THE UNIVERSITY *of* EDINBURGH
School of Engineering

SCEE09002
Control & Instrumentation Engineering 3 Laboratory

PRACTICAL 1:
Parameter estimation and system modelling

Topics Covered

- First and second order transfer functions.
- Obtaining the model of electro-mechanical systems using the bump test (step input) method.
- Validation of the models using first principles.

Prerequisites

Material covered in lectures on:

- Laplace transform;
- transfer functions;
- response of first and second order systems.

1 Bump test of a 1st order system

1.1 Background

The bump test is a simple test based on the step response of a stable system. A step input is given to the system and its response is recorded. As an example, consider a system given by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \tag{1.1}$$

The step response shown in Figure 1.1 is generated using this transfer function with $K = 5$ rad/V-s and $\tau = 0.05$ s.

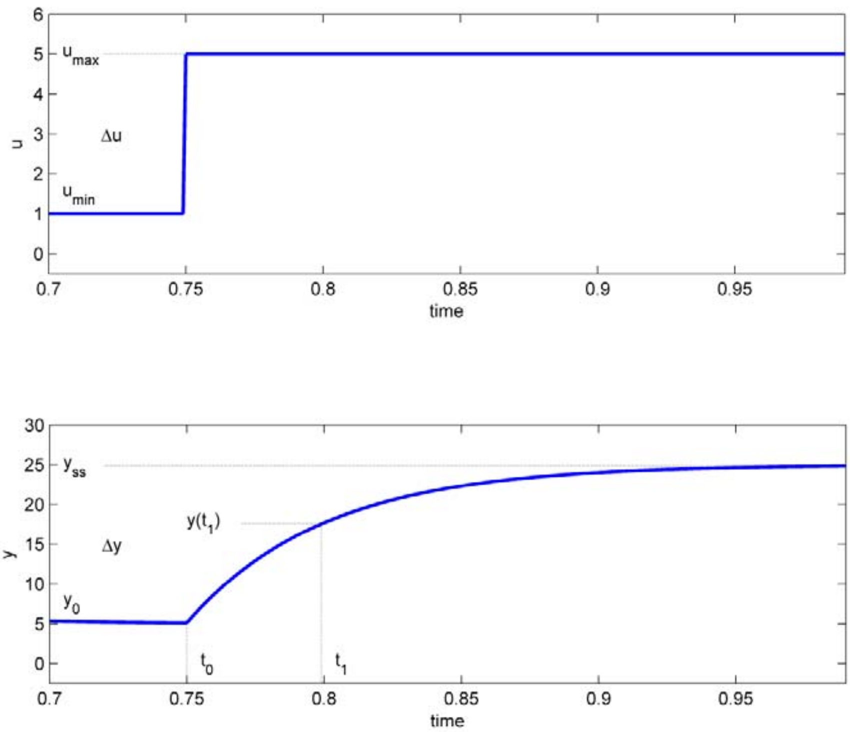

Figure 1.1: Input and output signal used in the bump test method

The step input begins at time t_0. The input signal has a minimum value of u_{min} and a maximum value of u_{max}. The resulting output signal is initially at y_0. Once the step is applied, the output tries to follow it and eventually settles at its steady-state value y_{ss}. From the output and input signals, the steady-state gain is

$$K = \frac{\Delta y}{\Delta u} \tag{1.2}$$

where $\Delta y = y_{ss} - y_0$ and $\Delta u = u_{max} - u_{min}$. Note that step signals do not necessarily have to start from zero and finish at one! The time constant of a system, τ, is defined as the time it takes the system to respond to the application of such a step input to reach $1 - 1/e \approx 63.2\%$ of the difference between the final (or steady-state) and the initial value, i.e. for Figure 1.1:

$$t_1 = t_0 + \tau,$$

where:

$$y(t_1) = 0.632\Delta y + y_0. \tag{1.3}$$

Then, we can read the time t_1 that corresponds to $y(t_1)$ from the response data in Figure 1.1. From the figure we can see that the time t_1 is equal to:

$$t_1 = t_0 + \tau$$

From this, the model time constant can be found as:

$$\tau = t_1 - t_0 \tag{1.4}$$

1.2 Applying this to the QUBE-Servo

Focusing on the QUBE-Servo system available in the lab, the s-domain representation of a step input voltage with a time delay t_0 is given by

$$V_m(s) = \frac{A_v e^{(-st_0)}}{s}, \tag{1.5}$$

where A_v is the amplitude of the step and t_0 is the step time (i.e. the delay).

