Compare RLC ladder approximation to NILT0 and NILTcv

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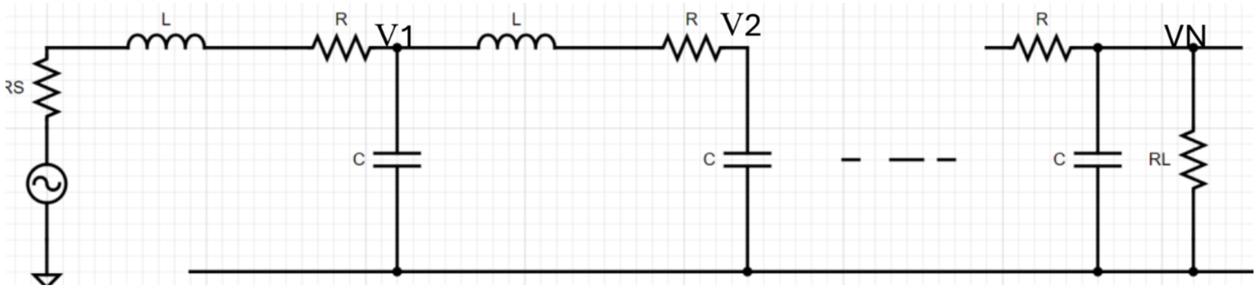
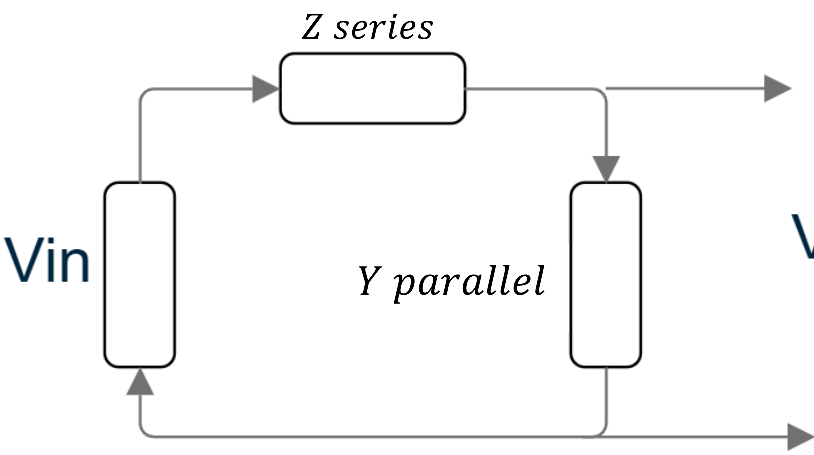
- A transmission line can be approximated using the following exact solation:



This can be represented as the following to obtain the exact solution:



Then,

$$Z\ series = Zo \sinh(\gamma l)$$

$$Zo = \sqrt{\frac{z}{y}}\,, \qquad Y = G + SC, \qquad Z = (R + SL), \quad \gamma = \sqrt{ZY}$$

$$Y\ parallel = Yo \tanh\left(\frac{\gamma l}{2}\right), \quad Yo = \frac{1}{Zo}\,, \quad Z\ parallel = \frac{1}{Y\ parallel}$$

