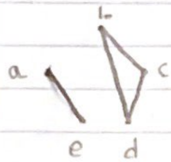
Mo Felonda felonda@wisc.edu Disc. 325

1a) $R = \{(a,e), (b,c), (b,d), (a,a), (b,b), (c,c), (d,d), (e,e), (e,a), (c,b), (d,b), (c,d), (d,c)\}$

b) 

1. $[b] = \{b,c,d\} = [d], [c] \to \{c \in A \mid (b,c),(c,b) \in R\}, \{d \in A \mid (b,d),(d,b) \in R\}$
2. $[a] = \{a,e\} = [e] \to_a \{c \in A \mid (a,c) \in R\}, {}_e\{a \in A \mid (e,a) \in R\}$

c.i) No, because S is not symmetric. (i.e. $|y| \le |x|$ is false and $(y,x) \notin R$)

ii) No, because S is not anti-symmetric. (i.e. $x = \{a,b\}, y = \{c,d\}$, if $|x| \le |y|$ and $|y| \le |x|$, then $x = y$; but $\{a,b\} \neq \{c,d\}$)

2a) We must show that R is both antisymmetric and transitive.

Antisymmetric: If $((a,b),(c,d)) \in R$ and $((c,d),(a,b)) \in R$, then $a \le c \wedge a \cdot b \le c \cdot d$ and $c \le a \wedge c \cdot d \le a \cdot b$. This can only be true if $a = c$ and $b = d$, which shows that $(a,b) = (c,d)$, and R is antisymmetric.

Transitive: If $((a,b),(c,d)) \in R$ and $((c,d),(e,f)) \in R$, then $a \le c \wedge a \cdot b \le c \cdot d$ and $c \le e \wedge c \cdot d \le e \cdot f$. Since $a \le c$ and $c \le e$, we find that $a \le e$. In the same manner, since $a \cdot b \le c \cdot d$ and $c \cdot d \le e \cdot f$, we find that $a \cdot b \le e \cdot f$. This means that $((a,b),(e,f)) \in R$ and that R is transitive.

R is antisymmetric and transitive and is thus an order relation.

b) Total order means if $(\forall a,b \in A)\ x \neq y \to (xRy \vee yRx)$. Since we have 2 statements using comparing, each element in set can be compared and then ordered. Thus, R is a total order.

c) Consider $((a,a),(a,a))$. $a \le a \wedge a \cdot a \le a \cdot a$ holds, so R is reflexive. Therefore, R is a partial order.

d) R cannot be a strict order because R was proven to be reflexive above and thus R cannot be antireflexive as well. Therefore, R is not a strict order.

Mo Felonda felonda@wisc.edu Disc. 325

3a) $R_1 \cap R_2$ is an equivalence relation.

Reflexive: Since R_1 and R_2 are both equivalence relations, and $R_1 \cap R_2$, then xRy iff xR_1y AND xR_2y. Since R_1 and R_2 are both equiv rels, xR_1x AND xR_2x $\forall x \in A$, therefore xRx $\forall x \in A$ and R is reflexive.

Symmetric: Since R_1 and R_2 are both equiv. rel, xR_1y, yR_1x AND xR_2y, yR_2x, and we find that if xRy for some $x, y \in A$, then yRx and R is symmetric.

Transitive: Consider $x, y, z \in A \mid xRy$ and yRz. Since R_1 and R_2 are equiv rel, xR_1y, yR_1z and xR_2y, yR_2z showing that xR_1z and xR_2z. Therefore xRz and R is transitive.

$R_1 \cap R_2$ is reflexive, symmetric, and transitive and is thus an equivalence relation.

b) $R_1 \cup R_2$ is NOT an equivalence relation.

Not transitive: If R_1 and R_2 are both equiv rel, and $R_1 \cup R_2$, then xRy iff xR_1y OR xR_2y. Counter-example is that $R_1 \cup R_2$ is not transitive. Consider: A is the set $\mathbb{N}$

R_1 is the relation xR_1y iff $2 \mid (x-y)$

R_2 is the relation xR_2y iff $3 \mid (x-y)$

$2R_14$ holds, and $4R_27$ holds

Transitivity would lead us to believe that $2R7$ holds, but $2R_17$ and $2R_27$ are both false, so $R_1 \cup R_2$ is not transitive because $2R7$ does not in fact hold.

Since $R_1 \cup R_2$ is not transitive, it cannot be an equivalence relation.