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A Final Project report in

Simulation and exploration of THz TRNASMISSION LINES

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MAJORING IN

THE INTERNET OF THINGS

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Acknowledgements

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Abstract

This final report presents the simulation and exploration of terahertz (THz) transmission lines, focusing on the development and validation of numerical models for high-frequency applications. The project addresses the challenges of accurately modelling THz transmission lines, which are essential for next-generation technologies such as 6G networks, wireless data centres, and biomedical imaging. The primary goal is to create computationally efficient and precise models capable of simulating time-domain behaviour at THz frequencies. Three key methods were employed: the Finite-Difference Time-Domain (FDTD) approach for initial approximations, the Numerical Inverse Laplace Transform (NILT) for exact s-domain solutions, and RLC ladder approximations for efficient time-domain modelling. The FDTD simulations provided a baseline for understanding transient and steady-state behaviours, while the RLC ladder method, combined with NILT, demonstrated the ability to closely match exact solutions when sufficient sections were used. Additionally, Y-parameters were derived to analyse transmission line behaviour, and Asymptotic Waveform Evaluation (AWE) was implemented to refine approximations and extract dominant system responses. The final model was obtained through iterative addition of small responses at high frequencies and comparison with exact solutions. The results highlight the importance of optimising the number of sections in the model to balance accuracy and computational efficiency. This work contributes to the advancement of THz communication systems by providing reliable modelling tools for future research and development.

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Chapter 1 - Introduction

The growth of communication technologies has led to the exploration of terahertz (THz) frequencies (0.1 to 10 THz) for applications like 6G networks, wireless data centres, and biomedical imaging, but accurate THz transmission line models remain a challenge. The motivation comes from the demand for high-speed THz communication systems, which are crucial for future technologies, but currently lack reliable and efficient modelling tools.

This project aims to develop and validate efficient numerical models for THz transmission lines to predict signal behaviour and optimize system performance. The study addresses key propagation challenges at THz frequencies, such as high attenuation and dispersion, which are critical for the design of next-generation communication systems. To achieve this, the project evaluates original methods and techniques proposed between 2000 and 2010, assessing their suitability for THz applications. In addition, the research focuses on developing an accurate model using Y-parameters and Asymptotic Waveform Evaluation (AWE). The developed model is then compared with the original methods to determine their effectiveness and limitations at THz frequencies

Three primary methods are employed in this study: Finite-Difference Time-Domain (FDTD) for transient analysis, Numerical Inverse Laplace Transform (NILT) for exact s-domain solution, and RLC ladder approximations for computational efficiency. The FDTD method provides a foundation for simulating time-domain electromagnetic wave propagation, while NILT offers precise frequency-domain insights. The RLC ladder approximations, on the other hand, are used to simplify complex transmission line models, making them computationally tractable for large-scale simulations. These methods are compared against each other and the obtained model using Y-parameters and Asymptotic Waveform Evaluation (AWE) to evaluate their effectiveness for THz applications. Factors affecting their performance, such as the number of sections, computational cost, and accuracy, are analysed to determine the most efficient approach for THz transmission line modelling.

The report includes a literature review, implementation details of these modelling methods, simulation results, challenges faced, and future work plans. By integrating Y-parameters and AWE into the modelling framework, this research aims to establish a validated numerical approach for THz transmission line modelling. The proposed model not only addresses the limitations of earlier methods but also offers a computationally efficient and accurate solution for THz signal analysis. By comparing the developed model with original methods from 2000 to 2010, this study provides valuable insights into the evolution of THz modelling techniques and their applicability to modern communication systems. This research aims to advance THz communication technologies by designing high-performance systems capable of operating at extreme frequencies.

1.2.4 Summary

This project develops and validates numerical models for THz transmission lines to optimize system performance and address propagation challenges. Three primary methods—FDTD, NILT (exact solution), and RLC ladder approximations—are evaluated for accuracy, computational efficiency, and effectiveness compared to the obtained model using AWE. MATLAB is used for implementation, but these methods can be applied using any coding language or software.

Chapter 2 - Technical Background

Modelling THz (0.1–10 THz) transmission lines require understanding wave propagation, transmission line theory, and numerical methods. At THz frequencies, the behaviour of transmission lines is governed by the Telegrapher's equations (1), which describe the relationship between voltage and current along the line. These equations are derived from Maxwell's equations and are given by:

$$\frac{dv(x,t)}{dx} = -R(x)\,i(x,t) - L(x)\frac{di(x,t)}{dt} \qquad (1)$$
$$\frac{di(x,t)}{dx} = -G(x)\,v(x,t) - C(x)\frac{dv(x,t)}{dt}$$

where $v(x,t)$ and $i(x,t)$ represent the voltage and current at position x and time t, respectively. R, L, G, and C are the per-unit-length resistance, inductance, conductance, and capacitance of the transmission line. At THz frequencies, these parameters become highly frequency-dependent, making accurate modelling more complex.

