# Quantum Communication using Quantum Repeater

***Abstract*—In 1687 Sir. Isaac Newton published his book *Philosophiæ Naturalis Principia Mathematica* for the first time. As the time goes by, we have employed the laws of nature, the Classical Physics, to build sky scraper, suspension bridge on the water's surface, satellite and other great applications. Quantum mechanics is the bigger picture which Classical Mechanics can be derived from it. Quantum Mechanics passed a long road to reach this point and now in its way to inter useful application.**

***Keywords*— communication, entanglement, Qubit, repeater**

## I. Introduction

From Latin communicare, the word communication is an ancient concept in the human history. From the early smoke signals into optical fibers, we were always searching for a fast and sufficient way of communication.

Communication is the tool for exchanging information – the only concept Which is conserve or should be! – and through its history the basic unit of information has been changed.

Nowadays our famous unit of information is "Bit". When you think of information theory, the picture in your mind would be a hand full of 0 and 1s carrying a concept called information. The early ideas of communication in terms of bits belongs to Samuel Morse. The idea behind Morse code was to convert each alphabet letter into a series of ditz and dahs (dots and dashes). So the substitute for Samuel would be • • • • – – – • • – • • – • • , quite long isn't it? Morse dominated normal communication in the case of confidentiality, one of the most important feature of every communication. We will discuss more about confidentiality later.

The beginning of bits carved the way for the birth of digital world. The bit is the representation of a state which is either "0" or "1", "on" or "off", "light" or "darkness" … or simply any two level logical state. The bitwise communication was faster and more efficient and began to rule our world and make it the digital world around us.

Open your browser, Google the word bit. Open Wikipedia page and you would probably see this sentence :"As a unit of information, the bit is also known as a *Shannon*, named after Claude E. Shannon."

Claude Shannon –the father of information theory – made a revolution in communication and information world. If I want to talk about Shannon works in Communication I have to write a whole essay but as we are concerned about confidentiality in communication let's get to the point.

To meet confidentiality, the basic solution would be encryption. Nowadays information communication is lied upon encryption algorithms and they themselves rely on mathematics, especially the problem of factoring.

Factoring a number into its prime roots is one of the hardest mathematical computation inquiries. The truth is that we actually base our security on the amount of effort, time and energy we possess to solve a NP problem. RSA cryptosystem which is now widely applied for asymmetric encryption is based on the factoring problem. Shor's algorithm for integer factorization, shows the advantages of Quantum Computers over Classical Computer.

Shor algorithm will be Completed

Another algorithm that shows the advantages of quantum computer over classical computer is the algorithm called Bernstein-Vazirani algorithm.

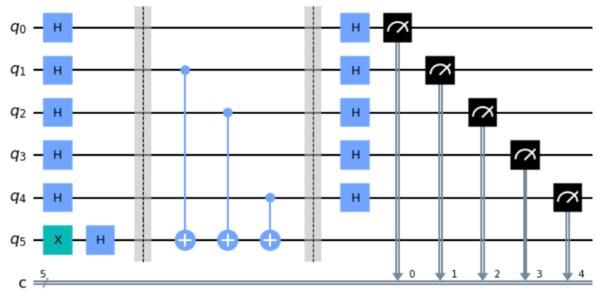



Figure 1. Bernstein-Vazirani algorithm

This algorithm is used to find a number in a black box. Consider black box as a hardware which you can connect input and get the output out of it but you cannot see the inside.

Consider a "n" bit number is stored in that hardware, the goal is to find that number without looking into the hardware.

Classical scheme:

Consider we have a 5-bit number - - - - -. what we can do is to do "AND" operation of the output with {10000,01000,00100,00010,00001}. Here after 5 trial, a classical scheme can guess the number correctly.

Quantum scheme:

The difference here is that we can (in one shot) guess the number correctly and we do that by considering all the possibilities of the result. If our inputs were in the state (|0> + |1>) / √2 then the output would be all the possible answers including the correct number. You can find the codes for Figure1. in Appendix A. And the mathematical explanation in Appendix B.

- Confidentiality

$$e.d \overset{\varphi(n)}{\equiv} 1 \qquad \varphi(n) = (p-1)(q-1)$$

Here the calculation of "d" without knowing {p, q} requires great deal of time which makes RSA algorithm secure.

