Implement AWE

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AWE involves 4 main steps:

1. Form a state – space representation
2. Form the moments
3. Find the poles of the system
4. Find the residues

And then form the impulse response as:

$$h(t) = k_0\delta(t) + k_1 e^{p_1 t} + \ldots + k_n e^{p_n t} \tag{1}$$

The following code implements AWE with first and second approximation:

```
clear all
clc
A = [-2 1 0 0;1 -2 1 0;0 1 -2 1;0 0 1 -1];
B = [1;0;0;0];
C = [1;0;0;0];
q = length(B);
moments = zeros(1,2*q-1);
for i=1:length(moments)
    moments(i) = -1*transpose(C)*A^-i*B;
end
%for first order approximation (Case 1)
b = -moments(2)/moments(1);
%the poles
p = -1/b;
% residues
k=-moments(1)*p;
%t = 0:0.01:0.5
%hense
ht1 = k*exp(p*t);
%Case 2
m2 = [moments(1),moments(2);moments(2),moments(3)];
m2_2 = -1*[moments(3);moments(4)];
b_case2 = inv(m2)*m2_2;
p_case2 = roots([b_case2(1),b_case2(2),1]);
%residues
V = [1 1 ;1/p_case2(1) 1/p_case2(2)];
A_case2 = [1/p_case2(1) 0;0 1/p_case2(2)];
k_case2 = -1* inv(A_case2) * inv(V) * [moments(1);moments(2)];
% hence the final expersion is
ht_case2 = k_case2(1)*exp(p_case2(1)*t)+k_case2(2)*exp(p_case2(2)*t);

```

This code takes the input as matrices A, B and C. For example let:

$$A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \; and \; C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

First one must find the moments as follows:

$$m_i = -C^T A^{-i+1} B$$

Where (i) goes from 0 to 2q-1 and q is the order of the transfer function associated with the equation or the state space.

In example 1 we can find that,

$$m_0 = 1\,, m_1 = -4, m_2 = 30, m_3 = -246, m_4 = 2037, m_5 = -16886, m_6 = 140000$$

Next, find b (coefficients of s in the Laplace expression) as follows:

$$\begin{bmatrix} m_0 & \cdots & m_{q-1} \\ m_1 & & m_q \\ \cdot & & \cdot \\ m_{q-1} & & m_{2q-1} \end{bmatrix} \begin{bmatrix} b_q \\ b_{q-1} \\ \cdot \\ b_1 \end{bmatrix} = - \begin{bmatrix} m_q \\ m_{q+1} \\ \cdot \\ m_{2q-1} \end{bmatrix}$$

Then solve for B(s)=0 to obtain the poles of the system where:

$$b_q s^q + b_{q-1} s^{q-1} \ldots\ldots + b_1 s + 1 = 0$$

Next, finding the residues as:

$$k = -\Lambda V^{-1} \begin{bmatrix} m_0 \\ \cdot \\ m_{q-1} \end{bmatrix}$$

So, back to example 1, since q = 4, we can find that for a first order approximation:

$$b_1 = 4, Thus, p = -0.25 \; and \; k = 0.5$$

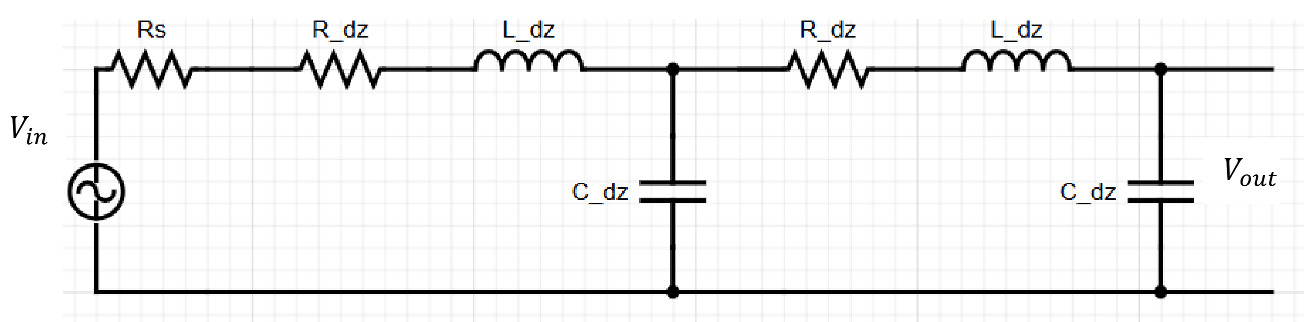
Hence using the general expression in (1) we obtain,

$$h(t) = 0.25 e^{-0.25t}$$

in the same manner we can then find second, third and fourth order approximations.