The voltage-to-speed transfer function is

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1} \tag{1.6}$$

where K is the model steady-state gain, τ is the model time constant, $\Omega_m(s) = L[\omega_m(t)]$ is the load gear rate, and $V_m(s) = L[v_m(t)]$ is the applied motor voltage.

If we substitute input 1.5 into the system transfer function 1.6, we get:

$$\Omega_m(s) = \frac{K A_v e^{(-st_0)}}{(\tau s + 1)s}.$$

We can then find the QUBE-Servo motor speed step response in the time domain, $\omega_m(t)$, by taking inverse Laplace of the above equation:

$$\omega_m(t) = K A_v \left(1 - e^{\left(-\frac{t-t_0}{\tau}\right)}\right) + \omega_m(t_0),$$

noting the initial conditions $\omega_m(0^-) = \omega_m(t_0)$.

2 In-Lab Exercise 1

In this exercise you will obtain the step response of a motor with a rotating mass (similar to the turntable example covered in the lectures), you will derive its parameters and then will confirm their values by simulating the model. You will be given four different aluminum disks that can be easily attached on the motor shaft using magnetic coupling. You will repeat the steps below for all three disks and will record your results.

2.1 Experiment

The front panel of a *Virtual Instrument* (VI) shown in Figure 1 below has been developed in LabVIEW and can be used to apply a 2V step to the motor and then read the motor angular velocity using the onboard incremental encoder.

- Open project '**Practical 1.lvproj**' (if not already open) from folder '**CIE3 Lab 1**' on your desktop.
- In the project explorer, right click '**NI-myRIO**' and click '**Deploy All**'. Note it may take a couple of minutes for the process to finish and it may ask you a few times to save some files.
- Double click VI '**P1E1.vi**' under '**NI-myRIO**' (NOT under 'My Computer') to open it. You should now see the front panel shown in Figure 2.1.

Using the step response, the model parameters can be found as discussed in the background section of this exercise. The top left graph shows the step input (motor voltage in volts), while the top right shows the system output (motor angular velocity in rad/s). The bottom chart also shows the response and also has cursors that can be used to read values at specific time instances (shown in the table on the bottom right). The cursors can either be moved by dragging the thick yellow lines in the chart, or by pressing the arrows .

