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Assignment 8

Problem 1: a)

n	0	1	2	3	4
A_n	1	6	11	16	21

b) The proposed solution is $A_n = 5n+1 \ \forall n \in \mathbb{N}^+$

c. Proof: Let $P(n)$: $A_n = 5n+1 \ \forall n \in \mathbb{N}^+$

Base Case: Show $P(0)$ holds

$A_0 = 1$ by recurrence relation

$5(0)+1 = 0+1 = 1$, so $A_0 = 5(0)+1$, which is in the form of $P(n)$

so, $P(0)$ holds

Inductive step: Show that $P(k) \to P(k+1)$

Inductive hypothesis: Assume $P(k)$ holds, i.e. $A_k = 5k+1$

Consider A_{k+1}: $A_{k+1} = A_k + 5$ by original recurrence relation

$= 5k+1+5$ by IH

$= 5k+5+1$ rearrangement

$= 5(k+1)+1$ factoring out 5

$5(k+1)$ is in the form of $P(n)$, so $P(k+1)$ holds

$\therefore$, by induction, $A_n = 5n+1 \ \forall n \in \mathbb{N}^+$ ■

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Problem 2: a) Let A_n represent how many different ways we can create a string of length n inches.

A_0 has a length of 0 inches, and there is only one way for 0 length $\rightarrow A_0 = 0$
A_1 has a length of 1 inch, and can only create a string using 1 inch letters (i.e. f, i, or t.) There are 3 options only $\rightarrow A_1 = 3$
A_2 has a length of 2 inches, and can creat a string using a single 2 inch letter or two consecutive 1 inch letters. There are 10 options for 2 inch letters. The number of ways for two 1 inch letters can be found by taking the number of options (i.e. 3) and raising it to the power of how many places in the combination (i.e. 2). So, $3^2 = 9$. $10 + 9 = 19$ options $\rightarrow A_2 = 19$.
A_3 has a length of 3 inches, and can create a string using three consecutive 1 inch letters, a 2 inch letter followed by a 1 inch letter, or a 1 inch letter followed by a 2 inch letter. $3^3 = 27$. $10^1 \cdot 3^1 = 30$. $3^1 \cdot 10^1 = 30$. $30 + 30 + 27 = 87$ options $\rightarrow A_3 = 87$
A_4 has a length of 4 inches, and can create a string using four consecutive 1 inch letters, two consecutive 2 inch letters, one 2 inch letter followed by two 1 inch letters, one 2 inch letter preceded by two 1 inch letters, or one 2 inch letter between two 1 inch letters. $3^4 = 81$. $10^2 = 100$. $10^1 \cdot 3^2 = 90$. $3^2 \cdot 10^1 = 90$. $3^1 \cdot 10^1 \cdot 3^1 = 90$. $81 + 100 + 90 + 90 + 90 = 451$ options $\rightarrow A_4 = 451$.

b) $A_0 = 1$ $A_1 = 3$ $A_2 = 19$
Consider A_3: There are two options; use either a 1 inch letter or 2 inch letter to start with.
Case 1 When using a 1 inch letter first, we have 3 options (i.e. f, i, or t). We still have 2 inches left, so we are left with how many ways you can create a 2 inch string, which is the same as A_2. So, $3 \cdot A_2$.
Case 2 When using a 2 inch letter first, we have 10 options. We still have 1 inche left, so we are left with how many ways you can create a 1 inch string, which is the same as A_1. So, $10 \cdot A_1$

So, $A_3 = 3 \cdot A_2 + 10 \cdot A_1$
This also applies to A_4, i.e. $A_4 = 3 \cdot A_3 + 10 \cdot A_2$.
Therefore, the recurrence relation is $A_n = 3 \cdot A_{n-1} + 10 \cdot A_{n-2}$ for $n > 2$. $A_0 = 1$, $A_1 = 3$, and $A_2 = 19$.