OUTLINE EXAMINATION MARKING SCHEME

EXAM DIET: CA Semester 1 2021-2022
COURSE: B.Eng. in Electronic and Computer Engineering
COURSE: B.Eng. in Mechatronic Engineering

MODULE: EE458 Control Systems Analysis

QUESTION 1

[Q1 - Read Question $\approx$ 5 mins]

(a) (i) The first way to address this question is to use first principles, which is most straight forward. However, block diagram reduction techniques can still be applied to solve this question, i.e. by moving the signal line after the G_3 block connected to the right most summing point to the left most summing point. By doing so, it is required to change the original G_3 block to $\frac{G_3}{G_2}$ to make the block diagram mathematically equivalent. Thus, the forward path transfer function consists of $G_1+\frac{G_3}{G2}$ and $\frac{G_2}{1+G_2G_4}$ connected in series. The feedback path is H, three blocks are then connected in the classic negative feedback loop. The final expression for the transfer function can be obtained as follows:

$$T(s) = \frac{C(s)}{R(s)} = \frac{(G_1 + \frac{G_3}{G2})\frac{G_2}{1+G_2G_4}}{1 + H(G_1 + \frac{G_3}{G2})\frac{G_2}{1+G_2G_4}} = \frac{G_1G_2 + G_3}{1 + G_2G_4 + HG_1G_2 + G_3H}$$

[4 marks]

(ii) Substituting the expressions for each component, we can easily find out that

$$T(s) = \frac{2}{s^2 + 4s + 5}$$

[2 marks]

(iii) Accordingly, the characteristic equation is $s^2+4s+5=0$. The type of the system is 0 as there are no roots of the CE located at $s=0$.

[2 marks]

(iv) The system is stable as all roots of the CE have negative real parts.

[2 marks]

[Q 1(a) 10 marks]

[Q 1(a) $\approx$ 20 mins]

(b) (i) Using the Final Value Theorem, we have

$$C_{ss} = \lim_{t\to\infty} C(t) = \lim_{s\to 0} s\frac{1}{s}\frac{2}{s^2 + 4s + 5} = \frac{2}{5}$$

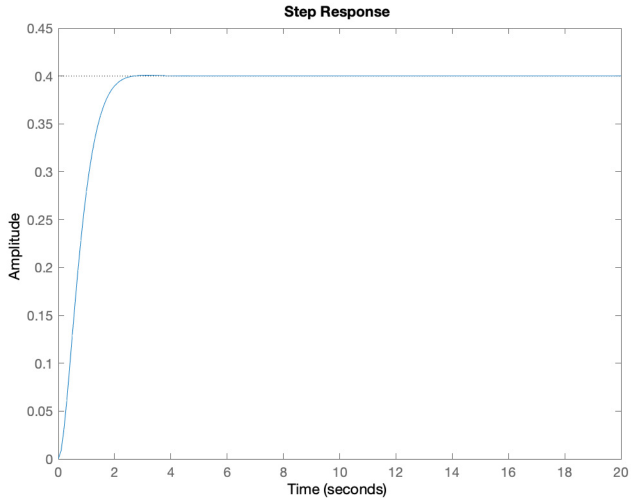
[3 marks]

(ii) The plot from Simulink is shown below.

[3 marks]

[Q 1(b) 6 marks]

[Q 1(b) $\approx$ 15 mins]



(c) (i) Using the formula for sensitivity, we can first find out that

$$E_{ss} = \frac{ab}{ab+K}$$

Then,

$$S_a^{E_{ss}} = \frac{a}{E_{ss}} \frac{\delta E_{ss}}{\delta a} = \frac{a}{\frac{ab}{ab+K}} \frac{(ab+K)b - ab^2}{(ab+K)^2} = \frac{K}{ab+K}$$

[2 marks]

(ii)

$$S_K^{E_{ss}} = \frac{K}{E_{ss}} \frac{\delta E_{ss}}{\delta k} = \frac{K}{\frac{ab}{ab+K}} \frac{-ab}{(ab+K)^2} = \frac{-K}{ab+K}$$

[2 marks]

(iii) According to the final expression $S_a^{E_{ss}} = \frac{K}{ab+K}$, fixing a, K, increasing b implies that the sensitivity is reduced.

[2 marks]

(iv) Looking at the characteristic equation of the system, we have

$$(s+a)(s+b) + K = 0$$

Letting $a = K = 1$, we have:

$$s^2 + s(b+1) + (b+1) = 0$$

Let s_1 and s_2 be two roots for the equation, then we have:

$$s_1 = \frac{-(b+1)}{2} + \frac{(b+1)}{2} j \sqrt{\frac{4}{(b+1)} - 1}, \; s_2 = \frac{-(b+1)}{2} - \frac{(b+1)}{2} j \sqrt{\frac{4}{(b+1)} - 1}$$

Since $0 < b < 3$, the real parts are always negative, and it will always have the imaginary parts, so system is stable.

[3 marks]

[Q 1(c) 9 marks]

[Q 1(c) $\approx$ 20 mins]

[Total: 25 marks]

[END OF Q1 ANSWERS]