W8

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- Complex frequency

Consider 4 models as follows using AWE_CHF with M = 6, where $f_{max} = 9x10^5$

w	Poles
AWE_$W(1) = 0$, Y_W(1:20)	1.6906 + 0.0000i -1.1379 + 0.0000i 0.3416 + 0.7967i 0.3416 - 0.7967i -0.1938 + 0.8052i -0.1938 - 0.8052i$x10^6$
AWE_$W(21) = 1.142x10^6$, Y_W(21:30)	-4.1884 + 0.2317i 1.9618 + 3.1320i -1.3208 - 0.0000i -0.4097 + 0.0000i 0.2323 + 1.8643i -0.0000 + 1.1424i $x10^6$
AWE_$W(26) = 1.713x10^6$, Y_W(31:40)	-0.2437 - 2.3025i 0.5619 + 2.0314i -0.2437 + 2.3025i -0.6176 + 1.3746i 0.0937 + 1.4062i -0.0000 + 1.7136i $x10^6$
AWE_$W(41) = 2.284x10^6$, Y_W(41:50)	-0.2194 - 2.3773i -0.2194 + 2.3773i 0.0847 + 2.4142i 0.0792 + 2.1397i -0.0595 + 2.1806i 0.0000 + 2.2848i $x10^6$

1. Consider the first 3 model and remove all unstable poles we get:

-0.2437 - 2.3025i -0.2437 + 2.3025i -0.6176 + 1.3746i -0.0000 + 1.7136i -4.1884 + 0.2317i -1.3208 - 0.0000i -0.4097 + 0.0000i -0.0000 + 1.1424i -1.1379 + 0.0000i -0.1938 + 0.8052i -0.1938 - 0.8052i

Since there is no overlapping, we assume all these poles are valid and dominant.

2. Find the residues,

We first need to determine the moments,

w	moments
AWE_$W(1) = 0$, Y_W(1:20)	1.0874, $-9.852x10^{-7}$, $-1.0285x10^{-12}$,….
AWE_$W(21) = 1.142x10^{6}$, Y_W(21:30)	-1.1042 - 0.4466i, $1.6x10^{-6}$ $2.43x10^{-7}i$,…
AWE_$W(26) = 1.713x10^{6}$, Y_W(31:40)	-0.8856 + 0.1380i, $6.2\,x10^{-7} + 1x10^{-6}i$, ..

Using the moments of the first model, generated residues are:

$-5.02x10^{-8} + 1.04x10^{-7}i$, $-8.47x10^{-6} + 2.36x10^{-6}i$, $3.48x10^{-6} + 5.51x10^{-7}i$
$2.00x10^{-6} - 4.91x10^{-6}i$, $4.10x10^{-6} + 8.63x10^{-7}i$, $1.80x10^{-7} - 8.71x10^{-7}i$,
$7.27x10^{-12} - 2.83x10^{-12}i$, $4.14x10^{-7} + 1.52x10^{-8}i$, $-1.55x10^{-7} + 3.31x10^{-7}i$,
$-1.27x10^{5} + 4.94x10^{5}i$, $-1.27x10^{5} - 4.94x10^{5}i$

All of these residues are close to 0 except for the one highlighted and these are the one associated with the poles from the first model. Hence, the final model is almost the same as the first model. The resultant mode highly depends on the moments' matrix.