$$T(s) = \frac{Vo}{Vin} = \frac{Z\ Parallel}{Zseries + Z\ parallel}$$

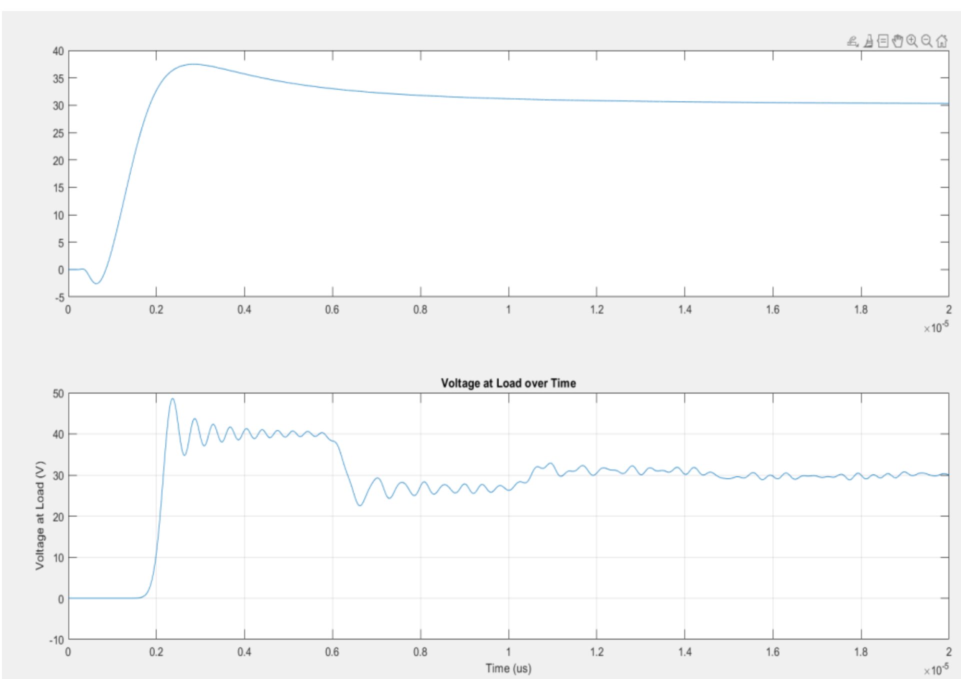
$$T(s)=\frac{Vo}{Vin}=\frac{\dfrac{\sqrt{\dfrac{R+SC}{G+SC}}}{\tanh\left(l*\dfrac{\sqrt{(R+SC)(G+SC)}}{2}\right)}}{\sqrt{\dfrac{R+SC}{G+SC}}\sinh\left(l*\sqrt{(R+SC)(G+SC)}\right)+\dfrac{\sqrt{\dfrac{R+SC}{G+SC}}}{\tanh\left(l*\dfrac{\sqrt{(R+SC)(G+SC)}}{2}\right)}}$$

$$T(s)=\frac{Vo}{Vin}=\frac{\dfrac{1}{\tanh\left(l*\dfrac{\sqrt{(R+SC)(G+SC)}}{2}\right)}}{\sinh\left(l*\sqrt{(R+SC)(G+SC)}\right)+\dfrac{1}{\tanh\left(l*\dfrac{\sqrt{(R+SC)(G+SC)}}{2}\right)}}$$

