Tentamen i kursen **Datastrukturer och algoritmer**	Kurskod	D0041D
Totalt antal uppgifter: 12 st	Tentamensdatum	2019-08-30
Lärare: Jingsen Chen (0920-49 20 44)	Skrivtid	4 timmar

Tillåtna hjälpmedel: språk ordbok

1. Sort numbers $44, 18, 17, 34, 15, 27, 25$ *step by step* using *mergesort.* **(5p)**

2. What are the best-case and worst-case running times of Countingsort on an input of n integers from $[1..k]$ (using Θ-notation). **(5p)**

3. Show the result of inserting the numbers $10, 20, 4, 30$ *in that order* into an initially empty hash table of size 10 with the hash function $h(x, i) = (x \bmod 10 + i \cdot (1 + x \bmod 9)) \bmod 10$, $i = 0, 1, \cdots, 9$. **(5p)**

4. Design an algorithm for checking whether a given an array $A = \langle a_1, a_2, \cdots, a_n \rangle$ is a max-heap. **(5p)**

5. Design an algorithm for checking whether a given binary tree of size n is a binary search tree. **(5p)**

6. Briefly justify your answers to the following questions (no formal proof required).

   (a) Express the function $200 \cdot n^2 + 9 \cdot n^3 - 1 + 12 \cdot n$ in terms of Θ-notation. Do not have any extraneous constants in your expression. **(4p)**

   (b) How to prove that the worst-case running time of an algorithm is $\Omega(n^2 \cdot \log n)$? **(4p)**

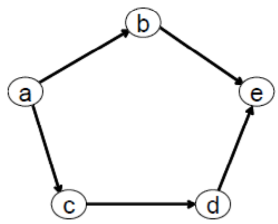
   (c) For a given problem with inputs of size n, the worst-case running times of algorithms A, B, and C are $\Omega(n \log n)$, $O(n^3)$, and $\Theta(n^2)$, respectively.

      i. Is algorithm A asymptotically faster than algorithm B? Why? **(3p)**

      ii. Is algorithm B asymptotically faster than algorithm C? Why? **(3p)**

7. Consider the following comparison-based sorting algorithm. If the length n of the input sequence is small, say $n \leq 4$, just sort the input using some sorting method. Otherwise, one divides the input into three parts, two of size $\frac{n}{4}$ and one of size $\frac{n}{2}$. After sorting each part recursively, one merges these three sorted parts into a sorted sequence of size n with at most n comparisons. Find a recurrence describing the number of comparisons used by this algorithm in the worst case. You may assume that n is a power of 4. **(6p)**

8. Given the graph G shown below:



   (a) Represent G by its adjacency matrix and adjacency list, respectively. **(5p)**

   (b) How many topological orderings does G have? Describe them. **(5p)**

   (c) Draw a depth-first search and a breadth-first search tree of G starting at the node a, respectively. **(5p)**

9. Let $A = \{a_1, a_2, \cdots, a_n\}$ be distinct numbers and assume that n is a power of 2. Design a worst-case linear-time algorithm to compute the sum $b_1 + b_2 + b_4 + \cdots + b_{\frac{n}{2}}$, where b_i the i^{th} smallest element in A for $i = 1, 2, \cdots, \frac{n}{2}$. For example, if $A = \{24, 8, 20, 5, 7, 16, 15, 2\}$, then the sum to be computed is $2 + 5 + 8 = 15$. **(10p)**

10. Design a divide and conquer algorithm that, given an array $A = \langle a_1, a_2, \cdots, a_n \rangle$ of positive numbers, determines two indices $1 \leq i < j \leq n$ such that $a_j - a_i$ is maximized over all such indices. Your algorithm should run in $O(n \log n)$ time in the worst case. For example, if $A = \langle 20, 2, 4, 10, 1 \rangle$, then $j = 4$ ($a_4 = 10$) and $i = 2$ ($a_2 = 2$) are the indices to be computed. **(10p)**

11. Show how to determine in $O\left(n^2 \log n\right)$ time whether any three points in a given set of n points in the plane are colinear (that is, they lie on the same line). **(10p)**

12. Let G be a connected graph with m edges and n vertices, where edges are colored either red or blue. Design an algorithm to find a path from a start node s to a destination node t containing as few blue edges as possible. Your algorithm should run in $O(m + n)$ time in the worst case. **(10p)**