This is not automated and not good implementation, it's just to obtain an understanding before automating it.

```
clear
clc
% Generate frequency points
f = linspace(0, 9e5, 100);
w = 2*pi*f;
s = 1i * w;
M = 6;
t = 50e-6;
first_idx = 1:20;
vo =1./(cosh(400.*(0 + 1e-10.*s).^(1/2).*(0.1 + 2.5e-7.*s).^(1/2)));
[H1,num,deno] = generate_yp2(real(vo(first_idx)),imag(vo(first_idx)),w(first_idx));
[A,B,C,D] = create_state_space(num,deno);
[p1c,np1c,r1c,m1c] = AWE_CFH_poles(A,B,C,D,M,w(first_idx(1)));
[p1,np1,r1,m1] = AWE_poles(A,B,C,D,w(first_idx(1)));
%second model -------------------------------------------------------
idx = 21:30;
%H_diff = vo(idx)-H(s(idx));
[H2,num,deno] = generate_yp2(real(vo(idx)),imag(vo(idx)),w(idx));
[A,B,C,D] = create_state_space(num,deno);
[p2c,np2c,r2c,m2c] = AWE_CFH_poles(A,B,C,D,M,w(idx(1)));
[p2,np2,r2,m2] = AWE_poles(A,B,C,D,w(idx(1)));
% Third model --------------------------------------------
idx = 31:40;
%H_diff = vo(idx)-H(s(idx));
%[Hi,num,deno] = generate_yp2(real(H_diff),imag(H_diff),w(idx));
[H3,num,deno] = generate_yp2(real(vo(idx)),imag(vo(idx)),w(idx));
[A,B,C,D] = create_state_space(num,deno);
[p3c,np3c,r3c,m3c] = AWE_CFH_poles(A,B,C,D,M,w(idx(1)));
[p3,np3,r3,m3] = AWE_poles(A,B,C,D,w(idx(1)));
% Forth model --------------------------------------------------
idx = 41:50;
```

```
%H_diff = vo(idx)-H(s(idx));
%[Hi,num,deno] = generate_yp2(real(H_diff),imag(H_diff),w(idx));
[H4,num,deno] = generate_yp2(real(vo(idx)),imag(vo(idx)),w(idx));
[A,B,C,D] = create_state_space(num,deno);
[p4c,np4c,r4c,m4c] = AWE_CFH_poles(A,B,C,D,M,w(idx(1)));
[p4,np4,r4,m4] = AWE_poles(A,B,C,D,w(idx(1)));
poles_c = [p1c,p2c,p3c,p4c];% poles from AWE_CFH with many moments
poles_nc = [np1c,np2c,np3c,np4c];% shifted poles
poles = [p1,p2,p3,p4]; % AWE with q = lenght(B)
polesn = [np1,np2,np3,np4]; %shifted poles
pt = [p3c',p2c',p1c'];
ptest = 0;
% remove unstable poles
for i=1:length(pt)
if real(pt(i))<0
    ptest = [ptest,pt(i)];
end
end
ptest = ptest(2:end);
mtest = m1c; %% moments from the first model has 1 value and zeros
[hs,r]= generate_hs(ptest,length(ptest),mtest);
%plot(f,hs(s),f,vo,f,H1(s),'r*');
```

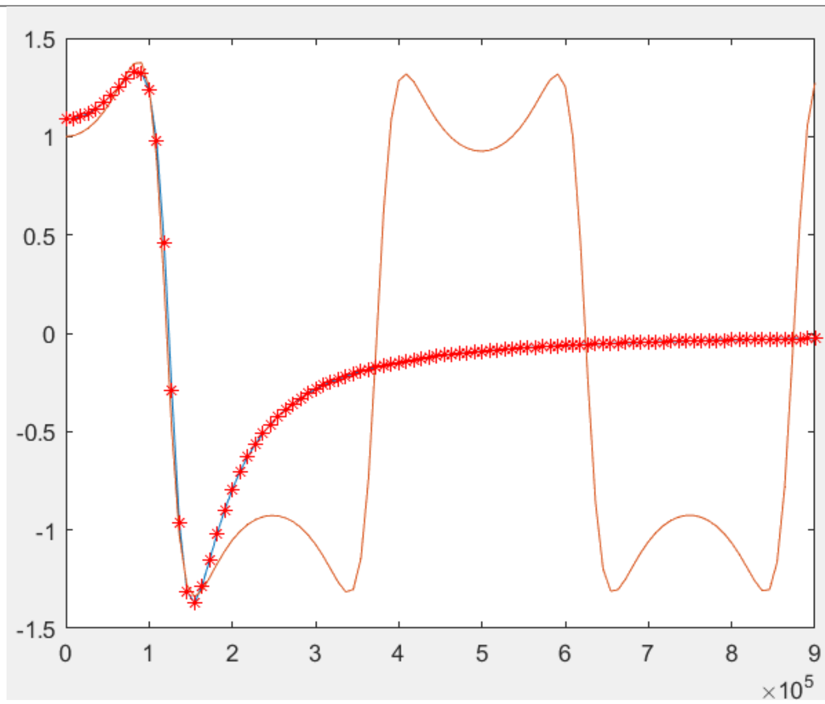


Figure 1: first model Vs resultant model

Case 2, largest residues:

1. Increase the dimensions of the A matrix to 3x3, currently it's a 2x2 matrix.

Rounding issue with tol = 8.589176e-02., so these results might not be accurate.

w	Poles	Residues
AWE_$W(1) = 0$, Y_W(1:20)	-2.2814 + 0.0000i -0.2105 + 0.7606i -0.2105 - 0.7606i10^6	-8.8177 - 0.0000i 0.0032 - 5.6259i 0.0032 + 5.6259i $x10^5$
AWE_$W(21) = 1.142x10^6$, Y_W(21:30)	-4.1540 - 0.4381i 0.2075 + 0.9514i -0.0003 + 1.1422i$x10^6$	-4.8632 - 0.9176i 0.1442 + 0.1181i -0.0000 - 0.0000i $x10^6$
AWE_$W(26) = 1.713x10^6$, Y_W(31:40)	-0.6004 - 0.0610i -0.2231 + 2.3680i -0.0065 + 1.7156i$x10^6$	3.1082 - 3.9632i 0.3882 + 5.7094i -0.0000 - 0.0000i $x10^5$
AWE_$W(41) = 2.284x10^6$, Y_W(41:50)	-0.5261 - 0.0711i -0.2396 + 2.3472i -0.0000 + 2.2849i$x10^6$	6.4974 - 4.9537i 0.4054 + 6.5539i 0.0000 + 0.0000i$x10^5$

Considering the first 2 models and removing unstable poles, then considering the one with the largest residue we get:

-0.2105 - 0.7606i10^6, -0.0003 + 1.1422i$x10^6$

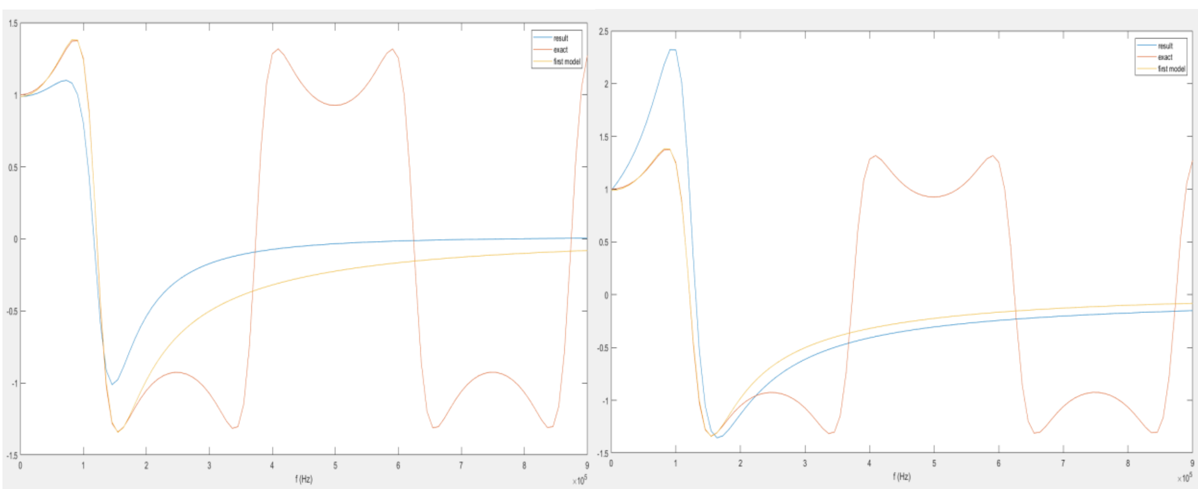


Figure 2 :impulse response of the first model Vs exact model Vs resultant model (unshifted vs shifted poles).

It can be seen that the resultant model is not more accurate than the first model which we want to achieve.

Consider model 3 and 4.

