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Multi-objective Firefly Algorithm Based on Compensation Factor and Elite Learning

Li Lv*, Jia Zhao, Jiayuan Wang, Tanghuai Fan*
School of Information Engineering, Nanchang Institute of Technology, Nanchang 330099, China
lvli@nit.edu.cn; zhaojia@nit.edu.cn; wangjiayuan1201@163.com; fantanghuai@nit.edu.cn
*Corresponding author

Multi-objective Firefly Algorithm Based on Compensation Factor and Elite Learning

Abstract: Aimed at early maturing and poor accuracy of multi-objective firefly algorithms, we propose a multi-objective firefly algorithm based on compensation factor and elite learning (CFMOFA). Based on iterations by introducing a compensation factor into the firefly learning formula, constraints by population can be overcome and the Pareto optimal solution can be approached in a reduced period. The non-inferior solutions produced in iterations were stored in the external archive and a random external archive particle was employed as the elite particle for population evolution. In this way, the detection range of firefly was extended and diversity and accuracy of non-inferior solution set were enhanced. The conventional algorithms, the improved algorithms and the proposed multi-objective optimization algorithm were tested and compared with each other. The results indicated great advantages of the proposed algorithm in convergence, diversity, and robustness and the proposed algorithm is an effective multi-objective optimization method.

Keywords: firefly algorithm, multi-objective optimization, Pareto optimality, compensation factor, elite learning

1. Introduction

Multi-objective optimization problems(MOP) refer to those containing multiple objectives that are in mutual contradiction with each other[1]. To achieve optimizations multi-objective evolutionary algorithms (MOEA) are usually employed for MOPs[2].

MOEAs can be categorized as: (a). Pareto dominate based MOEA. Examples include the multi-objective genetic algorithm(MOGA)[3], the non-dominated sorting genetic algorithm(NSGA)[4] and its improved version (NSGA-II) [5]. These algorithms are characterized by straightforward principles and few parameters. However, these algorithms are not suitable for MOPs with complicated Pareto front and high dimension MOPs due to their intrinsic limitations. (b). Elite reservation scheme based MOEA. These algorithms retain elite by establishing EA to increase the population diversity so that improved Pareto front can be obtained. Examples include the strength Pareto evolutionary algorithm(SPEA)[6] and its improved version (SPEA2)[7], the Pareto archived evolution strategy(PESA)[8], and PESA-II[9]. These algorithms can obtain elegant solutions, but is limited by accumulation of solutions (i.e., non-uniform solution distribution). (c). Decomposition based MOEA. In these algorithms, a MOP is divided into various sub-problems and solved synergistically using the neighborhood information of sub-problems. Examples include the multi-objective evolutionary algorithm based on decomposition(MOEA/D)[10] and the self-adaptive constrained sub-problems in a decomposition based multi-objective evolutionary algorithm (MOEA/D-ACD)[11].These algorithms can achieve solution set with uniform distribution, but this solution set may not be close to the real Pareto front. (d). Novel evolution scheme based MOEA. These algorithms introduce novel evolution mechanisms and heuristic algorithms into multi-objective optimizations. Examples include particle swarm optimization(PSO) based MOPSO algorithm[12] and fireworks explosion optimization(FEO) based MOFEO algorithm[13]. The introduction of new evolution cases provides more options for MOP and has attracted increasing attentions in multi-objective optimization field. (e). MOEA with mixed schemes. Examples include the hyper multi-objective evolutionary algorithm(HMOEA)[14] and the hybrid evolutionary algorithm with adaptive multi-population strategy (HMOEA-AMP)[15]. The mixed MOEA combines advantages of each MOEA or meta-heuristic algorithms to overcome intrinsic limitations of each MOEA or meta-heuristic algorithms, thus further enhancing the efficiency and effectiveness of searching in the solution domain.

Yang[16] proposed the firefly algorithm (FA), which is a Meta-Heuristic algorithm, based on simulations and simplification of population behavior of firefly. Compared with EA, FA exhibits advantages in concept, procedures, the parameters involved, and applicability and has been used for optimizations in different fields. Owing to its population searching characteristic and good performances, FA has been employed to solve complicated MOPs. Nevertheless, solutions of MOPs are essentially different from those of single objective optimization problems and FA needs to be expanded before being used for MOPs. Yang proposed the multi-objective firefly algorithm(MOFA)[17]. In this algorithm, the formula of firefly movement was improved so that the random terms in this formula decreased nonlinearly during iterations, and the current Pareto solution was determined using the weight ratio strategy. Leandro *et. al.*[18]introduced random coefficients that vary with the probability distribution into random terms in the formula of firefly movement and the diversity of archive populations was maintained using the crowding distance strategy. Tsai *et. al.* proposed the non-dominated sorting based MOFA[19]. In this algorithm, fireflies move randomly or approach the optimal individual in the current population in the absence of dominances, and population regeneration and archive maintenance were achieved by non-dominated sorting of the NSGA-II and crowding distance method. However, current MOFA needs to be further improved, although they are applicable to certain MOPs. This article proposes a MOFA based on compensation factor and elite learning (CFMOFA). Specifically, a compensation factor is added into the firefly learning formula for iterations. The non-inferior solutions generated during iterations were stored in the EA and a random EA particle was selected as the elite particle for population evolution. Introduction of a compensation facto helps to eliminate constraints by population so that Pareto optimal solutions can be approached rapidly. Additionally, firefly movements are guided by elite solutions in the archive so that convergence is accelerated while balancing global exploration and local mining. These two strategies are executed accordingly to synergistically improve the overall performance of MOFA.

Section 2 introduces the concept of MOP, Section 3 describes MOFA, Section 4 presents CFMOFA, Section 5 illustrates simulations, and Section 6 gives a summary.

2. MOP

Generally, the mathematical description of MOP is:

$$\begin{aligned} &\boldsymbol{x} = [x_1, x_2, \cdots, x_n] \\ &\min \boldsymbol{y} = f(x) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})] \end{aligned} \tag{1}$$

Where n is the quantity of decision variables, m is the quantity of objective functions, and $\boldsymbol{x}$ is the decision vector, $\boldsymbol{y}$ is the objective vector.

Unlike single objective optimization problems, MOPs involve multiple objective functions that are in mutual contradiction with each other and it is impossible to find a solution that optimizes all objective functions simultaneously. Hence, the objective functions should be balanced to obtain an optimal solution set. Therefore, concepts of Pareto dominate and Pareto optimal solution set have been proposed [20]. The following definitions are given:

Definition 1 Pareto dominate

If and only if $\begin{cases} \forall i \in \{1, 2, \cdots, m\}: f_i(\boldsymbol{x}_k) \le f_i(\boldsymbol{x}_l) \\ \exists j \in \{1, 2, \cdots, m\}: f_j(\boldsymbol{x}_k) < f_j(\boldsymbol{x}_l) \end{cases}$, $\boldsymbol{x}_k$ Pareto dominate $\boldsymbol{x}_l$ (denoted as $\boldsymbol{x}_k \prec \boldsymbol{x}_l$).