Finite-Difference Time-Domain (FDTD) Method

The FDTD method is a widely used numerical technique for solving electromagnetic problems, particularly in the time domain. It discretizes the transmission line into small segments as in Figure 1, allowing for the simulation of voltage and current over time. The FDTD method is based on approximating the derivatives in the Telegrapher's equations using finite differences [3][5]. In this approach voltages (v_n) are calculated at the ends of each section, while currents (i_n) are computed at the middle of each section as illustrated in Figure 1 and 2. Then, v_n and i_n can be derived as.

(2)

$$v_k^{n+1} = v_k^n - \frac{\Delta\mathrm{t}}{\Delta\mathrm{xC}}\left(i_k^{n+\frac{1}{2}} - i_{k-1}^{n+\frac{1}{2}}\right) \qquad (3)$$

$$i_{k-1}^{n+3/2} = i_k^{n+1/2} - \frac{\Delta\mathrm{t}}{\Delta\mathrm{xl}}\left(v_{k+1}^{n+1} - v_k^{n+1}\right)$$

This method provides a foundation for simulating transient and steady-state behaviors of transmission lines, but it can be computationally intensive, especially for long lines or high frequencies.

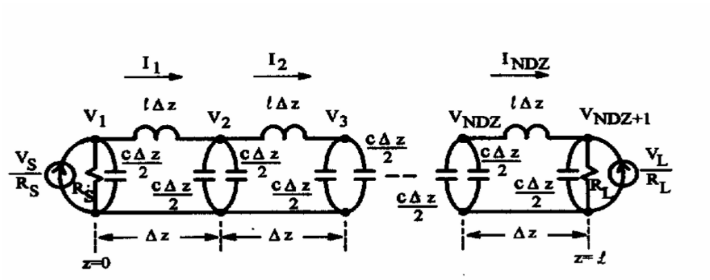

Figure 1: Equivalent representation of the transmission line using the Lumped Pi circuit model, illustrating the discretization of inductance and capacitance along the line [5].

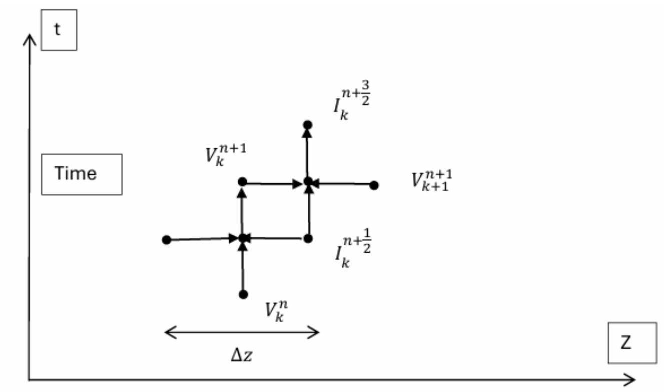

Figure 2: Staggered grid representation for the FDTD method, illustrating the spatial (z) and temporal (t) discretization. The voltage (V) is defined at grid points, while the current (I) is defined at the midpoints between the grid points [5].

Numerical Inverse Laplace Transform (NILT)

The NILT method is a powerful tool for convert frequency-domain solutions into time-domain solutions for simulating transient phenomena in multiconductor transmission line (MTL) systems [7]. NILT0, the baseline variant, approximates time-domain responses using residues and poles of a Padé rational function, enabling sparse time points with L-stability. NILTn (for $n \geq 1$) extends this by recursively computing high-order derivatives of Laplace-domain solutions, reducing truncation errors by $(n+1)N+M$ while maintaining stability [6]. Furthermore, NILTcv is based on the Bromwich integral, which is numerically evaluated using the Fast Fourier Transform (FFT) and the quotient-difference (q-d) algorithm. The time-domain function $f(t)$ is approximated using a discrete form derived from the Laplace transform $F(s)$ as in (4). The approximation involves a finite sum evaluated by the FFT and an infinite sum accelerated by the q-d algorithm, which uses a continued fraction to improve convergence [7].

$$f(t) = \frac{1}{2\pi j}\int_{c-\infty}^{c+\infty} F(s)e^{st}ds \tag{4}$$

This approach allows for the exact solution of the transmission line's behaviour in the s-domain, which can then be compared with approximate methods such as the RLC ladder to validate accuracy.

RLC Ladder Approximations

The RLC ladder method approximates a transmission line by dividing it into multiple sections, each represented by lumped resistive (R), inductive (L), and capacitive (C) elements as shown in Figure 3. This discretization simplifies the transmission line into a network of interconnected RLC circuits, making it easier to model and simulate

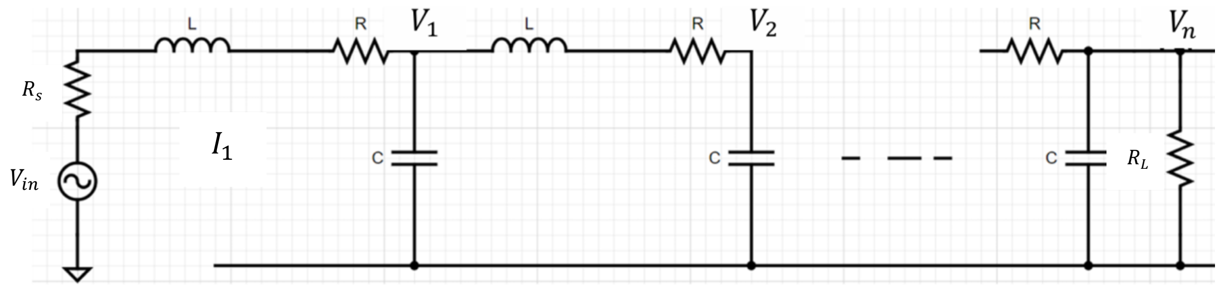

Figure 3: RLC ladder network approximates a transmission line with N sections and lumped elements (R), (L), and (C).

Governing equations:

Considering one section of the RLC ladder, the following equations are derived:

$$V_s - V_1 = (R + R_s)I_1\, dz + L\frac{dI_1}{dt}dz \tag{5}$$

$$I_1 - I_2 = C\frac{dV_1}{dt}dz \tag{6}$$

Here, dz represents the length of a small segment of the transmission line and is defined as:

$$dz = \frac{l}{N} \tag{7}$$

where l is the total length of the line, and NN is the number of sections or RLC circuits used to model the line.

$$\frac{dI_1}{dt} = -\frac{1}{L}V_1 - \frac{R_s + R}{L}I_1 + \frac{1}{L}V_s \tag{8}$$

$$\frac{dV_1}{dt} = \frac{1}{C}I_1 - \frac{1}{C}I_2 \tag{9}$$

$$\frac{dI_n}{dt} = -\frac{1}{L}V_n - \frac{R_s + R}{L}I_n + \frac{1}{L}V_{n-1} \tag{10}$$

$$\frac{dV_n}{dt} = \frac{1}{C}I_n - \frac{1}{C}I_{n+1} \tag{11}$$

The impedances R, L, and C are defined per unit length (i.e., per dz) in Equations (5) and (6). By rearranging Equations (5) and (6), Equations (8) and (9) are obtained. For the n-th section, the generalised forms are given by Equations (10) and (11). The accuracy of the RLC ladder approximation depends on the number of sections used; more sections generally lead to higher accuracy but at the cost of increased computational complexity. The RLC ladder method is particularly useful for simulating long transmission lines or systems with complex terminations.

Asymptotic Waveform Evaluation (AWE):

Asymptotic Waveform Evaluation (AWE) is a computationally efficient technique used to approximate the transient response of large linear systems, such as electrical interconnects, by reducing their high-order state-space models into lower-order approximations. Traditional methods such as SPICE become prohibitively slow for circuits with hundreds of nodes (e.g., PEEC models), but AWE addresses this by extracting dominant poles and residues from the system's transfer function using Padé approximation and moment matching [9]. By focusing on these critical poles, AWE converts the state-space representation—which describes the system's dynamics through differential equations—into a simplified time-domain model.

A general definition of moments in AWE:

The q^{th} moment is defined as:

$$H(s=0)=\int_0^\infty h(t)dt \tag{12}$$

$$H^{(1)}(s=0)=-\int_0^\infty h(t)tdt$$

$$H^{(2)}(s=0)=\int_0^\infty h(t)t^2dt$$

$$H^{(3)}(s=0)=-\int_0^\infty h(t)t^3dt$$

Application of moments to represent the transfer function $H(s)$:

$$\begin{aligned} H(s) &= \int_0^\infty h(t)(1-st+\frac{1}{2}s^2t^2-\frac{1}{6}s^3t^3+\frac{1}{24}s^4t^4dt \\ &= H(0)+sH^{(1)}(0)+\frac{1}{2}s^2H^{(2)}(0)+\frac{1}{6}s^3H^{(3)}(0)+\cdots \\ &= m_0+m_1s+m_2s^2+m_3s^3+\cdots \\ &= \sum_{k=0}^{\infty}\frac{s^k}{k!}H^{(k)}(s=0)=\sum_{k=0}^{\infty}m_ks^k \end{aligned} \tag{13}$$

Where

$$m_k = \frac{1}{k!} H^{(k)}(s = 0) = \frac{(-1)^q}{q!} \int_0^{\infty} t^q h(t) dt \tag{14}$$

Y-parameters:

In the analysis of transmission lines, the Y-parameters (admittance parameters) are commonly used to characterize the relationship between the currents and voltages at the input and output ports. These parameters are typically determined experimentally and can be approximated using rational functions of the complex frequency variable (s) as explained in (15) [9].

$$\begin{bmatrix} I_s \\ -I_R \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_s \\ V_R \end{bmatrix} \tag{15}$$

Each one is approximated with a rational function as in (16):

$$Y_{ij} = \frac{(a_{nij} s^{n-1} + .. + a_{0ij})}{s^n + .. + b_{0ij}} \tag{16}$$

In the open-circuit condition, where the output current (I_R) is zero, the voltage transfer function $(\frac{V_R}{V_s})$ is derived as a ratio of these rational functions (17). This transfer function is then converted into a state-space representation, to facilitate further analysis. Finally, AWE is applied to the state-space model to obtain a time-domain representation, enabling the study of the system's transient and steady-state behavior. This approach provides a systematic method for modeling and simulating the response of transmission lines.

$$\begin{aligned} \frac{V_R}{V_s} = -\frac{Y_{21}}{Y_{22}} = f\left(\frac{(a_{n21} s^{n-1} + .. + a_{021})}{s^n + .. + b_{021}}, \frac{(a_{n22} s^{n-1} + .. + a_{022})}{s^n + .. + b_{022}}\right) = -\frac{\frac{(a_{n21} s^{n-1} + .. + a_{021})}{s^n + .. + b_{021}}}{\frac{(a_{n22} s^{n-1} + .. + a_{022})}{s^n + .. + b_{022}}} \\ = \frac{f_{n-1} s^{n-1} + \ldots + f_0}{s^n + g_{n-1} s^{n-1} + .. + g_0} \end{aligned} \tag{17}$$

2.2 Summary

This section summarises methods for modelling transmission lines, governed by Telegrapher's equations. Key approaches include the FDTD method for transient simulations via staggered discretization, Numerical Inverse Laplace Transform (NILT) variants (e.g., NILTcv using FFT) for frequency-to-time-domain conversion, and RLC ladder networks RLC ladder approximations (lumped-element networks). Furthermore, AWE simplifies high-order systems via dominant poles and Pade approximations, while Y-parameters characterize admittance relationships using rational functions. In addition, complex frequency hopping is implicitly integrated via pole-residue analysis and Bromwich contour methods to enhance stability and convergence in transient simulations [9]. These methods address challenges in simulating transient and steady-state behaviours of THz lines, prioritizing trade-offs between accuracy, stability, and computational efficiency.

Chapter 3 - **Design of Transmission lines model**

The exact solution of Transmission lines (RLCG approach):

The exact solution of the transmission line can be derived from the RLCG ladder in the frequency domain as follows.

Writing Telegrapher equations (1) in state space

$$\frac{d}{dx}\begin{bmatrix} v(x,t) \\ i(x,t) \end{bmatrix} = \begin{bmatrix} 0 & -R(x) \\ -G(x) & 0 \end{bmatrix}\begin{bmatrix} v(x,t) \\ i(x,t) \end{bmatrix} - \begin{bmatrix} 0 & -L(x) \\ -C(x) & 0 \end{bmatrix}\frac{d}{dt}\begin{bmatrix} v(x,t) \\ i(x,t) \end{bmatrix} \quad (18)$$

Moving (18) to Laplace domain,

$$\frac{d}{dx}\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = \begin{bmatrix} 0 & -Z(x,s) \\ -Y(x,s) & 0 \end{bmatrix}\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} + \begin{bmatrix} 0 & L(x) \\ C(x) & 0 \end{bmatrix}\begin{bmatrix} V(x,0) \\ I(x,0) \end{bmatrix} \quad (19)$$

Where $Z\,(x,s) = R(x) + sL(x)$, and $Y\,(x,s) = G(x) + sC\,(x)$ are series impedance. Now, let:

$$W(x,s) = \begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix}, M = \begin{bmatrix} 0 & -Z(x,s) \\ -Y(x,s) & 0 \end{bmatrix}, and\ N = \begin{bmatrix} 0 & L(x) \\ C(x) & 0 \end{bmatrix}$$

Where

$$\frac{dW(x,s)}{dx} = M\,W(x,s) + N\,W(x,0) \quad (20)$$

Then, $W(l,s)$ at the end of the transmission line equal to,

$$W(l,s) = \Phi\,\mathrm{W}(0,\mathrm{s}) + \int_0^l e^{M(l-x)} N W(x,0)dx \quad (21)$$

Where

$$\Phi = e^{Ml}, so\ \phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \quad (22)$$

Considering zero initial conditions ($i.e, at\ W(x,0) = 0$) then,

$$W(l,s) = \Phi\,\mathrm{W}(0,\mathrm{s}) \quad (23)$$

Or

$$W(l,s) = \begin{bmatrix} V(l,s) \\ I(l,s) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \cdot \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}$$

Considering open voltage transmission line (i.e, $I(l,s) = 0$) then,

$$\begin{bmatrix} V(l,s) \\ 0 \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \cdot \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}$$

$$0 = \Phi_{21}V(0,s) + \Phi_{22}I(0,s)$$

$$I(0,s) = \Phi_{21}V(0,s) - \Phi_{22}^{-1}$$

$$V(l,s) = \Phi_{11}V(0,s) + \Phi_{12}\,I(0,s) \quad (24)$$

$$V(l,s) = \Phi_{11}V(0,s) - \Phi_{12}\,\Phi_{21}\,\Phi_{22}^{-1}V(0,s) \tag{25}$$

Re arrange (25) and simplify, the following is achieved in the s domain,

$$V(l,s) = \frac{2Vs(s)e^{l\sqrt{YZ}}}{e^{(2l\sqrt{YZ})} + 1}$$

Where $Vs(s)$ is the input in the s domain. Let,

$$x = l\sqrt{YZ}, \qquad so\;, V(l,s) = \frac{Vs(s)2e^{x}}{e^{(2x)} + 1}$$

Using the fact that,

$$cosh(x) = \frac{(e^{x} + e^{-x})}{2}$$

Then,

$$\frac{V(l,s)}{Vs(s)} = \frac{1}{\cosh\,(l\sqrt{YZ})} \tag{26}$$

One then can use NILTcv as in code 5, to simulate this with different values of R, L, C and G, and ultimately compare the output to the approaches mentioned earlier.

