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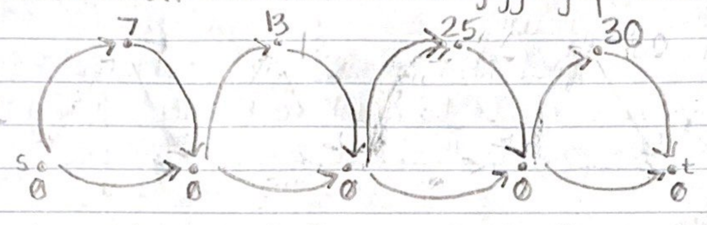
HW 12

Reduction: We do a reduction from the subset sum problem to our jogging problem. Let S be an instance of the subset sum problem with set $\{X_1, \dots, X_n\}$ where $X_i \in \mathbb{Z}^+$ and target sum T.

Construction: We construct an instance of our jogging problem as follows:

Figure (1)

K = T*2
K = 100



with set $\{7, 13, 30, 25\}$
T = 50

Let there be a checkpoint for every X_i in the set, where the elevation of each checkpoint is its corresponding X_i. Let there be n+1 additional checkpoints of elevation 0, where n is the number of elements in the set. For this set of checkpoints with elevation 0 $\{y_1, \dots, y_{n+1}\}$, for $y_1, \dots, y_n$ create a road from checkpoint y_i to checkpoint corresponding to X_i. Create roads for y_i to y_{i+1}. Finally, create roads from checkpoint x_i to checkpoint y_{i+1}. Let checkpoint s be y_1 and let checkpoint t be y_{n+1}. Set the total elevation target desired equal to 2T.

Soundness. Claim 1: The subset sum problem is only satisfiable if and only if the jogging problem is satisfiable.

$\Rightarrow$ Let I be the subset of numbers in S where this subset's sum is exactly T. Let B be the corresponding construction of the jogging problem where K = 2T. In this construction, the jogger visits every checkpoint that corresponds to elements in subset I, where the jogger incurs an elevation change of x_i. From this checkpoint, the jogger can only visit a checkpoint with elevation 0; this results in the jogger incurring another elevation change of x_i. This happens for all checkpoints corresponding to elements in subset I. When faced with a checkpoint that would incur elevation change but does not correspond to an element in I, and a checkpoint with an elevation of 0, choose the checkpoint with elevation 0. When the jogger reaches checkpoint t, this construction (B) will have resulted in the jogger undergoing a total elevation change of $\Sigma 2x_i$ which is equal to 2T. As stated above, 2T is equal to k, the total desired elevation change.

HW12 Cont.

Soundness Cont. <=

Suppose there's an instance of our jogging problem E where there is a path D from s to t where the total elevation change is equal to k. For each checkpoint in D with a nonzero elevation, there is a corresponding element in the solution to the subset sum problem. There are no elements in the solution that do not correspond to a checkpoint in D with a nonzero elevation. The total elevation change in path D is equal to k, so the sum of elements in the solution to the subset sum problem is equal to $k \div 2$, where $k = 2T$ and T is the target sum. Thus, the sum of all elements in the solution is equal to T.

Conclusion

Therefore the subset sum problem reduces to our jogging problem and our jogging problem in NP-Hard.

Time Complexity:

For each element in the subset sum set $1, \ldots, n$ we create one checkpoint in the jogging problem. We then create $n+1$ additional checkpoints in the jogging problem for each 0 elevation checkpoint. For each 0 elevation checkpoint $1, \ldots, n$ we create a road from the 0 elevation checkpoint to its corresponding elevated checkpoint. For 0 elevation checkpoint y_i, create a road to y_{i+1}. For each elevated checkpoint x_i, create a road to its next 0 elevation, checkpoint y_{i+1}. All other trivial actions are done in constant time. Thus our reduction runs in $O(5n+1)$ which is polynomial in n.