-0.6004 - 0.0610i ,-0.5261 - 0.0711i $x10^6$

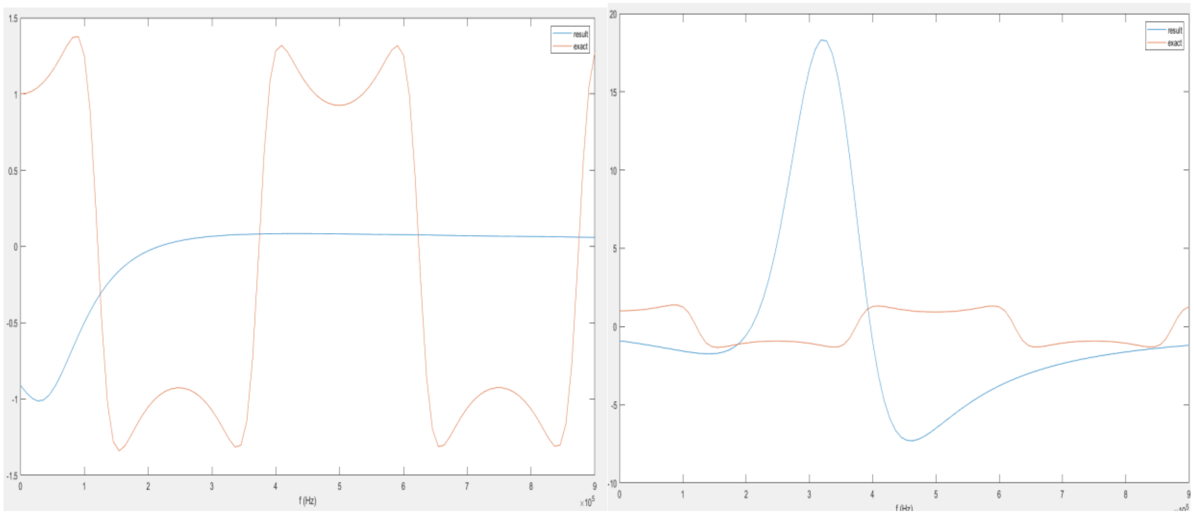


Figure 3: impulse response of the exact model Vs resultant model (unshifted vs shifted poles).

The same as in the previous case, now consider more poles from each model, say 2 poles from each model.

the first 2 models.

$-0.2105 - 0.7606i10^{6}$, $-0.2105 + 0.7606i10^{6}$, $-0.0003 + 1.1422ix10^{6}$

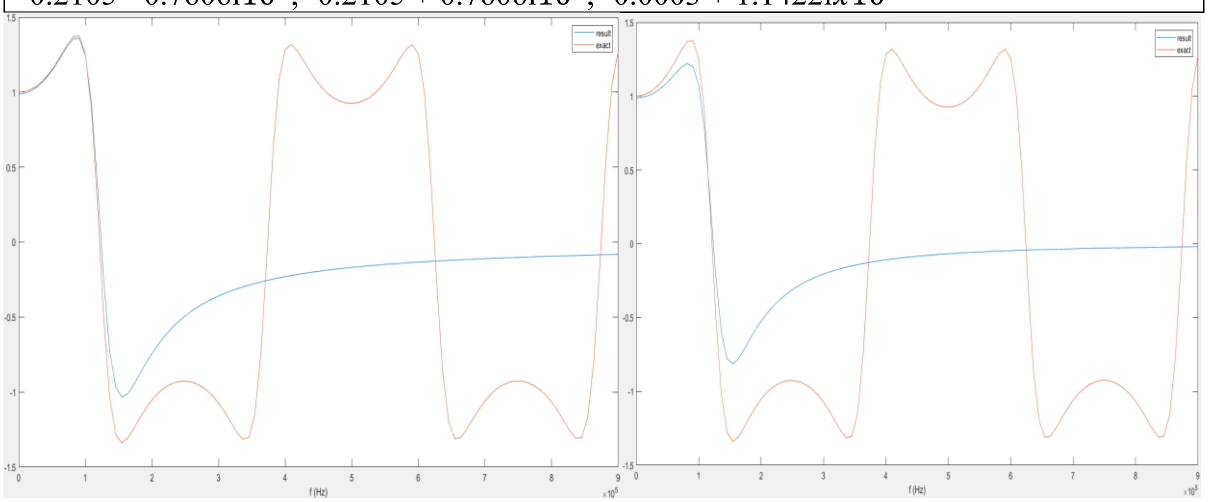


Figure 4: impulse response of the resultant model Vs the exact model (shifted and unshifted).

With RMSE 0.3972 compared to 0.0165 of the first model.