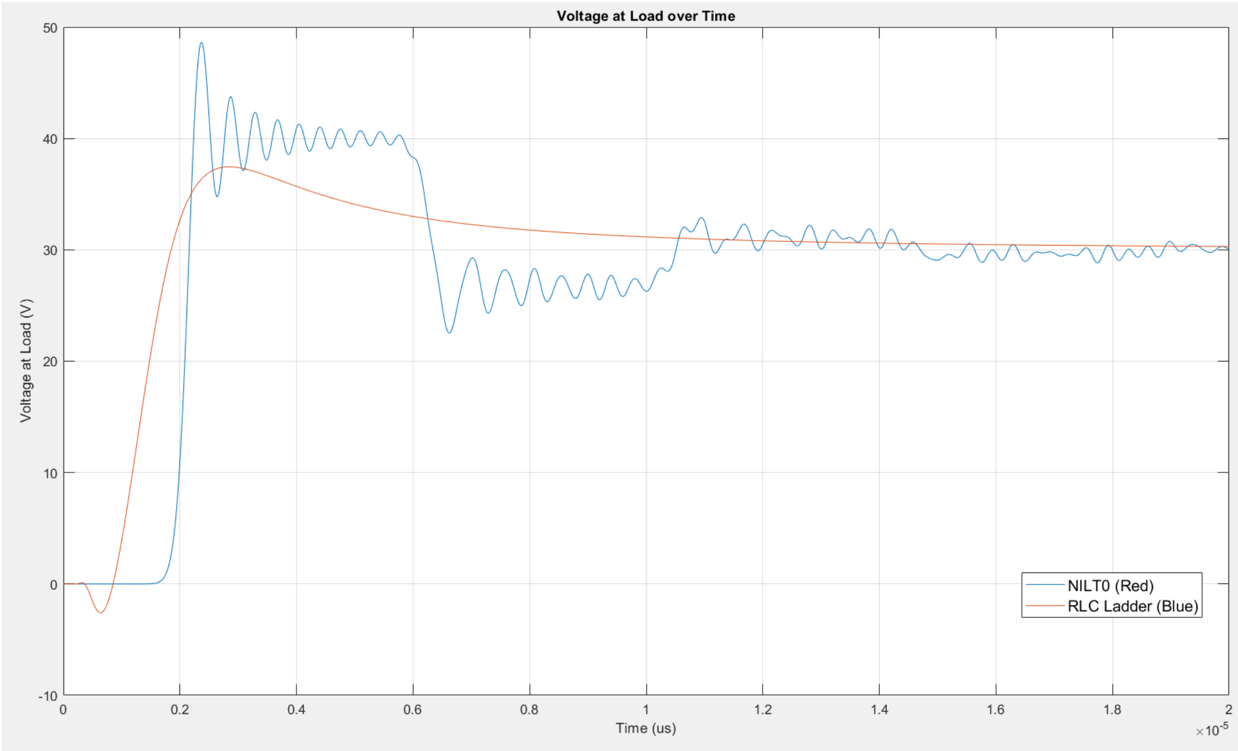
$$\tanh\left(\frac{x}{2}\right)=\frac{\cosh(x)-1}{\sinh(x)}$$

$$T(s)=\frac{Vo}{Vin}=\frac{1}{\cosh\left(l*\sqrt{(R+SC)(G+SC)}\right)}$$

Using NILT0, at M=2 and compare it with RLC ladder (not state space)



Or in one plot,



This with NILT0 without time stepping.

- RLC ladder using space state representation.

Telegrapher equations are known as.

$$\frac{dV(x,t)}{dx} = -R(x) * i(x,t) - L(x)\frac{di(xi,t)}{dt}$$
$$\frac{di(x,t)}{dx} = -G(x) * V(x,t) - C(x)\frac{dV(xi,t)}{dt}$$

in state space,

$$\frac{d}{dx}\begin{bmatrix} V(x,t) \\ i(x,t) \end{bmatrix} = \begin{bmatrix} 0 & -R(x) \\ -G(x) & 0 \end{bmatrix} * \begin{bmatrix} V(x,t) \\ i(x,t) \end{bmatrix} - \begin{bmatrix} 0 & -L(x) \\ -C(x) & 0 \end{bmatrix} * \frac{d}{dt}\begin{bmatrix} V(x,t) \\ i(x,t) \end{bmatrix}$$

Moving this to Laplace domain we get,

$$\frac{d}{dx}\begin{bmatrix} V(x,s) \\ i(x,s) \end{bmatrix} = \begin{bmatrix} 0 & -Z(x,s) \\ -Y(x,s) & 0 \end{bmatrix} * \begin{bmatrix} V(x,s) \\ i(x,s) \end{bmatrix} + \begin{bmatrix} 0 & L(x) \\ C(x) & 0 \end{bmatrix} * \begin{bmatrix} V(x,0) \\ i(x,0) \end{bmatrix}$$

Where Z (x, s) = R(x) + sL(x), and Y (x, s) = G(x) + sC (x, s) are series impedance.