As you see nowadays communication protocols met confidentiality or perhaps security is hard to be broken, but you can never be sure, can you? Let's see how quantum communication deals with security

We have talked about Bit but as we want to enter quantum world let's talk about quantum unit of information known as Qubit. A particle, consider electron has a two dimensional Hilbert state with poles labeled as |0> & |1> and the state of every particle with two dimensional Hilbert space is the super position of |0> & |1>.

Considering electron as a Qubit we can write its state as $\alpha|0> + \beta|1>$ yet for communication we need series of Qubits.

Suppose that $|\varphi>$ is a three dimensional ket and that we can write this state according to its subsystems as:

$$|\varphi> \ = |\alpha_1> \otimes |\alpha_2> \otimes |\alpha_3>$$

St. $|\alpha_j> \ = \ \alpha_{1j}|0> + \alpha_{2j}|1>$

Somehow similarly in three dimensional classical systems like:

$$B = \ \beta_1\beta_2\beta_3$$

St.

$$\beta_j = \Pr\left\{\frac{1}{2}\right\} 0\_\Pr\left\{\frac{1}{2}\right\} 1$$

This is the point where Quantum information steps away from what we know as Classical information and this is the border of security.

The point is that you can't always divide $|\varphi>$ into its subsystem tensor products. If you reach in a state which

$$|\varphi> \ \neq |\alpha_1> \otimes |\alpha_2> \cdots \otimes |\alpha_N>$$

then the state is entangled and cannot be divided.

Let's consider a famous entangled pair which is known as EPR pair – short for Einstein Podolsky Rosen – and call this state $|\varphi>$ , then :

$|\varphi> \ = \frac{1}{\sqrt{2}} (|0> \otimes |0> + |1> \otimes |1>)$ which its short form is:

$$|\varphi> \ = \frac{1}{\sqrt{2}} (|00> + |11>)$$

You can divide $|\varphi>$ into the tensor product of two state like $|\alpha_1> \otimes |\alpha_2>$ , and these two Qubits are in entangled state.

Consider one scenario for quantum communication and then evaluate confidentiality.

Consider two places called A and B (let's just get off Alice and Bob's back), imagine there is a station somewhere in between called S. the station task is to generate entangled photons (or simply entangled states for we didn't talk about photons yet) and send them towards A and B.

Take a moment!... There could be another scenario in which for example A generate these entangled photons and send one of them towards B, in the case of confidentiality the result would be the same.

The photon that A and B receives are entangled which means if A measure its photon in $|0>$ basis and get the result that $|<0|\varphi>|^2 = 1$ then B definitely will get the same result as A.

The result of A's measurement correlate with B's measurement (nothing travels faster than the speed of light) and so after analyzing the data from measurement both A and B can be sure (precisely) whether there is a third party (an eavesdropper) or not and this is not just the probability of confidentiality this is the exact security provided by quantum principles.

Two main factor here helps us with the case of confidentiality and that is

Firstly, Every Quantum state is unique and that means given a state $|\varphi_1>$ then after measurement, the state of the system is no longer $|\varphi_1>$ , it will collapse into the post-measurement state $|\varphi_2>$ . Thus the eavesdropper cannot get away with it easily.

Secondly, our photons are entangled and that's **just** these two photon which are entangled so as long as measurements data are correlated there is no need to worry about an Eve.

So far we've talked about what is communication and you saw how communication evolve through time and change into the modern communication of $21^{th}$ century. We talked about Bit and Qubit (Quantum Bit) and understand the nature of security in Quantum communication. Now it's time to examine all aspects of quantum communication and face with obstacles in the way of long distance quantum communication.

In Chapter One, an abstract concept of Quantum Communication will be discussed. There will be a discussion toward 'no cloning' theorem and entanglement as well as their role in Quantum Communication. At the end of this Chapter, best medium for Quantum Communication will be argued. In Chapter Two different Quantum modules like Quantum swapping, Quantum entanglement, Quantum Purification and etc., will be discussed using Qasm simulator in Qiskit. These modules have an important role in today's Quantum Repeaters. Chapter Three shows a rough design for Quantum Repeaters and finally in Chapter Four, one can find codes and other mathematical calculation in the following Appendix.

## II. CHAPTER ONE

As I mentioned earlier photonic communication is a vulnerable medium especially for long distance communication. One possible solution that probably came to our minds as a result of classical knowledge of communication is to divide our distance into shorter length and then use an amplifier to compensate the losses in the path. But I mean this is not as easy as you might think.