- RLC ladder implementation

Now, consider the open voltage RLC ladder for the transmission line with N=2 as follows:



From the circuit, we can say that:

$$v_{in} = (R_s + R_{dz})i_1 + L_{dz}\frac{di_1}{dt} + v_1 \tag{3}$$

$$v_1 = R_{dz}i_2 + L_{dz}\frac{di_2}{dt} + v_{out}$$

$$i_1 - i_2 = C\frac{dv_1}{dt}$$

$$i_2 = C\frac{dv_o}{dt}$$

So,

$$v_{in} = (R_s + R_{dz})i_1 + L_{dz}\frac{di_1}{dt} + \ R_{dz}i_2 + L_{dz}\frac{di_2}{dt} + v_{out}$$

Let,

$$x_1 = \ i_1\,,\quad x_2 = v_1\,,\ \ x_3 = i_2\ \ and\ , x_4 = v_o$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_o \end{bmatrix}$$

Rewriting the equations,

$$\frac{di_1}{dt} = -\frac{(R_s + R_{dz})}{L_{dz}}i_1 - \frac{v_1}{L_{dz}} + \frac{v_{in}}{L_{dz}}$$

$$\frac{di_2}{dt} = \frac{-R_{dz}i_2}{L_{dz}} - \frac{v_{out}}{L_{dz}} + v_1$$

$$\frac{dv_1}{dt} = \frac{1}{C}(i_1 - i_2)$$

$$\frac{dv_o}{dt} = \frac{1}{C}\ i_2$$

Now, express them in terms of the state variables.

1. For $\frac{dx_1}{dt}$:

$$\frac{dx_1}{dt} = -\frac{(R_s + R_{dz})}{L_{dz}}x_1 - \frac{x_2}{L_{dz}} + \frac{v_{in}}{L_{dz}}$$

2. For $\frac{dx_2}{dt}$:

$$\frac{dx_2}{dt} = \frac{1}{C}(x_1 - x_3)$$

3. For $\frac{dx_3}{dt}$:

$$\frac{dx_3}{dt} = \frac{-R_{dz}x_3}{L_{dz}} - \frac{x_4}{L_{dz}} + x_2$$

4. For $\frac{dx_4}{dt}$:

$$\frac{dx_4}{dt} = \frac{1}{C}\ x_3$$

Step 2: Write in State-Space Form

$$x^{\cdot} = Ax + Bu, \;\; y = Cx + Du$$

Where:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_o \end{bmatrix}, \quad u = v_{in}, \qquad y = v_{out}$$

Matrix A:

$$A = \begin{bmatrix} \frac{-R_s + R_{dz}}{L_{dz}} & -\frac{1}{L_{dz}} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & \frac{1}{L_{dz}} & -\frac{R_{dz}}{L_{dz}} & -\frac{1}{L_{dz}} \\ 0 & 0 & \frac{1}{C} & 0 \end{bmatrix}$$

Matix B:

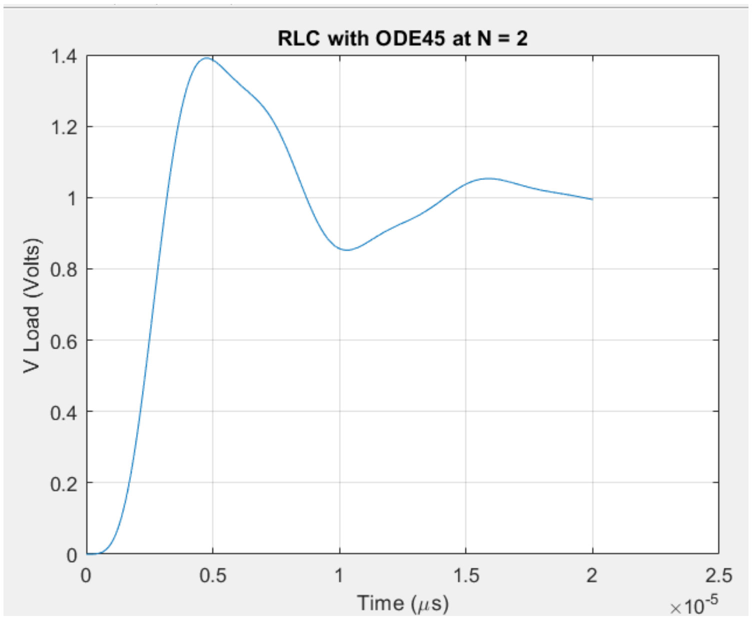
$$B = \begin{bmatrix} \frac{1}{L_{dz}} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrix C:

$$C = [0 \quad 0 \quad 0 \quad 1]$$

Now, let’s implement AWE with these on MATLAB and compare it to ode45 for validation.

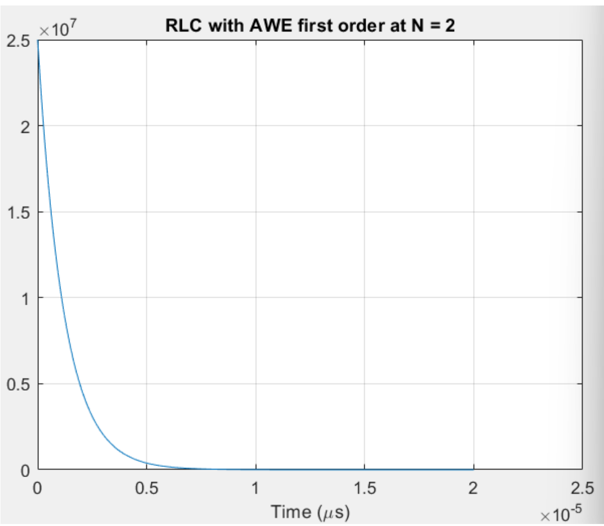
This is the expected output (from ode45).