Definition 2 Pareto optimal solution or non-inferior solution

If and only if $\neg\exists \boldsymbol{x} \in D$ (D is the definition domain of objective functions.) $\boldsymbol{x} \prec \boldsymbol{x}^*$ at $\boldsymbol{x}^* \in D$ is defined as Pareto optimal solution.

Definition 3 Pareto optimal solution set or non-inferior solution

The combination of all Pareto optimal solutions of a MOP is the Pareto optimal solution set of this MOP.

Definition 4 Pareto optimal front or non-inferior front

The area formed by all objective vectors corresponding to Pareto optimal solutions of a MOP is the Pareto optimal front of this MOP.

3. MOFA

Fireflies generate luminescence to attract opposite sex. Yang proposed a random optimization firefly algorithm based on the illumination behaviors of firefly. In the firefly algorithms, luminance and attractiveness are two key factors. The luminance determines the location and moving direction of an individual, while the attractiveness determines the moving distance. The objective is optimized by continuous updates of luminance and attractiveness. To simplify the MOP, we assume that the absolute luminance (I_i) of firefly i equals to the value of objective functions at $\boldsymbol{x}_i$, namely $I_i = f(\boldsymbol{x}_i)$. The firefly attractiveness is a relative value that varies with the distance between firefly i and firefly j and is related to the absorption factor. Due to light absorption by transmission medium, luminance degrades as its distance from the light source increases. Attractiveness(β) is defined as:

$$\beta = \beta_0 e^{-\gamma r_{ij}^2} \tag{2}$$

Here in, β_0 is the maximum attractiveness, which is the attractiveness at $r_{ij} = 0$ (usually = 1), γ is the light absorption coefficient that describes attractiveness variation, which is usually $\gamma \in [0.01,100]$ . .. is the Euclidean distance from firefly i to firefly j .

For MOPs, attractions between individuals are usually determined using the Pareto dominate. With firefly i as the object, if firefly $j \prec i$, then firefly j exhibits higher luminescence and firefly i is attracted by firefly j and the movement formula is:

$$\boldsymbol{x}_i(t+1) = \boldsymbol{x}_i(t) + \beta(\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)) + \alpha \cdot \boldsymbol{\varepsilon}_i \tag{3}$$

where t is the iteration number, $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ are space locations of firefly i and firefly j , α is the step factor, $\boldsymbol{\varepsilon}_i$ is the random number vector that follows uniform distribution, Gaussian distribution or other distributions.

If firefly i is independent from domination by any other firefly, its location is updated using Eq(4):

$$\boldsymbol{x}_i(t+1) = \boldsymbol{g}^* + \alpha \cdot \varepsilon_i \tag{4}$$

α and ε_i are defined by Eq(3) and their values can be determined according to actual situations [17]. $\boldsymbol{g}^*$ is the optimal obtained by transforming multi-objective functions into single objective functions in the form of random weighted sum as in Eq(5).

$$\begin{cases} \psi(x) = \sum_{k=1}^{K} w_k f_k \\ \sum_{k=1}^{K} w_k = 1 \end{cases} \tag{5}$$

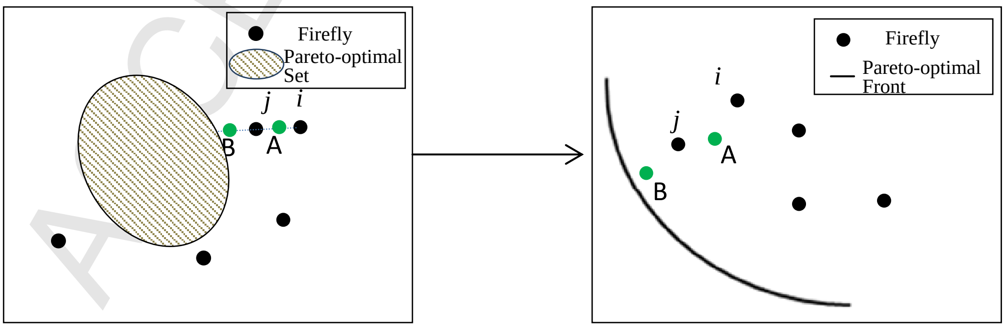
where K is the quantity of objective functions. Set $w_k \in (0,1)$ and each generation of w_k should be different.

At the end of each firefly evolution cycle, all non-inferior solutions were stored in the external archive (EA) and the EA is maintained so that all solutions are independent from dominance by other solutions. A maximum capacity of EA is designed to save calculation resources. If the quantity of non-inferior solutions exceeds that of EA, self-adaptive grid deletion method is employed to guarantee uniform distribution of Pareto non-inferior solutions obtained [21].

4. CFMOFA

4.1Compensation factor

Due to intrinsic limitations of Pareto optimality and defects of the MOFA learning formula, MOFA is readily exposed to local optimization. Therefore, a compensation factor was introduced into the firefly learning formulation to extend the learning step of firefly and overcome constraints by population, thus improving the performance of MOFA. A simplified dual-objective optimization problems model has been established, as shown in Fig. 1.



(a)Solution space (b)Objective space

Fig. 1 Simplified dual-objective optimization problems model.

Fig. 1(a) is the schematic of function solution domain. The dashed area refers to the Pareto optimal solution set. Fig. 1(b) illustrates the function objective domain. The dark curve is the Pareto optimal front and the dark dots refer to firefly particles. Generally, objective function values of solutions that are close to the Pareto optimal solution set are close to the Pareto optimal front. As observed, firefly $j \prec i$ and firefly i approaches firefly j. $\beta \in [0,1]$, firefly i reaches the middle point (Point A in Fig. 1).Although firefly j is superior to firefly i, it is still far from the optimal solution. Meanwhile, firefly i is constrained by firefly j, resulting in slow convergence, early maturing, and even local optimization. In the presence of a compensation factor (m, $m > 1$), firefly i overpasses firefly j (Point B in Fig. 1) and reaches a position closer to the Pareto optimal front. Based on that, the algorithm is improved to be:

$$\boldsymbol{x}_i(t+1) = \boldsymbol{x}_i(t) + m \cdot \beta(\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)) + \alpha \cdot \boldsymbol{\varepsilon}_i \tag{6}$$

Fig. 2 and Fig. 3 show trends of objective values during iterations in solving ZDT1 problems by MOFA with and without compensation factor, respectively. Herein, t refers to the iteration number, red circles refer to objective values corresponding to archive particles, black circles refer to objective values corresponding to firefly particles, and black segments refer to the real Pareto front. According to the definition of Pareto dominate, several Pareto optimal solutions in each iteration cycle will be the object of population learning. However, these Pareto optimal solutions are usually not the real optimal solutions. Indeed, the presence of these Pareto optimal solutions hinders population optimization and convergence. As observed, CFMOFA overcome constraints by population and are free from local optimizations and lead to improved Pareto solution set.

According to the definition of Pareto, there are several Pareto optimal solutions for each iteration to become the object of population learning. There are some differences between these Pareto optimal solutions and the real optimal solutions. These Pareto optimal solutions form a barrier that hinders the effect of population optimization and the speed of convergence. After adding the compensation factor, the algorithm quickly breaks through the constraints of the population, jumps out of the local optimum, and obtains a better Pareto solution set.