Amplification is not a feasible solution in quantum mechanics. This is due to a property called no cloning theorem.

Consider a state called |𝜅> = 𝜆_1 |0>+𝜆_2 |1> for copying the State $|\kappa>$ you need to be aware of both $\lambda_1 \ and \ \lambda_2$ which is not Feasible. Another explanation would be considering a unitary Operator called C which acts on state $|\kappa >$ and result into $C\,|\kappa > \otimes |0 >= |\kappa > \otimes |\kappa >$ .That unitary operator is a cloning operator if and only if it exists for every state $|\kappa >$.

Consider the previous state $|\kappa > \; = \lambda_1|0 > +\lambda_2|1 >$ ,If you want to amplify $|\kappa >$ you need $\lambda_1 \& \lambda_2$ and in the case of Quantum state you can't have them both. So the amplification is not an acceptable solution. One acceptable answer is repeater. (I have to note here that even still repeaters are not a feasible solution, scientists and quantum engineers are working on different aspects of quantum repeaters for instance Quantum memories hopping that a more efficient quantum memory will lead to a better and more accurate repeater)

A repeater works like an amplifier with this difference that it is not amplifying anything but it is building a new link. So first thing is to divide that long path into shorter path and then stablish a correlated link between them and then instead of amplifying the link and classically transfer it to another base station we can set another link with that station.

- PHOTONIC CHANNEL

Very properties that make light the ideal medium information, makes it hard to store the information. So far Quantum Optical Channel is the ideal medium for Quantum information. But a single photon is too low in energy to be lost in a complete light background. The fragile nature of Photons and the Noisy Quantum Channels makes it hard to reach high fidelity specially toward Long Distance Communication. Will be completed

## III. CHAPTER TWO

This is the design of a Quantum-Repeater using Quantum-Circuits and Evaluating its Performance on an IBM software tool, Qiskit.

- QUANTUM TELEPORTATION

The main aim of Quantum Communication is to transmit information (or entangled photons) over long distances. Due to the fragile nature of photons and the noisy channel, Propagating Quantum information is hard to do. Quantum entanglement allows Quantum Communication to perform some protocols which are somehow impossible in classical terms. Quantum teleportation is a protocol that allows us to transport the state of one Qubit to the other Qubit. For mathematical Explanation go to Appendix B.

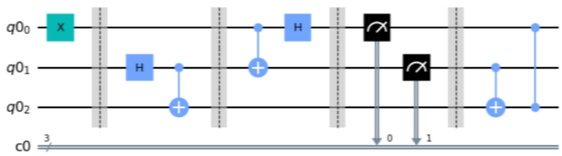



Figure 2. Quantum Teleportation

| | |
|---|---|
| Linear Operators | They act within the vector space |
| Identity Operators | I |
| Unitary operator | SS^† = I |
| Hermitian operator | H = H^† |
| normal operator | T T^† = T^† T |

Table 1. Operators

In this circuit the Qubit zero is transport to Qubit two. All the Qubits are at state |0> in the initial time.

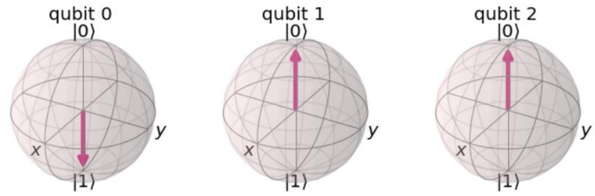



Figure 3. The state after first barrier

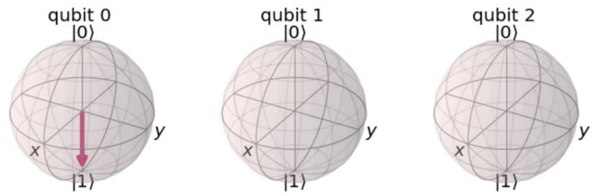



Figure 4. The state after second barrier

As we can see the state |1> must be transported to Qubit two. In Figure 4. Qubit 1 and Qubit 2 are entangled and that is why their state vectors are hidden. By using Histogram plot in 1024 trials, the results are the following.

