.647951 421.6993
Aug 2018       248.1736 126.87728 369.4699  62.666877 433.6804
Sep 2018       248.1736 119.51942 376.8278  51.414006 444.9332
Oct 2018       248.1736 112.56019 383.7870  40.770777 455.5765
Nov 2018       248.1736 105.94105 390.4062  30.647683 465.6995
Dec 2018       248.1736  99.61655 396.7307  20.975189 475.3720
Jan 2019       248.1736  93.55052 402.7967  11.697996 484.6492
Feb 2019       248.1736  87.71364 408.6336   2.771268 493.5760
Mar 2019       248.1736  82.08177 414.2655  -5.841947 502.1892
Apr 2019       248.1736  76.63469 419.7125 -14.172529 510.5198
May 2019       248.1736  71.35534 424.9919 -22.246601 518.5938
Jun 2019       248.1736  66.22911 430.1181 -30.086492 526.4337
Jul 2019       248.1736  61.24341 435.1038 -37.711468 534.0587
Aug 2019       248.1736  56.38727 439.9600 -45.138291 541.4855
Sep 2019       248.1736  51.65109 444.6961 -52.381651 548.7289
```

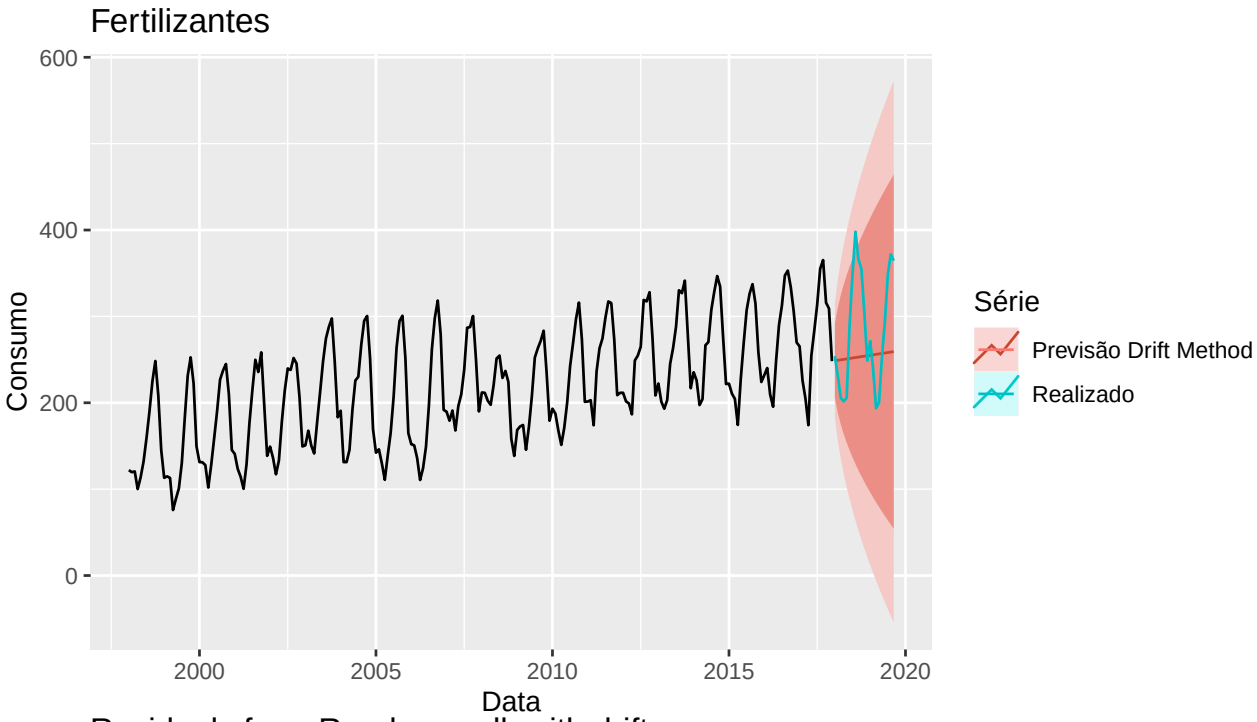
Seasonal Naive

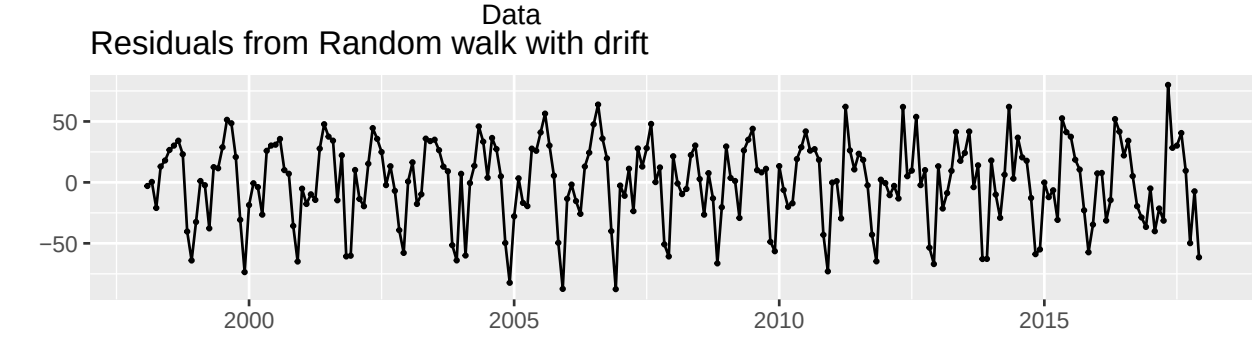


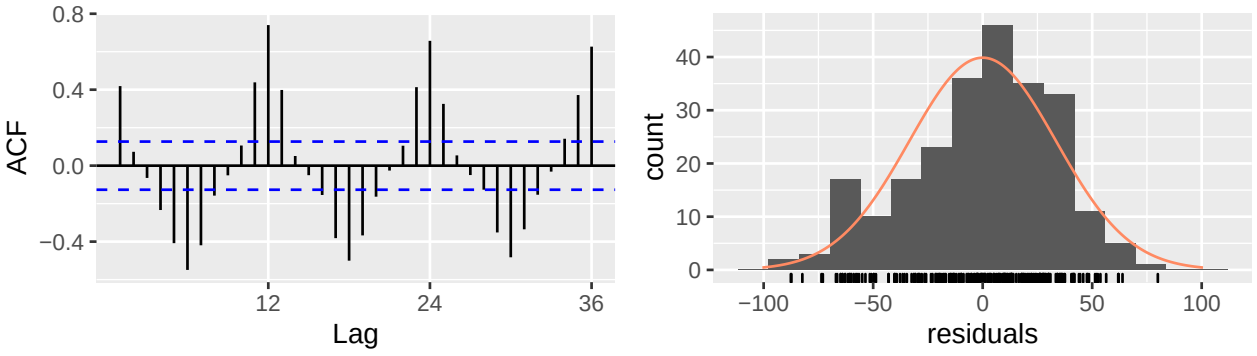

```
    Ljung-Box test

data:  Residuals from Seasonal naive method
Q* = 287.89, df = 24, p-value < 2.2e-16

Model df: 0.   Total lags used: 24
```

```
                    ME     RMSE      MAE      MPE      MAPE      MASE       ACF1
Training set 6.277218 29.22272 21.66952 1.929988 10.785796 1.0000000  0.7046447
Test set     9.022473 22.51227 16.43766 2.569095  6.017326 0.7585614 -0.1600989
             Theil's U
Training set        NA
Test set     0.5652217


Forecast method: Seasonal naive method

Model Information:
Call: snaive(y = ts.train, h = 21)

Residual sd: 28.6034

Error measures:
                    ME     RMSE      MAE      MPE    MAPE MASE      ACF1
Training set 6.277218 29.22272 21.66952 1.929988 10.7858    1 0.7046447

