Digital Signal Processing (Digital Filters and DFT)

Acknowledgment
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Let x[n] be a discrete time signal.

Its Discrete-Time Fourier Transform is defined as

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\omega T_s}$$

This is a continuous function of ω.

The Discrete-Time Fourier Transform is a continuous function of ω

The Discrete Fourier Transform (DFT) of a given signal, x[n] is a sampled version of the underlying continuous Fourier transform, $X(\omega)$.

Discrete Fourier Transform

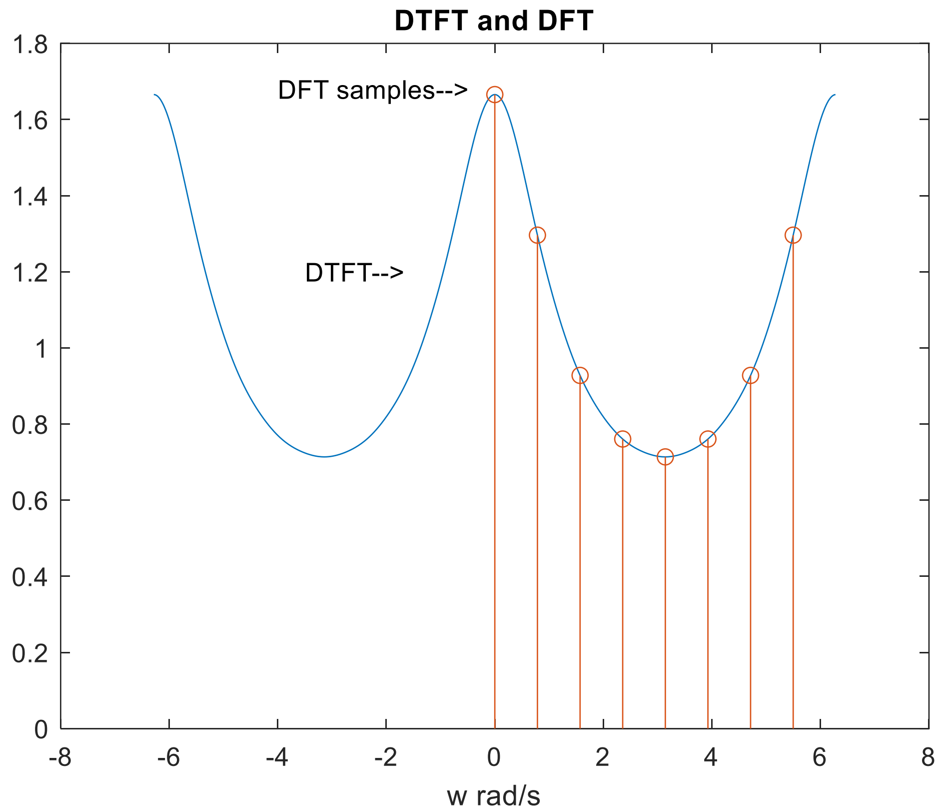
If $x[n]$ is an aperiodic, finite energy, finite duration sequence of length N, its DFT is defined as

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi kn}{N}}, \qquad k = 0,1,2,..,N-1$$

The inverse DFT is

$$x[n] = \sum_{n=0}^{N-1} X[k] e^{\frac{j2\pi kn}{N}}, \qquad n = 0,1,2,..,N-1$$

Consider the DTFT of $x[n] = 0.4^n u[n]$



The DFT gives samples of the DTFT at equally spaced frequency samples

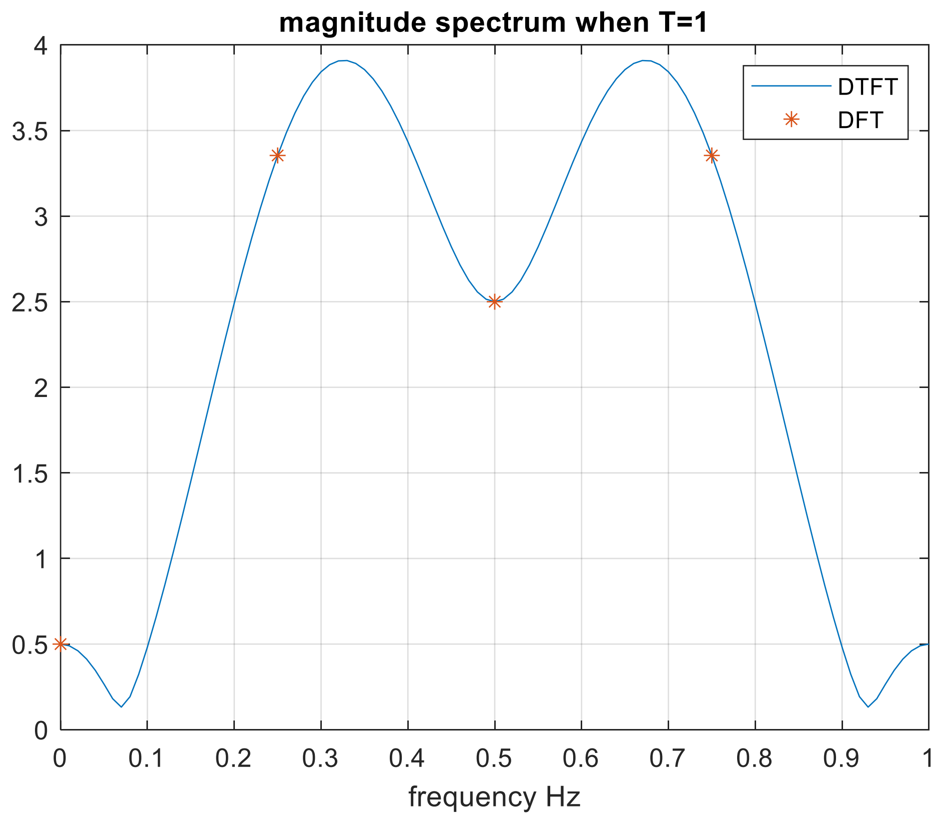
$$\omega_k = \frac{2k\pi}{NT}$$

Example

What is the DFT of $x[n] = [1, 0, -2,1.5]$?

Applying the formula with N=4 gives

$$X[k] = [1.5,3 + 0.5j, -3.5,3 - 0.5j]$$



magnitude spectrum when T=1
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
frequency Hz
DTFT
DFT

Properties of the DFT

Linearity

$$x_1[n] \rightarrow X_1[k]$$

$$x_2[n] \rightarrow X_2[k]$$

$$ax_1[n] + bx_2[n] \rightarrow aX_1[k] + bX_2[k]$$

Parseval's Theorem

The energy of a finite duration sequence $x[n]$ can be given in terms of the DFT coefficients

$$E = \sum_{N=0}^{n-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$