Let,

$$W(x,s) = \begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix}, M = \begin{bmatrix} 0 & -Z(x,s) \\ -Y(x,s) & 0 \end{bmatrix}, and\ N = \begin{bmatrix} 0 & L(x) \\ C(x) & 0 \end{bmatrix}$$

So,

$$\frac{dW(x,s)}{dx} = M * W(x,s) + N * W(x,0)$$

Then, W at the end of the transmission line (W (l , s)) should equal to,

$$Let\ \Phi = e^{Ml}, so\ \phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}, W(l,s) = \Phi\, \mathrm{W}(0,\mathrm{s}) + \int_0^l e^{M(l-x)} * N * W(x,0)dx$$

If the initial conditions are zero, (i.e. at W(x,0) = 0) then,

$$W(l,s) = \Phi\, \mathrm{W}(0,\mathrm{s})$$

Or matrix format,

$$W(l,s) = \begin{bmatrix} V(l,s) \\ I(l,s) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \cdot \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}$$

If I(l,s) = 0 , Then,

$$\begin{bmatrix} V(l,s) \\ 0 \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \cdot \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}$$

$$0 = \Phi_{21} V(0,s) + \Phi_{22} I(0,s)$$

$$I(0,s) = \Phi_{21} V(0,s) - \Phi_{22}^{-1}$$

$$V(l,s) = \Phi_{11} V(0,s) + \Phi_{12}\, I(0,s)$$

$$V(l,s) = \Phi_{11} V(0,s) - \Phi_{12}\, \Phi_{21}\, \Phi_{22}^{-1} V(0,s)$$

This is in the S domain, one can implement an NILT to solve it and get V(l,t) and to do this the following steps can be followed:

1. Find e^Ml, MATLAB has a function can do that very easily called expm(M*l).

Testing expm with the following example.

$$\frac{d}{dt}[x] = \begin{bmatrix} 0 & 2 \\ -3 & -5 \end{bmatrix} * [x] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} * xin$$

$$y = [1 \quad 0] * [x]$$

The answer to this is,

$$y(t) = C * e^{at} * B$$

$$e^{at} = M * e^{\Omega t} * M^{-1}$$

These can be obtained using the model matrix method, then y(t),

$$y(t) = 2 * e^{-2t} - 2e^{-3t}$$

Using the following code in MATLAB, we get the same result easily,

```
A = [0 2; -3 -5];
B = [0;1];
C= [1 0];
eat = expm(A*t);
y = mtimes(mtimes(C,eat),B);
```

Implemtning this to get an expression for V(l,s) we get,

$$V(l,s) = \frac{2 * Vs(s) * e^{l*\sqrt{Y*Z}}}{e^{(2*l*\sqrt{Y*Z})} + 1}$$

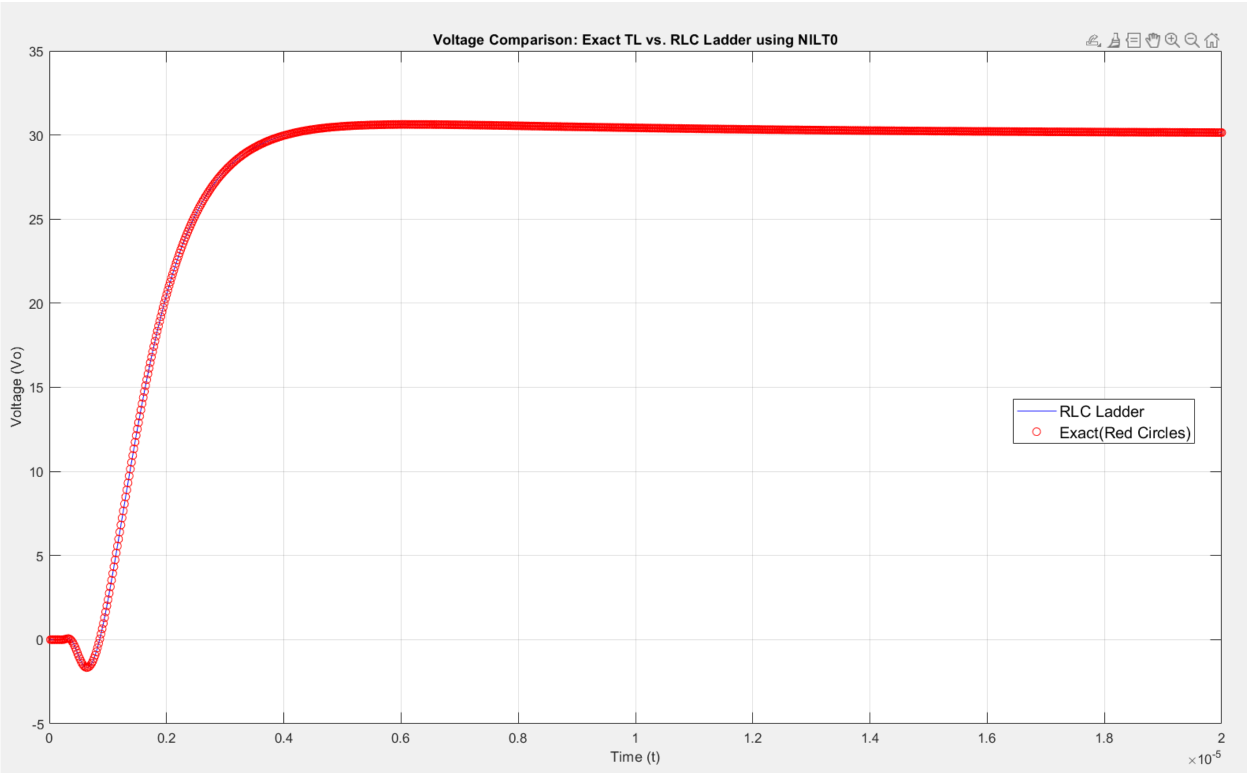
Where Vs(s) is the input in the s domain.

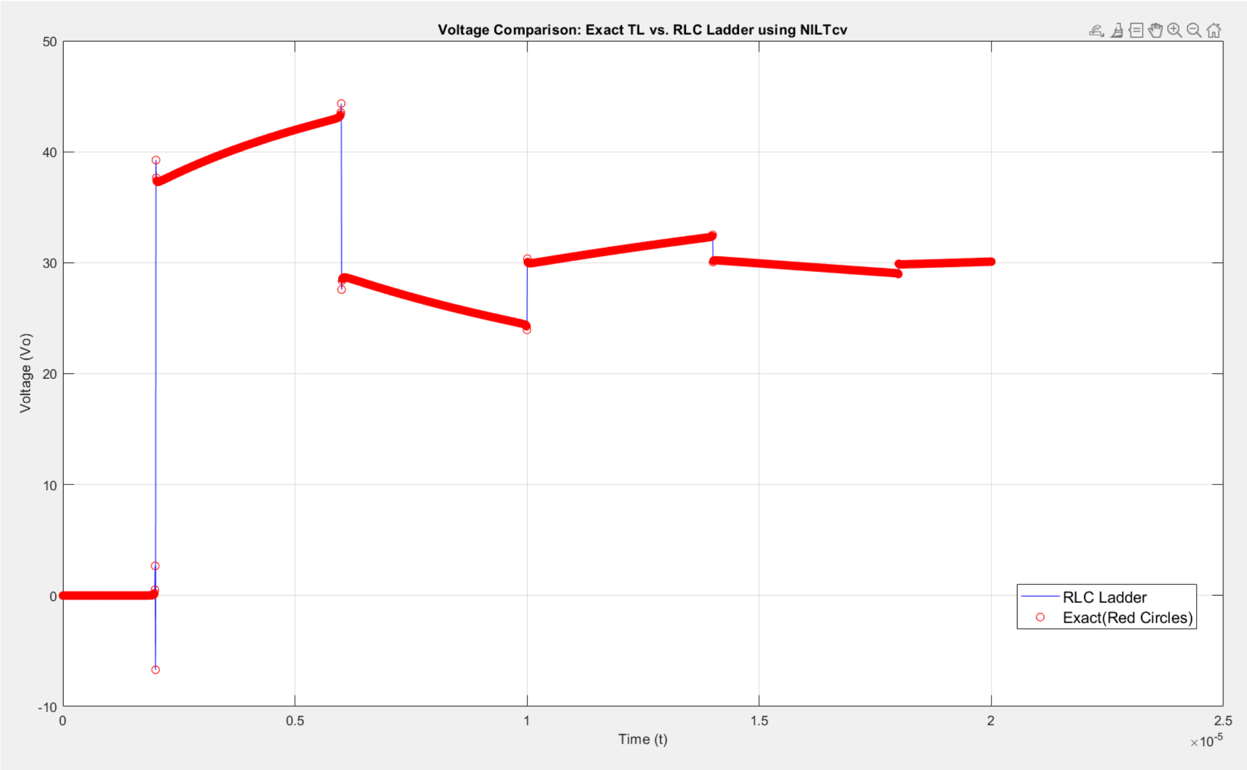
$$Let\ x = l * \sqrt{Y * Z}\,, \qquad so\ , V(l,s) = \frac{2 * Vs(s) * e^{x}}{e^{(2x)} + 1}$$

