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HW 7 577

Intuition: Suppose there are K bottles and n chemical substances (c).

Let $E(K,i)$ be the minimum energy required to ship the first i chemicals in K bottles.

Base Cases: $E(1,j)$ for some $j > 0$, i.e. there is only 1 bottle and at least 1 chemical. This base case results in $\sum e_{\ell m}$ where $1 \leq \ell \leq m \leq j$

$E(K,0)$ for any $K_j = 0$, since there are no chemicals left and thus no energy required.

DP: Consider all possible combinations of chemicals that could be in the last bottle (bottle K). Suppose we put chemicals $c_s, \dots, c_i$ in bottle K, where $s < n$

↳ The total energy at that level is $\sum e_{\ell m}$ where $s \leq \ell \leq m \leq i$

Once the last bottle is fixed, the minimum energy required to transport the remaining chemicals in the previous bottles is: $E(K-1, i-s) + \sum e_{\ell m}$ where $s \leq \ell \leq m \leq i$ (1).

To minimize the total energy required to transport the chemicals, we must enumerate all possible arrangements for the last bottle and account for the corresponding minimum energies for the remaining bottles and chemicals to transport, and then pick the arrangement that minimizes (1). That is, we evaluate the energies for each bottle using:
$E(K,i) = \min(E(K-1,j) + \sum e_{\ell m}$ where $i-j < \ell \leq m \leq i)$ where $j = 1, \dots, i$.
This will recursively evaluate each subproblem at each level for the minimum. We will have a memoization table of size $K \times n$ so that subproblems don't have to be calculated more than once. Accessing the memoization table takes constant time ($O(1)$).

Time Compl: Since we are doing our recursive calls while decrementing K, we can say that the "outer loop" iterates at most K times. Thus, there are K levels in the recursion tree. At each level, we evaluate n subproblems that have not yet been memoized. In addition, for each subproblem, the summation takes n time to evaluate. Thus, our algorithm's running time is $O(n^2)$ at each level, giving us an overall time complexity of $O(Kn^2)$.