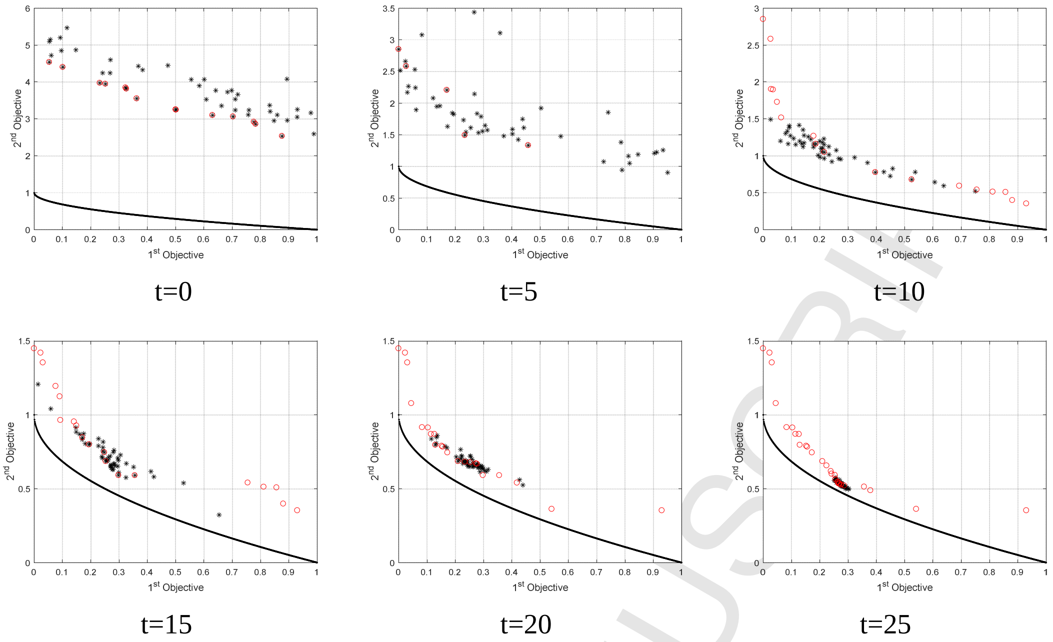


Fig. 2 Solving ZDT1 by MOFA.

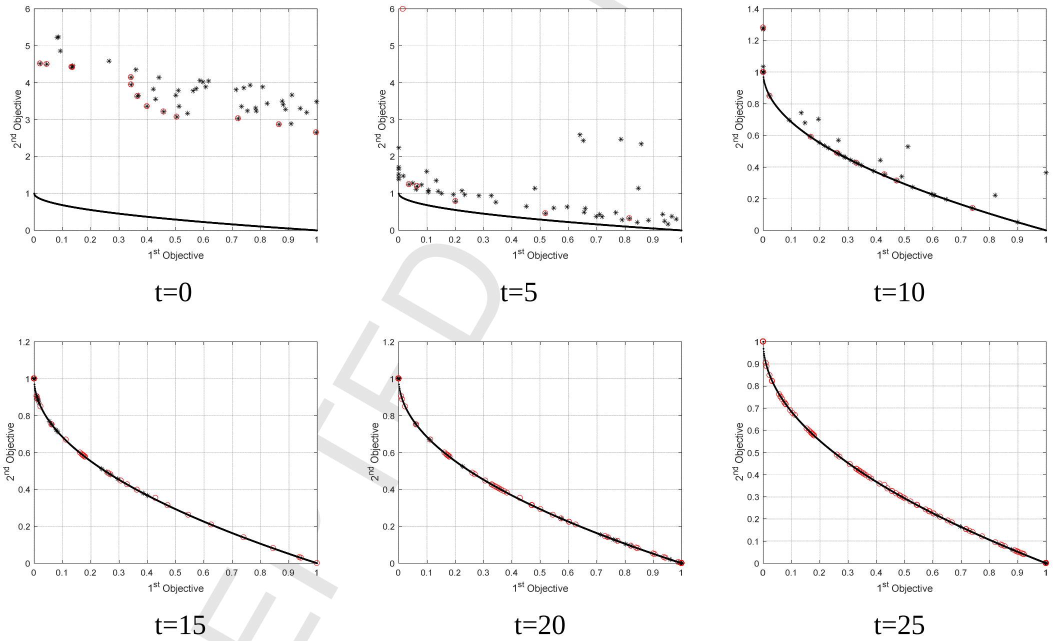


Fig. 3 Solving ZDT1 by CFMOFA.

4.2 Elite learning

In MOFA, non-inferior solutions are stored in the EA and particles in EA are elite particles, which contain significant population information that guides population evolutions. In other words, elite particles have not been fully utilized, resulting in slow population evolutions and poor diversity. In this study, elite particles are introduced into Eq(4) so that the searching area is increased and the distribution of solutions in the set is wide and uniform:

$$\boldsymbol{x}_i(t+1) = c_1\boldsymbol{g}^* + c_2\boldsymbol{leader} + \alpha \cdot \varepsilon_i \tag{7}$$

$$\boldsymbol{leader} = \boldsymbol{x}_i', \boldsymbol{x}_i' \in EA \tag{8}$$

Where $c_1, c_2 \in [0,1]$ and $\sum c_i = 1$, $\boldsymbol{leader}$ is a random firefly in the EA and its physical meaning is described in Fig. 4.

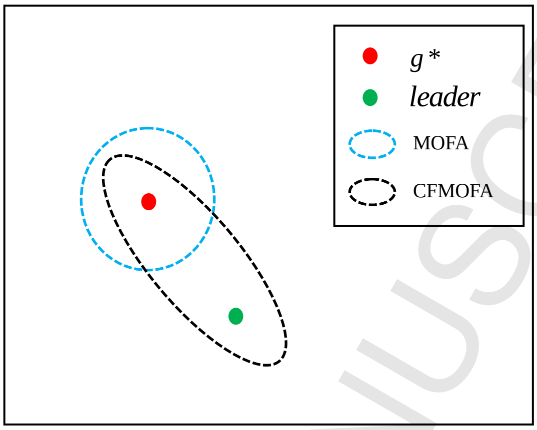


Fig. 4 Searching area of CFMOFA.

Here in, the solid area is the Pareto optimal solution set and the dashed area is the searching area; red dots are g^* and green dots are elite particles. As observed, MOFA searches around g^* only and the solutions obtained are limited. In CFMOFA, elite particles ($leader$) in the EA are employed and the searching area is extended. As a result, the directions of firefly movement are increased, as well as the range of firefly movement. In this way, the fitting of CFMOFA is enhanced.

4.3 Procedures

Table 1 summarizes procedures of CFMOFA that combines compensation factor and elite particle learning.

Table 1.CFMOFA.