Exact solution using (Lumped Element Model):

In this approach, the transmission line is represented as a cascade of small segments, each consisting of a series impedance (Z series) and a parallel admittance (Y parallel) as illustrated in Figure 4. These elements correspond to the physical properties of the transmission line [10].

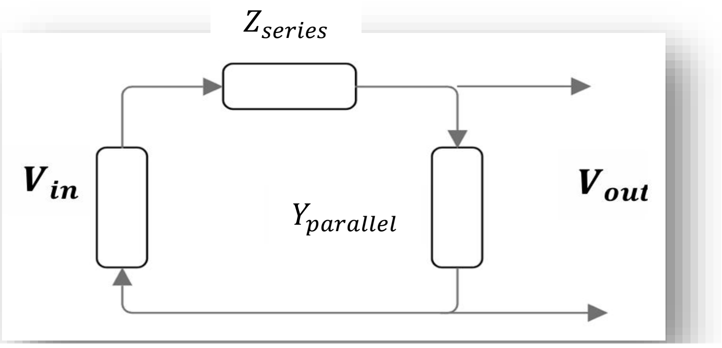

Figure 4: Equivalent circuit representation of a transmission line segment with series impedance (Z_{series}) and parallel admittance ($Y_{\{parallel\}}$).

Where,

$$Z_{series} = Z_o \sinh(\gamma l)$$

$$Z_o = \sqrt{\frac{Z}{Y}}\,, \qquad Y = G + sC, \qquad Z = (R + sL), \quad \gamma = \sqrt{ZY}$$

$$Y_{parallel} = Y_o \tanh\left(\frac{\gamma l}{2}\right), \quad Y_o = \frac{1}{Z_o} \ , \quad Z_{parallel} = \frac{1}{Y_{parallel}}$$

From Figure 4, the following equation of the transfer function is obtained:

$$T(s) = \frac{V_o}{V_{in}} = \frac{Z_{parallel}}{Z_{series} + Z_{parallel}} \tag{27}$$

$$T(s) = \frac{V_o}{V_{in}} = \frac{\dfrac{\sqrt{\dfrac{R+sC}{G+sC}}}{\tanh\left(l\dfrac{\sqrt{(R+sC)(G+sC)}}{2}\right)}}{\sqrt{\dfrac{R+sC}{G+sC}}\sinh\left(l\sqrt{(R+sC)(G+sC)}\ \right) + \dfrac{\sqrt{\dfrac{R+sC}{G+sC}}}{\tanh\left(l\dfrac{\sqrt{(R+sC)(G+sC)}}{2}\right)}}$$

$$T(s) = \frac{V_o}{V_{in}} = \frac{\dfrac{1}{\tanh\left(l\dfrac{\sqrt{(R+sC)(G+sC)}}{2}\right)}}{\sinh\left(l\sqrt{(R+sC)(G+sC)}\ \right) + \dfrac{1}{\tanh\left(l\dfrac{\sqrt{(R+sC)(G+sC)}}{2}\right)}}$$

Using the fact that,

$$\tanh\left(\frac{x}{2}\right) = \frac{\cosh(x) - 1}{\sinh(x)}$$

$$T(s) = \frac{V_o}{V_{in}} = \frac{1}{\cosh\left(l\sqrt{(R+sC)(G+sC)}\ \right)} \tag{28}$$

Which is the same as in (26).

AWE implementation:

AWE involves 4 main steps:

1. Form a state – space representation
2. Form the moments
3. Find the poles of the system
4. Find the residues

And then form the impulse response as:

$$h(t) = k_0\delta(t) + k_1 e^{p_1 t} + \ldots + k_n e^{p_n t} \tag{29}$$

Step 1: Form a state – space representation out of a model (RLC ladder or general TF):

Consider 2 sections of the RLC ladder in Figure 3, the following equations are derived:

$$v_{in} = (R_s + R_{dz})i_1 + L_{dz}\frac{di_1}{dt} + v_1 \tag{30}$$

$$v_1 = R_{dz}i_2 + L_{dz}\frac{di_2}{dt} + v_{out}$$

$$i_1 - i_2 = C\frac{dv_1}{dt}$$

$$i_2 = C\frac{dv_o}{dt}$$

Let,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_o \end{bmatrix} \tag{31}$$

