OUTLINE EXAMINATION MARKING SCHEME

EXAM DIET: In-Class Test 2020-2021
COURSE: B.Eng. in Electronic and Computer Engineering
COURSE: B.Eng. in Mechatronic Engineering

MODULE: EE458 Control Systems Analysis

QUESTION 1

[Q1 - Read Question $\approx$ 5 mins]

(a) (i) To solve this problem, it is easier to use the block diagram reduction by rearranging the branched signal line before $G_1(s)$ block, which connects to the $G_2(s)$ to the position after the $G_1(s)$ block. However, after this change the original $G_2(s)$ block has to be changed to $G_2(s)/G_1(s)$ for mathematical equivalence. Given this idea, the new structured block diagram can be easily resolved, i.e. G_1 and H_1 are in the feedback loop, and G_2/G_1 and G_3 are in parallel, and then it is in series with the feedback loop. Now, the transfer function can be obtained as follows.

$$T(s) = \frac{G_1(s)}{1 + G_1(s)H_1(s)}(G_3(s) + \frac{G_2(s)}{G_1(s)}) = \frac{G_2(s) + G_1(s)G_3(s)}{1 + G_1(s)H_1(s)}$$

(ii) By simply substituting the transfer functions G_1, G_2, G_3 and H_1, we can find that:

$$T(s) = \frac{2}{s^2 + 4s + 4}.$$

If using the Matlab approach, a student can first find out that $\frac{G_2}{G_1} = \frac{1}{s+2}$ and then type in the transfer function for further calculation using series, parallel and feedback, which will give the same final result above.

(iii) Given the result in the last step, the system $T(s)$ is 2rd order and the type is 0.

(iv) Using the Matlab isstable command, we can find out that the system is a stable system. Matlab will return 1 to indicate stability.

[Q 1(a) 9 marks (3 + 2 + 2 + 2)]

[Q 1(a) $\approx$ 25 mins]

(b) (i) By using the Final Value Theorem (FVT), i.e.

$$E_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s(R(s) - Y(s)) = 1 - \lim_{s \to 0} sT(s)R(s) = 1 - \lim_{s \to 0} T(s) = 0.5.$$

(ii) Using the 1 - dcgain(Ts), where Ts is the transfer function in Matlab for T(s) we can find out that the steady state error is indeed 0.5. This validates the result.

(iii) The input has been multiplied by 5, so the steady-state error will also be multiplied by 5 in this case, giving the final steady-state error 3. The result can also be obtained by reusing the formulae above, which gives 6 - T(0)*6 = 6 - 3 = 3.

[Q 1(b) 6 marks (2 + 2 + 2)]

[Q 1(b) $\approx$ 7 mins]

(c) (i) The closed-loop transfer function of the system for even number is

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)H(s)} = \frac{2k}{(s+1)(s+2)(s+a)+4k}.$$

for odd number is

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)H(s)} = \frac{2k}{(s+1)(s+2)(s+a)+2k}.$$

(ii) By using the quotient rule, the sensitivity value for the even number case is:

$$S_k^T = \frac{(s+1)(s+2)(s+a)}{(s+1)(s+2)(s+a)+4k}$$

for the odd number case is:

$$S_k^T = \frac{(s+1)(s+2)(s+a)}{(s+1)(s+2)(s+a)+2k}$$

(iii) By setting $s{=}0$, the result for the even case turns out to be:

$$S_k^T = \frac{2a}{2a+4k}$$

for the odd number case is:

$$S_k^T = \frac{2a}{2a+2k}$$

It is not difficult to find that fixing a, increasing k will decrease the sensitivity value for both cases. Note both a and k are positive parameters as mentioned in the question.

[Q 1(c) 7 marks (2 + 3 + 2)]

[Q 1(c) $\approx$ 8 mins]

(d) For both cases, we can find the equation to be solved is:

$$\lim_{s\to 0} T(s) = 0$$

For the even number case, this turns out to be:

$$\frac{2k}{4k+2} = 0.3$$

which gives $k{=}\frac{3}{4}$. For the odd number case, this turns out to be:

$$\frac{2k}{2k+2} = 0.3$$

which gives $k{=}\frac{3}{7}$.

[Q 1(d) 3 marks]

[Q 1(d) $\approx$ 15 mins]

[Total: 25 marks]

[END OF Q1 ANSWERS]