Forecasts:
         Point Forecast    Lo 80    Hi 80     Lo 95    Hi 95
Jan 2018       265.3239 227.8735 302.7743 208.04840 322.5993
Feb 2018       225.8062 188.3558 263.2567 168.53077 283.0817
Mar 2018       204.9195 167.4691 242.3699 147.64403 262.1950
Apr 2018       174.0318 136.5814 211.4822 116.75630 231.3072
May 2018       254.5293 217.0789 291.9797 197.25383 311.8048
Jun 2018       283.4108 245.9604 320.8612 226.13536 340.6863
Jul 2018       314.1044 276.6540 351.5548 256.82890 371.3798
Aug 2018       355.1285 317.6781 392.5790 297.85307 412.4040
Sep 2018       365.2126 327.7622 402.6630 307.93712 422.4881
Oct 2018       315.8860 278.4355 353.3364 258.61048 373.1614
Nov 2018       309.1001 271.6497 346.5505 251.82463 366.3756
Dec 2018       248.1736 210.7232 285.6240 190.89814 305.4491
Jan 2019       265.3239 212.3610 318.2868 184.32412 346.3236
Feb 2019       225.8062 172.8434 278.7691 144.80649 306.8060
Mar 2019       204.9195 151.9566 257.8824 123.91975 285.9192
Apr 2019       174.0318 121.0689 226.9947  93.03202 255.0315
May 2019       254.5293 201.5664 307.4922 173.52955 335.5290
Jun 2019       283.4108 230.4479 336.3737 202.41108 364.4106
Jul 2019       314.1044 261.1415 367.0673 233.10462 395.1041
Aug 2019       355.1285 302.1657 408.0914 274.12879 436.1283
Sep 2019       365.2126 312.2497 418.1755 284.21285 446.2123
```

Drift Method



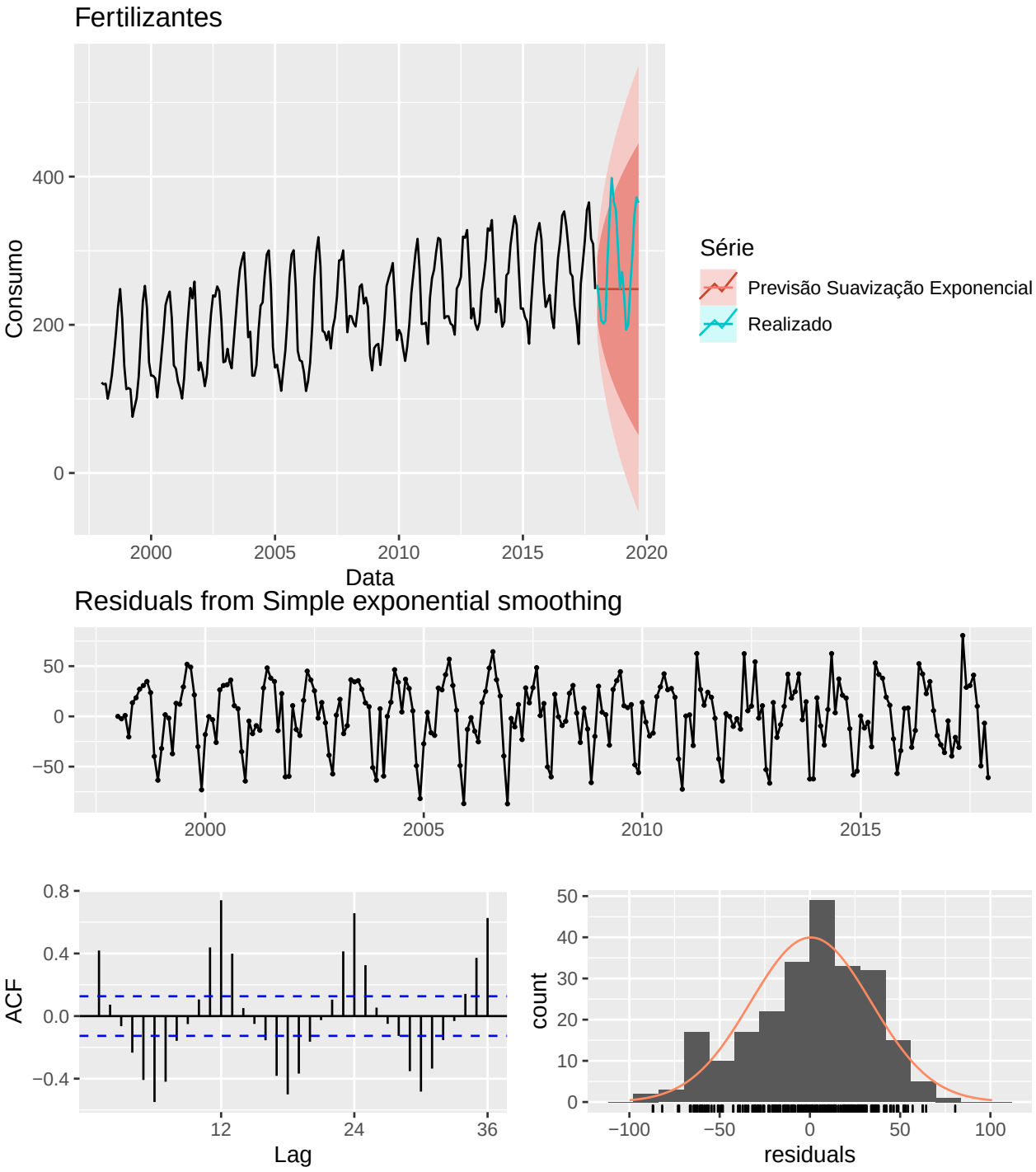





```
    Ljung-Box test

data:  Residuals from Random walk with drift
Q* = 772.42, df = 23, p-value < 2.2e-16

Model df: 1.   Total lags used: 24
```

```
                       ME     RMSE      MAE       MPE     MAPE     MASE
Training set -1.605730e-15 33.45897 26.85422 -1.338443 13.30201 1.239262
Test set      2.923936e+01 70.97605 57.91351  5.401372 19.35765 2.672579
                  ACF1 Theil's U
Training set 0.4189777        NA
Test set     0.7782601  1.482231


Forecast method: Random walk with drift

Model Information:
Call: rwf(y = ts.train, h = 21, drift = TRUE)

Drift: 0.5277  (se 2.1688)
Residual sd: 33.5292

Error measures:
                      ME     RMSE      MAE       MPE     MAPE     MASE
Training set -1.60573e-15 33.45897 26.85422 -1.338443 13.30201 1.239262
                  ACF1
Training set 0.4189777

Forecasts:
         Point Forecast     Lo 80    Hi 80      Lo 95    Hi 95
Jan 2018       248.7013 205.73190 291.6707 182.985283 314.4173
Feb 2018       249.2290 188.33408 310.1239 156.098273 342.3597
Mar 2018       249.7566 175.02073 324.4926 135.457923 364.0554
Apr 2018       250.2843 163.80786 336.7608 118.029990 382.5386
May 2018       250.8120 153.92882 347.6952 102.641969 398.9820
Jun 2018       251.3397 144.99132 357.6880  88.693907 413.9854
Jul 2018       251.8673 136.76285 366.9718  75.830217 427.9045
Aug 2018       252.3950 129.09227 375.6978  63.819741 440.9703
Sep 2018       252.9227 121.87483 383.9706  52.502287 453.3431
Oct 2018       253.4504 115.03445 391.8663  41.761488 465.1393
Nov 2018       253.9780 108.51381 399.4423  31.509701 476.4464
Dec 2018       254.5057 102.26851 406.7429  21.678993 487.3325
Jan 2019       255.0334  96.26329 413.8035  12.215471 497.8513
Feb 2019       255.5611  90.46966 420.6525   3.075541 508.0466
Mar 2019       256.0888  84.86418 427.3133  -5.776639 517.9541
Apr 2019       256.6164  79.42730 433.8055 -14.370950 527.6038
May 2019       257.1441  74.14256 440.1456 -22.732609 537.0208
Jun 2019       257.6718  68.99587 446.3477 -30.883117 546.2267
Jul 2019       258.1995  63.97515 452.4238 -38.840982 555.2399
Aug 2019       258.7271  59.06989 458.3844 -46.622264 564.0765
Sep 2019       259.2548  54.27091 464.2387 -54.241005 572.7506
```

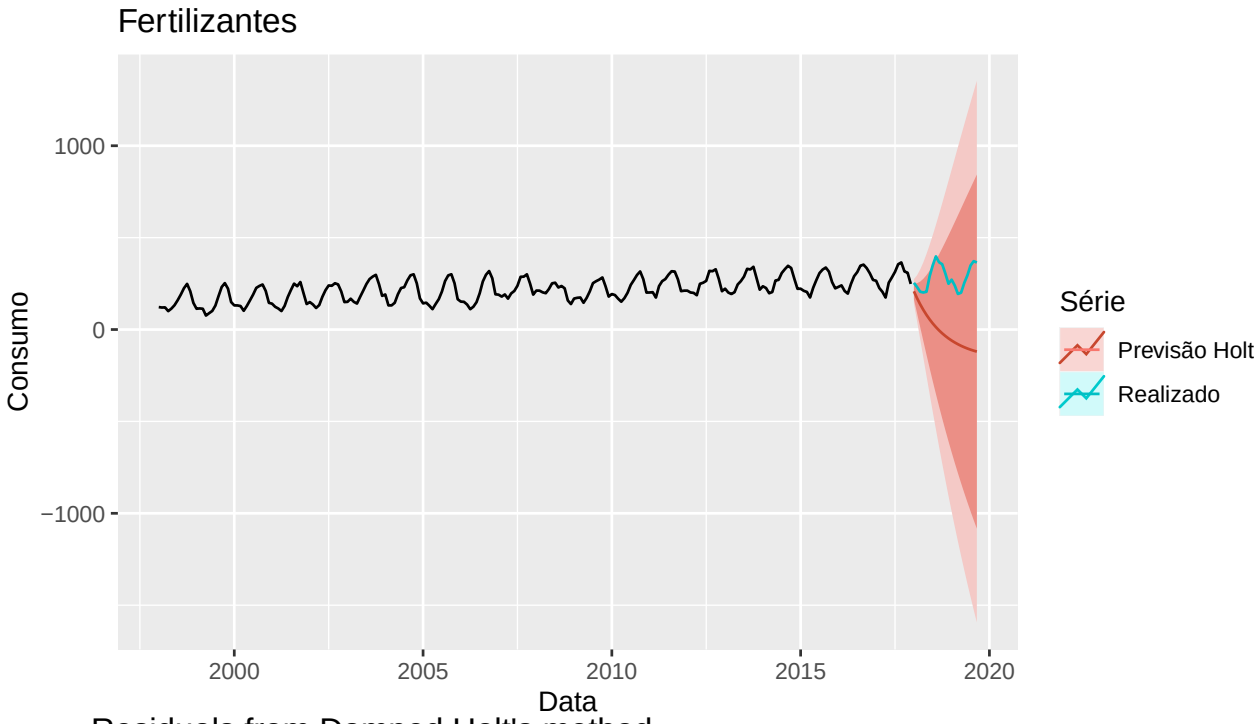
Suavização Exponencial

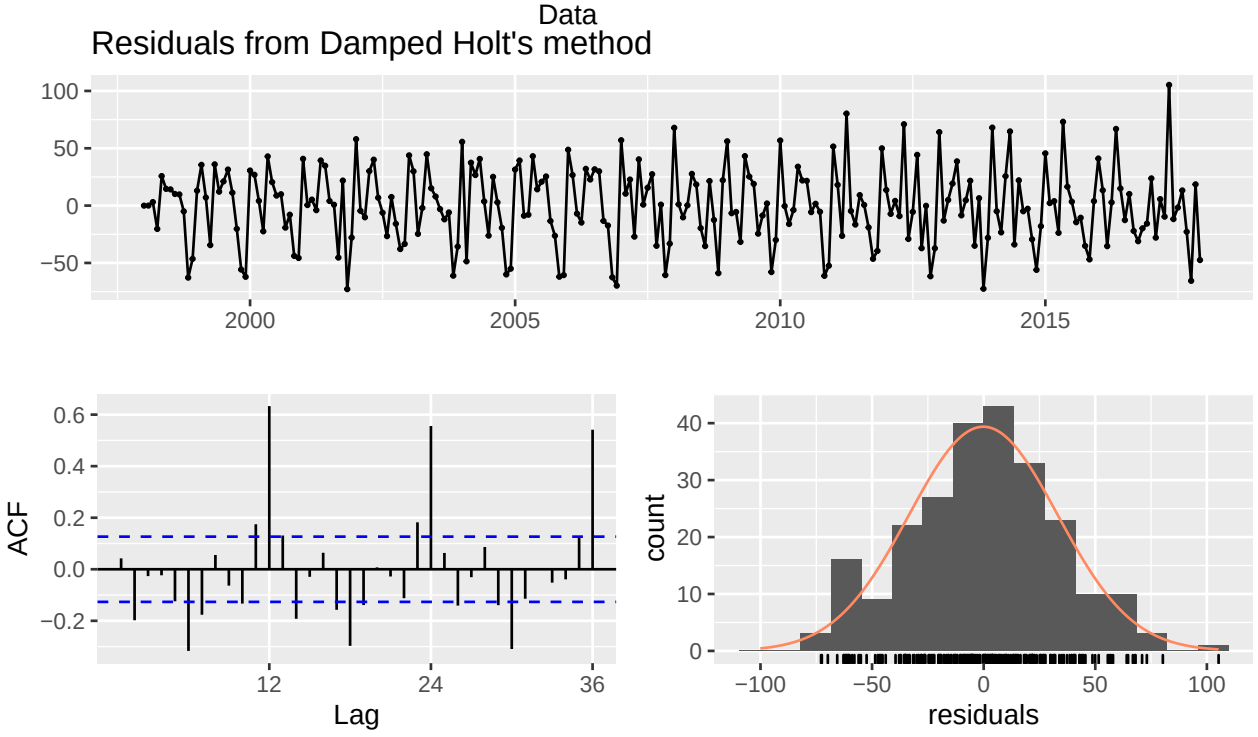



```
    Ljung-Box test

data:  Residuals from Simple exponential smoothing
Q* = 775.74, df = 22, p-value < 2.2e-16

Model df: 2.   Total lags used: 24
```

```
                    ME     RMSE      MAE       MPE     MAPE     MASE      ACF1
Training set  0.5255948 33.39475 26.80105 -1.066245 13.25068 1.236808 0.4190543
Test set     35.0377043 74.36985 59.82100  7.477730 19.64114 2.760605 0.7777121
             Theil's U
Training set        NA
Test set      1.525661


Forecast method: Simple exponential smoothing

Model Information:
Simple exponential smoothing

Call:
 ses(y = ts.train, h = 21)

  Smoothing parameters:
    alpha = 0.9999

  Initial states:
    l = 122.0496

  sigma:  33.5348

     AIC     AICc      BIC
3005.385 3005.486 3015.827

Error measures:
                    ME     RMSE      MAE       MPE     MAPE     MASE      ACF1
Training set 0.5255948 33.39475 26.80105 -1.066245 13.25068 1.236808 0.4190543

Forecasts:
         Point Forecast     Lo 80    Hi 80      Lo 95    Hi 95
Jan 2018       248.1797 205.20317 291.1562 182.452773 313.9066
Feb 2018       248.1797 187.40475 308.9547 155.232433 341.1270
Mar 2018       248.1797 173.74713 322.6123 134.344907 362.0145
Apr 2018       248.1797 162.23309 334.1263 116.735698 379.6237
May 2018       248.1797 152.08895 344.2705 101.221572 395.1378
Jun 2018       248.1797 142.91790 353.4415  87.195673 409.1637
Jul 2018       248.1797 134.48424 361.8752  74.297491 422.0619
Aug 2018       248.1797 126.63435 369.7251  62.292131 434.0673
Sep 2018       248.1797 119.26157 377.0978  51.016432 445.3430
Oct 2018       248.1797 112.28821 384.0712  40.351598 456.0078
Nov 2018       248.1797 105.65563 390.7038  30.207946 466.1515
Dec 2018       248.1797  99.31828 397.0411  20.515800 475.8436
Jan 2019       248.1797  93.23992 403.1195  11.219752 485.1397
Feb 2019       248.1797  87.39118 408.9682   2.274875 494.0845
Mar 2019       248.1797  81.74785 414.6116  -6.355855 502.7153
Apr 2019       248.1797  76.28969 420.0697 -14.703381 511.0628
May 2019       248.1797  70.99960 425.3598 -22.793878 519.1533
Jun 2019       248.1797  65.86294 430.4965 -30.649721 527.0091
Jul 2019       248.1797  60.86709 435.4923 -38.290214 534.6496
Aug 2019       248.1797  56.00107 440.3583 -45.732153 542.0916
Sep 2019       248.1797  51.25525 445.1042 -52.990257 549.3497
```

Holt



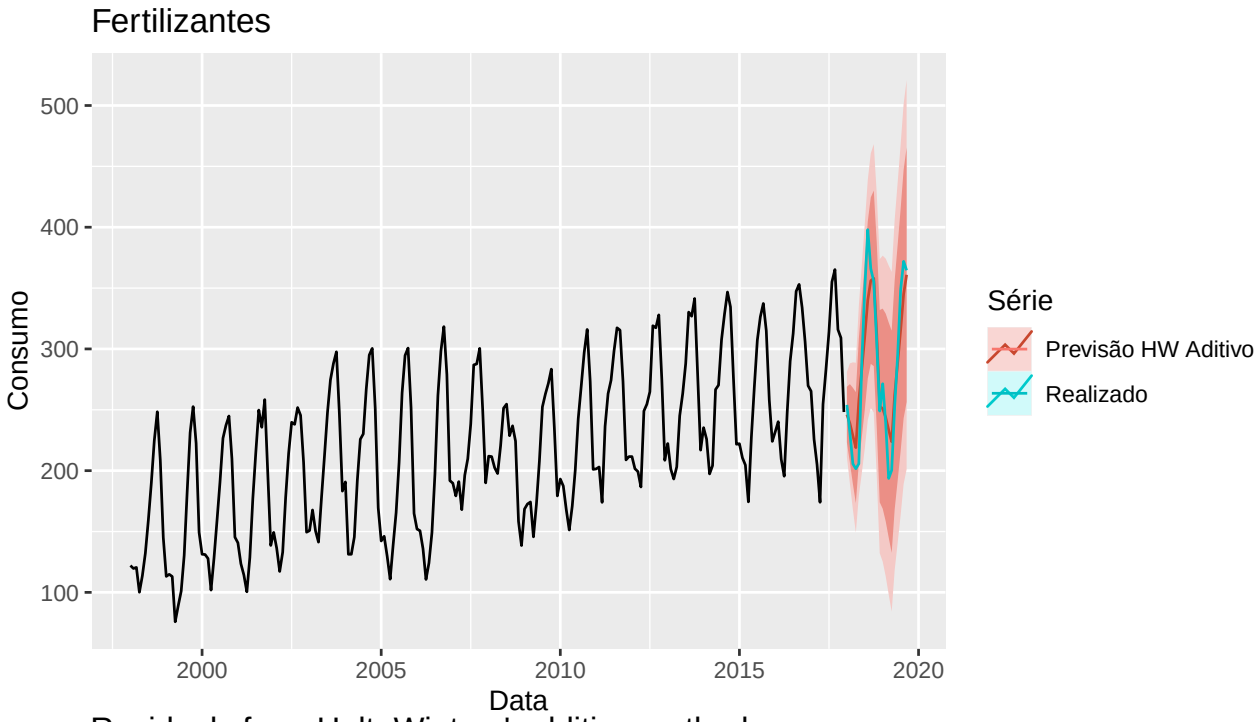


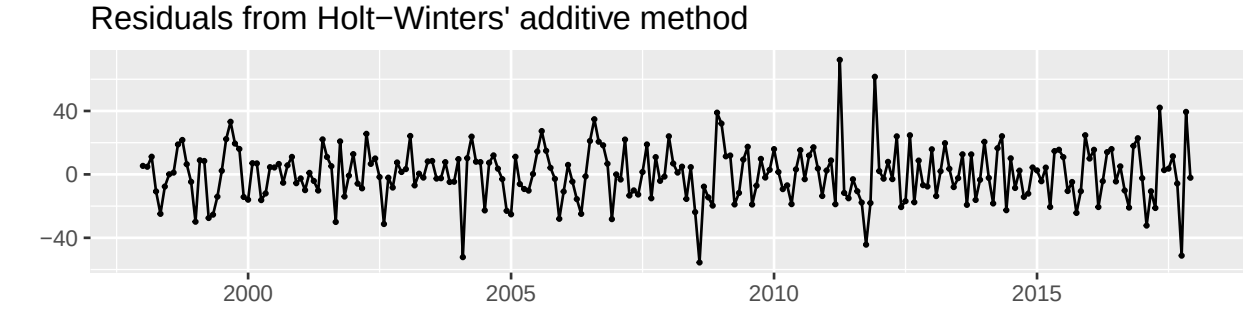

```
    Ljung-Box test

data:  Residuals from Damped Holt's method
Q* = 308.18, df = 19, p-value < 2.2e-16

Model df: 5.   Total lags used: 24
```

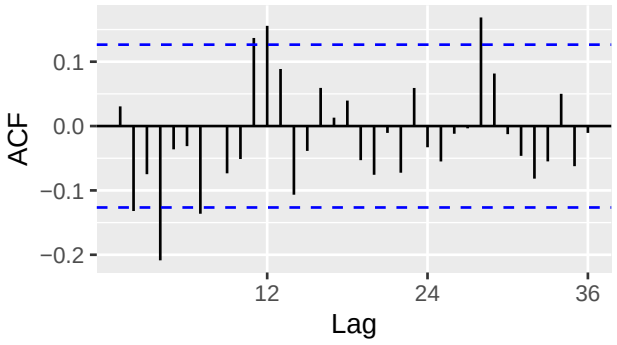
```
                     ME      RMSE       MAE        MPE     MAPE     MASE
Training set  -0.1510174  33.24857  26.31503  0.6792363 13.34938  1.21438
Test set     290.2985333 319.78976 290.29853 99.8978507 99.89785 13.39663
                   ACF1 Theil's U
Training set 0.04222199        NA
Test set     0.81526010  7.229639


Forecast method: Damped Holt's method

Model Information:
Damped Holt's method

Call:
 holt(y = ts.train, h = 21, damped = TRUE, alpha = 0.97, beta = 0.7,

 Call:
     phi = 0.9)

  Smoothing parameters:
    alpha = 0.97
    beta  = 0.7
    phi   = 0.9

  Initial states:
    l = 124.7858
    b = -2.9794

  sigma:  33.6004

     AIC     AICc      BIC
3003.279 3003.381 3013.721

Error measures:
                     ME     RMSE      MAE       MPE     MAPE    MASE       ACF1
Training set -0.1510174 33.24857 26.31503 0.6792363 13.34938 1.21438 0.04222199

Forecasts:
         Point Forecast       Lo 80    Hi 80       Lo 95     Hi 95
Jan 2018     208.085960   165.02529 251.1466   142.23034  273.9416
Feb 2018     170.724973    89.47826 251.9717    46.46885  294.9811
Mar 2018     137.100085    13.37358 260.8266   -52.12327  326.3234
Apr 2018     106.837685   -62.27526 275.9506  -151.79824  365.4736
May 2018      79.601525  -136.82664 296.0297  -251.39679  410.5998
Jun 2018      55.088982  -209.87862 320.0566  -350.14400  460.3220
Jul 2018      33.027692  -281.18320 347.2386  -447.51640  513.5718
Aug 2018      13.172532  -350.59458 376.9396  -543.16125  569.5063
Sep 2018      -4.697112  -418.03667 408.6424  -636.84542  627.4512
Oct 2018     -20.779792  -483.48177 441.9222  -728.42141  686.8618
Nov 2018     -35.254204  -546.93599 476.4276  -817.80397  747.2956
Dec 2018     -48.281175  -608.42911 511.8668  -904.95352  808.3912
Jan 2019     -60.005449  -668.00731 547.9964  -989.86405  869.8532
Feb 2019     -70.557295  -725.72790 584.6133 -1072.55427  931.4397
Mar 2019     -80.053957  -781.65549 621.5476 -1153.06091  992.9530
```

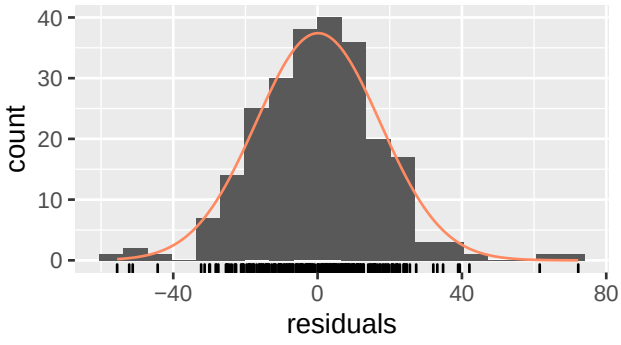
```
Apr 2019     -88.600952  -835.85910 658.6572 -1231.43368 1054.2318
May 2019     -96.293248  -888.41009 695.8236 -1307.73142 1115.1449
Jun 2019    -103.216314  -939.38058 732.9480 -1382.01921 1175.5866
Jul 2019    -109.447074  -988.84231 769.9482 -1454.36603 1235.4719
Aug 2019    -115.054758 -1036.86583 806.7563 -1524.84313 1294.7336
Sep 2019    -120.101673 -1083.51984 843.3165 -1593.52262 1353.3193
```

Holt Winther Aditivo





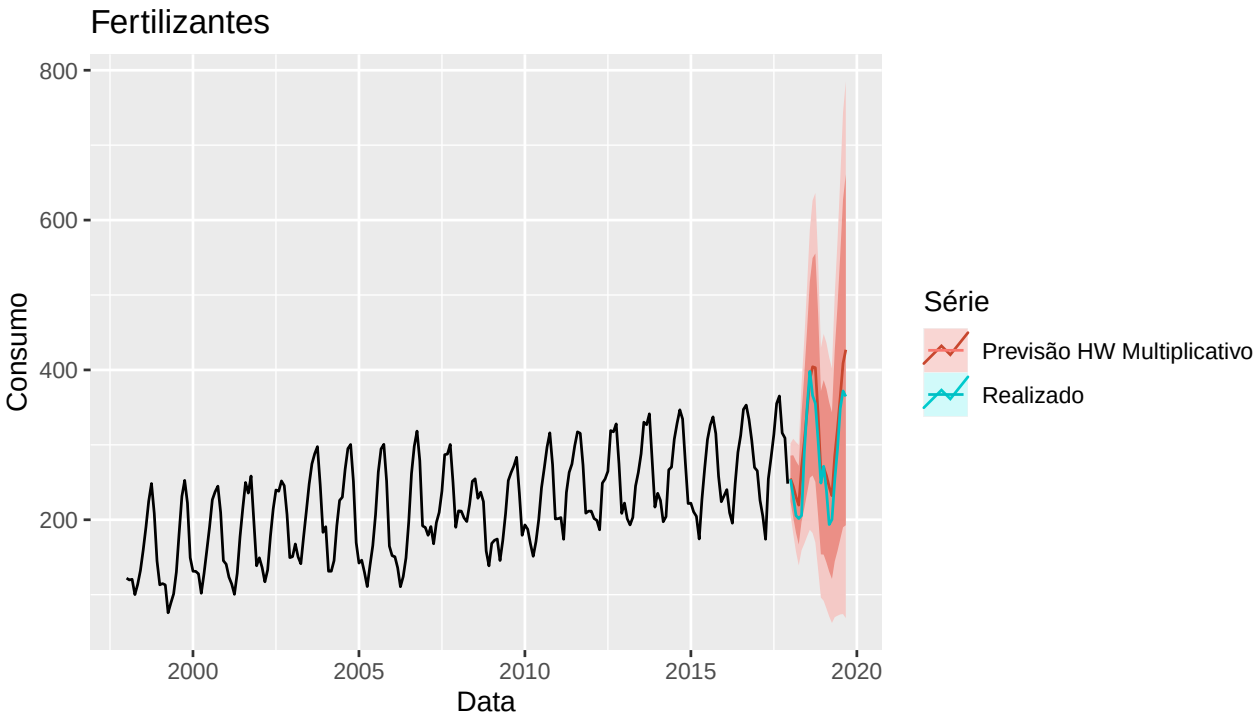


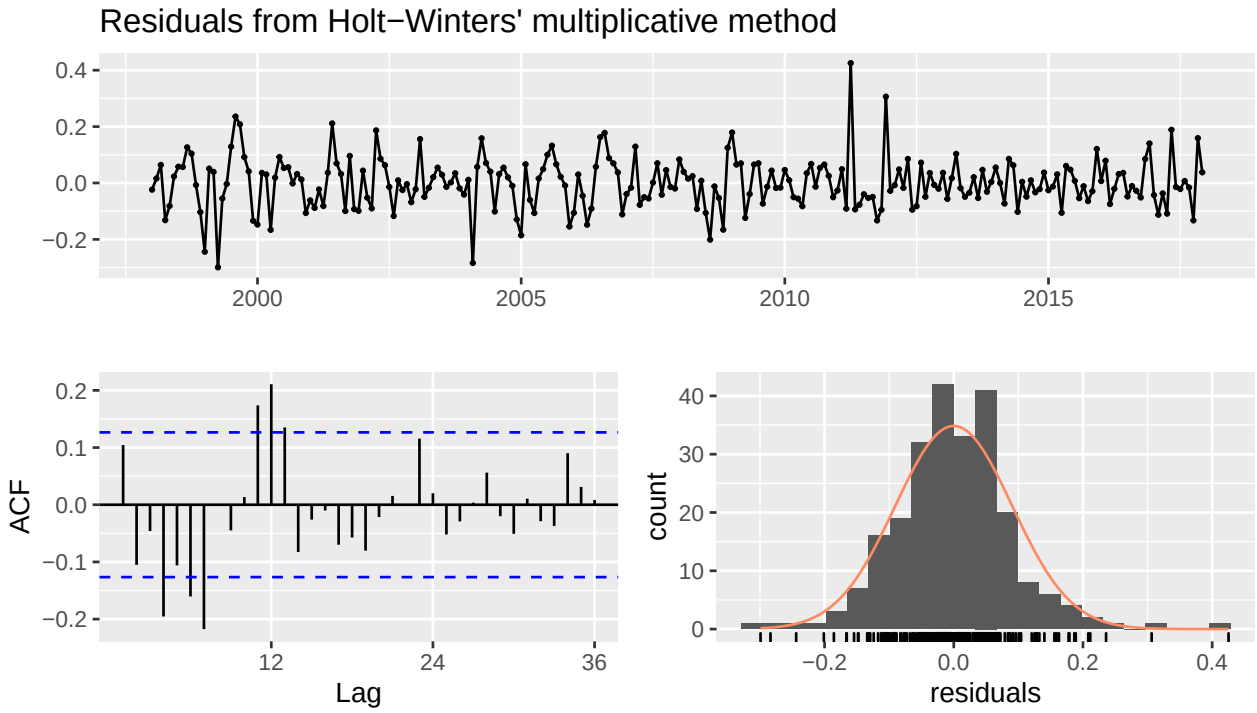


```
    Ljung-Box test

data:  Residuals from Holt-Winters' additive method
Q* = 46.306, df = 8, p-value = 2.079e-07

Model df: 16.   Total lags used: 24

                     ME     RMSE      MAE        MPE     MAPE      MASE
Training set 0.1386898 17.17784 13.12741 -0.5059202 6.645539 0.6058007
Test set     0.9741971 25.09012 19.12669 -1.4184047 7.227631 0.8826538
                   ACF1 Theil's U
Training set 0.03056295        NA
Test set     0.48862134 0.6320488


Forecast method: Holt-Winters' additive method

Model Information:
Holt-Winters' additive method

Call:
 hw(y = ts.train, h = 21, seasonal = "additive")

  Smoothing parameters:
    alpha = 0.9999
    beta  = 1e-04
    gamma = 1e-04

  Initial states:
    l = 151.5888
    b = 0.3987
    s = -32.819 26.3995 72.9962 71.4922 56.0314 26.8346
           0.1561 -25.6253 -63.6621 -53.5693 -42.9382 -35.2959

  sigma:  17.7808

     AIC     AICc      BIC
2714.291 2717.048 2773.462

Error measures:
                     ME     RMSE      MAE        MPE     MAPE      MASE
Training set 0.1386898 17.17784 13.12741 -0.5059202 6.645539 0.6058007
                   ACF1
Training set 0.03056295

