Tentamen i kursen **Datastrukturer och algoritmer**	Kurskod	D0041D
Totalt antal uppgifter: 8 st	Tentamensdatum	2019-05-27
Lärare: Jingsen Chen (0920-49 20 44)	Skrivtid	4 timmar

Tillåtna hjälpmedel: språk ordbok

1. Given an input $A = \langle 12, 19, 40, 22, 20, 29 \rangle$,

   (a) Sort A using *mergesort* step by step. **(5p)**

   (b) Draw the binary search tree that results after successively inserting all the elements into an initially empty binary search tree. **(5p)**

   (c) Insert all the elements above *in that order* into an initially empty max-heap. Draw the max-heap obtained. **(5p)**

   (d) Show the result of insert all the elements above *in that order* into an initially empty hash table of size 10 using linear probing. What is your hashing function? **(5p)**

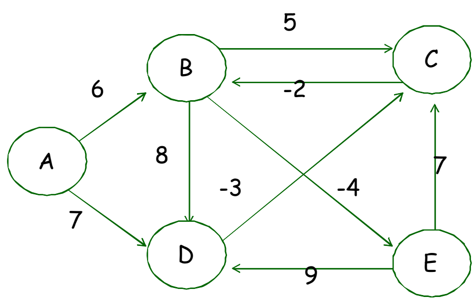
2. Prove (by using the definitions) or disprove (by giving counter examples) the following assertions. Let $f(n)$ and $g(n)$ be positive integer functions and $f(n) = O(g(n))$.

   (a) If $g(n) = \frac{1}{5}h(n)$, then $f(n) = O(h(n))$. **(5p)**

   (b) If $g(n) < \frac{1}{5}h(n)$, then $f(n) < h(n)$. **(5p)**

3. Consider the following algorithm that *sorts* an input sequence of length n. If $n \leq 4$, sort the input using any sorting method. Otherwise, divide first the input into three parts, two of size $\frac{n}{4}$ and one of size of $\frac{n}{2}$. Then sort each part *recursively*. After that, merge these three sorted sequences into a sorted output of size n with at most n comparisons. Find the *recurrence* that computes the running time of this algorithm in the worst case. You may assume that n is a power of 2. **(6p)**

4. Consider the graph G:



   (a) Represent G by its adjacency matrix and adjacency list, respectively. **(5p)**

   (b) Draw a depth-first search and a breadth-first search tree of this graph starting at the node A, respectively. **(5p)**

   (c) How many topological orderings does G have? Why? **(4p)**

5. For each of the following statements, indicate whether the statement is TRUE or FALSE, respectively. **Justify your answers**. *That is, if the statement is correct, state why; and if the statement is wrong, give a counter-example.* No credit will be given without justifications.

   Let G be a connected directed graph with edge-weights.

   (a) Adding a constant to every edge-weight in G does not change the solution to the single-source shortest-paths problem. **(5p)**

   (b) Adding a constant to every edge-weight in G does not change the solution to the minimum spanning tree problem. **(5p)**

6. Let $A = \langle x_1, x_2, ..., x_n \rangle$ be a sorted array of distinct integers (some of which may be negative). Design an $O(\log n)$-time algorithm to find an index i such that $1 \leq i \leq n$ and $x_i = i$, provided such an index exists. Show that your algorithm is correct. **(10p)**

7. Let A be an array of n positive numbers and x a number. Design a worst-case $O(n)$-time algorithm that determines whether there exists indexes i and j, where $1 \leq i \leq j \leq n$, such that $\sum_{k=i}^{j} A[k] = x$. **(10p)**

8. Let A be an array of n numbers and some numbers may appear more than once. Design and analyze algorithms to identify *all* numbers that appear at least $\alpha \cdot n$ times in A for a given constant $0 < \alpha < 1$.

   (a) For $\alpha = \frac{1}{2}$, your algorithm should run in $O(n)$ time in the worst case. **(10p)**

   (b) For any $0 < \alpha < 1$, your algorithm should run in $O(n \log \frac{1}{\alpha})$ time in the worst case. **(10p)**

Lycka till!