Using the fact that,

$$cosh(x) = \frac{(e^{x} + e^{-x})}{2}, then$$

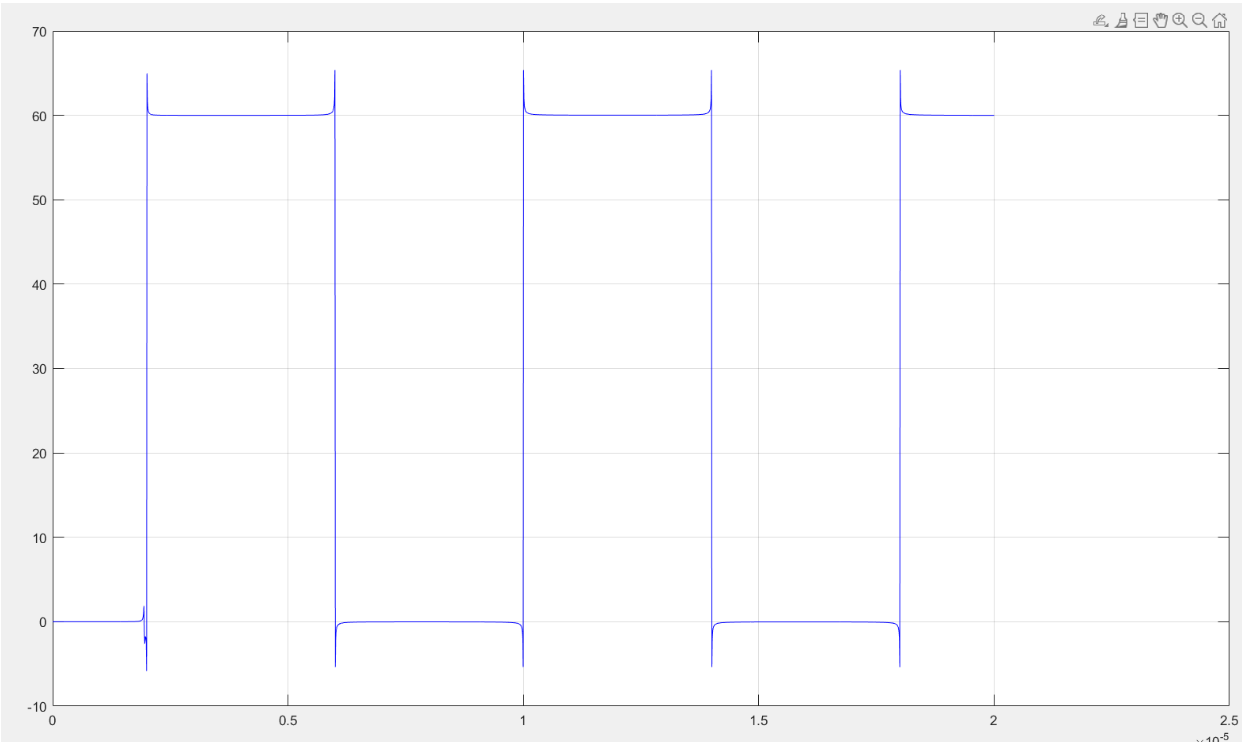
$$V(l,s) = \frac{Vs(s)}{\cosh\,(l * \sqrt{Y * Z}) + 1}\ which\ is\ the\ same\ as\ the\ exact\ soluation\ above.$$