Forecasts:
         Point Forecast    Lo 80    Hi 80     Lo 95    Hi 95
Jan 2018       246.1041 223.3171 268.8910 211.25443 280.9537
Feb 2018       238.8544 206.6290 271.0798 189.56993 288.1388
Mar 2018       228.6256 189.1566 268.0947 168.26289 288.9884
Apr 2018       218.9358 173.3590 264.5126 149.23205 288.6395
May 2018       257.3713 206.4126 308.3300 179.43667 335.3059
Jun 2018       283.5592 227.7342 339.3843 198.18221 368.9363
Jul 2018       310.6443 250.3436 370.9451 218.42225 402.8664
```

```
Aug 2018       340.2422 275.7749 404.7096 241.64793 438.8365
Sep 2018       356.0980 287.7167 424.4793 251.51787 460.6781
Oct 2018       358.0055 285.9218 430.0892 247.76295 468.2480
Nov 2018       311.8121 236.2064 387.4179 196.18309 427.4411
Dec 2018       252.9983 174.0268 331.9698 132.22176 373.7748
Jan 2019       250.9287 168.7278 333.1296 125.21329 376.6441
Feb 2019       243.6790 158.3710 328.9871 113.21159 374.1465
Mar 2019       233.4503 145.1437 321.7569  98.39703 368.5035
Apr 2019       223.7604 132.5533 314.9675  84.27115 363.2497
May 2019       262.1959 168.1771 356.2147 118.40658 405.9852
Jun 2019       288.3839 191.6346 385.1332 140.41856 436.3492
Jul 2019       315.4690 216.0636 414.8744 163.44156 467.4964
Aug 2019       345.0669 243.0740 447.0597 189.08230 501.0515
Sep 2019       360.9226 256.4059 465.4394 201.07808 520.7672
```

Holt Winther Multiplicativo





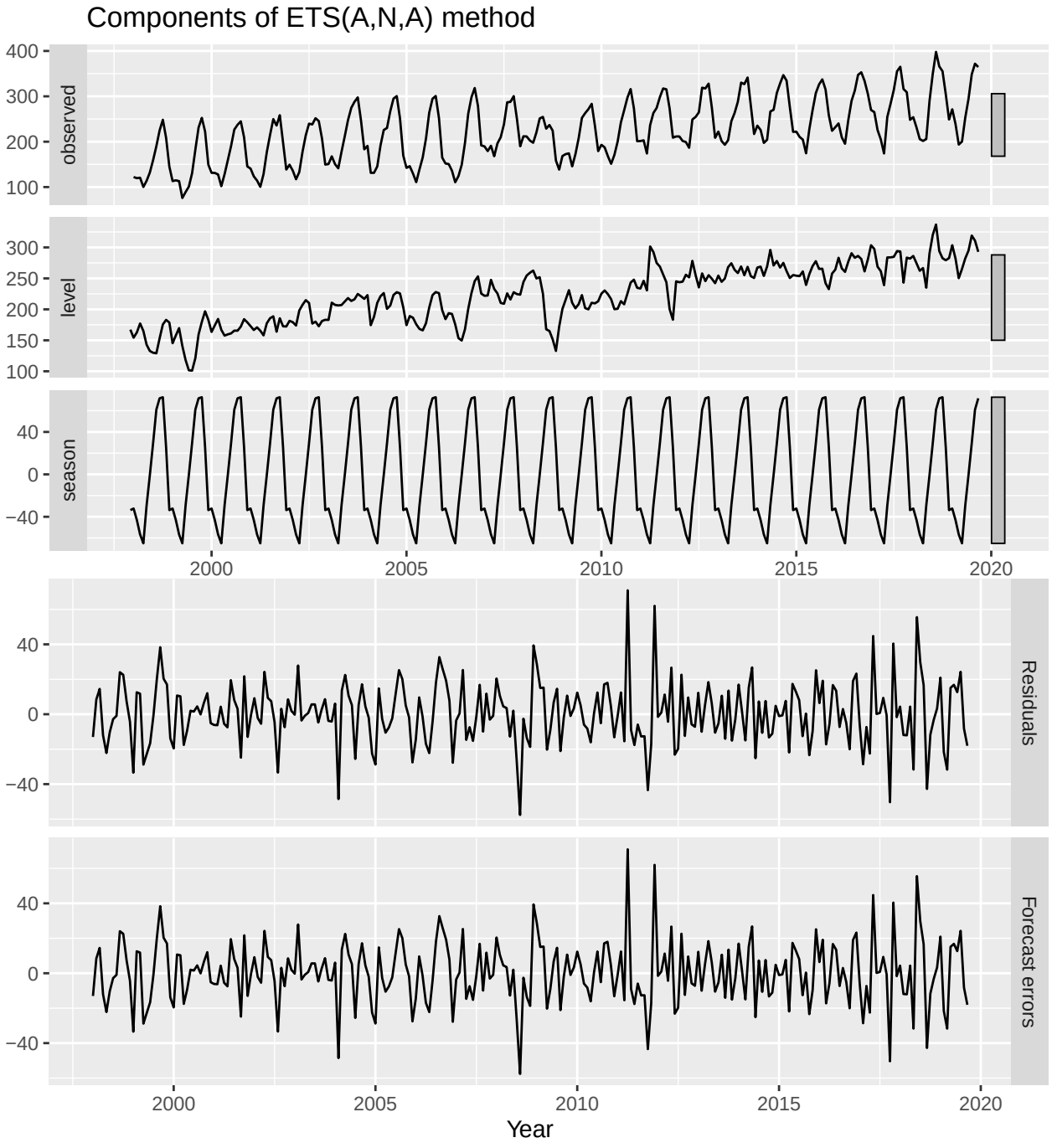

```
    Ljung-Box test

data:  Residuals from Holt-Winters' multiplicative method
Q* = 70.037, df = 8, p-value = 4.831e-12

Model df: 16.   Total lags used: 24

                      ME     RMSE      MAE        MPE      MAPE      MASE
Training set  -0.5385824 17.64582 13.67573 -0.8246467  6.853081 0.6311042
Test set     -25.2293039 32.32965 26.89050 -9.6822304 10.131861 1.2409362
                  ACF1 Theil's U
Training set 0.0872113        NA
Test set     0.2064641 0.7998586


Forecast method: Holt-Winters' multiplicative method

Model Information:
Holt-Winters' multiplicative method

Call:
 hw(y = ts.train, h = 21, seasonal = "multiplicative")

  Smoothing parameters:
    alpha = 0.9999
    beta  = 2e-04
    gamma = 1e-04

  Initial states:
    l = 144.0414
    b = 1.4777
```

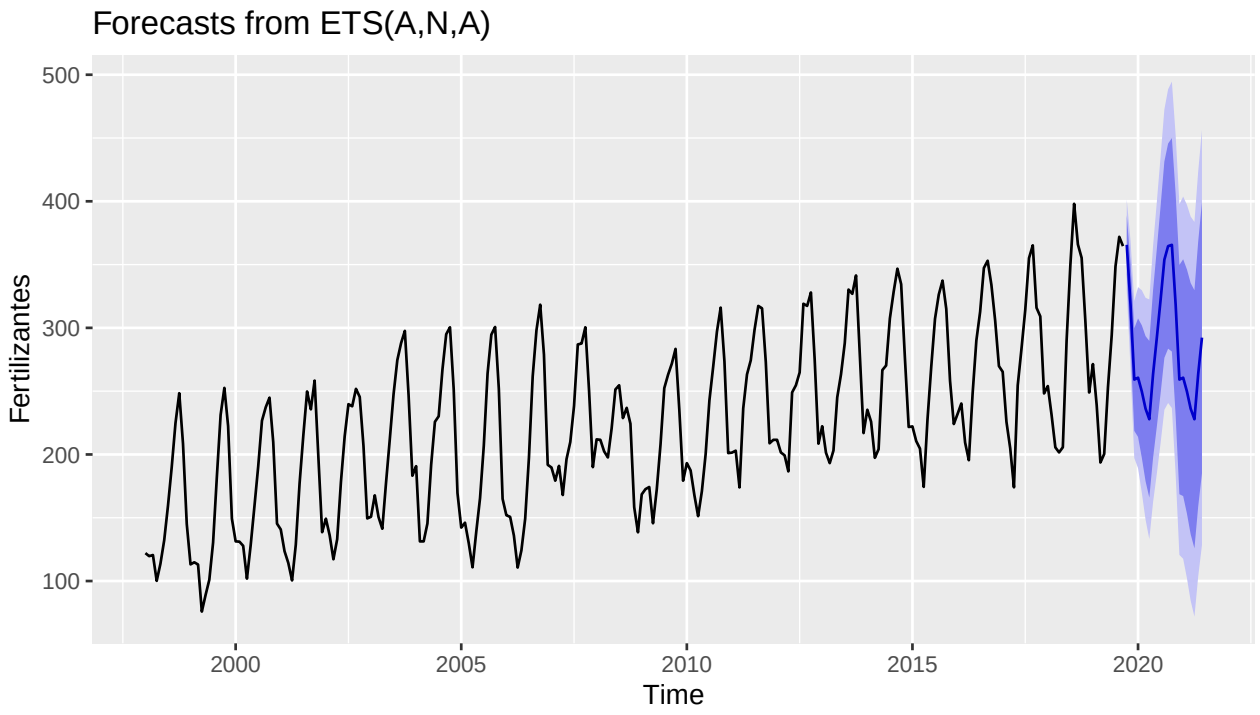
```
    s = 0.84 1.0912 1.3002 1.3104 1.2602 1.1283
           1.0013 0.8912 0.7288 0.7689 0.8202 0.8592

  sigma:  0.0935

     AIC     AICc      BIC
2757.699 2760.456 2816.870

Error measures:
                      ME     RMSE      MAE        MPE     MAPE      MASE
Training set -0.5385824 17.64582 13.67573 -0.8246467 6.853081 0.6311042
                   ACF1
Training set 0.0872113