Now, let's consider expansion points near the imaginary part of the poles of the first model (0.7606):

w	Poles	Residues
AWE_$W(1) = 0$, Y_W(1:20)	-2.2814 + 0.0000i -0.2105 + 0.7606i -0.2105 - 0.7606i10^6	-8.8177 - 0.0000i 0.0032 - 5.6259i 0.0032 + 5.6259i $x10^5$
AWE_$W(14) = 0.74256x10^6$, Y_W(14:30)	-1.6442 + 0.8550i -0.1184 + 0.6931i 0.0001 + 0.7425i$x10^6$	-1.3628 + 0.3149i 0.3148 - 0.2773i 0.0000 - 0.0000i $x10^6$

Considering the following poles:

-0.2105 - 0.7606i, -0.2105 + 0.7606i, -1.6442 - 0.1124i, -0.1184 + 0.0495i $x10^6$

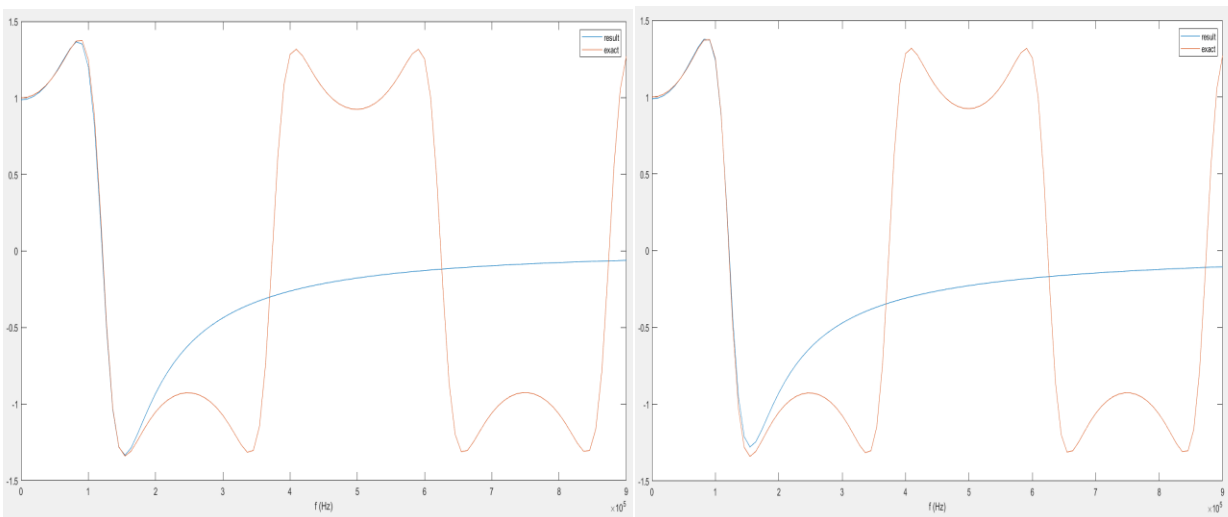


Figure 5 : shows the impulse response of the exact solution compared to resultant model (shifted and unshifted)

With RMSE 0.0507 or 0.0412 for shifted and unshifted poles respectively.

Now, let's consider evaluating the second model as the difference between the exact model and the first model at the range of frequencies.

w	Poles	Residues
AWE_$W(1) = 0$, Y_W(1:20)	-2.2814 + 0.0000i -0.2105 + 0.7606i -0.2105 - 0.7606i10^6	-8.8177 - 0.0000i 0.0032 - 5.6259i 0.0032 + 5.6259i $x10^5$
AWE_$W(21) = 1.142x10^6$, Y_W(21:30)	-0.0746 - 0.1820i -0.2505 + 1.6580i 0.0014 + 1.1530i$x10^6$	-1.2972 + 0.0725i -0.7350 - 0.0402i -0.0000 - 0.0000i $x10^6$

AWE_$W(26) = 1.713x10^6$, Y_W(31:40)	-0.7546 - 0.0160i -0.1737 + 2.2948i -0.0036 + 1.7130i$x10^6$	1.0823 - 2.3855i -0.5038 + 3.6336i 0.0000 - 0.0000i$x10^5$
AWE_$W(41) = 2.284x10^6$, Y_W(41:50)	-0.2502 - 0.0849i -0.2343 + 2.4916i 0.0000 + 2.2847ix10^6	4.5658 - 1.4445i 0.7652 + 1.9364i -0.0000 - 0.0000i$x10^5$