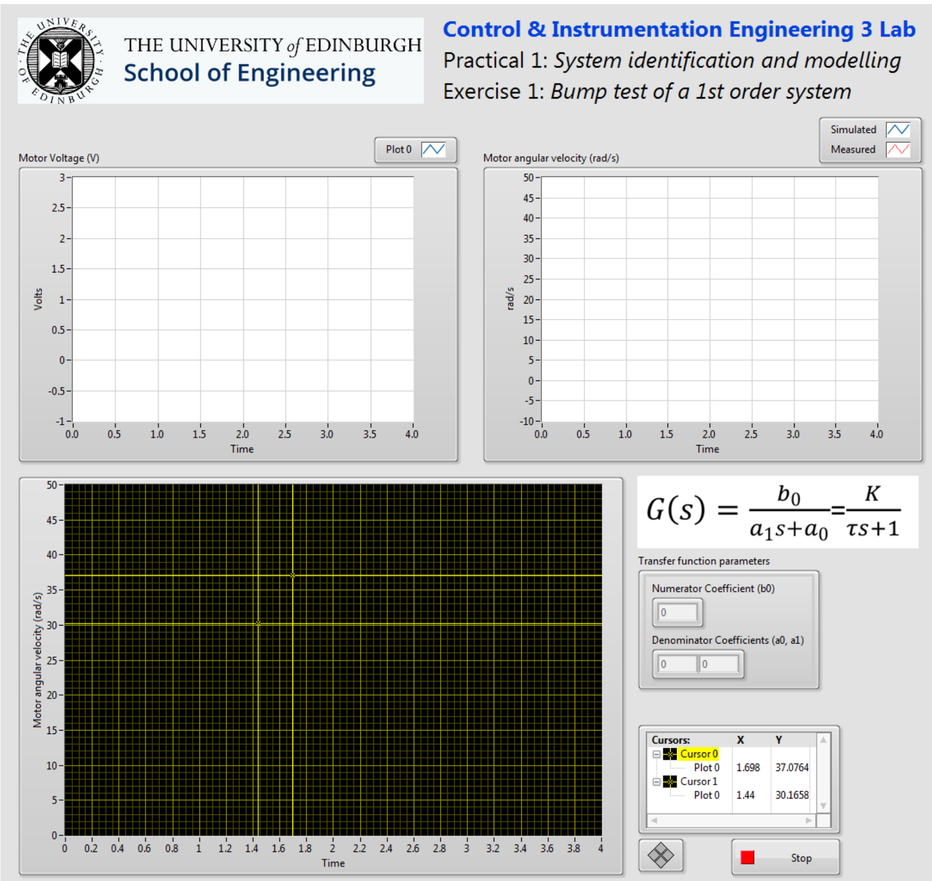

Figure 2.1: LabVIEW front panel for Exercise 1 (P1E1.vi)

Note that it is possible to store both the plots (as figures) as well as the data contained in them (e.g. in excel files). **This may be useful for post-processing the results at home as well as for presenting them in your report, therefore you are advised to do so at every step.**

Storing the plots as figures:

Right-click on the plot you want to save, select 'Export' and then 'Export simplified image…'. You now have the option either to save it as a bitmap file or to copy it to the clipboard and then paste it in a Word document. In both cases, save the file with an indicative filename and make sure you take it with you before you leave the lab. You are free to youse whichever method you prefer. Note though that this method only stores the plot as an image and the data will then not be available for processing.

Storing the data contained in a plot:

Right-click on the plot you want to save, select 'Export' and then 'Export Data to Excel'. This method will create a new Excel file with the data nicely arranged in columns. It does not automatically generate a figure, but you can then do so from within Excel. This is the preferred method as it allows post-processing of data in Excel. Save the Excel file with an indicative filename and make sure you take it with you before you leave the lab.

- Attach one of the disks to the motor shaft.
- Initialise the model transfer function by setting all the numerator and denominator coefficients to '1'.
- Run the VI by pressing button on the toolbar to apply a 2 V step to the servo. The obtained step response should be similar to Figure 2.2 below (but not identical as you may be using a different disk to the one of this example).

In the output plot note how the response of the simulated system (blue trace) differs from the actual one (red trace). Your next task is to make it match to the actual response of the real system.