Rewriting the equations,

$$\frac{di_1}{dt} = -\frac{(R_s + R_{dz})}{L_{dz}}i_1 - \frac{v_1}{L_{dz}} + \frac{v_{in}}{L_{dz}}$$

$$\frac{di_2}{dt} = \frac{-R_{dz}i_2}{L_{dz}} - \frac{v_{out}}{L_{dz}} + v_1$$

$$\frac{dv_1}{dt} = \frac{1}{C}(i_1 - i_2)$$

$$\frac{dv_o}{dt} = \frac{1}{C}\ i_2$$

Then the state space representation is:

$$A = \begin{bmatrix} \frac{-R_s + R_{dz}}{L_{dz}} & -\frac{1}{L_{dz}} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & \frac{1}{L_{dz}} & -\frac{R_{dz}}{L_{dz}} & -\frac{1}{L_{dz}} \\ 0 & 0 & \frac{1}{C} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L_{dz}} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad C = [0 \quad 0 \quad 0 \quad 1]\ and\ D = 0 \tag{32}$$

If 3 sections are considered then,

$$A = \begin{bmatrix} \frac{-R_s + R_{dz}}{L_{dz}} & -\frac{1}{L_{dz}} & 0 & 0 & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 & 0 & 0 \\ 0 & \frac{1}{L_{dz}} & -\frac{R_{dz}}{L_{dz}} & -\frac{1}{L_{dz}} & 0 & 0 \\ 0 & 0 & \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & 0 & 0 & \frac{1}{L_{dz}} & -\frac{R_{dz}}{L_{dz}} & -\frac{1}{L_{dz}} \\ 0 & 0 & 0 & 0 & \frac{1}{C} & 0 \end{bmatrix}$$

In general, the RLC ladder can be expressed as state space model following the same pattern where the dimensions of matrix A is $2N$ and this is coded on MTALB in code 4, where it generates matrix A, B and C based on N.

Now, consider a general form of a transfer function that is:

$$H(s) = \frac{V_o}{V_i} = \frac{(a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_0)}{(s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} \ldots + b_0)} \tag{33}$$

This can then be converted into a state space model as follows:

$$\frac{Z(s)}{V_i} \; and \; \frac{V_o}{Z(s)}$$

$$\boxed{\frac{1}{s^n + b_{n-1}s^{n-1} + \ldots + b_0}} \xrightarrow{Z(s)} \boxed{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} \ldots + a_0}$$

Then, converting this to time domain gives:

$$\frac{d^n z}{dt^n} + b_{n-1}\frac{d^{n-1}z}{dt^{n-1}} \ldots \ldots + b_0 Z = V_i$$

And,

$$a_{n-1}\frac{d^{n-1}z}{dt^{n-1}} + \cdots \ldots + a_0 Z = V_o$$

Now, let

$$x_1 = Z \,, x_2 = \frac{dz}{dt}, x_3 = \frac{d^2 z}{dt^2}, \ldots \ldots, x_{n+1} = \frac{d^n z}{dt^n} \tag{34}$$

This gives the A matrix as:

$$A = \begin{bmatrix} 0 & 1 & \dots. & 0 & 0 \\ 0 & 0 & 1 & .. & 0 \\ .. & .. & .. & .. & 1 \\ -b_0 & -b_1 & -b_2 & \dots.. & -b_{n-1} \end{bmatrix} \tag{35}$$

The B matrix:

$$B = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix}, (nx1) entries$$

The C matrix:

$$C = [a_0 \quad a_1 \quad \dots.. \quad a_{n-1}]$$

One can make the rational fraction in (33) proper (numerator degree < denominator degree) by preforming polynomial division as follows.

$$H(s) = \frac{N(s)}{D(s)}$$

Where,

$$N(s) = Q(s)D(s) + R(s)$$

Then the proper form is:

$$H(s) = Q(s) + \frac{R(s)}{D(s)} \tag{36}$$

Where $deg(R) < deg(D)$, and the D matrix in the state space equal to $Q(s)$.

Step 2: Compute the moments associated with the system:

Looking at the general form of Y(s):

$$sX(s) = AX(s) + BU(s) \tag{37}$$

$$Y(s) = C^T X(s)$$

For impulse input, U(s) = 1.

$$Y(s) = C^T (sI - A)^{-1} B \tag{38}$$

If (38) is expanded about $s = 0$.

$$Y(s) = C^T (-A)^{-1} B - C^T (-A)^{-2} B(s) + C^T (-A)^{-3} B s^2 \dots$$

But:

$$Y(s) = m_0 + m_1 s + m_2 s^2 + .. + m_n s^n \tag{39}$$

So,

$$m_0 = -C(A)^{-1} B$$

$$m_1 = Y'(s_0) = -C^T (A)^{-2} B$$

$$m_2 = \frac{Y''(s_0)}{2!} = -C^T (A)^{-3} B$$

$$m_k = \frac{Y^{(k)}(s_0)}{k!} = -C^T(A)^{-(k+1)}B \tag{40}$$

However, if $s = s_0$, then expanding (38)

$$Y(s) = C^T(s_0I - A)^{-1}B - C^T(s_0I - A)^{-2}B(s - s_0). + \cdots \ldots \ldots$$

So,

$$m_0 = C(s_0I - A)^{-1}B$$

$$m_1 = Y'(s_0) = -C^T(s_0I - A)^{-2}B$$

$$m_2 = \frac{Y''(s_0)}{2!} = C^T(s_0I - A)^{-3}B$$

$$m_k = \frac{Y^{(k)}(s_0)}{k!} = (-1)^k C^T(s_0I - A)^{-(k+1)}B \tag{41}$$

Step 3: Calculate the poles of the system

In AWE, the poles and residues are determined using the system's moments in order to obtain the time-domain model as outlined below.