Forecasts:
         Point Forecast    Lo 80    Hi 80     Lo 95    Hi 95
Jan 2018       255.0765 224.5225 285.6306 208.34819 301.8049
Feb 2018       244.7159 203.2711 286.1607 181.33153 308.1003
Mar 2018       230.5108 182.7064 278.3153 157.40021 303.6215
Apr 2018       219.5231 166.9623 272.0840 139.13827 299.9080
May 2018       269.7415 197.5429 341.9402 159.32322 380.1598
Jun 2018       304.5421 215.2586 393.8256 167.99481 441.0895
Jul 2018       344.7748 235.6077 453.9419 177.81813 511.7314
Aug 2018       386.9171 255.9579 517.8762 186.63231 587.2018
Sep 2018       404.2165 259.1121 549.3210 182.29841 626.1347
Oct 2018       402.9335 250.4729 555.3942 169.76510 636.1020
Nov 2018       339.7365 204.9185 474.5546 133.55006 545.9230
Dec 2018       262.7281 153.8355 371.6207  96.19130 429.2649
Jan 2019       269.9638 153.5031 386.4245  91.85247 448.0751
Feb 2019       258.9293 143.0113 374.8474  81.64796 436.2107
Mar 2019       243.8347 130.8404 356.8290  71.02484 416.6446
Apr 2019       232.1511 121.0389 343.2633  62.21969 402.0825
May 2019       285.1842 144.4816 425.8869  69.99808 500.3704
Jun 2019       321.8944 158.4653 485.3235  71.95123 571.8376
Jul 2019       364.3266 174.2712 554.3820  73.66189 654.9913
Aug 2019       408.7555 189.9643 627.5468  74.14318 743.3679
Sep 2019       426.9246 192.7410 661.1082  68.77168 785.0776
```

ETS

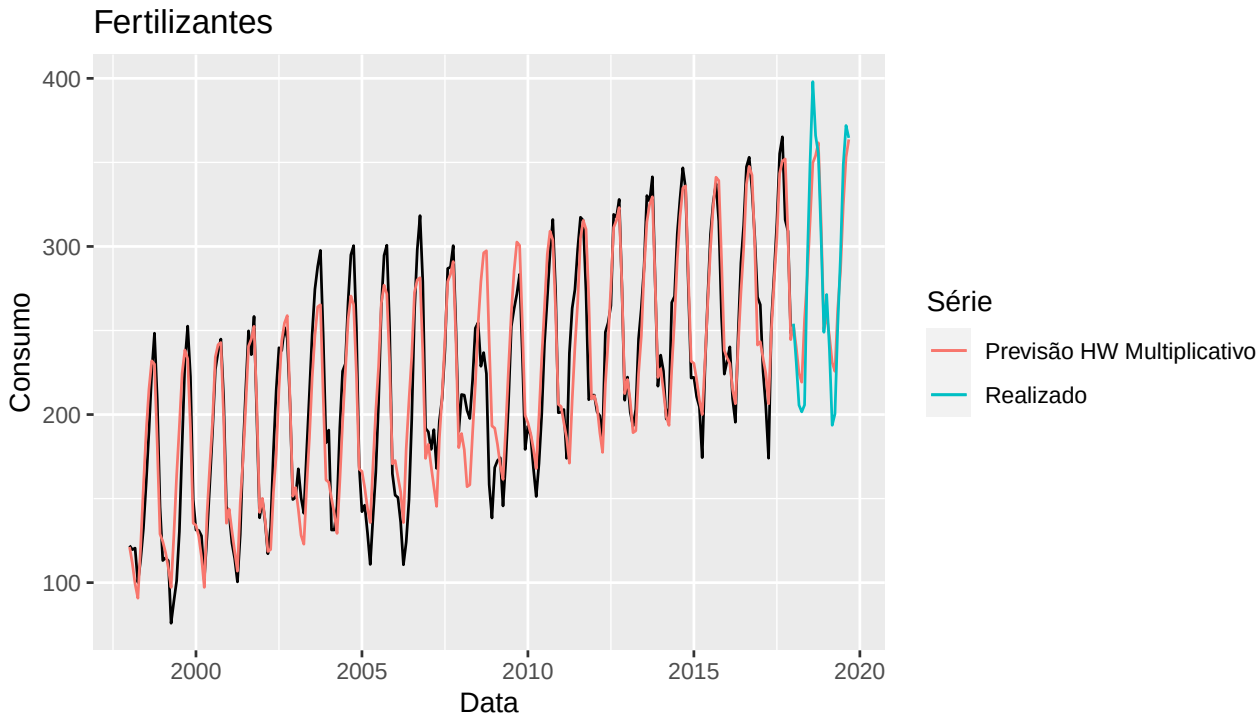
```
ETS(A,N,A)

Call:
 ets(y = ts.total)

  Smoothing parameters:
    alpha = 0.9999
    gamma = 1e-04

  Initial states:
    l = 167.2623
    s = -33.704 25.563 72.7694 71.7423 61.052 29.381
           -0.5442 -29.2881 -65.0344 -56.6661 -43.0889 -32.1819
```

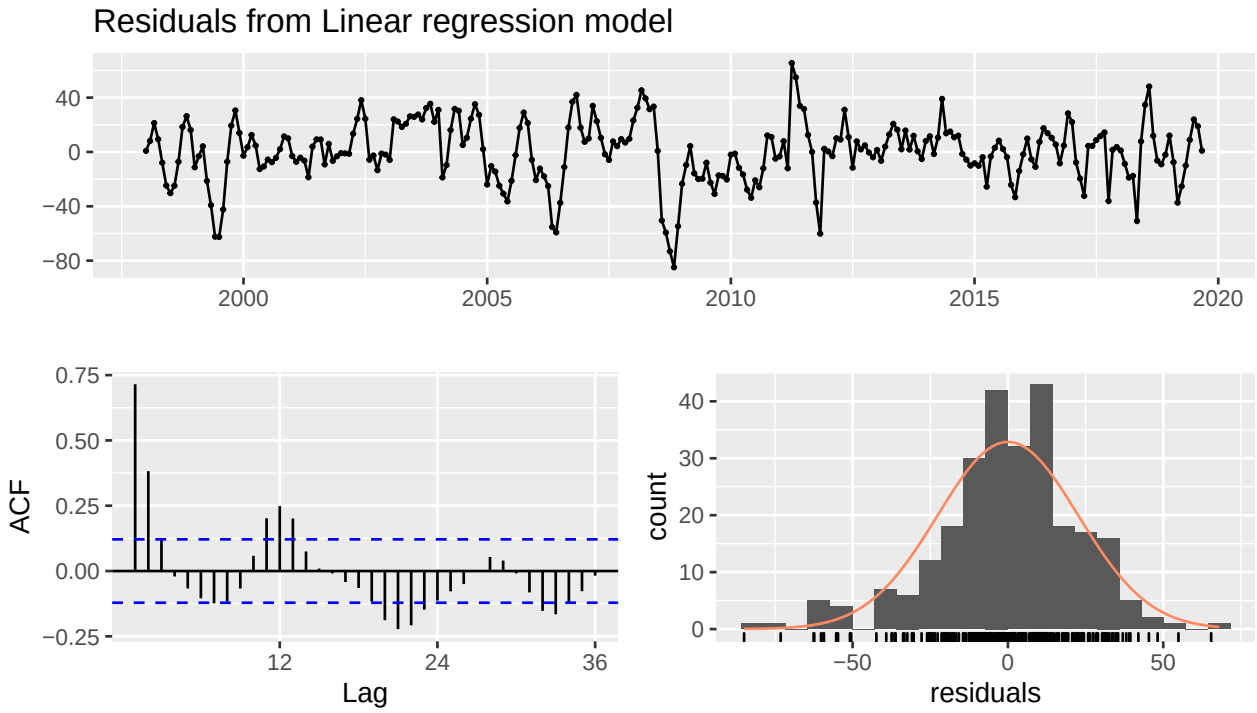
```
  sigma:  18.2498

