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1. We must show that G is reflexive, symmetric, and transitive.

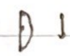
Reflexive) Arbitrarily, $\forall u \in V$, if you take a path of length 0, u is reachable from u (i.e. $(u,u) \in E$). Since u and u are mutually reachable in G, we know that G is reflexive.

Symmetric) Arbitrarily considering $u, v \in V$ where v is reachable from u (i.e. $(u,v) \in E$), since u and v are mutually reachable in G, we know that there is a directed path from u to v ((u,v)) and from v to u ((v,u)). Therefore, G is symmetric.

Transitive) Arbitrarily consider that $(u,v), (v,w) \in G$. By the transitive property, there is a path from u to w (u,w). Since u, v, w are mutually reachable in G, there is also a path from w to u (w,u). Therefore, G is transitive.

$\therefore$ $\{(u,v) \mid$ u and v are mutually reachable in directed graph $G=(V,E)\}$ is an equivalence relation because it is reflexive, symmetric, and trasitive.

2a) i. $C_6 =$  $\bar{C}_6 =$ 

ii $K_{3,2} =$  $\bar{K}_{3,2} =$ 

The complement of a bipartite graph results in the two subsets each being unconnected from one another and also each being complete. That is, there is an edge between every vertex in m, there is an edge between every vertex in n, and, there are no edges between m and n. Effectively $\bar{K}_{m,n} = K_m$ and K_n (unconnected)

iii Since K_n is a complete graph, $\bar{K}_n$ has no edges (and n vertices).

$K_4 =$  $\bar{K}_4 =$ 

2b) Proof: Suppose $G=(V,E)$ is a simple graph on n vertices with no self-loops with two connected components. Prove that $\overline{G}$ is connected.

Case 1: Vertices u and v are not in the same connected component.
If u and v are not in the same connected component, then no edge exists between them.
This means that is G's complement, $\overline{G}$, there must be an edge between u and v. Therefore $\forall$ vertices u,v in $\overline{G}$ there exists a path between u and v and $\overline{G}$ is connected.

Case 2: Vertices u and v are in the same connected component
Consider vertex w in the other connected component.
If u and v are in the same connected component, there is an edge between them. However, there is no edge between u and w, or v and w, since w is in the other connected component.
This means that in G's complement, $\overline{G}$ there must be edges between both u and w and v and w. By the the transitive property, u and v are connected in $\overline{G}$, a path is between them - since both are adjacent to w. Therefore $\forall$ vertices u,v in $\overline{G}$ there exists a path between u and v and $\overline{G}$ is connected.

$\therefore$ In all cases there is exists a path between any two vertices so G's complemen $\overline{G}$ is connected.