Consider the first 2 models with the largest residue, we get

-0.2105 - 0.7606i, -0.2105 + 0.7606i, -0.2505 - 0.5156i

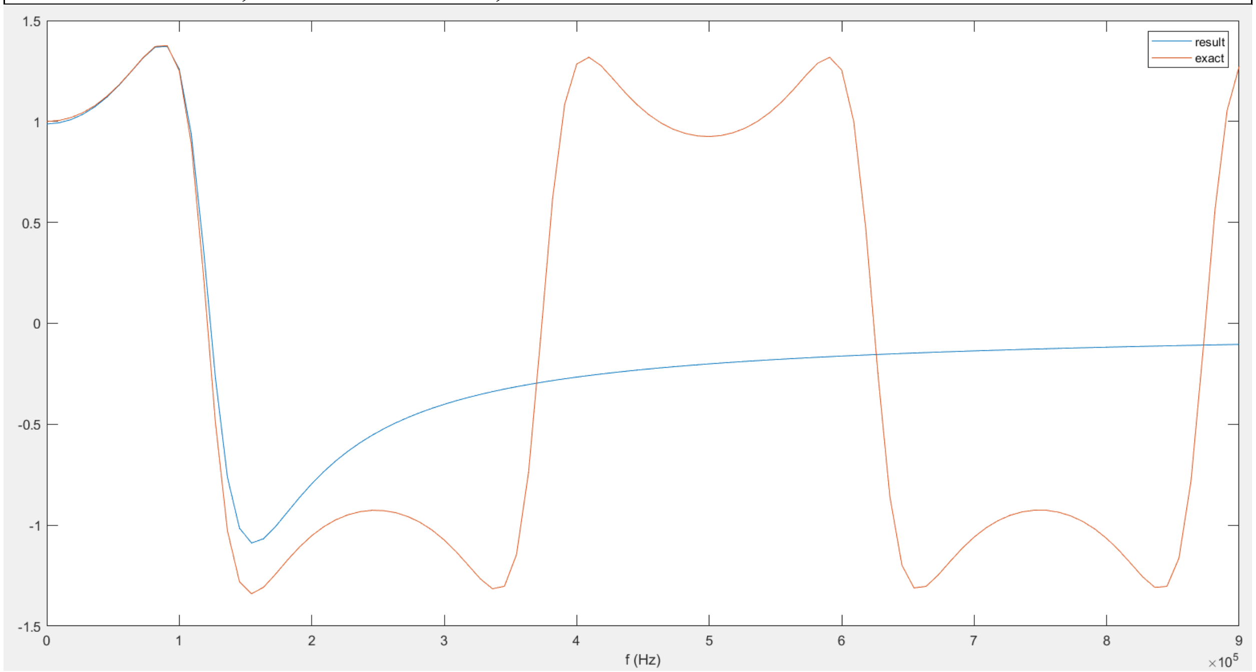
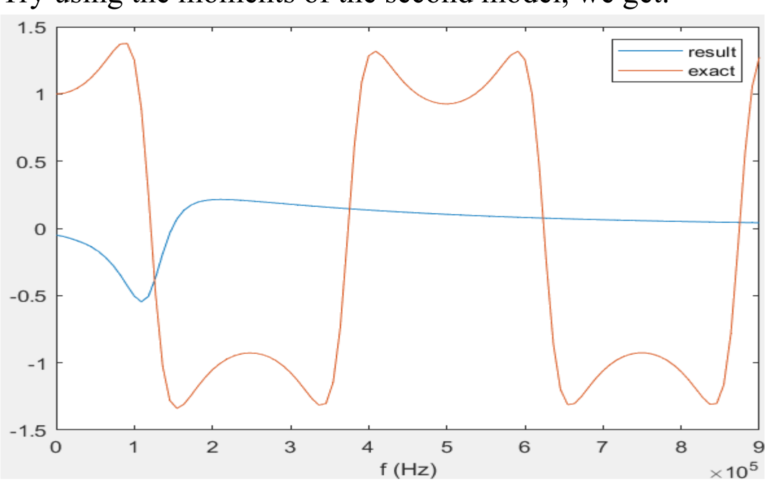


Figure 6 : shows the exact response compared to the result.

With RMSE 0.1560,

Try using the moments of the second model, we get.



Now, let's consider the poles and their residues.

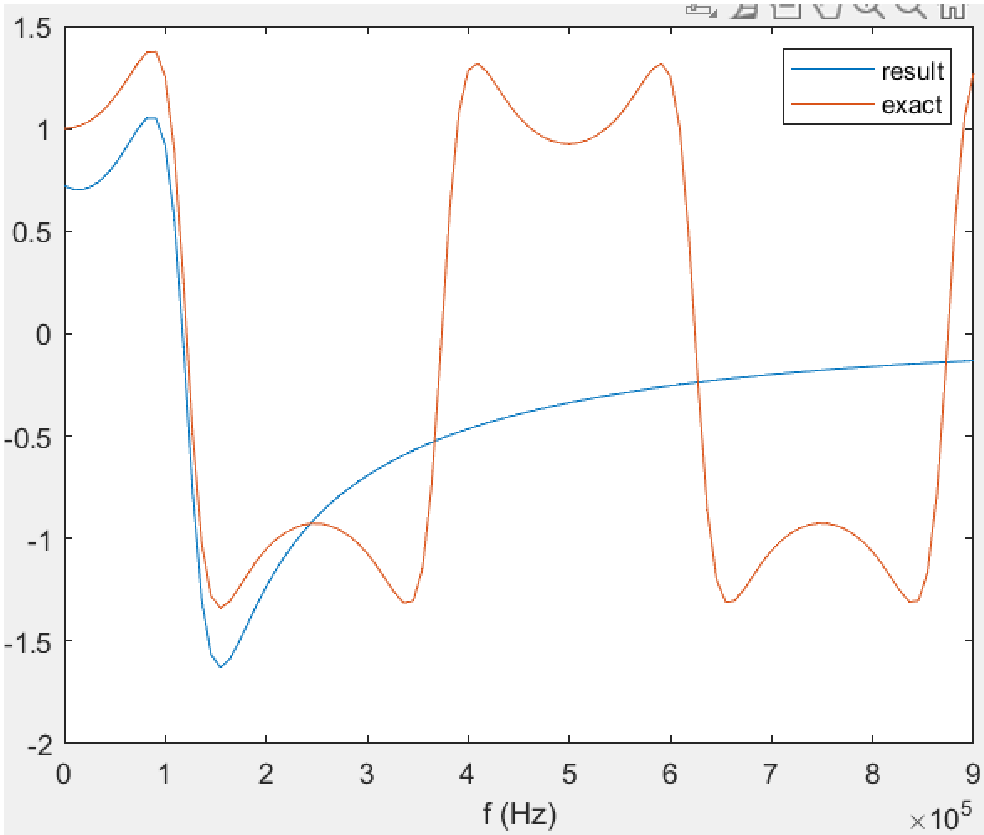


Figure 7: shows the sum of poles and their residues.

The following code is used for testing purpose only:

```
%%
clear
clc
% Generate frequency points
f = linspace(0, 9e5, 100);
w = 2*pi*f;
s = 1i * w;
M = 6;
t = 50e-6;
first_idx = 1:20;
vo =1./(cosh(400.*(0 + 1e-10.*s).^(1/2).*(0.1 + 2.5e-7.*s).^(1/2)));
[H1a,num,deno] = generate_yp(real(vo(first_idx)),imag(vo(first_idx)),w(first_idx)); %3 order TF
[A,B,C,D] = create_state_space(num,deno); % will generate 3x3 A matrix
[~,H1]=AWE2(A,B,C,D,w(1),30,t); % this is for comparison
[p1,np1,r1,m1] = AWE_poles(A,B,C,D,w(first_idx(1)));
%second model ----------------------------------------------------------
idx = 14:30;
%H_diff = vo(idx)-H1a(s(idx));
%[H2a,num,deno] = generate_yp2(real(H_diff),imag(H_diff),w(idx));
[H2a,num,deno] = generate_yp(real(vo(idx)),imag(vo(idx)),w(idx));
[A,B,C,D] = create_state_space(num,deno);
```

```
[~,H2]=AWE2(A,B,C,D,w(idx(1)),30,t);
[p2,np2,r2,m2] = AWE_poles(A,B,C,D,w(idx(1)));
% Third model ---------------------------------------------
idx = 31:40;
%H_diff = vo(idx)-H2a(s(idx));
[H3s,num,deno] = generate_yp(real(vo(idx)),imag(vo(idx)),w(idx));
%[H3s,num,deno] = generate_yp(real(H_diff),imag(H_diff),w(idx));
[A,B,C,D] = create_state_space(num,deno);
[~,H3]=AWE2(A,B,C,D,w(idx(1)),30,t);
[p3,np3,r3,m3] = AWE_poles(A,B,C,D,w(idx(1)));
% Forth model ---------------------------------------------------
idx = 41:50;
%H_diff = vo(idx)-H3s(s(idx));
[Hi,num,deno] = generate_yp(real(H_diff),imag(H_diff),w(idx));
[H4a,num,deno] = generate_yp(real(vo(idx)),imag(vo(idx)),w(idx));
[A,B,C,D] = create_state_space(num,deno);
[p4,np4,r4,m4] = AWE_poles(A,B,C,D,w(idx(1)));
[~,H4]=AWE2(A,B,C,D,w(idx(1)),30,t);
poles = [p1,p2,p3,p4]; % AWE with q = lenght(B)
polesn = [np1,np2,np3,np4]; %shifted poles
pt = [p1',p2'];
rt = [r1',r2'];
ptest = 0;
rtest = 0;
% remove unstable poles
for i=1:length(pt)
if real(pt(i))<0
    ptest = [ptest,pt(i)];
    rtest = [rtest,rt(i)];
end
end
%ptest = [ptest(3:4),ptest(end)];
ptest = ptest(2:end);
rtest = rtest(2:end);
%ptest = [ptest(2:3),ptest(end)];
mtest = [m1,m2]; %% moments from the first model has 1 value and zeros
[hs,r]= generate_hs(ptest,length(ptest),mtest,w(35));
%hs = @(s) hs(s)+H1(s);
RMSE_idx = 1:20;
R1 = RMSE(hs(s(RMSE_idx)),vo(RMSE_idx),length(vo(RMSE_idx)));
R2 = RMSE(H1(s(RMSE_idx)),vo(RMSE_idx),length(vo(RMSE_idx)));
plot(f,hs(s),f,vo);
legend('result','exact');
xlabel('f (Hz)')
```

AWE_poles

```
function [poles,poles_unshifted,residues,moments]= AWE_poles(A, B, C, D, w)
   q = length(B);
    num_moments = 2 * q;
    s0 = 1i * w;
    moments = zeros(1, num_moments);
    [r, c] = size(C);
    if r ~= 1
        C = C';
    end
    for k = 1:num_moments
        moments(k) = (-1)^(k-1) * C * (s0 * eye(size(A)) - A)^-(k) * B;
    end
    moments(1) = moments(1) + D;  % Include D in the zeroth moment
```

```
    approx_order = q;

    % Construct moment matrix and vector for denominator coefficients
    moment_matrix = zeros(approx_order);
    Vector_c = -moments(approx_order+1 : 2*approx_order)';
    for i = 1:approx_order
        moment_matrix(i, :) = moments(i : i + approx_order - 1);
    end

    % Solve for denominator coefficients
    b_matrix = moment_matrix \ Vector_c;
    poles_unshifted = roots([b_matrix; 1]);  % Unshifted poles (s' = s - s0)

    % Compute residues using unshifted poles
    V = zeros(approx_order);
    for i = 1:approx_order
        for j = 1:approx_order
            V(i, j) = 1 / (poles_unshifted(j))^(i-1);
        end
    end
    A_diag = diag(1 ./ poles_unshifted);
    r_moments = moments(1:approx_order);
    residues = -A_diag \ (V \ r_moments(:));

    % Shift poles to s-plane
    poles = poles_unshifted + s0;
end
```

generate_hs

```
function [h_s,residues]= generate_hs(poles,q,moments,w)
    s0=1i*w;
    approx_order =q;

    % Compute residues using given poles and moments
    V = zeros(approx_order);
    for i = 1:approx_order
        for j = 1:approx_order
            V(i, j) = 1 / (poles(j))^(i-1);
        end
    end
    A_diag = diag(1 ./ poles);
    r_moments = moments(1:approx_order);
    residues = -A_diag \ (V \ r_moments(:));

    % Transfer function in s-domain
    h_s = @(s)0;
    for i =1:length(poles)
        h_s = @(s) h_s(s)+residues(i) ./ (s - poles(i));
    end
    % this for testing only
    poles = poles+s0;
    hs = @(s)0;
    for i =1:length(poles)
        hs = @(s) hs(s)+residues(i) ./ ((s) - poles(i));
    end
end
```