     AIC     AICc      BIC
2983.920 2985.879 3037.387

Training set error measures:
                    ME     RMSE     MAE        MPE    MAPE      MASE       ACF1
Training set 0.4812123 17.75362 13.5058 -0.3440582 6.67491 0.6365459 0.03075803
```




Forecasts from ETS(A,N,A)
Fertilizantes
500
400
300
200
100
2000
2005
2010
2015
2020
Time

Dummy


Fertilizantes
Consumo
400
300
200
100
2000
2005
2010
2015
2020
Data
Série
Previsão HW Multiplicativo
Realizado

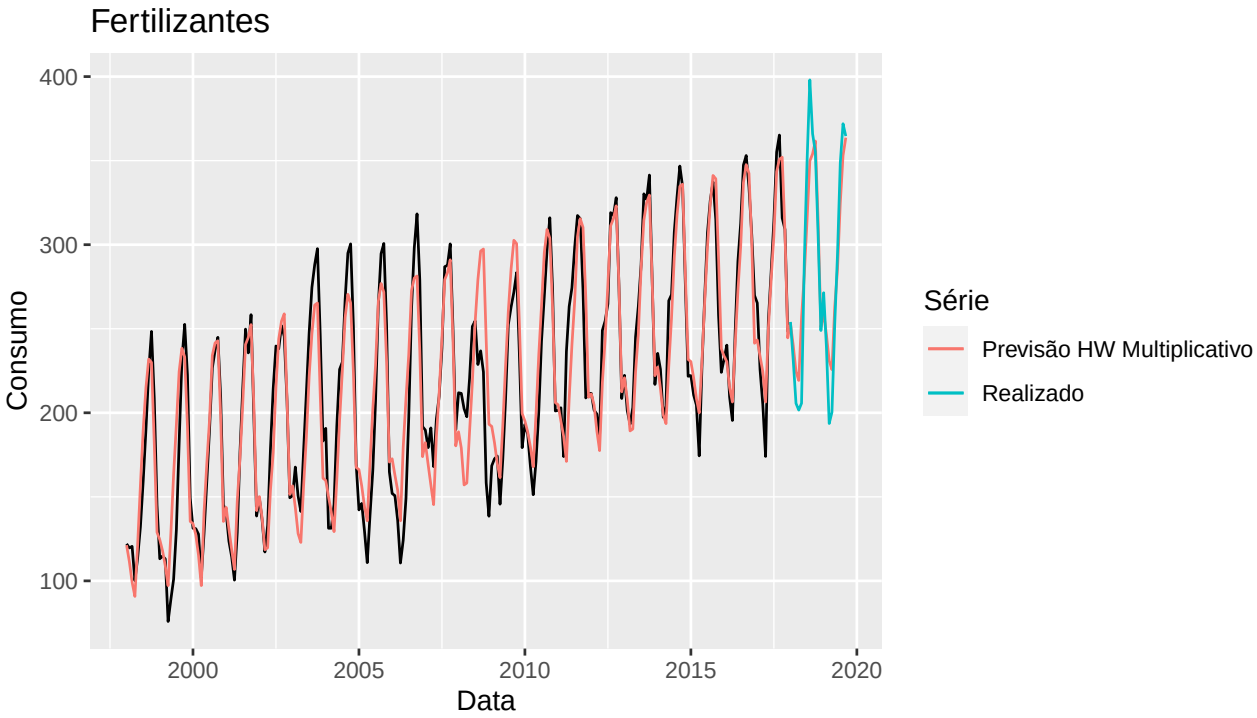


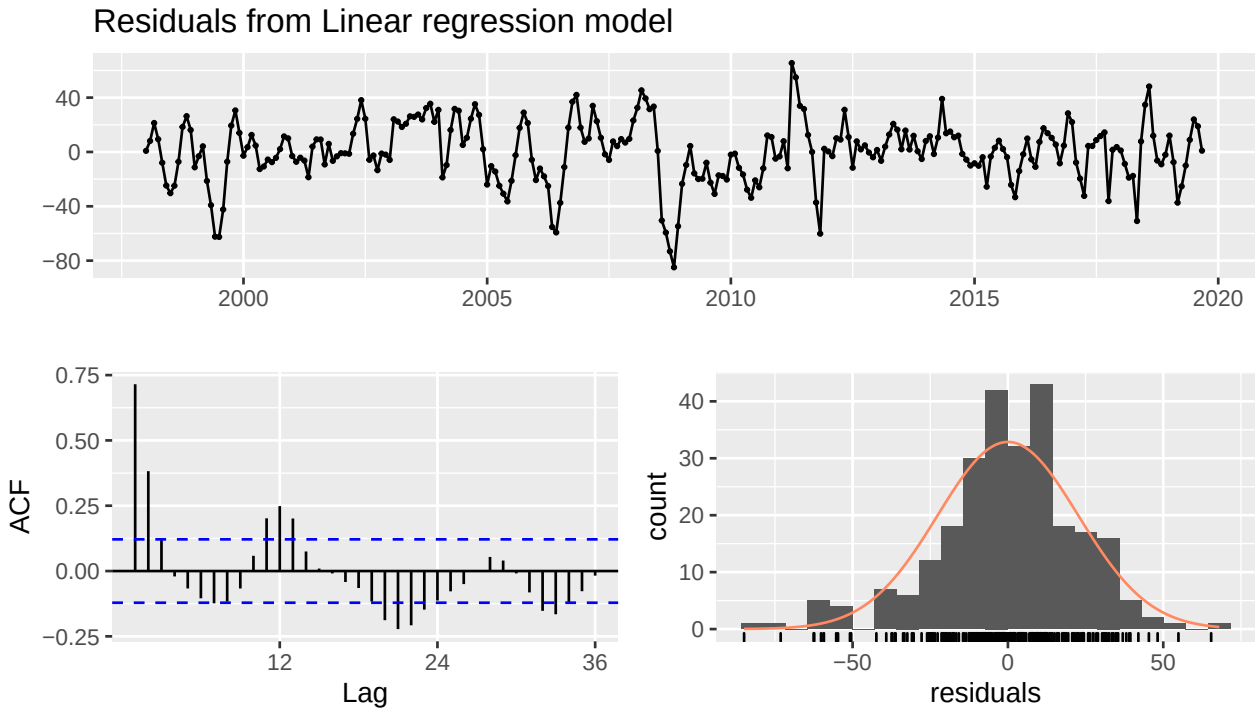

```
    Breusch-Godfrey test for serial correlation of order up to 24

data:  Residuals from Linear regression model
LM test = 155.86, df = 24, p-value < 2.2e-16

                ME     RMSE      MAE       MPE     MAPE      ACF1 Theil's U
Test set -1.214474 22.39923 17.24986 -2.049079 6.626138 0.5171863 0.5855158


Call:
tslm(formula = ts.total ~ trend + season + bizdays(ts.total))

Residuals:
    Min      1Q  Median      3Q     Max
-84.924 -10.719   0.916  12.510  65.420

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       56.57296   37.26048   1.518 0.130215
trend              0.53538    0.01913  27.989  < 2e-16 ***
season2           -7.05247    7.29087  -0.967 0.334340
season3          -29.49324    7.58848  -3.887 0.000131 ***
season4          -35.31609    7.05602  -5.005 1.06e-06 ***
season5           -1.77560    7.20876  -0.246 0.805646
season6           26.58467    7.31469   3.634 0.000339 ***
season7           58.78892    7.19088   8.175 1.55e-14 ***
season8           86.70926    7.82836  11.076  < 2e-16 ***
season9          103.21620    7.02013  14.703  < 2e-16 ***
season10          97.48104    7.87078  12.385  < 2e-16 ***
season11          55.63757    7.10959   7.826 1.48e-13 ***
season12          -4.64139    7.25585  -0.640 0.522976
```

```
bizdays(ts.total)   3.20739    1.81636   1.766 0.078658 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 23.27 on 247 degrees of freedom
Multiple R-squared:  0.8872,    Adjusted R-squared:  0.8813
F-statistic: 149.5 on 13 and 247 DF,  p-value: < 2.2e-16
```

```
[1] 2399.14
```

Fourier






```
    Breusch-Godfrey test for serial correlation of order up to 24

data:  Residuals from Linear regression model
LM test = 155.86, df = 24, p-value < 2.2e-16
```

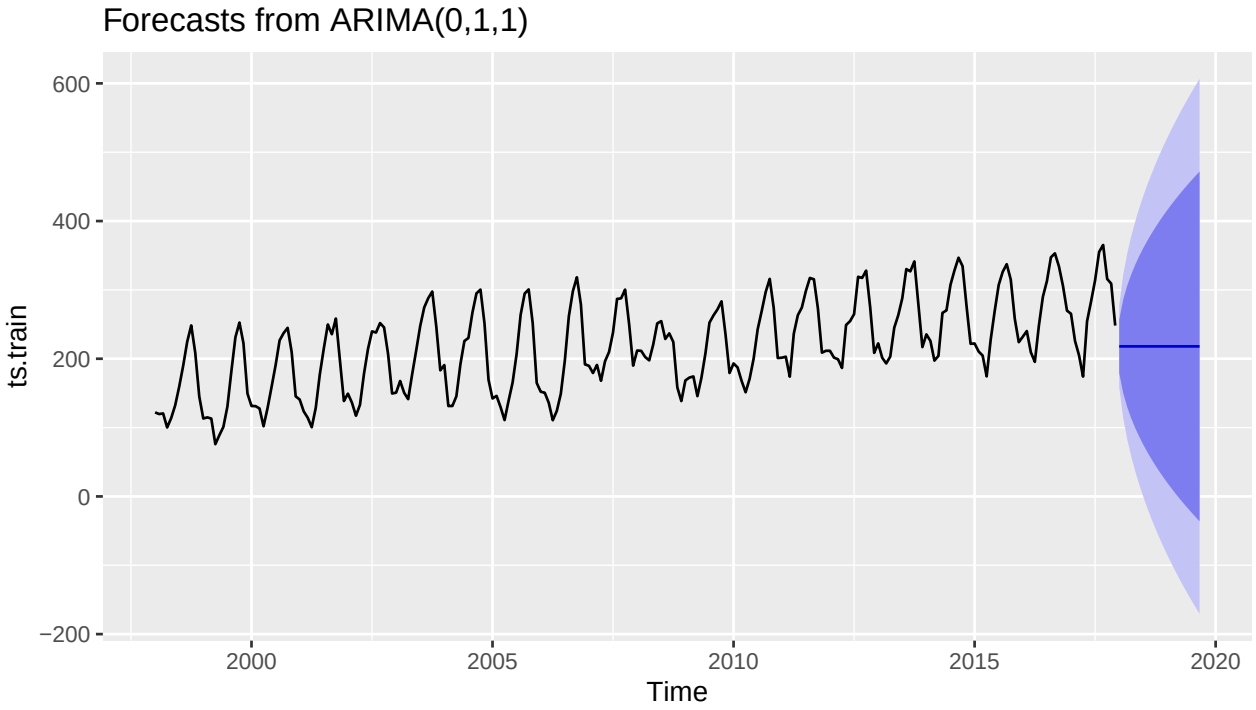
```
                ME     RMSE      MAE       MPE     MAPE      ACF1 Theil's U
Test set -1.214474 22.39923 17.24986 -2.049079 6.626138 0.5171863 0.5855158
```

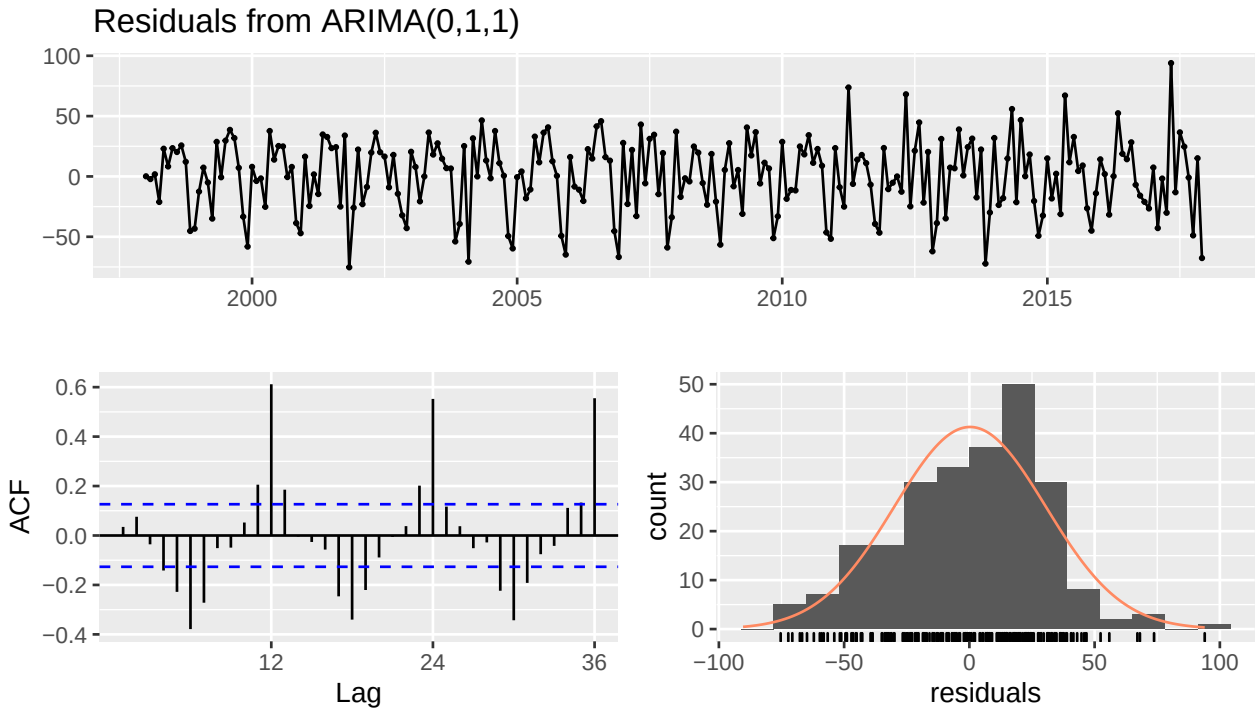
```
Call:
tslm(formula = ts.total ~ trend + bizdays(ts.total) + fourier(ts.total,
    K = 6))

