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Problem 1

If 36 days are picked at random from a 2018 calendar, then at least 6 days must fall on the same day of the week. Prove by contradiction.

Assume: 36 days are picked at random from a calendar & it is not true that 6 days fall on the same day of the week.

Proof: (1) There are 7 days in one week. So there are 7 different cases when a day is picked
(2) If the days picked are spread out evenly across the week, then the eighth day will fall on a day of the week that already has a pick.
(3) Using this logic, since $5 \times 7 = 35$, 35 days spread out evenly will result in each day of the week having 5 picks.
(4) Now, once the 36^{th} day is picked, all possible cases will result in at least one day of the week having 6 picks.
(5) This is inconsistent with the second part of the assumption.

∴ by contradiction, if 36 days are picked at random from a 2018 calendar, then at least 6 days must fall on the same day of the week. ∎

Problem 2

Prove: $\min(x,y) = \frac{x+y-|x-y|}{2}$; $x,y \in \mathbb{R}$

Proof: Assume x and y are real numbers. Show $\min(x,y) = \frac{x+y-|x-y|}{2}$. Consider the following cases.

Case 1: $x-y>0$

This implies $x>y$

$\min(x,y) = \frac{x+y-|x-y|}{2}$

$y = \frac{x+y-(x-y)}{2}$; definition of absolute value, $|x-y| = x-y$ (since $(x-y)>0$)

$y = \frac{2y}{2}$

$y = y$ □

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Problem 2 cont.

Case 2: $x-y<0$

This implies $x<y$

$\min(x,y) = \frac{x+y-|x-y|}{2}$

$x = \frac{x+y-(-(x-y))}{2} = \frac{x+y+(x-y)}{2}$; definition of absolute value, $|x-y| = -(x-y)$ (since $x-y<0$)

$x = \frac{2x}{2}$

$x = x$ $\square$

Case 3: $x-y=0$

This implies $x=y$

$\min(x,y) = \frac{x+y-|x-y|}{2}$

$x = \frac{x+y-|0|}{2}$ ($|x-y| = |0|$) OR $y = \frac{x+y-|0|}{2}$ ($|x-y| = |0|$)

$x = \frac{x+x-0}{2}$ (sub y with x) | $y = \frac{y+y-0}{2}$ (sub x with y)

$x = \frac{2x}{2}$ | $y = \frac{2y}{2}$

$x = x$ $\square$ | $y = y$ $\square$

$\therefore$ for all possible cases, $\min(x,y) = \frac{x+y-|x-y|}{2}$ ∎

Problem 4

Prove by contrapositive: If $x \notin \mathbb{Q}$, then $\frac{1}{x} \notin \mathbb{Q}$

Contrapositive: If $\frac{1}{x} \in \mathbb{Q}$, then $x \in \mathbb{Q}$

Assume: $\frac{1}{x} \in \mathbb{Q}$

Proof: Since $\frac{1}{x} \in \mathbb{Q}$, then by definition, $\frac{1}{x} = \frac{a}{b}$ for some $a \in \mathbb{Z}$, $b \in \mathbb{Z}$, $b \neq 0$

Let $1 = a$, and $x = b$

Then, $1 \in \mathbb{Z}$, $x \in \mathbb{Z}$, $x \neq 0$.

$x = \frac{x}{1}$

Since $x \in \mathbb{Z}$, $1 \in \mathbb{Z}$, and $1 \neq 0$, then by definition, $\frac{x}{1} \in \mathbb{Q}$.

Since $\frac{x}{1} = x$, $x \in \mathbb{Q}$.

We have proven that if $\frac{1}{x} \in \mathbb{Q}$, then $x \in \mathbb{Q}$

$\therefore$ by contrapositive, if $x \notin \mathbb{Q}$, then $\frac{1}{x} \notin \mathbb{Q}$. ∎