Voltage Comparison: Exact TL vs. RLC Ladder using NILT0
Voltage (Vo)
Time (t)
$\times 10^{-5}$
RLC Ladder
Exact(Red Circles)
35
30
25
20
15
10
5
0
-5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

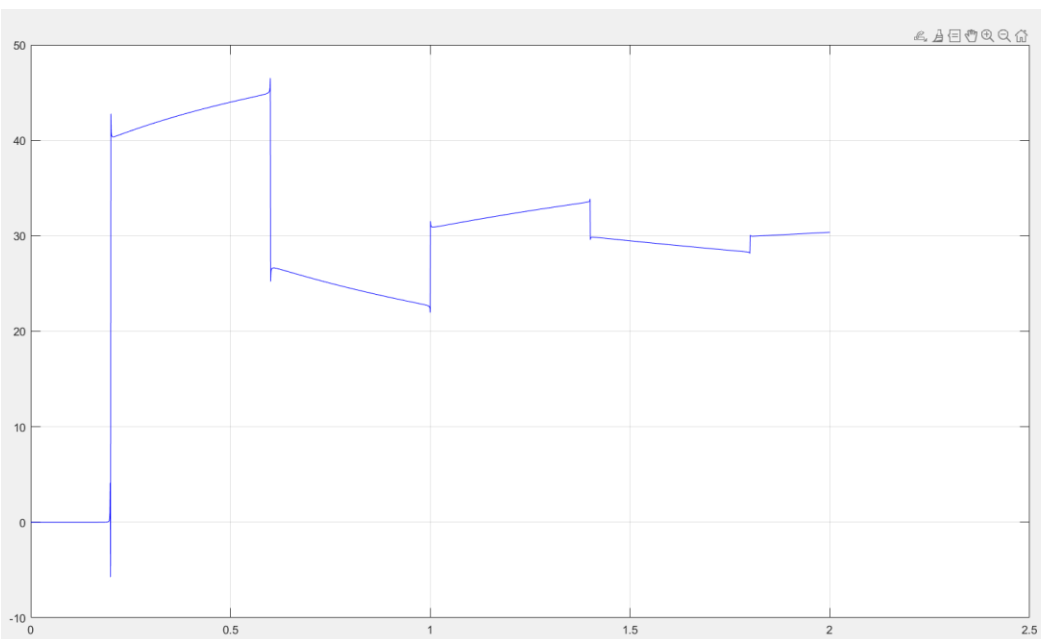

Voltage Comparison: Exact TL vs. RLC Ladder using NILTcv
Voltage (Vo)
Time (t)
$\times 10^{-5}$
RLC Ladder
Exact(Red Circles)
50
40
30
20
10
0
-10
0
0.5
1
1.5
2
2.5

- NILTcv

When passing the exact solution to NILTcv with R = G = 0, L= 2.5e-7, C=1e-10 and a unit step input, the output was like the following.



However, changing the value of either G or R, get me a good result as below:



- Further reading includes

1. Understand how Niltcv works.
2. Look at the derived expression for the transmission Line in MATLAB for Engineers – Applications in Control, Electrical Engineering, IT and Robotics.

$$V(s,x) = Vi(s)\frac{Z_c(s)}{Z_c(s) + Z_i(s)} * \frac{e^{-\gamma(s)x} + \rho_2(s)e^{-\gamma(s)[2l-x]}}{1 - \rho_1(s)\rho_2(s)e^{-\gamma(s)l}}$$

Where,

$$Z_c(s) = \sqrt{\frac{z}{y}}, \qquad Y = G + SC, \qquad z = (R + SL),, \quad \gamma = \sqrt{ZY}$$

$$\rho_1(s) = \frac{Z_i(s) - Z_c(s)}{Z_i(s) + Z_c(s)}, \qquad \rho_2(s) = \frac{Z_L(s) - Z_c(s)}{Z_L(s) + Z_c(s)}$$