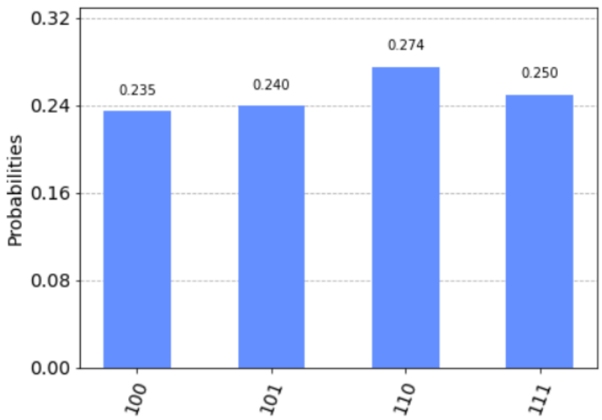



Figure 5. Histogram plot for Teleportation
(Read the results from left to right, C2, C1, C0.)

As we can see C2 is 1 which means the circuit successfully transported the state |1> from the first Qubit to the third Qubit. Find the codes for this circuit in Appendix A.

After measuring the first two Qubits, there is a final state which depends on the measurement result. If the measurement results showed correlation, then the state of third Qubit is the correct result. But if the measurement results were not correlated the final state must pass through a phase shift Gate (after fourth barrier). For mathematical details go to Appendix B.

- QUANTUM PURIFICATION

Quantum Repeaters are not the only answer to Long Distance Quantum Communication. The trick of Quantum

Repeaters is that the path is divided into smaller length so the path loss is negligible but still the purification is necessary.

Before stablishing new entanglement, we have to check whether or not photons transmitted correctly.

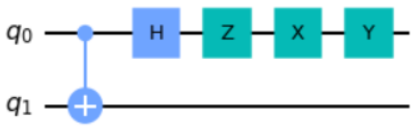
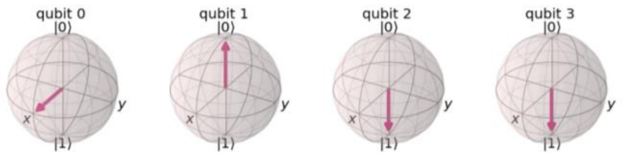



Figure 6. Universal Gates




(a) (b) (c) (d)

Figure 7. From right to left H, Z, X and Y Gate

Here there will be two approaches for Quantum Purification. One approach is the ideal Quantum Simulators and the second one is the NISQ simulation using ‘Qasm’ simulator with noise.

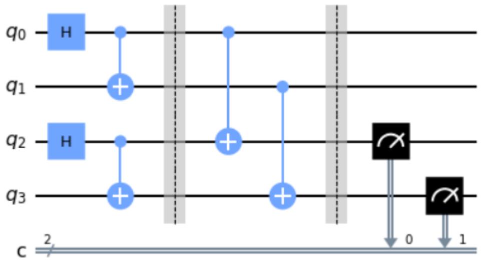
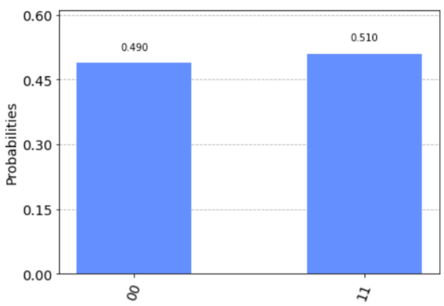
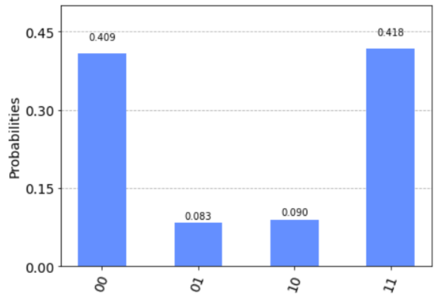



Figure 8. Quantum Circuit for Purification

Here Q0 and Q2 are Qubits on the Alice’s side and Q1 and Q3 are Bob’s. After stablishing two entanglements these Qubits will be sent through channel and then both these Qubits will be measured and results must be compared. If the results of measurements were equal the adjacent node would be created but if the results were not matched, Entangled Qubits must be stablished again.




