# Compare RLC ladder approximation to NILT for the exact solution and find t domain approximation

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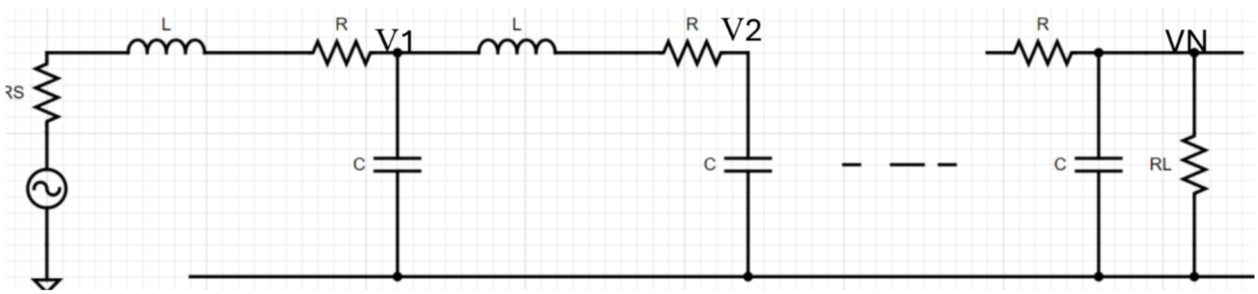
- The objective is to use the exact soluation and compare it to the RLC ladder to find an approximation in the time domain.
- The reference is the following equation.

$$T(s) = \frac{Vo}{Vin} = T(s) = \frac{Vo}{Vin} = \frac{1}{\cosh\left(l * \sqrt{(R+SC)(G+SC)}\right)}$$




In an RLC ladder approximation, the transmission line is modeled by dividing it into **N** sections. Increasing the value of **N** improves the accuracy of the approximation.

If the load is RL and Rs = 0; then

$$Vs - V1 = (R + Rs) * I1 * \ dz + L\frac{dI1}{dt} * dz\,, \qquad where\ dz\ is\ unit\ per\ length = \frac{l}{N}.$$

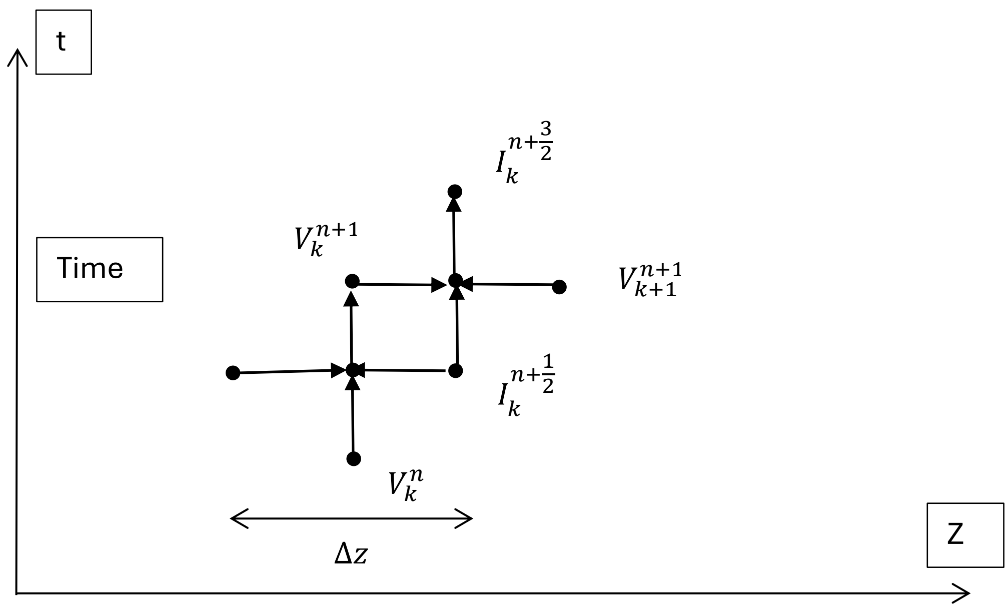
$$-\frac{dV1}{dz} = R * I1 * \ dz + L\frac{dI1}{dt} * dz\,, \qquad where\ dz\ is\ unit\ per\ length = \frac{l}{N}.$$