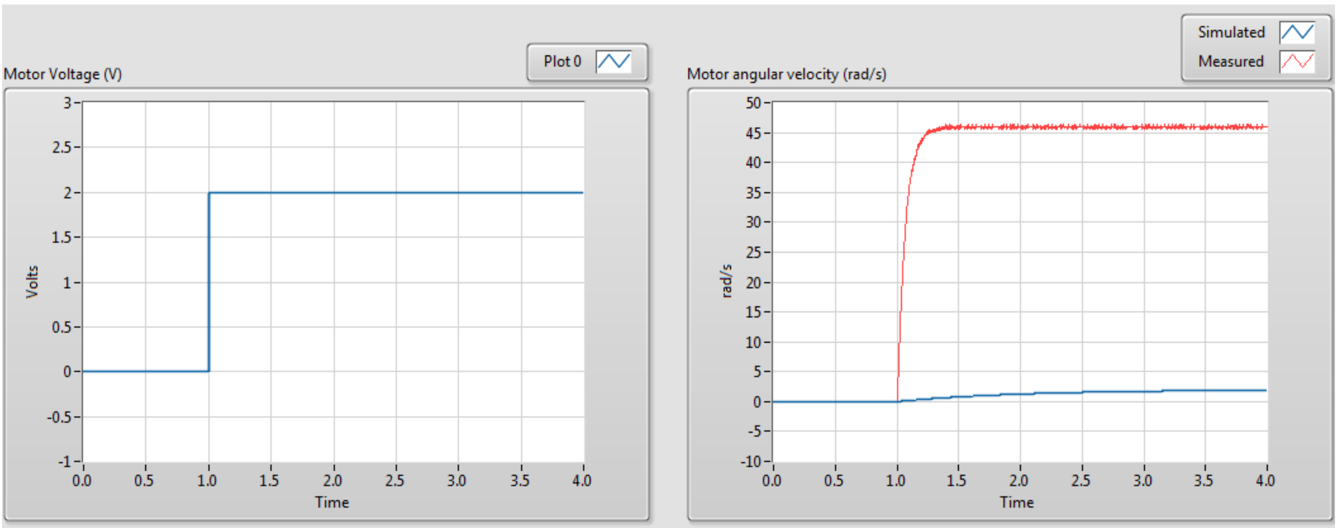

Figure 2.2: Simulated and actual step response of the 1^{st} order motor-mass system.

- Using the cursors in the X-Y chart, identify the gain, K, and the time constant, τ, of the system.
- Update the transfer function coefficients using the values for K and τ you obtained in the previous step and run again the VI.
- Measure and record the dimensions of the disk (diameter D and height H).

Does the simulated response match closely enough the one of the physical system? If not, you can further refine your design by slightly tweaking K and τ.

- Repeat the experiment for all three disks and record the data in the following table. It is advised that within your team you take turns in running the experiments.

Disk information			**Measured parameters**		**Refined parameters**	
No.	Diameter (mm)	Height (mm)	Gain K	Time const. τ (s)	Gain K	Time const. τ (s)
1						
2						
3						

Table 1.1: Disk data and step response parameters

2.2 Questions to take home

Your report should include a detailed description of the experimental process as well as tables and figures of all the results. Additionally, you are asked to discuss the following questions.

1. Write the transfer function of the system for each disk.

2. What are the possible reasons for the non-perfect match of the response of the simulated model with the one of the actual system?

3. What is the impact of the physical size of the rotating mass on the transient and steady state response of the system? What are the implications of this on the required characteristics of the motor?

3 Validation using first principles

3.1 Background

The Quanser QUBE-Servo is a direct-drive rotary servo system. Its motor armature circuit schematic is shown in Figure 1.1 and the electrical and mechanical parameters are given in Table 1.1. The DC motor shaft is connected to the *load hub*. The hub is a metal disk used to mount the disk or rotary pendulum and has a moment of inertia of J_h. A disk load is attached to the output shaft with a moment of inertia of J_d .

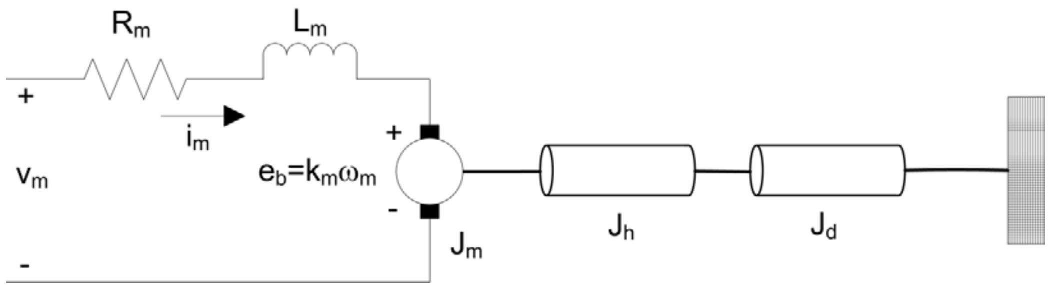

Figure 1.1: QUBE-Servo DC motor and load

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, ω_m, and the back-emf constant of the motor, k_m. It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t)$$