(a) Ideal simulator




(b)Noisy simulator

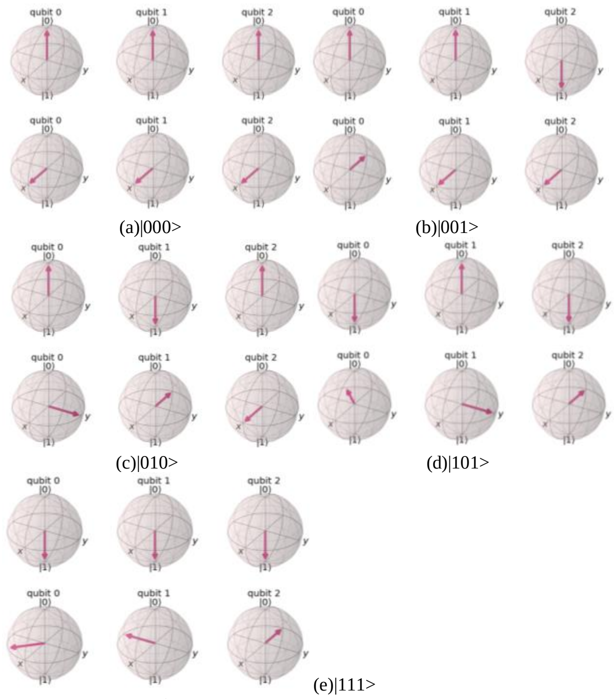
Figure 9. Purification result

We can see here that for a low noise channel (0.01 error), about 20 percent of the time the entire entanglement must be stablish again which shows the sensitivity of Quantum result with respect to the infrastructure noise.

The noisy simulation shows that due to decoherence the state |1> can be toward |0>. See Appendix A for Qiskit codes.

- QFT: QUANTUM FOURIER TRANSFORM

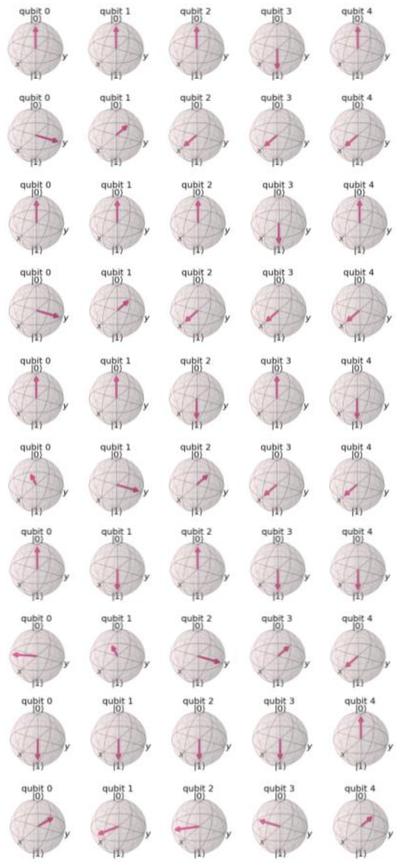
Fourier Transform is an important module in Classical Communication. Fourier Transform makes Signal Processing much easier and Frequency modulation has a great application in Signal Broadcasting, telecommunication and computing. In classical term, CFT transform a classical signal from time basis into Frequency basis. Similarly, QFT, Transform a Quantum State from computational basis into Fourier basis.



(a)|000> (b)|001> (c)|010> (d)|101> (e)|111>

Figure 10. QFT of a 3-Qubit system

As shown in Figure 10. before QFT all the Qubits were either |0> or |1> but after QFT, state vectors possess different angle with respect to X. but every Qubit has different freedom. As in figure 10. Qubit0 could have $n\pi/4$ phases and Qubit1 could have $n\pi/2$ phases and Qubit2 could have $n\pi$ phases with natural n. The number of states will not change, but the freedom in possessing different phases will increase by using QFT. The $m^{th}$ Qubit's angle in the m-Qubit system is $n\pi/2^{m-1}$ for $n \in N$. Find the QFT Qiskit codes in Appendix A.



(a)|00000>

(b)|00010>

(c)|00101>

(d)|01011>

(e)|11110>

Figure 11. QFT of a 5-Qubit system

## IV. CHAPTER THREE

After purification if the channel is safe enough to send Qubits, Alice can send one pair of her Entangled Qubits to the Bob using Swapping protocol. Figure 9, shows the transmission of Qubit 1 over Quantum Channel to Bob.