But in order to obtain a good approximation, we must consider as section as well. Where

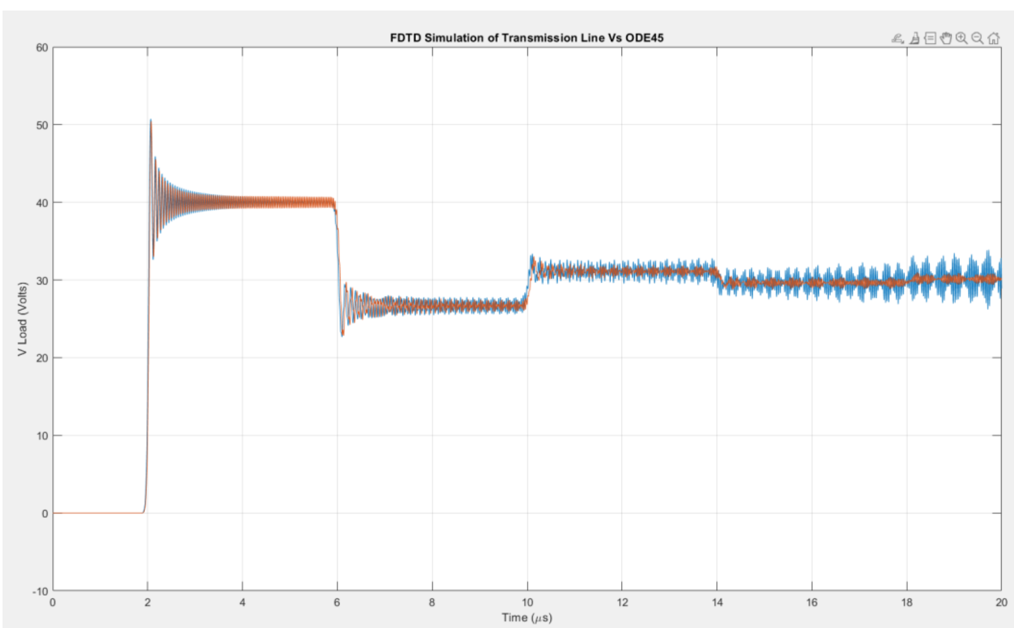
$$\Delta t \leq \frac{\Delta z}{v}.$$

We can use the approximation in Incorporation of Terminal Constraints in the FDTD Analysis of Transmission Lines 1994 by Clayton R. Paul, Fellow, IEEE. (week 4).

Where the line is divided into sections and Vn is at the end of each section and In is in the middle of each section.




Compare this to solving RLC Ladder using ode45. Set the step size to 1e-11. And N,NDZ = 200.



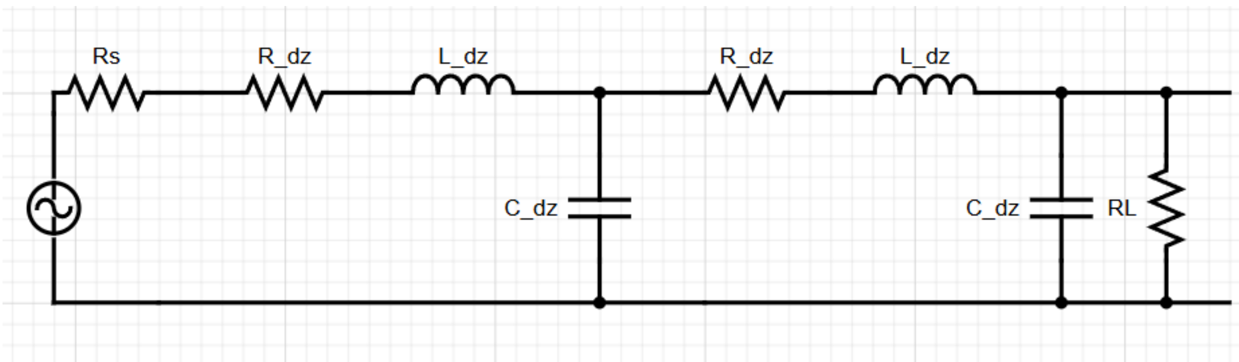
It can be seen that these two approximations are almost exactly the same. So, using the ode 45 with varying N (i.e NDZ) to get the best one with lower N but close to the exact solution. We have:

$$\frac{dI1}{dt} = -\frac{1}{L}V1 - \frac{(R+Rs)}{L} * I1 + \frac{1}{L} * Vs$$

$$\frac{dV1}{dt} = \frac{1}{C} * I1 - \frac{1}{C} * I2$$

$$\frac{dVn}{dt} = \frac{1}{C} * I(n-1) - \frac{1}{C} * \frac{Vn}{RL}$$

So, in general the number of differential equations will be 2*N.  Now assuming RL is at the end of the TL and N = 2 so,




Assuming Rs =0, then convert all components to impedances in the s domain.

$$L_{dz} = S * L_{dz},\ \ C_{dz} = \frac{1}{C_{dz} * S}\ \ , R_{dz} = R_{dz}$$

Starting from the right side, RL is in parallel with C_dz so,

$$Z_1 = \frac{RL}{RL * S * C_{dz} + 1}$$

Z1 is in series with R_dz and L_dz,

$$Z_2 = \ Z_1 + (R_{dz} + S * L_{dz})$$

Z2 is in parallel with C_dz so ,

$$Z_3 = \frac{Z_2}{Z_2 * S * C_{dz} + 1}$$

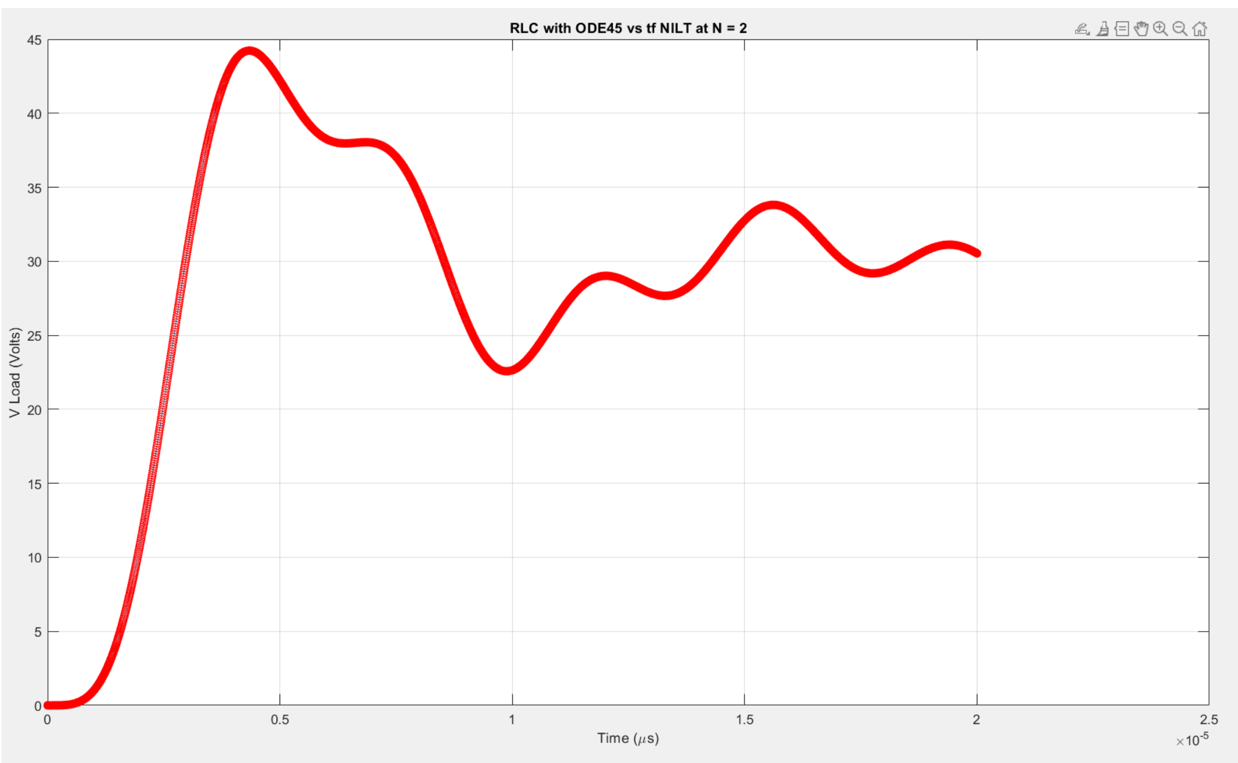
Now apply voltage divider rule to get V1.

