B505: Applied Algorithms

HW3 (Due: **March 5, 2018 5:00 PM EDT**)

http://darwin.informatics.indiana.edu/col/courses/B505-18

Please submit your completed homework through canvas using pdf file format

1. (10 pts) For the set of {1, 4, 5, 10, 16, 17, 21} of keys, draw binary search trees of height 2, 3, 4, 5, and 6.

2. (15 pts) Suppose that the search for the key *k* in a binary search tree ends up in a leaf. Consider three sets: 1) the keys to the left of the search path; 2) the keys on the search path; 3) the keys to the right of the search path. One claims that any three keys a, b and c from these three sets, respectively (say a from 1, b from 2 and c from 3), it is always true that $a \leq b \leq c$. Is this claim true? Explain your answer.

3. (15 pts) We can sort a given set of n numbers by first building a binary search tree containing these numbers (using Tree_insert algorithm repeatedly to insert the numbers on by one) and then printing the number in an inorder tree walk. What are the worst-case and best-case running times for this sorting algorithm?

4. (20 pts) Suppose two teams A and B are playing a match to see who is the first to win n games (from given n). Assume that A and B are equally competent, so each has a 50% chance of winning any particular game. Suppose they have already played i+j games, of which A won i and B has won j. Given an efficient algorithm to compute the probability that A will go on to win the match. For example, if i=n-1 and j=n-3 then the probability that A will win the match is 7/8, since it must win any of the next three games.

5. (20 pts) Give an O(nt) algorithm for the following task.

   Input: a list of n positive integers $a_1, a_2, \ldots, a_n$; a positive integer t.
   Question: does some subset of the a_i's add up to t?

6. (20 pts) Professor Midas drives an automobile from Newark to Reno along Interstate 80. His car's gas tank, when full, holds enough gas to travel n miles, and his map gives the distances between gas stations on his route. The professor wishes to make as few gas stops as possible along the way. Give an efficient method by which Professor Midas can determine at which gas stations he should stop, and prove that your strategy yields an optimal solution.