Symbol	Description	Value
DC Motor		
R_m	Terminal resistance	6.3 Ω
k_t	Torque constant	0.036 N-m/A
k_m	Motor back-emf constant	0.036 V/(rad/s)
J_m	Rotor inertia	4.0×10^{-6} kg-m^2
L_m	Rotor inductance	0.85 mH
m_h	Load hub mass	0.0087 kg
r_h	Load hub mass	0.0111 m
J_h	Load hub inertia	1.07×10^{-6} kg-m^2
Load Disk		
ρ_d	Mass density of disk load	2.7 g/cm^3

Table 1.2: QUBE-Servo system parameters

Using Kirchoff's Voltage Law, we can write the following equation:

$$v_m(t) - R_m i_m(t) - L_m \frac{di_m(t)}{dt} - k_m \omega_m(t) = 0.$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the equation becomes

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0.$$

Solving for $i_m(t)$, the motor current can be found as:

$$i_m(t) = \frac{v_m(t) - k_m \omega_m(t)}{R_m}. \tag{1.1}$$

The motor shaft equation is expressed as

$$J_{eq}\,\dot{\omega}_m(t) = \tau_m(t), \tag{1.2}$$

where J_{eq} is total moment of inertia acting on the motor shaft and τ_m is the applied torque from the DC motor. Based on the current applied, the torque is

$$\tau_m = k_m i_m(t)$$

The moment of inertia of a disk about its pivot, with mass m and radius r, is

$$J_d = \frac{1}{2} m r^2. \tag{1.3}$$

The equivalent total moment of inertia for the complete system is

$$J_{eq} = J_m + J_h + J_d\,. \tag{1.4}$$

3.2 Questions to take home

1. The motor shaft of the QUBE-Servo is attached to a *load hub* and a disk load. Based on the parameters given in Table 1.2, and the disk dimensions measured in Section 2, calculate the equivalent moment of inertia that is acting on the motor shaft for each of the three disks you used.
2. Based on the analysis in Section 3 and using the Laplace transform, derive the transfer function of the complete system, between the input voltage and the angular velocity, for each disk. Do these transfer functions match the ones derived in Section 2? If not, can you suggest any reasons for this mismatch?
3. Assuming that the gain K of the servo system is equal to 1, can you design and derive the transfer function of an appropriate resistor-capacitor (RC) circuit that can mimic the behaviour of your system? If the capacitor size is 47 μF, what would the resistor value be in order to get the same response as the one you got with each of the three discs?

4 Second Order Systems

4.1 2nd Order System Step Response

The *standard 2nd order* transfer function has the form:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{4.1}$$

where ω_n is the natural frequency and ζ is the damping ratio. The properties of its response depend on the values of the parameters ω_n and ζ.

Consider a second-order system as shown in Equation 4.1 subjected to a step input given by

$$R(s) = \frac{R_0}{s},$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 4.1, where the red trace is the output response $y(t)$ and the blue trace is the step input $r(t)$.