Consider a general form of a transfer function.

$$H(s) = \frac{a_0 + a_1s + a_2s^2 + a_3s^3}{1 + b_1s + b_2s^2 + b_3s^3 + b_4s^4} \tag{42}$$

This can be equated to: $H(s) = m_0 + m_1s + m_2s^2 + m_3s^3 + \cdots$ and,

$$(1 + b_1s + b_2s^2 + b_3s^3 + b_4s^4)(m_0 + m_1s + m_2s^2 + m_3s^3 + \cdots) = a_0 + a_1s + a_2s^2 + a_3s^3 \tag{16}$$

Multiply and equate powers of s in (16) to obtain,

$$s^0: a_0 = m_0 \tag{43}$$

$$s^1: a_1 = m_0b_1 + m_1$$

$$s^2: a_2 = m_0b_2 + m_1b_1 + m_2$$

$$s^3: a_3 = m_0b_3 + m_1b_2 + m_2b_1 + m_3$$

Higher powers of s:

$$s^4: 0 = m_0b_4 + m_1b_3 + m_2b_2 + m_3b_1 + m_4$$

$$s^5: 0 = m_1b_4 + m_2b_3 + m_3b_2 + m_4b_1 + m_5$$

$$s^6: 0 = m_2b_4 + m_3b_3 + m_4b_2 + m_5b_1 + m_6$$

$$s^7: 0 = m_3b_4 + m_4b_3 + m_5b_2 + m_6b_1 + m_7$$

Putting this in matrices to solve for b_i coefficients in general.

$$\begin{bmatrix} m_0 & m_1 & \dots. & m_{q-1} \\ m_1 & m_2 & \dots. & m_q \\ m_3 & m_4 & \dots & .. \\ .. & \dots & \dots. & .. \\ m_{q-1} & \dots & \dots & m_{2q-1} \end{bmatrix} \begin{bmatrix} b_q \\ b_{q-1} \\ b_{q-2} \\ .. \\ b_1 \end{bmatrix} = - \begin{bmatrix} m_q \\ m_{q+1} \\ m_{q+2} \\ \dots \\ m_{2q-1} \end{bmatrix}$$

Gaussian elimination is applied to determine the coefficients ($b_1, b_2, b_3, .., b_q$) with ($q = 4$) in this case. The poles of the system are found by solving ($B(s) = 0$). That is, solve:

$$b_q s^q + b_{q-1} s^{q-1} \dots .. + b_1 s + 1 = 0 \tag{44}$$

Which gives the poles of the system.

Step 4: find the residues:

Generalised approach to determining the residues:

$$h(t) = \sum_{j=1}^{q} k_j e^{p_j t} \tag{45}$$

in the s domain:

$$H(s) = \sum_{j=1}^{q} \frac{k_j}{s - p_j} = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_{q-1} s^{q-1}}{1 + b_1 s + b_2 s^2 + b_3 s^3 + \dots + b_q s^q}$$

$$\frac{k_j}{s - p_j} = k_j \left(\frac{1}{s - p_j} \right) = -\frac{k_j}{p_j} \left(\frac{1}{1 - \frac{s}{p_j}} \right)$$

Let: $x = \frac{s}{p_j}$, Thus

$$\left(1 - \frac{s}{p_j} \right)^{-1} = 1 + \left(\frac{s}{p_j} \right) + \left(\frac{s}{p_j} \right)^2 + \left(\frac{s}{p_j} \right)^3 + \cdots \tag{46}$$

Hence:

$$H(s) = \sum_{j=1}^{q} -\frac{k_j}{p_j} \left(1 + \left(\frac{s}{p_j} \right) + \left(\frac{s}{p_j} \right)^2 + \left(\frac{s}{p_j} \right)^3 + \cdots \right) \tag{47}$$

But $H(s)$ is as in (42), then

$$m_0 = -\left(\frac{k_1}{p_1} + \frac{k_2}{p_2} + \dots + \frac{k_q}{p_q} \right)$$

$$m_1 = -\left(\frac{k_1}{p_1^2} + \frac{k_2}{p_2^2} + \dots + \frac{k_q}{p_q^2} \right)$$

$$m_{2q-1} = -\left(\frac{k_1}{p_1^{2q}} + \frac{k_2}{p_2^{2q}} + \dots + \frac{k_q}{p_q^{2q}} \right)$$