Initialize the population of fireflies and set the parameter.
Calculate the fitness values of each firefly.
while(t<MAX_GEN) do
    for $i = 1:n$ do
        for $j = 1:n$ do
            if $\boldsymbol{f}(\boldsymbol{x}_j) \prec \boldsymbol{f}(\boldsymbol{x}_i)$ do
                Move firefly $\boldsymbol{x}_i$ towards $\boldsymbol{x}_j$ according to Eq.(6)
            end if

if no one dominates $\boldsymbol{f}(\boldsymbol{x}_i)$
 Move firefly $\boldsymbol{x}_i$ using Eq.(7)
end if
end for
end for
Update and save Pareto non-dominates solutions in EA
Gen++
end while
Post-process results and visualization

5. Results and discussion

5.1 Test functions

In this study, CFMOFA was used for 11 typical MOPs [22-24], as shown in Table 2. Herein, the first eight test functions have two objective functions each and the last 3 test functions have 3 objective functions each. The variable dimensions of SCH and SCH2 functions is 1, the variable dimensions of Viennet1-Viennet3 are 2, the variable dimension of KUR is 3, the variable dimensions of ZDT4and ZDT6 are 10,andthe variable dimensions of ZDT1-ZDT3 are 30. Moreover, the Pareto optimal fronts of this test function set show different configurations, including convex configuration (SCH and ZDT1), discontinuous configuration (KUR and ZDT3), and mixed degeneration configuration (Viennet3), and the performances of the tested algorithms for different problems can be fully investigated.

Table 2.Test function set for MOP.

Problem	Definition	Constraints
SCH	$f_1(x)=x^2$, $f_2(x)=(x-2)^2$	$-10^5 \le x \le 10^5$
SCH2	$f_1(x)=\begin{cases}-x, x\le 1\\ -2+x, 1<x\le 3\\ 4-x, 3<x\le 4\\ -4+x, x>4\end{cases}$, $f_2(x)=(x-5)^2$	$-5 \le x \le 10$
KUR	$f_1(x)=\sum_{i=1}^{n-1}(-10e^{(-0.2)*\sqrt{x_i^2+x_{i+1}^2}})$, $f_2(x)=\sum_{i=1}^{n}(\lvert x_i\rvert^{0.8}+5\sin(x_i)^3)$	$n=3, -5\le x_i \le 5, i=1,\cdots,n$

ZDT1	$f_1(x)=x_1,\ f_2(x)=g(x)[1-\sqrt{x_1/g(x)}],$ $g(x)=1+9\sum_{i=2}^{n}x_i/(n-1)$	$n=30, 0\le x_i\le 1, i=1,\cdots,n$
ZDT2	$f_1(x)=x_1,\ f_2(x)=g(x)[1-(x_1/g(x))^2],$ $g(x)=1+9\sum_{i=2}^{n}x_i/(n-1)$	$n=30, 0\le x_i\le 1, i=1,\cdots,n$
ZDT3	$f_1(x)=x_1,$ $f_2(x)=g(x)[1-\sqrt{x_1/g(x)}-x_1\sin(10\pi x_1)/g(x)],$ $g(x)=1+9\sum_{i=2}^{n}x_i/(n-1)$	$n=30, 0\le x_i\le 1, i=1,\cdots,n$
ZDT4	$f_1(x)=x_1,\ f_2(x)=g(x)[1-\sqrt{x_1/g(x)}],$ $g(x)=1+10(n-1)+\sum_{i=2}^{n}[x_i^2-10\cos(4\pi x_1)]$	$n=10, 0\le x_1\le 1, -5\le x_i\le 5, i=2,\cdots,n$
ZDT6	$f_1(x)=1-e^{(-4x_1)}\sin^6(6\pi x_1),$ $f_2(x)=g(x)[1-(f_1(x)/g(x))^2],$ $g(x)=1+9[\sum_{i=2}^{n}x_i/(n-1)]^{0.25}$	$n=10, 0\le x_i\le 1, i=1,\cdots,n$
Viennet1	$f_1(x,y)=x^2+(y-1)^2,$ $f_2(x,y)=x^2+(y+1)^2+1,$ $f_3(x,y)=(x-1)^2+y^2+2$	$-2\le x,y\le 2$
Viennet2	$f_1(x,y)=(x-2)^2/2+(y+1)^2/13+3,$ $f_2(x,y)=(x+y-3)^2/36+(2y-x)^2/8-17,$ $f_3(x,y)=(x+2y-1)^2/175+(2y-x)^2/17-13$	$-4\le x,y\le 4$
Viennet3	$f_1(x,y)=0.5(x^2+y^2)+\sin(x^2+y^2),$ $f_2(x,y)=(3x-2y+4)^2/8+(x-y+1)^2/27+15,$ $f_3(x,y)=1/(x^2+y^2+1)-1.1e^{(-x^2-y^2)}$	$-3\le x,y\le 3$

5.2 Comparison with conventional algorithms

In this study, five typical multi-objective optimization algorithms (MOPSO[12], NSGA-III[25], MOEA/D[10], PESA-II[9], and MOFA[17]) were analyzed and Table 3 summarizes parameters involved. All algorithms were generated using Matlab 2016b and executed on the same platform. In all cases, the maximized iteration number (MAX_GEN) was300, the population size (n_{Pop}) was 50, and the maximized EA number (n_{Rep}) was 200. Meanwhile, all algorithms were run on each test function for 30 times and the average was regarded as the value.

Table 3.Parameters of all algorithms.

Algorithm	Parameter setting	Reference
MOPSO	$w=0.4, c_1, c_2=Rand[0,1]$	Colleo 2004[12]

NSGA-III	$pCrossover = 0.5,$ $nCrossover = 2 * round[pCrossover * nPop / 2]$	Mkaouer 2015[25]
MOEA/D	$\gamma = 0.5$	Zhang 2007[10]
PESA-II	$pCrossover = 0.5, \beta = 1, \gamma = 2,$ $nCrossover = 2 * round[pCrossover * nPop / 2]$	Corne 2001[9]
MOFA	$\alpha = 0.2, \beta_0 = 1, \gamma = 1$	Yang 2013[17]
CFMOFA	$\alpha = 0.2, \beta_0 = 1, \gamma = 1, m = 2$	–

The real Pareto optimal fronts of all MOPs involved were known. The generation distance(GD), the spacing (SP), and the maximum spread (MS), all of which reflect the performances of multi-objective optimization algorithms in certain way, were used as evaluation parameters [26-28]. Detailed description is as follows.

(1) Generation Distance(GD)

$$GD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \tag{9}$$

Where n is the number of non-inferior solutions obtained by the algorithm, and d_i is the minimum distance from the i th solution to the real Pareto optimal solution set. If $GD = 0$, the resulting non-inferior solution belongs to the real Pareto optimal solution set. It reflects the degree of approximation of the optimal solution set obtained by the algorithm and the real Pareto optimal solution set.

(2) Spacing (SP)

$$SP = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\overline{d'} - d_i')^2} \tag{10}$$

Where n is the number of non-inferior solutions obtained by the algorithm, d_i' is the distance between the i th non-inferior solution corresponding to the target vector and the nearest target vector, and $\overline{d'}$ is the mean of d_i'. If $SP = 0$, it means that the leading edge corresponding to the obtained non-inferior solution is completely uniform. This indicator reflects the uniformity of the frontier.

