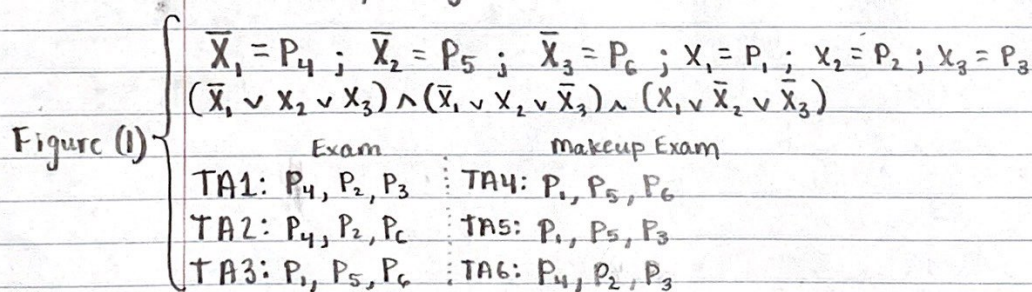


HW11

NP-Organization: Let E be an instance for 3-SAT with m clauses and 3 literals. For each clause in E there is a corresponding TA who is assigned to the exam. Additionally, for each TA there exists a set of questions corresponding to the literals of the clause from which it was made. For each clause in E there is also a corresponding TA who is assigned to the makeup exam. For each of these TAs there exists a set of questions corresponding to the negation of the literals of the clause from which it was made.

NP-Hardness: $3SAT \leq_m^P TA$ Problem. We claim that E is satisfiable if and only if the exams are disjoint in the TA problem. Suppose we have R , a satisfying assignment for E . For this to be the case, R must assign at least one literal in each clause of E to true. In the TA problem constructed from E , for each TA assigned to the exam, choose the questions corresponding to the literals set to true in the corresponding clauses. For each TA assigned to the makeup exam, choose the questions corresponding to the negations of the literals set to true in the corresponding clauses of R . See Figure (1) for clarification.



In R ; \bar{X}_1, X_2 , and \bar{X}_3 are set to true, so P_4, P_2 , and P_6 are chosen for the exam. Accordingly, the negations of those set to true are X_1, \bar{X}_2 , and X_3 , so we pick P_1, P_5 , and P_3 for the makeup exam. Thus we have sets of $\{P_4, P_2, P_6\}$ for the exam and $\{P_1, P_5, P_3\}$ for the makeup exam; these sets are disjoint. E will fulfill the requirements of the TA problem because each TA assigned to either exam will have at least one of their questions chosen. This is because each clause in E has at least one literal set to true. The sets of questions chosen for each exam will be disjoint as the questions chosen for the makeup exam are derived from the negations of the literals corresponding to the questions chosen for the exam. Thus there will be no common questions between exams and the sets are disjoint.

HW11

NP-Hardness: Consider a disjoint set in E' ; we know from the TA problem specification that all TAs will have at least one of their questions chosen. Each TA corresponds to a clause in the 3SAT and each question corresponds to a literal in the clause corresponding to the associated TA. As such, if each TA has at least one question chosen, then each clause in the 3SAT will have at least one literal set to true.

Time Complexity: We traverse m clauses with exactly three literals in each clause. In the TA problem, each clause becomes 2 TAs (which can be done in $O(1)$) and their associated 3 questions. Thus the reduction is done in $O(3m) = O(m)$ and is with the required runtime necessary to achieve polynomial reduction.

Conclusion: Therefore, the 3SAT problem reduces to the TA problem. As stated in the scribe notes, the 3SAT problem is NP-Hard. Thus, the TA problem is NP-Hard.