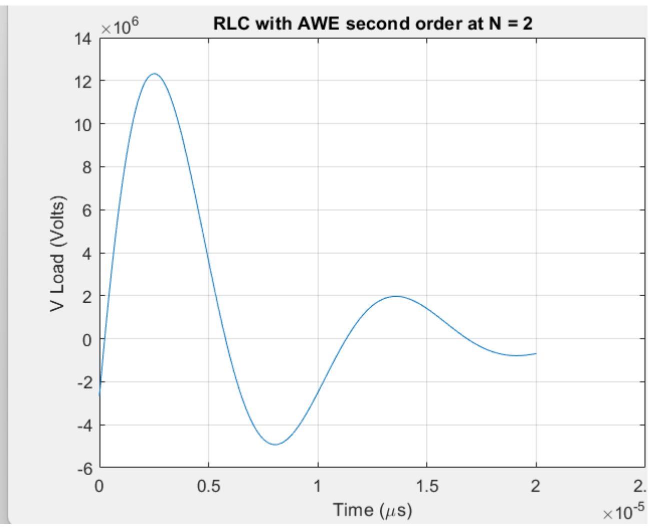


The cod:

```
clear all
clc
l = 400;
N = 2;
dz = l/N;
R = 0.1*dz;
L = 2.5e-7*dz;
C = 1e-10*dz;
Rs = 0;
Vs  = 30; % this is u
A = [-(Rs+R)/L, -1/L, 0    , 0    ;
       1/C     ,  0  , -1/C, 0    ;
    0          , 1/L , -R/L, -1/L;
    0          , 0   , 1/C , 0   ];
B = [1/L;0;0;0].*Vs;
C = [0;0;0;1];
q = length(B);
moments = zeros(1,2*q-1);
for i=1:length(moments)
    moments(i) = -1*transpose(C)*A^-i*B;
end
%for first order approximation (Case 1)
b = -moments(2)/moments(1);
%the poles
p = -1/b;
% residues
k=-moments(1)*p;
%t =
%hense
t = 0:1e-10:20e-6;
ht1 = k*exp(p*t);
%Case 2
m2 = [moments(1),moments(2);moments(2),moments(3)];
```

```
m2_2 = -1*[moments(3);moments(4)];
b_case2 = m2^-1*m2_2;
p_case2 = roots([b_case2(1),b_case2(2),1]);
%residues
V = [1 1 ;1/p_case2(1) 1/p_case2(2)];
A_case2 = [1/p_case2(1) 0;0 1/p_case2(2)];
k_case2 = -1* inv(A_case2) * inv(V) * [moments(1);moments(2)];
% hence the final expersion is
ht_case2 = k_case2(1)*exp(p_case2(1)*t)+k_case2(2)*exp(p_case2(2)*t);
figure(1)
plot(t,ht1);
grid on
xlabel('Time (\mus)');
ylabel('V Load (Volts)');
title(['RLC with AWE first order at N = ',num2str(N)]);
figure(2)
plot(t,ht_case2);
grid on
xlabel('Time (\mus)');
ylabel('V Load (Volts)');
title(['RLC with AWE second order at N = ',num2str(N)]);
```





Looking at the first and second order approximations, they don't match the expected outputs from ode45.

Let's investigate the third order approximation.

$$\begin{bmatrix} m_0 & m_1 & m_2 \\ m_1 & m_2 & m_3 \\ m_2 & m_3 & m_4 \end{bmatrix} \begin{bmatrix} b_3 \\ b_2 \\ b_1 \end{bmatrix} = -\begin{bmatrix} m_3 \\ m_4 \\ m_5 \end{bmatrix}$$

Using the matrices above, the moments can be found as follows:

$$m_0 = 1\,, m_1 = -1.2e-6, m_2 = -1.72e-12, m_3 = \ 5.05e-18, m_4 = -6.72e-25, m_5 = -1.2595e-29$$