(3)Maximum Spread (MS)

$$MS = \sqrt{\frac{1}{k}\sum_{l=1}^{k}\delta_l}, \delta_l = \left(\frac{\min(f_l^{\max}, F_l^{\max}) - \max(f_l^{\min}, F_l^{\min})}{F_l^{\max} - F_l^{\min}}\right)^2 \tag{11}$$

Where f_l^{max} and f_l^{min} are the maximum and minimum values of the function value of the l th objective function of the Pareto optimal frontier obtained by the algorithm, $F_l \max$ and $F_l \min$ are the maximum and minimum values of the function value of the l th objective function of the real Pareto frontier . k is the number of objective functions. MS represents the coverage of the Pareto frontier of the algorithm to the true Pareto front, and $MS=1$ indicates that the true Pareto frontier is completely covered by the Pareto frontier obtained by the algorithm.

Table 4, 5, and 6 summarizes means and standard deviations of GD, SP, and MS in all algorithms for the 11test functions. The significances of all algorithms were determined using double tail t tests with horizon of 5%. Herein, + means the proposed algorithm is superior to the reference algorithm, − means the proposed algorithm is inferior to the reference algorithm, ~ means the proposed algorithm is similar to the reference algorithm. For instance, w/t/l refers to the case where the CFMOFA is superior in w functions, similar to t functions, and inferior to one function. The Score refers to the result of t tests (superior = 1, similar = 0, inferior = -1) and the bold parts denote optimal in this test function. Table 7 summarizes the number of optimal values that each algorithm obtains on each of the three evaluation functions, the more the number, the better the algorithm. which is convenient for the performance of the comprehensive evaluation algorithm.

Table 4.Comparison of algorithms in GD.

Problem		MOPSO	NSGA－III	MOEA/D	PESA－II	MOFA	CFMOFA
SCH	Mean	6.74E-04	6.44E-04	**5.85E-04**	6.67E-04	6.70E-04	6.75E-04
	Std.	1.56E-05	4.40E-05	4.39E-05	4.01E-05	4.29E-05	1.87E-05
	T-test	~	~	−	~	~	
SCH2	Mean	3.07E-04	2.41E-04	**6.28E-05**	3.29E-04	2.21E-04	2.34E-04
	Std.	2.82E-04	9.48E-05	1.81E-05	2.48E-04	1.61E-04	1.81E-04
	T-test	+	~	−	+	~	
KUR	Mean	1.31E-01	6.01E-02	9.86E-02	5.17E-02	9.98E-02	**1.99E-02**
	Std.	3.34E-02	4.33E-03	6.01E-02	3.98E-03	3.01E-02	9.21E-03
	T-test	+	+	+	+	+	
ZDT1	Mean	8.16E-04	9.84E-03	2.24E-02	8.25E-03	2.36E-03	**3.53E-05**
	Std.	1.97E-03	1.67E-03	1.87E-02	3.27E-03	8.57E-04	8.51E-06
	T-test	+	+	+	+	+	
ZDT2	Mean	8.37E-01	7.30E-03	1.08E-01	1.45E-02	6.19E-03	**3.02E-05**
	Std.	2.23E+00	2.30E-03	3.95E-02	4.92E-03	5.35E-03	2.21E-06
	T-test	+	+	+	+	+	
ZDT3	Mean	3.27E-04	1.97E-02	3.26E-02	8.88E-03	1.24E-03	**8.37E-05**
	Std.	1.76E-03	5.72E-03	2.79E-02	2.82E-03	6.31E-03	9.91E-06

	T-test	+	+	+	+	+	
ZDT4	Mean	2.84E-01	1.84E-01	7.45E-01	1.69E-01	5.37E-02	**2.04E-04**
	Std.	4.63E-01	1.47E-01	1.35E+00	3.22E-02	2.21E-02	2.97E-05
	T-test	+	+	+	+	+	
ZDT6	Mean	2.77E-02	**3.61E-05**	3.43E-01	2.27E-03	2.00E-02	1.54E-04
	Std.	5.22E-02	1.41E-05	2.09E-01	3.51E-03	1.31E-01	4.41E-04
	T-test	+	−	+	+	+	
Viennet1	Mean	8.32E-03	5.21E-03	**1.65E-03**	5.89E-03	7.66E-03	7.63E-03
	Std.	7.92E-04	9.60E-04	5.21E-04	7.65E-04	1.61E-04	6.19E-04
	T-test	+	−	−	−	~	
Viennet2	Mean	6.38E-04	4.46E-04	**3.48E-04**	5.46E-04	1.02E-03	6.24E-04
	Std.	2.49E-04	4.83E-04	1.50E-03	3.72E-04	7.68E-04	1.34E-04
	T-test	~	−	−	−	+	
Viennet3	Mean	3.11E-04	4.35E-04	4.96E-03	**2.62E-04**	4.19E-04	**2.53E-04**
	Std.	8.49E-05	2.60E-04	1.16E-02	1.05E-04	1.75E-04	4.53E-05
	T-test	+	+	+	~	+	
T-test	w/t/l	9/2/0	6/2/3	7/0/4	7/2/2	8/3/0	
Score		7	3	3	5	5	

Table 5.Comparison of algorithms in SP.

Problem		MOPSO	NSGA－III	MOEA/D	PESA－II	MOFA	CFMOFA
SCH	Mean	2.23E-02	3.20E-02	**9.48E-03**	2.24E-02	2.90E-02	2.24E-02
	Std.	1.54E-03	9.83E-03	1.84E-02	2.91E-03	6.92E-03	1.87E-05
	T-test	~	+	−	~	+	
SCH2	Mean	4.18E-02	3.24E-02	**2.27E-02**	4.47E-02	5.41E-02	3.79E-02
	Std.	8.77E-03	5.00E-03	8.02E-02	1.50E-02	1.44E-02	5.72E-03
	T-test	+	−	−	+	+	
KUR	Mean	1.09E-01	7.47E-02	**2.80E-03**	8.54E-02	9.83E-02	7.66E-02
	Std.	4.59E-03	1.41E-02	1.19E-02	5.65E-03	8.81E-02	4.48E-02
	T-test	+	~	−	+	+	
ZDT1	Mean	5.19E-03	1.72E-02	3.62E-03	5.81E-03	**1.38E-03**	6.48E-03
	Std.	6.87E-04	5.63E-03	2.02E-03	1.59E-03	4.74E-03	7.28E-04
	T-test	−	+	−	−	−	
ZDT2	Mean	3.39E-03	1.11E-02	3.62E-03	6.72E-03	**2.11E-03**	5.99E-03
	Std.	2.50E-03	6.36E-03	4.31E-03	2.62E-03	4.67E-03	5.94E-04
	T-test	−	+	−	+	−	
ZDT3	Mean	4.32E-03	2.33E-02	**1.96E-03**	8.36E-03	1.63E-02	7.88E-03
	Std.	1.76E-03	1.15E-02	3.09E-03	8.97E-03	1.78E-02	1.90E-03
	T-test	−	+	−	+	+	
ZDT4	Mean	7.28E-03	5.53E-03	**4.96E-03**	1.19E-02	1.62E-01	6.49E-03
	Std.	3.03E-03	3.59E-03	1.14E-02	2.35E-01	1.88E-01	7.81E-04
	T-test	+	−	−	+	+	
ZDT6	Mean	5.45E-03	4.18E-02	**1.91E-03**	6.67E-03	4.21E-02	7.51E-03
	Std.	2.88E-03	4.01E-02	5.63E-03	6.41E-03	3.18E-02	5.35E-03

