OUTLINE EXAMINATION MARKING SCHEME

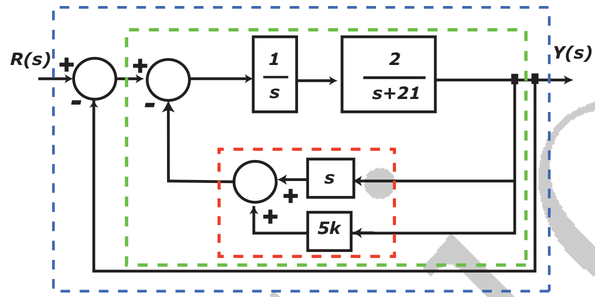
EXAM DIET: In-Class Test 2016-17
COURSE: B.Eng. in Electronic and Computer Engineering
COURSE: B.Eng. in Electronic Engineering
COURSE: B.Eng. in Mechatronic Engineering

MODULE: EE458 Control Systems Analysis

QUESTION 1

[Q1 - Read Question ≈ 5 mins]

(a) **(i)** In order to determine the overall closed loop transfer function $T(s)=\frac{Y(s)}{R(s)}$, the noise disturbance can be ignored and the inner loop in the block diagram must be identified.



It should be clear that combination of blocks in the red dashed box contains a system $s+5k$. Looking at the system within the green dashed box, $G_1=\frac{2}{s(s+21)}$ and $H_1=s+5k$, the overall transfer function for this is:

$$T_{inner}(s) = \frac{G_1}{1+G_1H_1} = \frac{\frac{2}{s(s+11)}}{1+\frac{2(s+5k)}{s(s+11)}}$$
$$= \frac{2}{s^2+23s+10k}$$

The outer loop (blue dashed box) consists of a block T_{inner} in a unity feedback loop. Hence, the overall CLTF is:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{T_{inner}}{1+T_{inner}}$$
$$\frac{2}{s^2+23s+10k+2}$$

(ii) Once the overall CLTF has been found, the sensitivity of this transfer function to changes in k can be determined.

$$S_k^T = \frac{k}{T}\frac{\partial T}{\partial k}$$
$$= \frac{k}{\frac{2}{denT}}.\left(\frac{denT.(0)-2.(10)}{denT^2}\right)$$
$$= \frac{-10k}{s^2+23s+10k+2}$$

[Q 1(a) 6 marks]

[Q 1(a) ≈ 14 mins]

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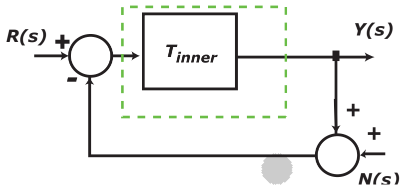
QUESTION 1 CONTINUED

(b) **(i)** The expression for $E(s)$ can be established using a simplified block diagram from Q 1(a) (featuring T_{inner}) and putting each of the inputs equal to zero. Thus,



$$E_R(s) = R(s) - Y(s)$$
$$Y(s) = T(s).R(s)$$
$$\Rightarrow E_R(s) = (1 - T(s)).R(s)$$

(ii)

$$E_N(s) = R(s) - Y(s) = -Y(s)$$
$$Y(s) = T_{inner}.(-(Y(s) + N(s))) = -T(s).N(s)$$
$$\Rightarrow E_N(s) = +T(s).N(s)$$

(iii) The Final Value Theorem is then applied to find a value for k to give an overall steady-state error of 1.8.

$$E_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s.E(s)$$
$$E(s) = E_R(s) + E_N(s) = (1 - T(s)).R(s) + T(s).N(s)$$
$$E_{ss} = \lim s \to 05 - \frac{5.(2)}{s^2 + 23s + 10k + 2} + \frac{2.(1)}{s^2 + 23s + 10k + 2}$$
$$\Rightarrow E_{ss} = 1.8 = 5 - \frac{8}{10k + 2}$$
$$k = \frac{8 - 6.4}{32} = \frac{1}{20}$$

[Q 1(b) 8 marks]

[Q 1(b) ≈ 15 mins]

(c)

(i),(ii),(iii) MATLAB is used to predict the steady-state error using the expression for $E(s)$ developed in Q 1(b). The closed loop TF $T(s)$ is found using the **feedback** command and the limit as $s\to 0$ is found using the **dcgain** command. Note: the steady-state output is required, **NOT** the steady-state error.

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QUESTION 1 CONTINUED

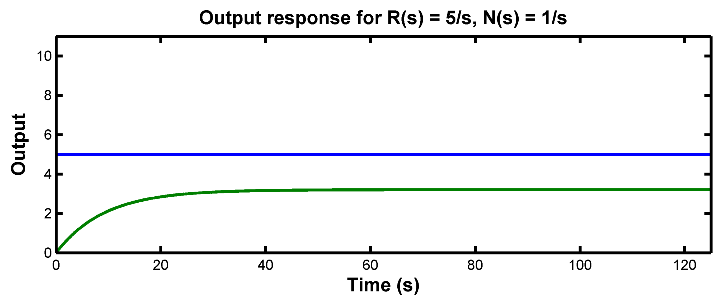
$$\begin{aligned} T(s) &= \frac{2}{s^2 + 213s + 2.5} \\ Ess_R &= 1 \\ Ess_N &= 0.8 \\ Y_{ss} &= R_{ss} - E_{ss} = 5 - (1.8) = 3.2 \end{aligned}$$

[Q 1(c) 3 marks]

[Q 1(c) $\approx$ 6 mins]

(d) **(i)** The system was set up in SIMULINK, taking care to set the final value as 5 in the step input block for $R(s)$ and to 1 in the step input block for $N(s)$. The simulation parameters must also be set correctly for a simulation stop time of 125 s. A *Sum* block and a *To Workspace* block can be used to capture the error signal which must be $E(s) = R(s) - Y(s)$

(ii) This simulation produced the output response plot:



(iii) The steady-state output and steady-state error are both measured in MATLAB. The values are found to be Ess_{meas}=1.7947 and Yss_{meas}=3.2053. They do match the predicted/designed values and comments on why they match should refer to the the actual system being used in the FVT and/or the length of the simulation time on the accuracy of the steady-state values.

[Q 1(d) 8 marks]

[Q 1(d) $\approx$ 20 mins]

[Total: 25 marks]

[END OF SOLUTIONS]