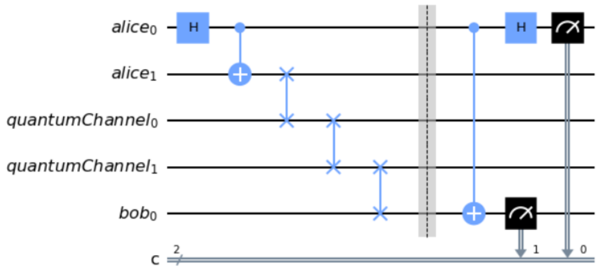



Figure 12. Entanglement transmission using swapping

Pay attention that after the barrier, entangled photons are not Alice0 and Bob0, so the entanglement measurement must be apply on these Qubits. The results from the measurement has been taken in two scenarios. First one with an ideal Quantum channel and the second one a noisy channel with 0.01 error for one port Quantum Gates. The error of multi-ports Quantum Gates can be derived respectively. Figure 10, shows the results of different measurements. As the result shows here, for the same low noise Channel (0.01 error), Entanglement Swapping is more Fragile than other modules. Roughly half the time (0.6) the results will be incorrect. For Qiskit codes of this part see Appendix A.

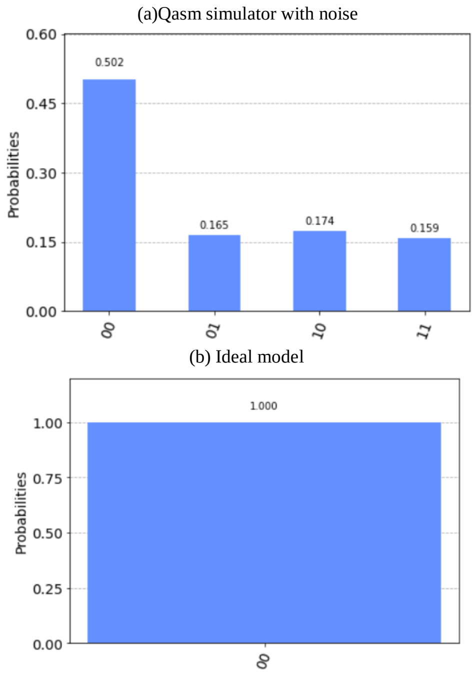



Figure 13. measurement results for Quantum Swapping

## V. CHAPTER FOUR

- APPENDIX A

Bernstein-Vazirani algorithm:

```
#import libraries
from qiskit import *
from qiskit.tools.visualization import plot_histogram
number = 'number'

#Creating Quantum Circuit
Circuit = QuantumCircuit(len(number)+1,len(number))
%matplotlib inline

#Implementing Quantum Gate
Circuit.h(range(len(number)))
Circuit.x(len(number))
Circuit.h(len(number))
Circuit.barrier()

for i,k in enumerate(reversed(number)):
    if k == '1':
        Circuit.cx(i,len(number))

Circuit.barrier()

Circuit.h(range(len(number)))
#measuring the result
Circuit.measure(range(len(number)),range(len(number)))

simulator = Aer.get_backend('qasm_simulator')
result = execute(Circuit, backend=simulator, shots=1).result()
counts = result.get_counts()
Circuit.draw(output = 'mpl')
#print the result
print(counts)
```

Quantum Teleportation:

```
from qiskit import * #importing library
Qb = QuantumRegister(3)
Cb = ClassicalRegister(3)
circuit = QuantumCircuit(Qb,Cb)
%matplotlib inline  #here Q0 form Quantum Register
circuit.x(Qb[0])    #now we want to apply X operator to our first Qubit
from qiskit.tools.visualization import plot_bloch_multivector
circuit.barrier()   #we can put a barrier in our draw plot so that we can divide different stages
circuit.h(Qb[1])
circuit.cx(1,2)     # first we need to make the control qubit (|0> + |1>)/sqrt(2)
circuit.barrier()   #Q[1] and Q[2] are now entangled
circuit.cx(0,1)
circuit.h(Qb[0])
circuit.barrier()
circuit.measure(Qb[0],Cb[0])
circuit.measure(Qb[1],Cb[1])
circuit.barrier()
circuit.cx(1,2) #flip
circuit.cz(0,2) #phase - shift
simulator = Aer.get_backend('statevector_simulator')
result = execute(circuit,backend=simulator).result()
statevector =result.get_statevector()  #plot_bloch_multivector(statevector)
circuit.measure(Qb[2],Cb[2])
circuit.measure(Qb[1],Cb[1])
circuit.measure(Qb[0],Cb[0])
simulator = Aer.get_backend('qasm_simulator')
result = execute(circuit,backend = simulator, shots= 1024).result()
counts = result.get_counts()
from qiskit.tools.visualization import plot_histogram   #plot_histogram(counts)
```