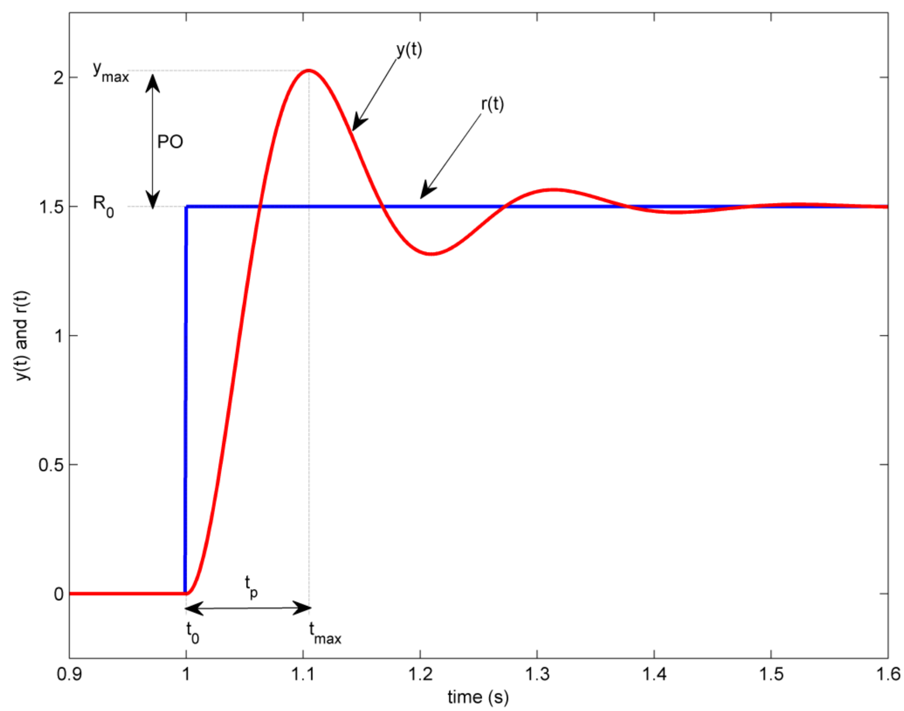

Figure 4.1: Standard second-order step response

4.2 Peak Time and Overshoot

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 4.1, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0}. \tag{4.2}$$

From the initial step time, t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0. \tag{4.3}$$

This is called the *peak time* of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}. \tag{4.4}$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}. \tag{4.5}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

4.3 Unity Feedback

The unity-feedback control loop shown in Figure 4.2 will be used to control the position of the QUBE-Servo.

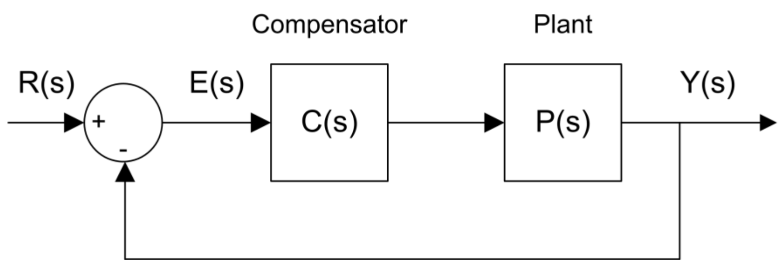

Figure 4.2: Unity feedback loop

The QUBE-Servo voltage-to-position transfer function is

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}.$$

where K is the model steady-state gain in rad/(V-s), τ is the model time constant in seconds, $\Theta_m(s) = L[\theta_m(t)]$ is the motor / disk position, and $V_m(s) = L[v_m(t)]$ is the applied motor voltage. In the next exercise you will conduct an experiment to find the model parameters, K and τ, for your particular servo system.

The controller is denoted by $C(s)$. Here, the controller is set as a unity gain block therefore

$$C(s) = 1.$$

Based on the above analysis, the closed-loop transfer function of the QUBE-Servo position control from the reference input $R(s) = \Theta_d(s)$ to the output $Y(s) = \Theta_m$ using unity feedback and a simple unity gain controller as shown in Figure 4.2 is:

$$\frac{\Theta_d(s)}{V_m(s)} = \frac{\frac{K}{\tau}}{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}}. \tag{4.6}$$

5 In-Lab Exercise 2

In this part of the practical you will study the response of a 2nd order system and will determine its transfer function parameters. Here, the input and output of the system are the angle reference (setpoint) and output.

5.1 2nd Order System Step Response: Experiment

The front panel of a *Virtual Instrument* (VI) shown in Figure 5.1 below has been developed in LabVIEW and can be used to apply a step input to the motor and then read the motor position using the onboard incremental encoder.

- Open project '**Practical 1.lvproj**' (if not already open) from folder '**CIE3 Lab**' on your desktop.
- In the project explorer, right click '**NI-myRIO**' and click '**Connect**'. Note it may take a couple of minutes for the process to finish and it may ask you a few times to save some files.
- Double click VI '**P1E2.vi**' under '**NI-myRIO**' to open it. You should now see the front panel shown in Figure 5.1.