Which can be solved using $V\Lambda k = -m$ where

$$V = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \frac{1}{p_1} & \frac{1}{p_2} & \dots & \frac{1}{p_q} \\ \frac{1}{p_1^2} & \frac{1}{p_2^2} & .. & \frac{1}{p_q^2} \\ . & . & . & . \\ \frac{1}{p_1^{q-1}} & \frac{1}{p_2^{q-1}} & . & \frac{1}{p_q^{q-1}} \end{bmatrix}, \Lambda = \begin{bmatrix} \frac{1}{p_1} & 0 & .. & .. & 0 \\ 0 & \frac{1}{p_2} & 0 & .. & 0 \\ 0 & 0 & . & . & 0 \\ 0 & 0 & .. & \frac{1}{p_{q-1}} & 0 \\ 0 & 0 & 0 & .. & \frac{1}{p_q} \end{bmatrix}, k = \begin{bmatrix} k_1 \\ k_2 \\ . \\ . \\ k_q \end{bmatrix}, m = \begin{bmatrix} m_0 \\ m_1 \\ . \\ . \\ m_{q-1} \end{bmatrix} \quad (48)$$

Thus $k = -\Lambda^{-1}V^{-1}m$ which are the residues of the system. Finaly, the poles obtained in (44) are combined with the residues in (48) to obtain the time domain impulse response in equation (29).

Adapting AWE to Obtain a Response to a Unit Step Input:

To avoid computationally expensive explicit convolution, recursive convolution is applied based on the pole-residue representation of the transfer function [10]. For a system described in the Laplace domain as:

$$Y(s) = \frac{k_i}{s + p_i}X(s) \dots \dots . \frac{d}{dt}y(t) + p_i y(t) = k_i x(t) \quad (49)$$

Assume $x(t)$ is piecewise constant over each time interval $[t_n - t_{n-1}]$. Solving the above over the time interval using the recursive solution is:

$$y(t_n) = k_\infty x(t_n) + \sum_{i=1}^{q} y_i'(t_n) \quad (50)$$

Where

$$y_i'(t_n) = k_i\left(1 - e^{p_i(t_n - t_{n-1})}\right)x(t_{n-1}) + e^{-p_i(t_n - t_{n-1})}y_i'(t_{n-1}). \quad (51)$$

The equation in (51) updates the output at t_n using only the previous state $y_i'(t_{n-1})$ and the input $x(t_{n-1})$ eliminating the need to store the entire history of $x(t)$ [10].

Implementation of Y parameters:

Considering the general form derived in equation (17) with $n = 2$. One can generate rational expression from a given Y values of a transmission line as follows:

$$Y_R + jY_i = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0}$$

At $s = jw$

$$(Y_R + jY_i)(-w^2 + b_1 jw + b_0) = a_1 jw + a_0$$

$$(-w^2 Y_R + b_1 jwY_R + b_0 Y_R - w^2 jY_i - b_1 wY_i + jb_0 Y_i) = a_1 jw + a_0$$

Re-write this as:

$$(b_1 jwY_R + b_0 Y_R - b_1 wY_i + jb_0 Y_i) - a_1 jw - a_0 = w^2 Y_R + jw^2 Y_i$$

To find the coefficients the following is applied:

$$A = \begin{bmatrix} -1 & Y_R^1 & 0 & -Y_i^1 w \\ 0 & Y_i^1 & -jw^1 & jwY_R^1 \\ \dots & .. & .. & .. \\ -1 & Y_R^N & 0 & -Y_i^N w \\ 0 & Y_i^N & -jw^N & -w^2 Y_i^N \end{bmatrix}, B = \begin{bmatrix} a_0 \\ b_0 \\ a_1 \\ b_1 \end{bmatrix} \text{ and } C = \begin{bmatrix} Y_R^1 w^2 \\ jY_i^1 w^2 \\ \dots \\ Y_R^N w^2 \\ jY_i^N w^2 \end{bmatrix}$$

$where\ A\ B = C\ and\ B = A^{-1}B.$

Once the rational fraction is derived, AWE is used to obtain the response of the transmission line and compare it to the exact results.

(next few weeks) this will depend on what's achieved.

Filter the given Y values based on the frequency and smoothness, so the first model has points at low frequencies and combine it with the smaller models at high frequencies.

The approach to obtain an efficient model is as follows:

consider 2 models of Y parameters with 100 points of given Y values and follow the same steps for as many Y models.

1. $Y_1 = generated\ from\ the\ first\ 50\ pints.$
2. $evaluate\ Y_1 at\ the\ last\ 50\ points\ (w_1).$
3. $subtract\ the\ obtain\ values\ from\ the\ exact\ given\ value\ at\ (w_1)\ (i.e, Y_1(w_1) - exact(w_1).$
4. $use\ the\ results\ to\ obtain\ Y_2\ ans\ so\ on.$

Once the model obtained and most factors affecting it are considered, it can be compared to other methods in term of accuracy and speed.

Complex frequency hopping:

Chapter 5 – Testing, Results and Discussion

Analyse all figures and results corresponding to specific values, explaining their significance and whether they align with expectations. Discuss any discrepancies, potential sources of error, and the reasoning behind why the results may or may not be valid.

- FDTD
- RLC ladder
- Exact solution compared with FDTD and RLC
- AWE testing
- AWE unit step response
- Generating rational expression testing
- Generating transmission line testing (considering different factors)
- Comparing all methods in terms of accuracy and speed.
- Applying complex frequency hopping.

Chapter 6 – Ethics

Chapter 7 - Conclusions and Further Research

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Appendix 1