Purification, ideal

```
circuit = QuantumCircuit(4,2)
%matplotlib inline #circuit.draw(output = 'mpl')
circuit.h(0)
circuit.h(2)
circuit.cx(0,1)
circuit.cx(2,3)
simulator = Aer.get_backend('statevector_simulator')
result = execute(circuit,backend=simulator).result()
statevector =result.get_statevector() #plot_bloch_multivector(statevector)
circuit.barrier()
circuit.cx(0,2)
circuit.cx(1,3)
circuit.barrier()
circuit.measure(2,0)
circuit.measure(3,1)
circuit.measure(2,0)
simulator = Aer.get_backend('qasm_simulator')
result = execute(circuit,backend = simulator, shots= 1024).result()
counts = result.get_counts()
from qiskit.tools.visualization import plot_histogram #plot_histogram(counts)
```

Purification, with noise

```
import qiskit.providers.aer.noise as noise # Error probabilities
prob_1 = 0.01  # 1-qubit gate
prob_2 = 0.1   # 2-qubit gate
error_1 = noise.depolarizing_error(prob_1, 1) # Depolarizing quantum errors
error_2 = noise.depolarizing_error(prob_2, 2)
noise_model = noise.NoiseModel()  # Add errors to noise model
noise_model.add_all_qubit_quantum_error(error_1, ['u1', 'u2', 'u3'])
noise_model.add_all_qubit_quantum_error(error_2, ['cx'])
basis_gates = noise_model.basis_gates # Get basis gates from noise model
circuit = QuantumCircuit(4,2)
circuit.h(0)
circuit.h(2)
circuit.cx(0,1)
circuit.cx(2,3)
circuit.barrier()
circuit.cx(0,2)
circuit.cx(1,3)
circuit.barrier()
circuit.measure(2,0)
circuit.measure(3,1)
result = execute(circuit, Aer.get_backend('qasm_simulator'),  # Perform a noise simulation
                 basis_gates=basis_gates,
                 noise_model=noise_model,shots = 1024).result()
counts = result.get_counts()  #plot_histogram(counts)
```

Quantum Swapping with noise

```
#swapping with noise
A = QuantumRegister(2, 'alice')
C = QuantumRegister(2, 'quantumChannel')
B = QuantumRegister(1, 'bob')
cl = ClassicalRegister(2, 'c')
Circuit = QuantumCircuit(A, C, B, cl)
Circuit.h(A[0])  # perform entanglement algorithm
Circuit.cx(A[0],A[1])
Circuit.swap(A[1],C[0])
Circuit.swap(C[0],C[1])
Circuit.swap(C[1],B)
Circuit.barrier()  #Circuit.draw(output='mpl')
Circuit.cx(A[0],B) # measurement  # inverse entanglement
Circuit.h(A[0])
Circuit.measure(A[0],cl[0])
Circuit.measure(B,cl[1])   #Circuit.draw(output = 'mpl')
result = execute(Circuit, Aer.get_backend('qasm_simulator'),  # Perform a noise simulation
                 basis_gates=basis_gates,
                 noise_model=noise_model,shots = 1024).result()
counts = result.get_counts()  #plot_histogram(counts)
simulator = Aer.get_backend('statevector_simulator')
result = execute(Circuit,backend=simulator).result()
statevector =result.get_statevector()
```

Quantum Swapping without noise

```
A = QuantumRegister(2, 'alice')
C = QuantumRegister(2, 'quantumChannel')
B = QuantumRegister(1, 'bob')
cl = ClassicalRegister(2, 'c')
CircuitB = QuantumCircuit(A, C, B, cl)
CircuitB.h(A[0])  # perform entanglement algorithm
CircuitB.cx(A[0],A[1])
CircuitB.swap(A[1],C[0])
CircuitB.swap(C[0],C[1])
CircuitB.swap(C[1],B)
CircuitB.barrier()
CircuitB.cx(A[0],B) # measurement  # inverse entanglement
CircuitB.h(A[0])
CircuitB.measure(A[0],cl[0])
CircuitB.measure(B,cl[1])
CircuitB.measure(2,0)
simulatorB = Aer.get_backend('qasm_simulator')
resultB = execute(CircuitB,backend = simulatorB, shots= 1024).result()
countsB = resultB.get_counts()
simulatorB = Aer.get_backend('statevector_simulator')
resultB = execute(CircuitB,backend=simulatorB).result()
statevectorB =resultB.get_statevector()  #plot_histogram(countsB)
```