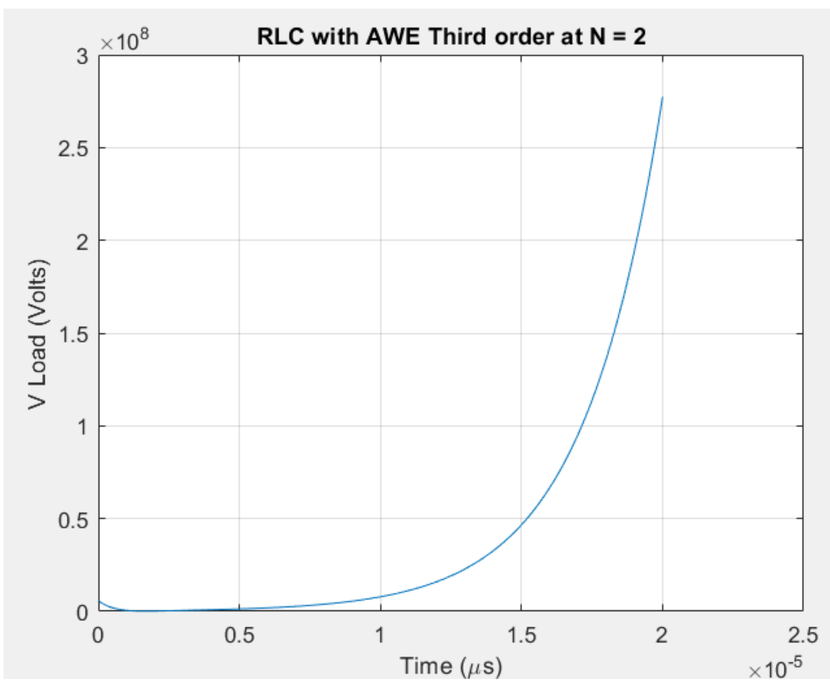
Thus,

$$b_3 = \ -1.3270e-18, b_2 = -3.1441e-12, b_1 = -1.5175e-6$$

And ,

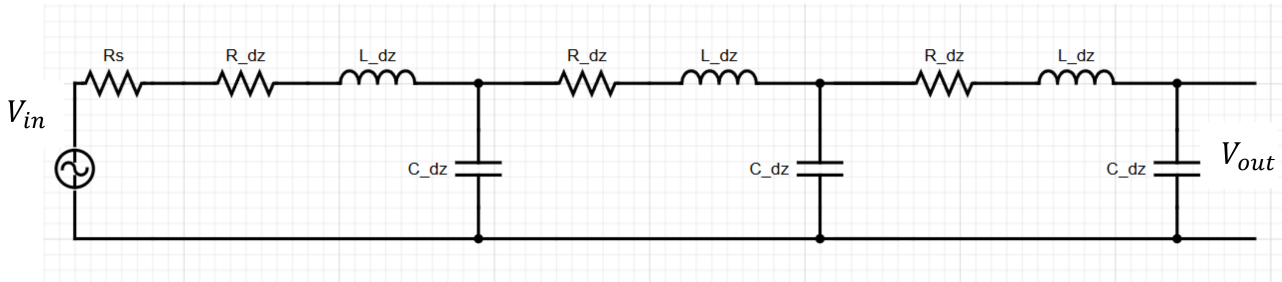
$$V = \begin{bmatrix} 1 & 1 & 1 \\ \frac{1}{p_1} & \frac{1}{p_2} & \frac{1}{p_3} \\ \frac{1}{p_1^2} & \frac{1}{p_2^2} & \frac{1}{p_3^2} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \frac{1}{p_1} & 0 & 0 \\ 0 & \frac{1}{p_2} & 0 \\ 0 & 0 & \frac{1}{p_3} \end{bmatrix}, \quad k = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$$

Hence,



This is still not close to the expected result as in ode45.

Now, Let's consider the following circuit (RLC ladder with N = 3 and open voltage) to find a pattern where we can link the number of sections to the state space model:



$$v_{in} = (R_s + R_{dz})i_1 + L_{dz}\frac{di_1}{dt} + v_1$$

$$v_1 = R_{dz}i_2 + L_{dz}\frac{di_2}{dt} + v_2$$

$$v_2 = R_{dz}i_3 + L_{dz}\frac{di_3}{dt} + v_{out}$$

$$i_1 - i_2 = C\frac{dv_1}{dt}$$

$$i_2 - i_3 = C\frac{dv_2}{dt}$$

$$i_3 = C\frac{dv_o}{dt}$$

$$v_{in} = (R_s + R_{dz})i_1 + L_{dz}\frac{di_1}{dt} + \ R_{dz}i_2 + L_{dz}\frac{di_2}{dt} + R_{dz}i_3 + L_{dz}\frac{di_3}{dt} + v_{out}$$

Let,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_2 \\ i_3 \\ v_o \end{bmatrix}, \quad u = v_{in}, \quad y = v_{out}$$

So, each section of the transmission line contributes two states to the state-space model:

What to do next (in the meantime):

1. Find what causing the issue for the AWE

Y and S parameters:

Y-parameters, or admittance parameters, characterize the electrical behavior of transmission lines in network analysis by relating port voltages and currents. They are used for modeling high-frequency circuits, especially in microwave design, as they describe how a network admits current in response to voltage.

For a two-port network:

- **Y11**: Input admittance at port 1 (port 2 shorted).
- **Y12**: Reverse admittance from port 2 to port 1.
- **Y21**: Forward admittance from port 1 to port 2.
- **Y22**: Output admittance at port 2 (port 1 shorted).

These parameters are key for understanding transmission line interactions with other components [1].

[1] A. Arrais and R. Levy, "Direct Y-Parameter Estimation of Microwave Structures Using TLM Simulation and Prony's Method," *ResearchGate*, [Online]. Available: https://www.researchgate.net/publication/230844927_Direct_Y-parameter_estimation_of_microwave_structures_using_TLM_simulation_and_Prony's_method. [Accessed: Jan. 25, 2025].

Further research can be done, at this time the IEEE website is down.