Mouhammadou Dabo

April 1, 2021

CS 1675: Intro to Machine Learning

Professor Milos Hauskrecht

Problem Assignment 8

Problem 1. Bayesian belief networks

Assume you want to compute $P(B = T, E = T)$

Part a.

$P(B = T, E = T)$

$$= \sum_{a \in T,F} \sum_{c \in T,F} \sum_{d \in T,F,X} \sum_{f \in T,F} P(A = a, B = T, C = c, D = d, E = T, F = f)$$

$$= P(B = T) \sum_{a \in T,F} \sum_{c \in T,F} \sum_{d \in T,F,X} \sum_{f \in T,F} \frac{P(E = T|C = c)P(D = d|A = a, B = T, C = c)P(F = f|D = d)}{P(A = a)\, P(C = c)}$$

# of additions: $2 * 2 * 3 * 2 = 24 - 1 = 23$

# of multiplications: $24 * 5 = 120$

Part b.

$P(B = T, E = T)$

$$= \sum_{a \in T,F} \sum_{c \in T,F} \sum_{d \in T,F,X} \sum_{f \in T,F} \frac{P(B = T)P(E = T|C = c)P(D = d|A = a, B = T, C = c)P(F = f|D = d)}{P(A = a)\, P(C = c)}$$

$$= P(B = T) \sum_{a \in T,F} P(A = a) \left[\sum_{c \in T \in,F} P(C = c)P(E = T|C = c) \right]$$

$$\left[\sum_{d \in T,F,X} P(D = d|A = a, B = T, C = c \left[\sum_{f \in T,F} P(F = f|D = d) \right] \right]$$

# of additions: $1 + (2 * 1) + (2 * 2 * 1) + (2 * 3 * 1) = 13$

# of multiplications: $1 + (2 * 2) + (2 * 1) + (2 * 2 * 3 * 1) = 19$

By interleaving the sums and products, the computational cost in terms of number of additions and multiplications both are reduced, especially the multiplication.

Problem 2. Pneumonia diagnosis

Part a.

P(Fever | Pneumonia)

Fe	**T**	**F**
T	0.9	0.1
F	0.6	0.4

P(Paleness | Pneumonia)

P	**T**	**F**
T	0.7	0.3
F	0.5	0.5

P(Cough | Pneumonia)

C	**T**	**F**
T	0.9	0.1
F	0.1	0.9

P(HighWBCcount | Pneumonia)

H	**T**	**F**
T	0.8	0.2
F	0.5	0.5

Part b.

$$P(Pneumonia = T | Fever = T, Paleness = F, Cough = T, HighWBCcount = F)$$

$$= 0.062021$$

Part c.

$$P(Pneumonia = T | Fever = T, Cough = T)$$

$$= 0.216$$

Part d.

From the data provided in the file “example.txt”:

- With the parameters 1, 0, 1, 0, the probability the patient has pneumonia is: 0.062021
- With the parameters 1, 0, 1, -1, the probability the patient has pneumonia is: 0.141856
- With the parameters 0, 1, -1, 0, the probability the patient has pneumonia is: 0.002849
- With the parameters 1, -1, -1, 0, the probability the patient has pneumonia is: 0.012097
- With the parameters -1, 0, 1, -1, the probability the patient has pneumonia is: 0.099265