Mo Felonda
Andrew Moy

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Input: n chemical substances; non-negative numbers e_{ij} where i and j are chemical substances and e_{ij} is the amount of energy generated from mixing i and j; and two disjoint sets L_1 and L_2 each containing the chemical substance that must be stored in the corresponding bottle (i.e. $L_2 = \{3, 4\}$ mean substances 3 and 4 must be in bottle 2).

Output: Two bottles, B_1 and B_2, each containing the chemical substances that should be placed in the corresponding bottle to achieve maximum energy generation.

Network Flow Structure: We construct the network as follows:
We have one node representing each chemical substance 1…n, except those that are in either L_1 or L_2. Let there be an edge between the source and each of the aformentioned nodes. Let the weight of these edges be equivalent the the sum of all energies generated by the mixture of all substances in L_1, in addition to the energies generated by mixing the chemical substance represented by the node with each of the substances in L_1. Let there also be an edge connecting each node with every other node in the network. Let these edge weights be equivalent to the energy generated by mixing the two sustances connected by the edge. Let there also be an edge between the sink and each node. Let the weight of these edges be equivalent to the sum of all energies generated by mixture of all substance in L_2, in addition to the energies generated by mixing the substance represented by the node with each of the substances in L_2.

Algorithm Analysis: We run the algorithm to find the minimum-cut of our network. The nodes (or substances) that are on the s side of the min-cut will be put into B_1 to maximize energy generation. The nodes on the t-side of the min-cut will be put into B_2 to maximize energy generation. The algorithm then returns B_1 and B_2 as output.

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Correctness: The reason that we use min-cut on our network is because it shows us the partitioning between bottles that will result in the maximum energy generated. This is because the maximum energy generated by the optimal partitioning between bottles is equal to the sum of all energies generated by all substances being mixed together minus the minimum of the sum of energies that are unable to be acheived by the partitioning of the bottles. This can be seen in the case in which there are at least 2 substances but only one bottle requires a substance. In this case, since all edge weights between the bottle with no required substances and all nodes will be 0, the min-cut will provide the correct partitioning in which all substances are placed in the bottle with a requred substance, resulting in the maximum possible generation of energy.

Time Complexity: As stated in the scribe notes, finding the min-cut of a network takes O(#nodes * #edges). The number of nodes in our network is at most $n+2$ (all substances + source + sink). The number of edges is at most $2n+\frac{n(n+1)}{2}$ (n edges between n substances and source + n edges between n substances and sink + $\frac{n(n+1)}{2}$ edges between all n substances with each other). The assignment of substances into bottles takes at most n time. Thus, our overall time complexity is $O(n+(n+2)(2n+\frac{n(n+1)}{2}))$ which is equivalent to $O((n+2)(2n+\frac{n(n+1)}{2}))$, and is polynomial in n.