QFT

```
from qiskit.circuit.library import QFT
from qiskit.quantum_info import Statevector
from qiskit import QuantumCircuit
from qiskit.visualization import plot_bloch_multivector
import warnings
warnings.filterwarnings('ignore')
state = 'State'
FourierCircuit = QuantumCircuit(len(state))
FourierCircuit.initialize(Statevector.from_label(state).data,FourierCircuit.qubits[::-1])
display(plot_bloch_multivector(Statevector.from_instruction(FourierCircuit).data))
FourierCircuit.append(QFT(len(state),do_swaps=True),FourierCircuit.qubits)
display(plot_bloch_multivector(Statevector.from_instruction(FourierCircuit).data))
```

## ▪ APPENDIX B

Bernstein-Vazirani algorithm:

Consider the case of a number with n=2

$step\ 1$:

$|\varphi> = |00>$

$step\ 2$:

$|\emptyset> = H\,|\varphi> = \frac{|00> + |01> + |10> + |11>}{\sqrt{2}}$

$step\ 3$:

$|\emptyset> \ = \frac{(-1)^{00\cdot number}|00> + (-1)^{01\cdot number}|01> + (-1)^{10\cdot number}|10> + (-1)^{11\cdot number}|1}{\sqrt{2}}$

$step\ 4$:

$number\ = 10$

$|\emptyset> \ = \frac{-|00> + |01> + |10> - |11>}{\sqrt{2}}$

$H\,|\emptyset> \ = |10>$

Teleportation Protocol:

*Alice* *Bob*

$A' = |m> = \alpha|0> + \beta|1>$

$A \leftrightarrow B$ $B \leftrightarrow A$

***Total Quantum System :*** $|m> \otimes |\Psi'>_{AB}$

$[\alpha|0> + \beta|1>] \otimes \left[\frac{|00> + |11>}{\sqrt{2}}\right]$

$\frac{1}{2\sqrt{2}}\,\alpha\,\Big|000> + \frac{1}{2\sqrt{2}}\,\beta\Big|\,001> + \frac{1}{2\sqrt{2}}\,\alpha\,\Big|110> + \frac{1}{2\sqrt{2}}\,\beta\Big|\,111> +$

$\frac{1}{2\sqrt{2}}\,\alpha\,\Big|000> - \frac{1}{2\sqrt{2}}\,\beta\Big|\,001> - \frac{1}{2\sqrt{2}}\,\alpha\,\Big|110> + \frac{1}{2\sqrt{2}}\,\beta\Big|\,111> +$

$\frac{1}{2\sqrt{2}}\,\alpha\,\Big|011> + \frac{1}{2\sqrt{2}}\,\beta\Big|\,010> + \frac{1}{2\sqrt{2}}\,\alpha\,\Big|101> + \frac{1}{2\sqrt{2}}\,\beta\Big|\,100> +$

$\frac{1}{2\sqrt{2}}\,\alpha\,\Big|011> - \frac{1}{2\sqrt{2}}\,\beta\Big|\,010> - \frac{1}{2\sqrt{2}}\,\alpha|101> + \frac{1}{2\sqrt{2}}\,\beta|100>$

$\frac{1}{2}|\Psi'_1> \otimes |m_1> + \frac{1}{2}|\Psi'_2> \otimes |m_2> + \frac{1}{2}|\Psi'_3> \otimes |m_3> + \frac{1}{2}|\Psi'_4> \otimes |m_4>$

$|\Psi'_1> = \frac{|00> + |11>}{\sqrt{2}}$ , $|\Psi'_2> = \frac{|00> - |11>}{\sqrt{2}}$ , $|\Psi'_3> = \frac{|01> + |10>}{\sqrt{2}}$ , $|\Psi'_4> = \frac{|01> - |10>}{\sqrt{2}}$

$|m_1> = \alpha|0> + \beta|1>$ , $|m_1> = \alpha|0> - \beta|1>$ , $|m_1> = \beta|0> + \alpha|1>$ , $|m_1> = \beta|0> - \alpha|1>$

## VI. CONCLUSION

Will be Completed

## VII. REFERENCE

Will be Completed