

EEN1047  
CONTROL SYSTEMS ANALYSIS  
Exercises

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# Lead Compensation

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1. (a) Given the process  $G(s) = \frac{30}{s(s+1)(s+6)}$ , use MATLAB to help design a **Phase Lead** compensator that satisfy the following:
- Overshoot = 29%.
  - Steady-state error of 3/5 for a unit ramp input.
  - A margin of safety of  $6^\circ$ .
- (b) Plot the uncompensated and compensated frequency responses on the same plot; save this plot.
- (c) Use SIMULINK to find the response of the compensated system to a unit step input *and* a unit ramp input; use Zero-Pole blocks for  $C(s)$  and  $G(s)$ . Plot the step response on the same plot as the step input, save this plot. Plot the error response to the unit ramp input, save this plot.
- (d) Use MATLAB to measure the overshoot and ramp steady-state error from the simulation responses. Do these match the design criteria? Explain why they do/do not match.

## Lag Compensation

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2. (a) Given the process  $G(s) = \frac{26}{(s+10)(s+2.4+j\sqrt{7})(s+2.4-j\sqrt{7})}$ , use MATLAB to help design a **Phase Lag** compensator to satisfy the following requirements:
- Overshoot = 16.3%
  - Steady-state error of 20% for a unit step input
  - A margin of safety of  $3.8^\circ$
- (b) Plot the uncompensated and compensated frequency responses on the same plot, save this plot.
- (c) Use SIMULINK to find the response of the compensated system for  $R(s) = 1/s$ . Use a LTI block for  $C(s)$  and a Zero-Pole block for  $G(s)$ . Plot the output response on the same plot as the step input, save this plot. Plot the step error response, save this plot.
- (d) Use MATLAB to measure the overshoot and steady-state error from the simulation responses. Do these match the design criteria? Explain why they do/do not match.

## Lag-Lead Compensation

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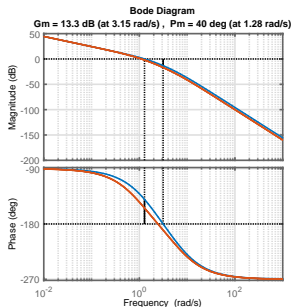
3. (a) Given the process  $G(s) = \frac{30}{s(s^2/6400 + s/50 + 1)}$ , use MATLAB to help design a **Phase Lag-Lead** compensator to achieve:
- Overshoot = 31.3%
  - Steady-state error of 1/180 for an input  $r(t) = 2tu(t)$
  - A Lag phase contribution of  $30.5^\circ$
- (b) Plot the uncompensated and compensated frequency responses on the same plot; save this plot.
- (c) Use SIMULINK to find the response of the compensated system to a unit step input *and* an input  $r(t) = 2tu(t)$ ; use a transfer function block for  $G(s)$ . Plot the step response on the same plot as the step input, save this plot. Plot the error response to the  $r(t) = 2tu(t)$  input, save this plot.
- (d) Use MATLAB to measure the overshoot and steady-state error from the simulation responses. Do these match the design criteria? Explain why they do/do not match.

# Indicative Answers - Part 1

a. Key values for the different Steps:

- $k = 1/3$
- Uncompensated PM =  $31.83^\circ$
- Desired  $\zeta = 0.367$  and  $PM = 39.9^\circ$
- Required maximum phase angle  $\phi_m = 14.117^\circ$
- $\alpha = 1.645$
- $\omega_m = 1.29 \text{ rad/s}$
- $\tau = 0.61 \text{ s}$
- Compensated PM =  $39.97^\circ$

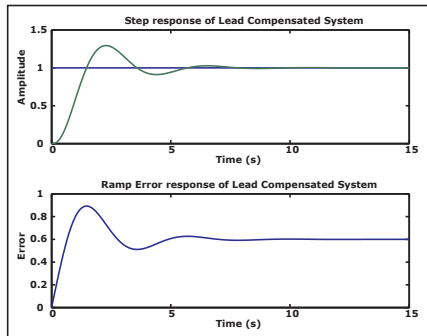
b. Plot:



# Indicative Answers - Part 1 Continued

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## c. Response Plots:



## d. Measurements:

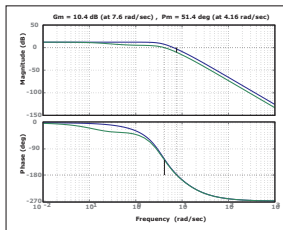
- $E_{ss_{ramp}} = 0.6002$
- Measured overshoot: 29.3649%

## Indicative Answers - Part 2

a. Key values for the different Steps:

- $k = 19.63$
- Uncompensated PM =  $13.7086^\circ$
- Desired  $\zeta = 0.5$  and  $PM = 51.83^\circ$
- Required phase from original =  $-124.37^\circ$
- $\omega_{cnew} = 4.15 \text{ rad/s}$
- $\beta = 0.4324$
- $\tau = 5.578 \text{ s}$
- Compensated PM =  $51.43^\circ$

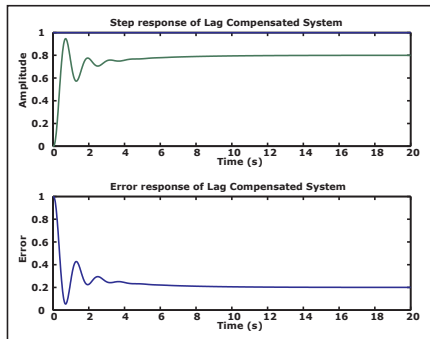
b. Plot:



## Indicative Answers - Part 2 Continued

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### c. Response Plots:



### d. Measurements:

- $E_{ss_{ramp}} = 0.2003$
- Measured overshoot: 18.3141%



## Indicative Answers - Part 3 Continued

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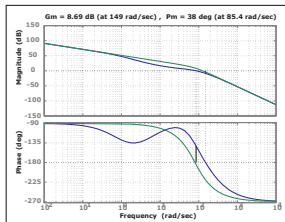
### 1. **Lag-Lead:** Key values for the different Steps:

- $k = 12$
- Uncompensated  $PM = -29.9204^\circ$
- Desired  $\zeta = 0.3468$  and  $PM = 38^\circ$
- Desired phase angle for stability  $= -149.5^\circ$
- $\omega_{ctemp} = 50.9 \text{ rad/s}$
- $\beta = 0.167$
- $\tau_2 = 1.1784 \text{ s}$
- Phase Margin after Lag compensation  $= 25.38^\circ$
- $\alpha = 6$
- $\phi_m = 45.58^\circ$
- $\omega_m = 86.1 \text{ rad/s}$
- $\tau_{u1} = 0.0047 \text{ s}$
- Compensated  $PM = 38^\circ$

# Indicative Answers - Part 3

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## 2. Frequency Plots:



## 3. Response Plots:

