OUTLINE EXAMINATION MARKING SCHEME

EXAM DIET: Semester 1 2018-19
COURSE: B.Eng. in Electronic and Computer Engineering
COURSE: B.Eng. in Mechatronic Engineering

MODULE: EE458 Control Systems Analysis

QUESTION 1

[Q1 - Read Question ≈ 5 mins]

(a) (i) In order to determine the **type** of the system, the forward path transfer function $T_F(s)$ must be found. The first step for finding $T_F(s)$ is to simplify the block diagram, this is done by recognising that the inner feedback loop can be represented as a single transfer function $G_{inner}(s)$.
Hence, the forward path transfer function $T_F(s)$ is:

$$T_F(s) = \frac{k_C}{0.2s+1}.\frac{0.25s+1}{1.375s+1.5}.\frac{1}{s}$$

It can be seen that the transfer function is **Type 1** as there is one pole at s=0. Since the input $R(s)$=2/s is also a **Type 1** system, there will be no steady-state error for the closed-loop system for a step input.

(ii) The transfer function between $Y_m(s)$ and $D(s)$:

$$Y(s) = K_D(s) + \frac{1}{s}[C(s)G_{inner}(s)(-Y(s))]$$
$$T_D(s) = \frac{1}{1+C(s)G_{inner}(s)}$$
$$T_D(s) = \frac{(0.275s^3+1.675s^2+1.5s)}{0.275s^3+1.675s^2+(1.5+0.25k_C)+k_C}$$

[Q 1(a) X marks]

[Q 1(a) ≈ X mins]

(b) (i) The expression for $E(s)$ can be established using the simplified block diagram from part 1 (a) (featuring $G_{inner}(s)$ and putting each of the inputs to zero. Thus,

$$E_R(s) = R(s) - Y(s)$$
$$Y(s) = T(s).R(s)$$
$$E_R(s) = (1-T(s))R(s)$$

$$E_D(s) = R(s) - Y(s) = -Y(s)$$
$$Y(s) = T_D(s).D(s)$$
$$E_D(s = -T_D(s).D(s)$$

$$E(s) = E_R(s) + E_D(s)$$
$$E(s) = (1-T(s))R(s) - T_D(s).D(s)$$

(ii) The Final Value Theorem is then applied to find a value for k_C to give an overall steady-state error of -15%. If the result from **part 1 (a) (i)** is used, this working can be simplified by ignoring the contribution due to the step input i.e. $E_R(s)$.

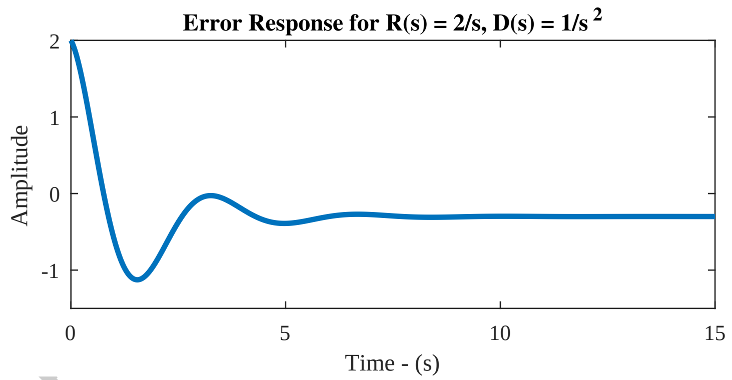
$$
\begin{aligned}
E_{ss} &= \lim_{s \to 0} s.E(s) \\
&= \lim_{s \to 0} sE_R(s) + \lim_{s \to 0} s.E_D(s) \\
&= \lim_{s \to 0} s.E_D(s) = \lim_{s \to 0} -s.T_D(s).D(s) \\
&= \frac{-1.5}{k_C} \\
k_C &= \frac{-1.5}{-0.3} = 5
\end{aligned}
$$

[Q 1(b) X marks]

[Q 1(b) $\approx$ X mins]

(c) The system was set up in SIMULINK, taking care to set the final value as 2 of the step input block for $R(s)$ and to set up the ramp block for $D(s)$. The simulation parameters must also be set correctly for a simulation stop time of 8 s.

The simulation produces the error response plot:



The steady-state error is measured in MATLAB and the value is found to be $E_{ss_{meas}} = -0.3$

[Q 1(c) X marks]
[Q 1(c) $\approx$ X mins]

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(d) The sensitivity of a closed-loop system, $T(s)$, to an open loop parameter, α, is given as:

$$S_\alpha^T = \frac{\alpha}{T} \cdot \frac{\partial T}{\partial \alpha}$$

The chain rule version of this definition for the system given where the parameter to vary is k_C is:

$$S_\alpha^T = S_C^T . S_\alpha^C$$

This rule is then used to find the sensitivity of the closed loop system to variations in k_C:

$$\begin{aligned}
S_{k_C}^C &= \frac{k_C}{C} \cdot \frac{\partial C}{\partial k_C} \\
&= \frac{k_C}{\frac{k_C}{0.2s+1}} \cdot \left[\frac{(0.2s+1)k_C - 0.k_C}{(0.2s+1)^2} \right] \\
&= 1 \\
S_C^T &= \frac{C}{T} \cdot \frac{\partial T}{\partial C} \\
&= \frac{1}{1+CG} \\
S_{k_C}^T &= S_C^T . S_{k_C}^C = \frac{1}{1+CG}
\end{aligned}$$

Students may fill in for the transfer function C but it is not strictly necessary.

[Q 1(d) X marks]
[Q 1(d) $\approx$ X mins]