	T-test	−	+	−	−	+	
Viennet1	Mean	9.32E-02	1.17E-01	**1.09E-02**	9.77E-02	1.41E-01	9.08E-02
	Std.	5.47E-03	2.63E-02	1.46E-02	1.04E-02	1.09E-02	6.60E-03
	T-test	~	+	−	~	+	
Viennet2	Mean	1.26E-02	2.71E-02	**2.51E-03**	1.21E-02	4.01E-02	1.69E-02
	Std.	2.41E-03	2.52E-02	5.59E-02	1.87E-03	1.44E-02	4.25E-03
	T-test	−	+	−	−	+	
Viennet3	Mean	4.46E-02	2.88E-02	**2.11E-02**	4.35E-02	3.97E-02	4.24E-02
	Std.	8.24E-03	2.05E-02	7.23E-02	5.79E-03	2.77E-02	5.75E-03
	T-test	~	−	−	~	~	
T-test	w/l/t	3/3/5	7/1/3	0/0/11	5/3/3	8/1/2	
Score		-2	4	-11	2	6	

Table 6.Comparison of algorithms in MS.

Problem		MOPSO	NSGA－III	MOEA/D	PESA－II	MOFA	CFMOFA
SCH	Mean	**9.98E-01**	9.69E-01	7.33E-01	9.87E-01	**9.97E-01**	**9.97E-01**
	Std.	2.57E-03	3.77E-02	7.57E-02	6.98E-03	3.46E-03	2.31E-03
	T-test	~	+	+	+	~	
SCH2	Mean	9.91E-01	9.84E-01	5.29E-01	9.83E-01	**9.95E-01**	**9.96E-01**
	Std.	1.26E-02	1.57E-02	2.81E-02	1.51E-02	7.71E-03	3.21E-03
	T-test	+	+	+	+	~	
KUR	Mean	9.35E-01	**9.55E-01**	4.73E-01	8.72E-01	9.43E-01	**9.65E-01**
	Std.	4.45E-02	4.14E-03	2.28E-01	8.72E-02	1.99E-02	1.97E-02
	T-test	+	~	+	+	+	
ZDT1	Mean	9.96E-01	6.28E-01	8.34E-01	9.41E-01	9.43E-01	**1.00E+00**
	Std.	1.41E-02	9.69E-02	8.04E-02	1.71E-02	1.38E-02	0.00E+00
	T-test	+	+	+	+	+	
ZDT2	Mean	6.85E-01	3.29E-01	8.09E-01	8.40E-01	9.23E-01	**1.00E+00**
	Std.	3.42E-01	9.36E-02	1.28E-01	7.94E-03	8.78E-02	0.00E+00
	T-test	+	+	+	+	+	
ZDT3	Mean	**9.81E-01**	7.08E-01	7.09E-01	9.46E-01	**9.88E-01**	**9.85E-01**
	Std.	1.94E-02	1.28E-01	1.17E-01	1.10E-02	9.02E-03	5.25E-02
	T-test	~	+	+	+	~	
ZDT4	Mean	1.87E-01	8.13E-01	3.61E-01	1.06E-01	7.65E-01	**1.00E+00**
	Std.	2.84E-01	2.56E-01	3.57E+00	4.00E-01	5.60E-03	0.00E+00
	T-test	+	+	+	+	+	
ZDT6	Mean	**1.00E+00**	9.17E-01	2.81E-01	5.96E-01	**1.00E+00**	**1.00E+00**
	Std.	2.69E-04	1.95E-01	1.02E+00	1.93E-01	1.77E-05	4.34E-04
	T-test	~	+	+	+	~	
Viennet1	Mean	**8.63E-01**	7.78E-01	6.57E-01	8.25E-01	**8.61E-01**	**8.60E-01**
	Std.	3.65E-03	5.03E-02	5.76E-02	2.93E-02	5.45E-03	6.63E-03
	T-test	~	+	+	+	~	
Viennet2	Mean	9.73E-01	5.16E-01	5.44E-01	9.40E-01	9.33E-01	**9.81E-01**
	Std.	2.17E-02	3.03E-01	7.03E-02	4.19E-02	5.97E-02	1.95E-02

	T-test	+	+	+	+	+	
Viennet3	Mean	**9.94E-01**	8.03E-01	4.84E-01	8.40E-01	8.70E-01	**9.93E-01**
	Std.	4.95E-03	2.56E-02	2.54E-01	7.94E-03	7.34E-02	6.76E-03
	T-test	~	+	+	+	+	
T-test	w/t/l	5/6/0	10/1/0	11/0/0	11/0/0	6/5/0	
Score		5	10	11	11	6	

Table 7.Summary of advantages of different algorithms.

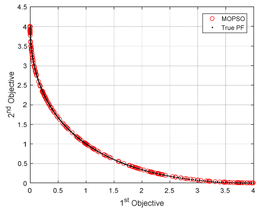
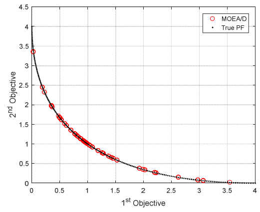
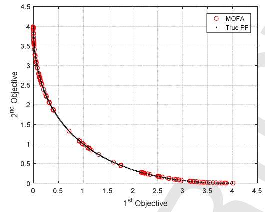
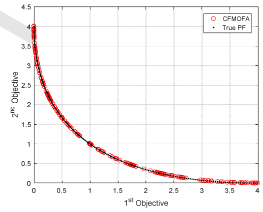
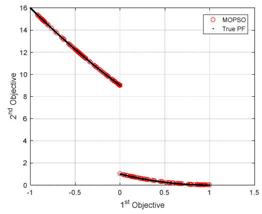
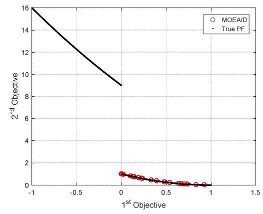
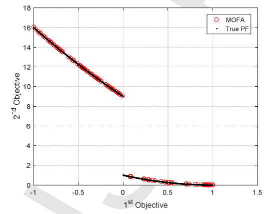
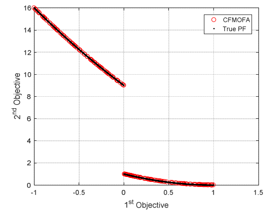
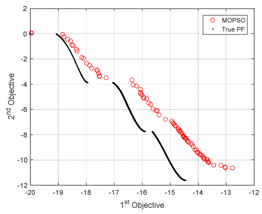
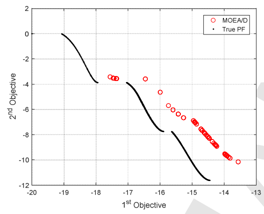
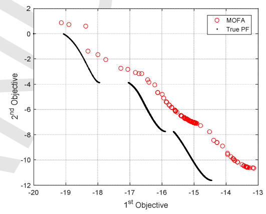
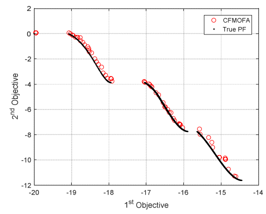
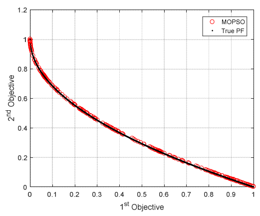
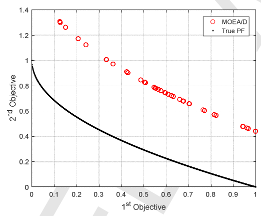
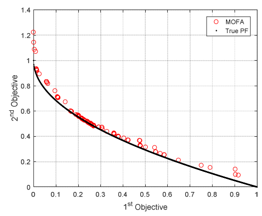
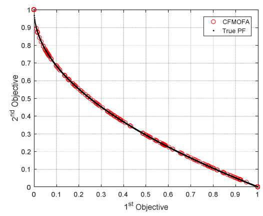
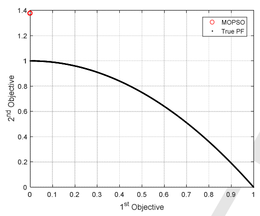
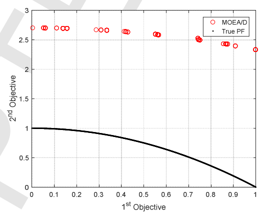
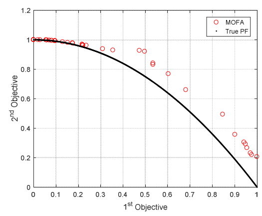
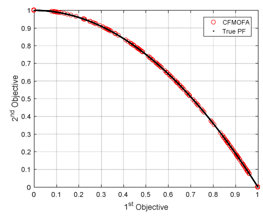
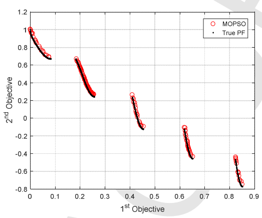
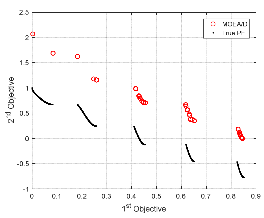
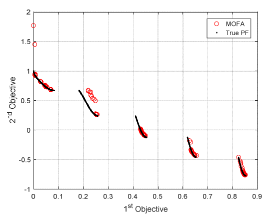
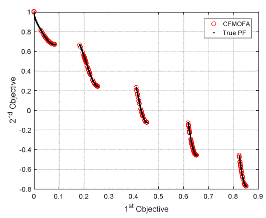
Evaluation indexes	MOPSO	NSGA-III	MOEA/D	PESA-II	MOFA	CFMOFA
GD	0	1	4	1	0	6
SP	0	0	8	0	2	0
MS	5	1	0	0	5	11
Total	5	2	12	1	7	17

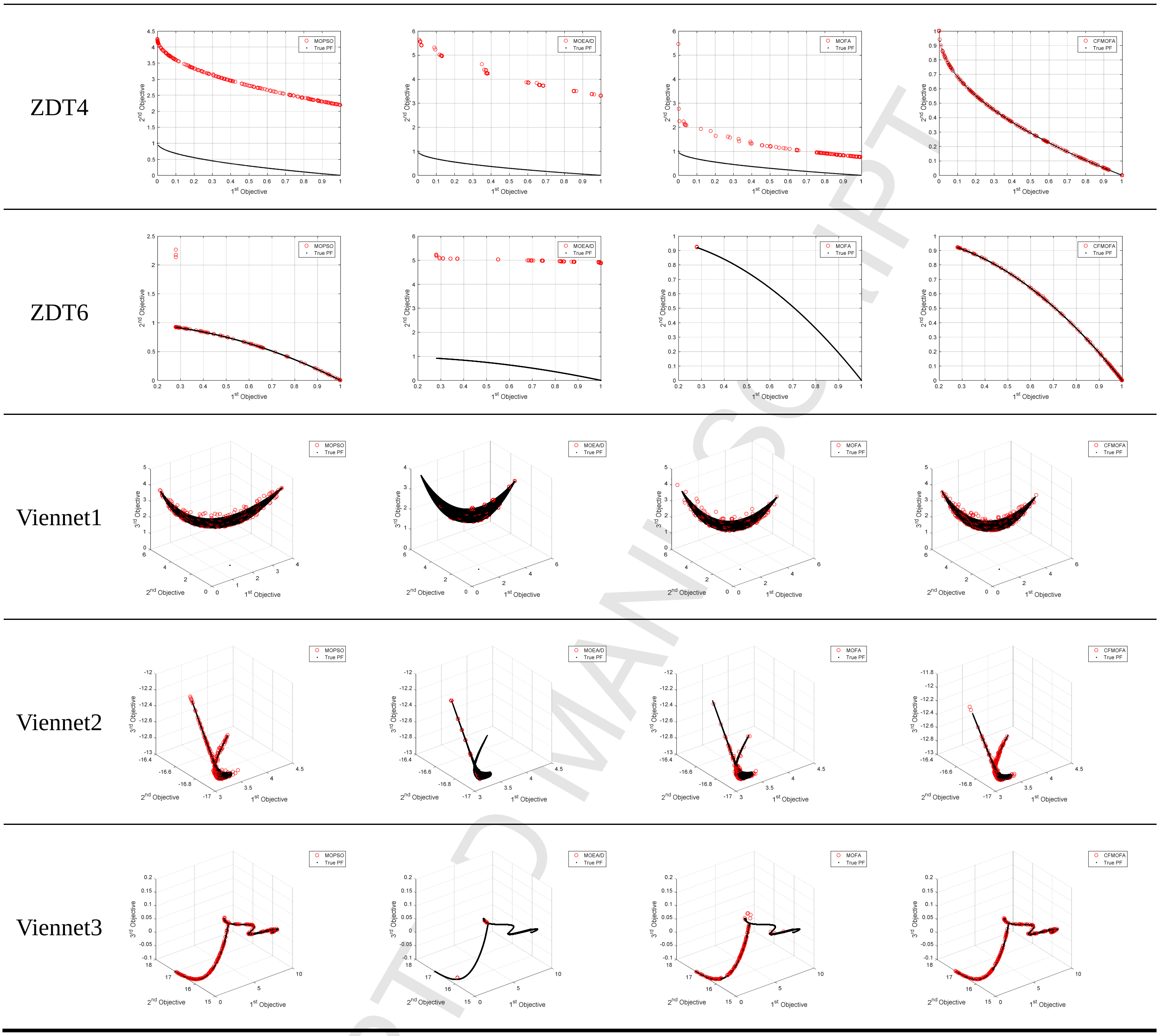
As observed, the CFMOFA exhibits significant advantages over other algorithms in GD (positive scores in all cases), especially for the ZDT1-ZDT6test function. In SP, the CFMOFA is inferior to the MOPSO and the MOEA, while superior to the other three algorithms. The differential evolution of the MOEA/D led to uniform distribution of Pareto front obtained. Indeed, the MOEA/D generated eight optimal out of eleven test functions. In SP, the CFMOFA generated optimal in all eleven test functions, in virtue of its wide searching range. Overall, the CFMOFA generated optimal for 17 times out of all 33 cases, the MOEA/D generated optimal for 17 times out of all 33 cases, and the PESA-II generated optimal for one time only. Despite its great performance in SP, the MOEA/D concentrates on distribution uniformity of solutions and shows negligible advantages in accuracy and distribution range of solutions.