Residuals:
    Min      1Q  Median      3Q     Max
-84.924 -10.719   0.916  12.510  65.420

Coefficients:
                              Estimate Std. Error t value Pr(>|t|)
(Intercept)                   85.75119   38.27783   2.240   0.0260 *
trend                          0.53538    0.01913  27.989  < 2e-16 ***
bizdays(ts.total)              3.20739    1.81636   1.766   0.0787 .
fourier(ts.total, K = 6)S1-12 -64.50270    2.16439 -29.802  < 2e-16 ***
fourier(ts.total, K = 6)C1-12  -2.14996    2.17151  -0.990   0.3231
fourier(ts.total, K = 6)S2-12   3.23587    2.08624   1.551   0.1222
fourier(ts.total, K = 6)C2-12 -11.06085    2.03931  -5.424 1.39e-07 ***
fourier(ts.total, K = 6)S3-12   2.75123    2.09453   1.314   0.1902
fourier(ts.total, K = 6)C3-12 -11.71024    2.07504  -5.643 4.56e-08 ***
fourier(ts.total, K = 6)S4-12  -1.81359    2.06215  -0.879   0.3800
fourier(ts.total, K = 6)C4-12  -5.26168    2.06485  -2.548   0.0114 *
fourier(ts.total, K = 6)S5-12   0.89921    2.58727   0.348   0.7285
```

```
fourier(ts.total, K = 6)C5-12  -1.75282    2.04322  -0.858   0.3918
fourier(ts.total, K = 6)C6-12  -1.88407    1.45620  -1.294   0.1969
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 23.27 on 247 degrees of freedom
Multiple R-squared:  0.8872,    Adjusted R-squared:  0.8813
F-statistic: 149.5 on 13 and 247 DF,  p-value: < 2.2e-16
```

```
[1] 2399.14
```

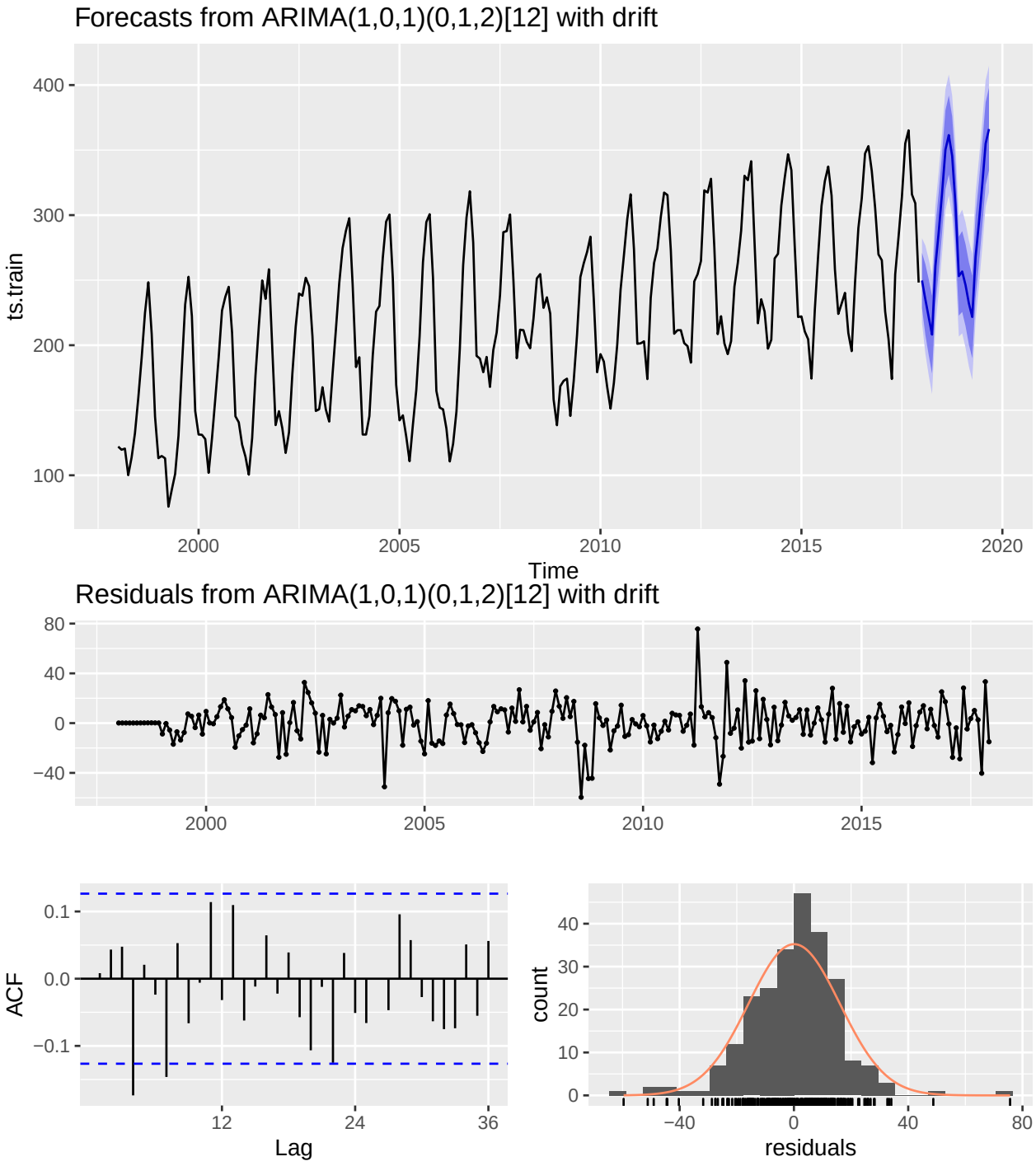
Arima nao sasonal






```
    Ljung-Box test

data:  Residuals from ARIMA(0,1,1)
Q* = 345.89, df = 23, p-value < 2.2e-16

Model df: 1.   Total lags used: 24

                     ME     RMSE      MAE        MPE     MAPE     MASE
Training set  0.2769404 30.14828 24.42535 -0.4628617 12.13153 1.127175
Test set     65.2932280 92.55506 73.17336 18.7570963 22.69551 3.376787
                   ACF1 Theil's U
Training set 0.03470858        NA
Test set     0.77771207  1.833074

Series: ts.train
ARIMA(0,1,1)

Coefficients:
         ma1
      0.4470
s.e.  0.0562

sigma^2 estimated as 916.6:  log likelihood=-1153.8
AIC=2311.6   AICc=2311.65   BIC=2318.56

Training set error measures:
                     ME     RMSE      MAE        MPE     MAPE     MASE
Training set 0.2769404 30.14828 24.42535 -0.4628617 12.13153 1.127175
                   ACF1
Training set 0.03470858
```

Arima sasonal




```
    Ljung-Box test

data:  Residuals from ARIMA(1,0,1)(0,1,2)[12] with drift
Q* = 34.256, df = 19, p-value = 0.01714

Model df: 5.   Total lags used: 24
```

```
Series: ts.train
ARIMA(1,0,1)(0,1,2)[12] with drift

Coefficients:
         ar1     ma1     sma1     sma2   drift
      0.6276  0.1693  -0.6984  -0.1545  0.5351
s.e.  0.0713  0.0916   0.0796   0.0781  0.0669

sigma^2 estimated as 273.9:  log likelihood=-968.35
AIC=1948.69   AICc=1949.07   BIC=1969.27

Training set error measures:
                     ME     RMSE     MAE        MPE     MAPE      MASE
Training set 0.08641124 15.95277 11.6103 -0.3792579 5.795372 0.5357892
                    ACF1
Training set 0.008179276

                      ME     RMSE      MAE        MPE     MAPE      MASE
Training set  0.08641124 15.95277 11.61030 -0.3792579 5.795372 0.5357892
Test set     -0.68440888 22.02509 15.75203 -1.7658648 6.058180 0.7269208
                    ACF1 Theil's U
Training set 0.008179276        NA
Test set     0.380699192 0.5806079

Warning in rbind(accuracy(fit.naive, ts.test)[2, ],
accuracy(fit.seasonal_naive, : number of columns of result is not a multiple of
vector length (arg 8)
```