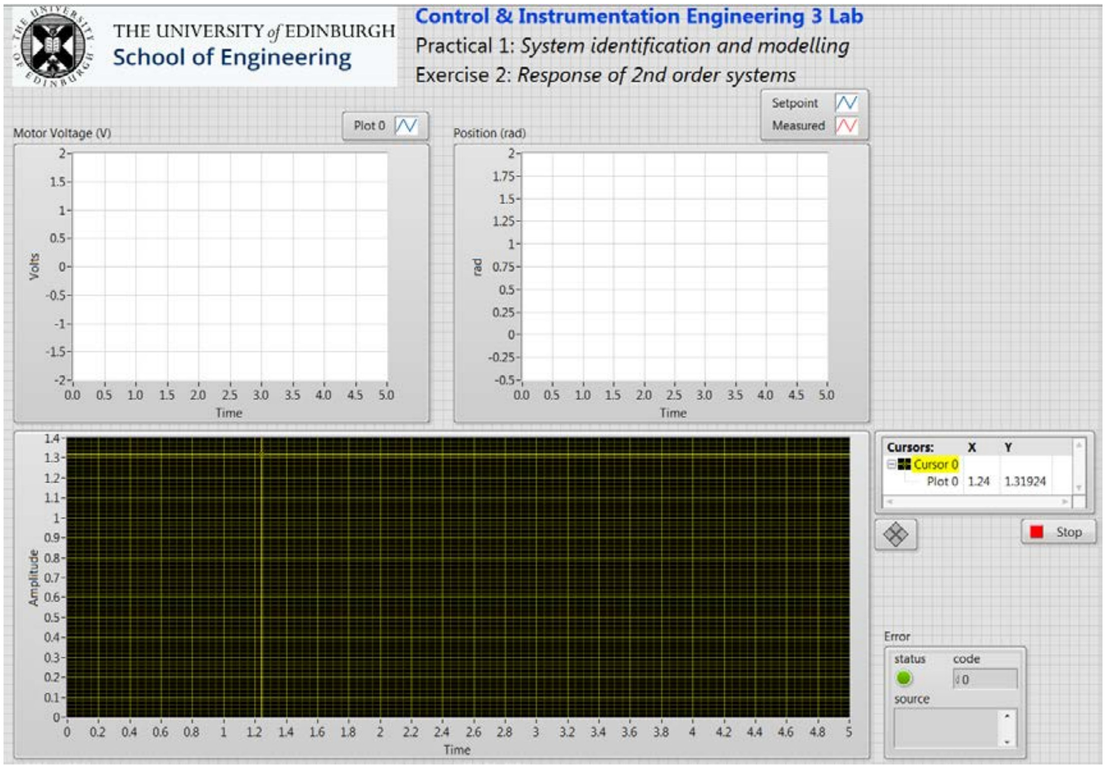

Figure 5.1: The VI front panel for the study of the response of a 2nd order system

Using the Virtual Instrument shown in Figure 5.1, you can measure the key step response parameters of the 2nd order system. We are particularly interested in the percentage overshoot PO and peak time t_p, as they allow us to calculate the undamped natural frequency ω_n damping ratio ζ.

- Mount the 1st of your three inertia discs on the servo.
- Run the experiment. You should observe the response of the system in the three plots.
- Using the cursor in the bottom plot, find the peak time, measure the percentage overshoot and record then in Table 5.1 below.
- Repeat the process for the other two discs.

Disc No	Response parameters	
	t_p	PO
1		
2		
3		

Table 5.1: PID controller parameters

5.2 Questions to take home

1. Based on the analysis shown in sections 4.1 and 4.2 and using your measurements obtained in this experiment, calculate the undamped natural frequency and damping ratio for each disc, and derive the corresponding transfer function.
2. In Sections 2 and 3 you derived the transfer functions of the 1st order system (angular velocity control). Using these, and the analysis shown in section 4.3, derive the transfer function of the 2nd order angle control system, for each of the three discs. Do they match with the ones you calculated in question 1?
3. Similarly, calculate the peak time and percentage overshoot from the transfer functions you calculated in question 2. How much do they differ from the ones you measured in this experiment?
4. Can you suggest any reasons for any discrepancies in the measured/calculated parameters?