In order to show the advantages and disadvantages of the algorithm more intuitively, Table 8 selects several typical and well-performing algorithms, including MOPSO, MOEA/D, MOFA and CFMOFA. The table shows the effect of the algorithm in 11 test functions. The distribution of the Pareto non-inferior solution and the real Pareto frontier is clearly shown. The dark dots refer to real Pareto front and the red circles refer to the Pareto front generated by the algorithm. Compare the graphs to visually show the pros and cons of the algorithm.

As observed, the results are consistent with data in Table 4-7. The Pareto fronts generated by the CFMOFA were close to real Pareto fronts in all eleven cases and its goodness of fit is higher than other algorithms. The MOEA/D exhibited poor goodness of fit.

Table 8.Fitting of Pareto fronts of certain algorithms.

MOPs	MOPSO	MOEA/D	MOFA	CFMOFA
SCH				
SCH2				
KUR				
ZDT1				
ZDT2				
ZDT3				



In practical problem solving, the priority is an optimal solution set closest to the real Paretofront and with optimized overall performance, instead of solution distribution uniformity. CFMOFA makes clever use of elite particle guidance and adds compensation factors to improve the convergence speed of the algorithm and improve the diversity and accuracy of non-inferior solution sets. Therefore, it can be concluded that the CFMOFA is superior to other reference algorithms tested in this study.

5.3Comparison with improved algorithms

To further investigate performances of the proposed algorithm, the algorithm was compared with some recent multi-objective optimization algorithms(e.g., SMSEMOA[29], TVMOPSO[30], DMSPSO[31], and HMOEA-AMP[15]). Test functions reported previously were employed[15] and the inverted generation distance(IGD)was used for algorithm evaluation [32]. In all cases, the evaluation was repeated for 10000 times and the maximized EA number (n_{Rep}) was 100. Meanwhile, all algorithms were run on each test function for 30 times and the average was regarded as the value. The results are shown in Table 9.

Table 9.IGD results of all algorithms involved.

Instances	NSGA-II	MOEA/D	SMSEMOA	TVMOPSO	DMSPSO	HMOEA-AMP	CFMOFA
FON	0.0054	0.0052	0.0053	0.0182	0.0102	0.0041	**0.0038**
KUR	0.0432	0.0485	0.0423	0.4301	0.2501	**0.0225**	0.2456
ZDT1	0.1651	0.3912	0.1213	0.0145	0.1699	0.013	**0.0053**
ZDT2	0.6152	0.4517	0.562	0.05	0.2209	0.0135	**0.0063**
ZDT3	0.1277	0.1924	0.1024	0.0573	0.198	0.0334	**0.0064**
ZDT4	0.882	2.1224	1.224	0.4299	1.4066	0.048	**0.0052**
ZDT6	1.7812	0.0612	0.8963	0.0036	0.0575	**0.0032**	0.0066
DTLZ2	0.6723	0.6702	0.6625	0.7856	0.7936	0.6448	**0.0648**
DTLZ4	0.1863	0.7124	0.331	0.4157	0.4536	**0.0792**	0.9253
DTLZ5	0.344	0.3104	0.3103	0.3593	0.4383	**0.2856**	0.5721
DTLZ6	4.7216	3.5382	3.8521	1.0955	3.5277	**0.3122**	0.8285
DTLZ7	0.13	0.3321	0.1274	0.0768	0.2202	0.0763	**0.0535**

Herein, the data in bold are optimal of all involved algorithms in corresponding test functions. As observed, the CFMOFA dominated in seven out of 12 test functions, especially in ZDT1-ZDT4 problems (one order of significance higher than other algorithms). However, the CFMOFA didn't show significant advantages over other algorithms in triple-objective continuous non-convex problems such as DTLZ5-DTLZ6. Because on the non-convex problem, the Pareto solution set has less effect on the movement of the firefly, so the effect of the compensation factor is not obvious, and the optimization effect is limited. In summary, the CFMOFA shows advantages in optimization over other algorithms and is reliable.

According to the results obtained, the CFMOFA overcome constraints by population, thus exhibiting improved resistance to local optimization. The CFMOFA shows good fitting for most testing problems and the results obtained are consistent with theoretical values. Additionally, archive elite particles are involved in population evolutions and the searching area is significantly enhanced, resulting in widened distribution of solutions.

6. Conclusions

In this article, we propose CFMOFA by introducing compensation factor into MOFA. The

CFMOFA exhibits improved resistance to local optimization and solution accuracy. The archive elite particles are involved in population evolutions to increase the searching area so that the Pareto non-inferior solution set obtained is close to the real Pareto front and the solution distribution is wide. The proposed algorithm was compared with other algorithms using various evaluation parameters. The results indicated that the CFMOFA is a multi-objective optimization algorithm with excellent performance and efficiency. Future studies may focus on improvements of uniformity of solution distribution by differential evolution of the MOEA/D.

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Author Biography

*Li Lv r*eceived an MA degree in Computer Architecture form Jiangxi Normal University, Nanchang China. She has been an assistant professor in the School of Information Engineering, Nanchang Institute of Technology, since 2013. Her research interests include evolutionary computation, embedded system, and images processing.

Jia Zhao received an MA degree in Technology of Computer Applications form Nanchang Hong Kong University, Nanchang,China. He has been a professor in the School of Information Engineering, Nanchang Institute of Technology, since 2016. His research interests include evolutionary computation, swarm intelligence, wireless sensor networks, images processing, and big data.

Jiayuan Wang is currently working toward his MSc degree at the School of Information Engineering, Nanchang Institute of Technology, China. His research interest focuses on swarm intelligence.

Tanghuai Fan received a BA degree, postgraduate certificate, and an MA degree from the Nanchang University, Nanchang China, in 1983, 1991, and 2010, respectively, and PhD degree from Hohai University, Nanjing, China. He has been a professor in the School of Information Engineering, Nanchang Institute of Technology, since 2009. His research interests include wireless sensor networks, signal gathering and processing, telemetry and remote control, and information fusion.

Author Photo

Li Lv

Jia Zhao

Jiayuan Wang

Tanghuai Fan

Highlights

Aimed at early maturing and poor accuracy of multi-objective firefly algorithms, we propose a multi-objective firefly algorithm based on compensation factor and elite learning (CFMOFA). Based on iterations by introducing a compensation factor into the firefly learning formula, constraints by population can be overcome and the Pareto optimal solution can be approached in a reduced period. The non-inferior solutions produced in iterations were stored in the external archive and a random external archive particle was employed as the elite particle for population evolution. In this way, the detection range of firefly was extended and diversity and accuracy of non-inferior solution set were enhanced. The conventional algorithms, the improved algorithms and the proposed multi-objective optimization algorithm were tested and compared with each other. The results indicated great advantages of the proposed algorithm in convergence, diversity, and robustness and the proposed algorithm is an effective multi-objective optimization method.