$$V_1 = \frac{Z_3}{Z_3 + R_{dz} + S * L_{dz}} * Vs$$

$$V_1 = \frac{Z_3}{Z_3 + R_{dz} + S * L_{dz}} * Vs$$

$$V_2 = V_o = \frac{Z_1}{Z_1 + R_{dz} + S * L_{dz}} * V1$$

This gives a transfer function with order 4 (i.e S^4). I tested with NILTC to check if the expression is correct.

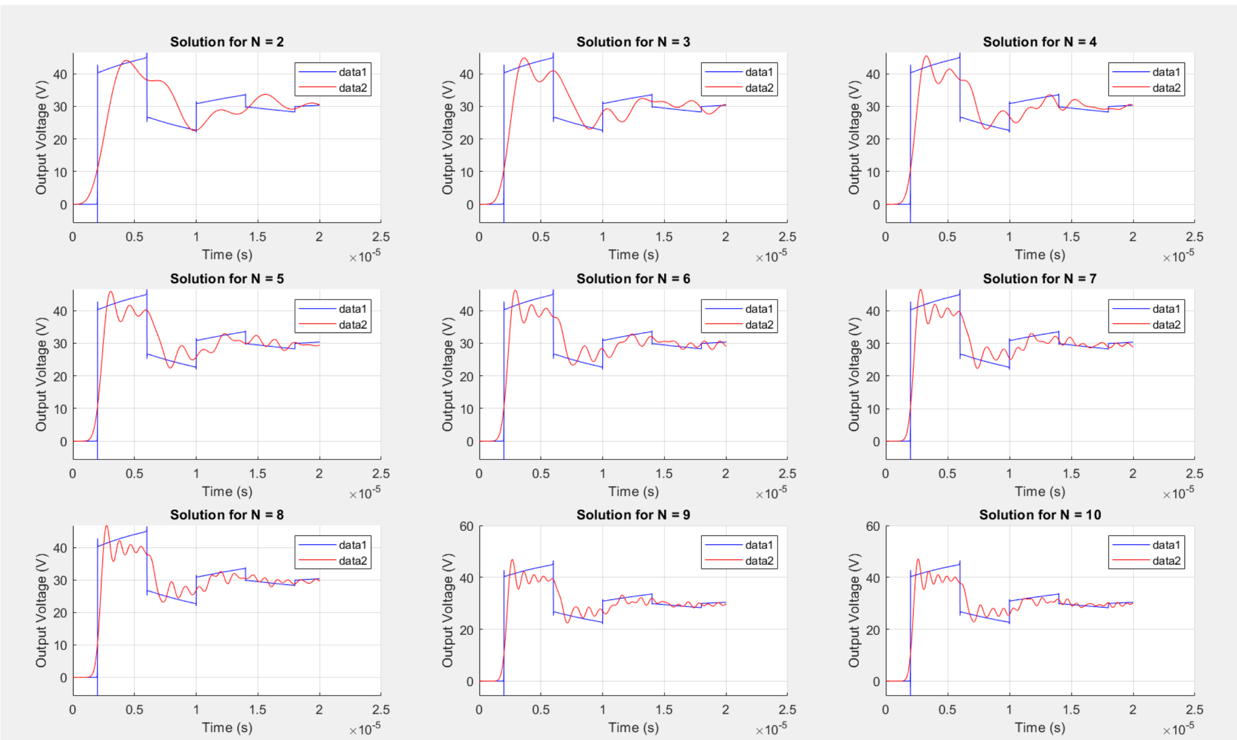


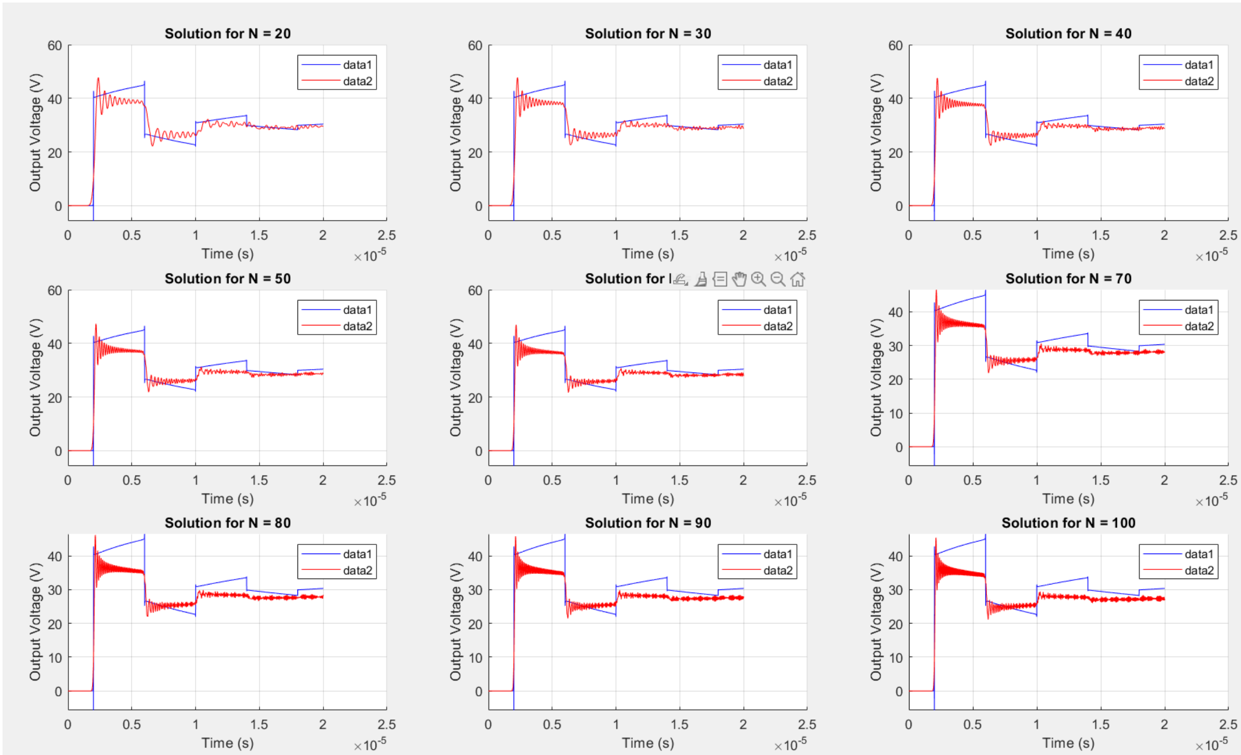

$$T(s) = \frac{RL}{RL * C_{dz}^2 * L_{dz}^2 * S^4 + 2 * RL * C_{dz}^2 * L_{dz} * R_{dz} * S^3 + RL * C_{dz}^2 * R_{dz}^2 * S^2 + C_{dz} * L_{dz}^2 * S^3 + 2 * C_{dz} * L_{dz} * R_{dz} * S^2 + 3 * RL * C_{dz} * L_{dz} * S^2 + C_{dz} * R_{dz}^2 * S + 3 * RL * C_{dz} * R_{dz} * S + 2 * L_{dz} * S + 2 * R_{dz} + RL}$$

The goal is to convert the given transfer function from the s-domain to the t-domain using partial fraction. However, this example uses N=2, which provides limited accuracy. To achieve better precision, N should be greater than 10. In other words, the order of the transfer function is 2*N.

I assume that there is another way to simplify this or to convert it to t domain, maybe by section, or maybe do some reading to find an approximation in the t domain. (I'll try to work on it in the meantime).

- Compare the exact solution to RLC ladder with different N values and find the best match. R = 0.1






At N =20, we can see that the RLC Ladder is very close to the exact solution and from there is just almost the same plot with more sections.