Actuarial, Statistical, and Financial Formulas Reference Document

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Aligned with IFOA Standards

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1. Financial Mathematics

Time Value of Money

1. **Compound Interest (Accumulation Factor)**

$$FV = PV \cdot (1+i)^n \quad \text{or} \quad FV = PV \cdot e^{\delta n}$$

   - i: Annual effective interest rate.
   - δ: Force of interest ($\delta = \ln(1+i)$).

2. **Discount Factor**

$$v = \frac{1}{1+i} = e^{-\delta}$$

3. **Present Value (PV) of Annuity-Certain**

   - *Annuity-immediate* (payments at end of period):

$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

   - *Annuity-due* (payments at start of period):

$$\ddot{a}_{\overline{n}|} = \frac{1 - v^n}{d} \quad \text{where } d = \frac{i}{1+i}$$

4. **Future Value (FV) of Annuity-Certain**

$$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}, \quad \ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}$$

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2. Actuarial Mathematics

Life Contingencies

1. **Present Value of Whole Life Annuity**

   - *Annuity-due* (IFOA notation):

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^k \cdot {}_kp_x$$

   - *Continuous annuity*:

$$\bar{a}_x = \int_0^{\infty} v^t \cdot {}_tp_x \, dt$$

2. **Present Value of Term Life Insurance**

$$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} \cdot {}_kp_x \cdot q_{x+k}$$

   - q_x: Probability of death within 1 year for age x.

3. **Net Premium Reserve**

$$ {}_tV = \ddot{a}_{x+t} - P \cdot \ddot{a}_{x+t:\overline{n-t}|} $$

- P: Premium payment.

4. **Endowment Insurance**

$$ A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + v^n \cdot {}_np_x $$

5. **Deferred Life Annuity**

$$ {}_m\ddot{a}_x = \sum_{k=m}^{\infty} v^k \cdot {}_kp_x $$

6. **Accumulated Cost of Insurance**

$$ \bar{A}_x = \int_0^{\infty} v^t \cdot {}_tp_x \cdot \mu_{x+t}\, dt $$

Mortality and Survival Functions

1. **Survival Probability**

$$ {}_tp_x = e^{-\int_0^t \mu_{x+s}\, ds} $$

- μ_x: Force of mortality.

2. **Deferred Mortality Probability**

$$ {}_t|_uq_x = {}_tp_x \cdot {}_uq_{x+t} $$

3. **Complete Expectation of Life**

$$ \mathring{e}_x = \int_0^{\infty} {}_tp_x\, dt $$

4. **Select Mortality**

$$ {}_tp_{[x]+s} = \frac{l_{x+s+t}}{l_{x+s}} $$

Commutation Functions (IFOA)

1. **Discounted Lives**

$$D_x = v^x l_x$$

2. **Accumulated Annuity Factors**

$$N_x = \sum_{k=0}^{\infty} D_{x+k}$$

3. **Death Benefit Factors**

$$C_x = v^{x+1} d_x, \quad M_x = \sum_{k=0}^{\infty} C_{x+k}$$

4. **Annuity-Due Commutation**

$$\ddot{a}_x = \frac{N_x}{D_x}$$

5. **Term Insurance Commutation**

$$A^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x}$$

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3. Probability and Statistics

Distributions

1. **Normal Distribution**

$$X \sim N(\mu, \sigma^2) \Rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

2. **Binomial Distribution**

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

3. **Poisson Distribution**

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

4. **Exponential Distribution**

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

5. **Gamma Distribution**

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x \geq 0$$

Expectation and Variance

1. **Law of Total Expectation**

$$E[X] = E[E[X|Y]]$$

2. **Variance Formula**

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

3. **Covariance**

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

Hypothesis Testing

1. **Z-Test Statistic**

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

2. **Chi-Square Test**

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

3. **t-Test Statistic**

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

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4. Financial Derivatives

1. **Black-Scholes Option Pricing**

$$C = S_0 N(d_1) - K e^{-rT} N(d_2)$$

   - $d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$.
   - *Macaulay Duration*:

$$D = \frac{\sum t \cdot PV(CF_t)}{\text{Price}}$$

   - *Convexity*:

$$C = \frac{\sum t(t+1) \cdot PV(CF_t)}{(1+i)^2 \cdot \text{Price}}$$

Additional Actuarial Formulas

1. **Joint Life Annuity**

$$\ddot{a}_{xy} = \sum_{k=0}^{\infty} v^k \cdot {}_k p_{xy}$$

2. **Last Survivor Annuity**

$$\ddot{a}_{\overline{xy}} = \ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy}$$

3. **Net Premium for Whole Life Insurance**

$$P_x = \frac{A_x}{\ddot{a}_x}$$

4. **Thiele's Differential Equation**

$$\frac{d}{dt}{}_t V = \delta \cdot {}_t V + P - \mu_{x+t}(S - {}_t V)$$

5. **Variance of Present Value**

$$\text{Var}(PV) = E[PV^2] - (E[PV])^2$$

Find out what this is.

$$\ddot{a}_{\overline{xy}}\ddot{a}_{\overline{xy}}$$