EXAMINATION MARKING SCHEME

EXAM DIET: Class Test 2013-14
COURSE: BEng in Electronic Engineering
COURSE: BEng in Mechatronic Engineering

MODULE: EE406 Systems

QUESTION 1

(a) The sensitivity of a closed loop system, $T(s)$, to an open loop parameter, α, is given as :

$$S_{\alpha}^{T} = \frac{\alpha}{T}.\frac{\delta T}{\delta \alpha}$$

The chain rule version of this definition for the system given where the parameter to vary is k_{PA} is:

$$S_{kPA}^{T} = S_{CPA}^{T}.S_{kPA}^{CPA}$$

This rule is then used to find the sensitivity of the closed loop system to variations in k_{PA}:

$$\begin{aligned}
S_{kPA}^{CPA} &= \frac{k_{PA}}{C_{PA}}.\frac{\delta C_{PA}}{\delta k_{PA}} \\
&= \frac{k_a}{\frac{k_{PA}}{0.15s+1}}.\left[\frac{(0.15s+1).1 - k_{PA}.0}{(0.15s+1)^2}\right] \\
&= 1 \\
S_{CPA}^{T} &= \frac{C_{PA}}{T}.\frac{\delta T}{\delta C_{PA}} \\
&= \frac{1}{1+C_{PA}GH} \\
S_{kPA}^{T} &= S_{CPA}^{T}.S_{kPA}^{CPA} = 1.\frac{1}{1+C_{PA}GH}
\end{aligned}$$

[Students can put in transfer function for C_{PA} but it's not strictly necessary]

[Part 1(a) 7 marks]

(b) The steady–state error is given as:

$$E_{ss} = \lim_{s\to 0} s.E(s)$$

This formula is used to calculate the value of k_{PA} to give a steady–state error of -30^o for a disturbance input of $D(s) = 2/s$ when $R(s) = 0$.

$$\begin{aligned}
E(s) &= R(s) - Y(s) = -Y(s) \\
&= -\frac{G}{1+C_{PA}GH}.D(s) \\
E_{ss} &= \lim_{s\to 0} s.E(s) = \lim_{s\to 0}\frac{-s.\frac{2}{s}.G}{1+C_{PA}GH} \\
&= \frac{-2(420.5)}{2k_{PA}(420.5)} \\
-30^o &= -30.\frac{\pi}{180} = \frac{-2}{2k_{PA}} \\
k_{PA} &= \frac{6}{\pi} = 1.9099
\end{aligned}$$

[Part 1(b) 6 marks]

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QUESTION 1 CONTINUED

(c) MATLAB can be used to find the overall closed loop transfer function and then the steady–state error for a unit step input using the *dcgain* function:

```
ess_pred = 1-dcgain(syscl)
% ess_pred =  0.500 rad
```

The closed loop transfer function is found to be:

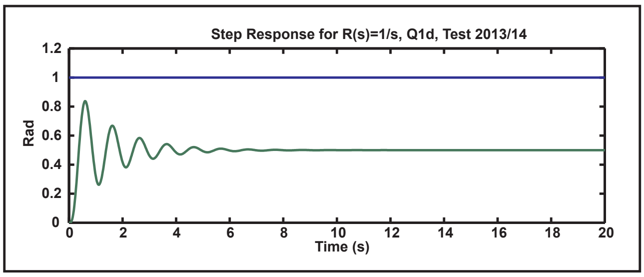
$$T(s) = \frac{5354}{(s^2 + 1.376s + 38.91)\,(s^2 + 26.29s + 275.2)} = \frac{5354}{s^4 + 27.67s^3 + 350.3s^2 + 1402s + 107079}$$

and the steady–state error for a unit step input is 0.5 *rad*.

[Part 1(c) 3 marks]

(d) SIMULINK is used to simulate the complete closed loop system; care must be taken when implementing $G(s)$ in Zero-Pole format to represent the complex poles correctly. The initial simulation has no disturbance.

The output from the simulation is plotted on the same plot as the input signal.



MATLAB is used to measure the steady–state error:

```
ess_meas = mean(error(901:end));
% ess_meas = 0.5000
```

The measured steady–state error is found to be 0.500 *rad* which matches that predicted in part **1(c)**.

[Part 1(d) 6 marks]

(e) EXAMINATION MARKING SCHEME

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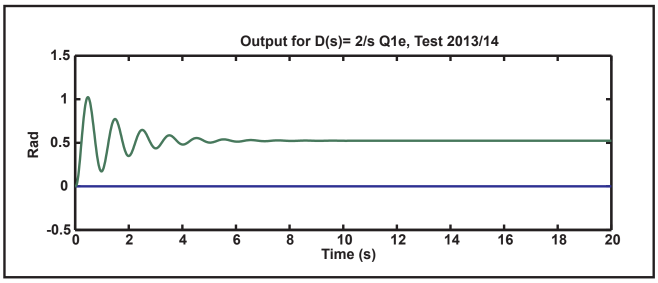
QUESTION 1 CONTINUED

SIMULINK is used to simulate the complete closed loop system; the step input must be set at output level 0 and a step disturbance with magnitude 2 is added to the simulation. The output from the simulation is plotted on the same plot as the input signal (which has magnitude zero).



MATLAB is used to measure the steady–state error:

```
% ess2_meas = -0.5236
% ess2_meas_deg =  -30.0000  % converted to degrees
```

The measured steady–state error is found to be -30^o which matches that designed for in part **1(b)**. The steady–state error matches very accurately because it does not rely on dominance of poles but on the steady–state values of the system.

[Part 1(e) 4 marks]

[Total: 25 marks]