EXAMINATION MARKING SCHEME

EXAM DIET: In-Class Test 2015-16
COURSE: B.Eng. in Electronic Engineering
COURSE: B.Eng. in Mechatronic Engineering
COURSE: Study Abroad (Engineering & Computing)

MODULE: EE406 Systems

QUESTION 1

[Q1 - Read Question $\approx$ 5 mins]

(a) The expression for the error signal $E(s)$ is found as follows:

$$\begin{aligned}
\text{Given}\quad Y(s) &= G(s).D(s) + C(s)G(s).R(s) - C(s)G(s)H(s).Y(s) \\
\left(1 + C(s)G(s)H(s)\right).Y(s) &= G(s).D(s) + C(s)G(s).R(s) \\
Y(s) &= \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}.R(s) + \frac{G(s)}{1 + C(s)G(s)H(s)}.D(s) \\
\text{Importantly:}\quad E(s) &= R(s) - Y(s) \\
\Rightarrow E(s) &= R(s) - \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}.R(s) - \frac{G(s)}{1 + C(s)G(s)H(s)}.D(s) \\
&= \left(1 - \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}\right).R(s) - \frac{G(s)}{1 + C(s)G(s)H(s)}.D(s)
\end{aligned}$$

[Q 1(a) 3 marks]

[Q 1(a) $\approx$ 5 mins]

(b) (i) The Final Value Theorem is:

$$E_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s.E(s)$$

(ii) The Final Value Theorem is then applied to find a value for K_C to give an overall steady-state error of 2/3.

$$\begin{aligned}
E_{ss} &= \lim_{s\to 0} s.E(s) \\
E_{ssR} &= \lim_{s\to 0} s.\left(1 - \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}\right).R(s) \\
&= \lim_{s\to 0} s.2/s.\left(1 - \frac{\frac{K_C}{(s+4)}.\frac{1}{(0.01s+1)}}{1 + \frac{K_C}{(s+4)}.\frac{1}{(0.01s+1)}\frac{2}{(s+2)}}\right) \\
&= 2 - \frac{2.\frac{K_C}{4}}{1 + \frac{K_C}{4}.\frac{2}{2}} \\
&= 2 - \frac{2K_C}{4 + K_C} = \frac{8 + 2K_C - 2K_C}{4 + K_C} \\
E_{ssR} &= \frac{8}{4 + K_C}
\end{aligned}$$

QUESTION 1 CONTINUED

$$
\begin{aligned}
E_{ssD} &= \lim_{s\to 0} s.\left(-\frac{C(s)}{1+C(s)G(s)H(s)}\right).D(s) \\
&= \lim_{s\to 0} s.0.5/s.\left(\frac{-\frac{1}{(0.01s+1)}}{1+\frac{K_C}{(s+4)}.\frac{1}{(0.01s+1)}\frac{2}{(s+2)}}\right) \\
&= \frac{-0.5}{1+\frac{K_C}{4}} \\
E_{ssD} &= \frac{-2}{4+K_C}
\end{aligned}
$$

$$
\begin{aligned}
E_{ss} &= E_{ssR} + E_{ssD} = \frac{2}{3} \\
\Rightarrow \frac{2}{3} &= \frac{8}{4+K_C} + \frac{-2}{4+K_C} \\
&= \frac{6}{4+K_C} \\
8+2K_C &= 18 \\
2K_C &= 10 \\
K_C &= 5
\end{aligned}
$$

[Q 1(b) 8 marks]

[Q 1(b) ≈ 20 mins]

(c) The matrices for $G(s)$, $C(s)$ and $H(s)$ are entered into MATLAB and the *feedback()* function is used carefully to determine the matrices given for $E(s)$ above. The *dcgain()* function is then used with the appropriate values for the reference and disturbance inputs to determine the E_{ss}.

```
kc = 5;
sysG = tf(1,[1/100 1]);
sysC = tf(kc,[1 4]);
sysH = tf([2],[1 2]);
sysCL = feedback(series(sysG,sysC),sysH);
stepAmp = 2;
distAmp = 0.5;
essR = stepAmp*(1 - dcgain(sysCL))
essD = - distAmp*dcgain(feedback(sysG,series(sysC,sysH)))
essTotal = essR+essD
```

The value for E_{ssR} is found to be 0.8889, the value for E_{ssD} is found to be -0.2222, giving a total $E_{ss} = 0.667$ which is the same as that prescribed in **Q 1(b)(ii)**.

[Q 1(c) 3 marks]

[Q 1(c) ≈ 7 mins]

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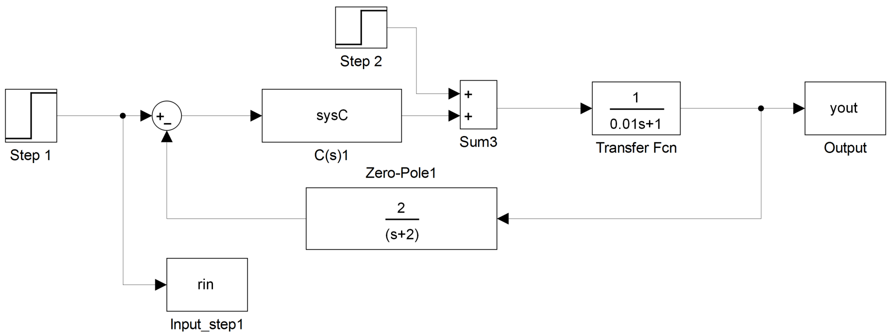
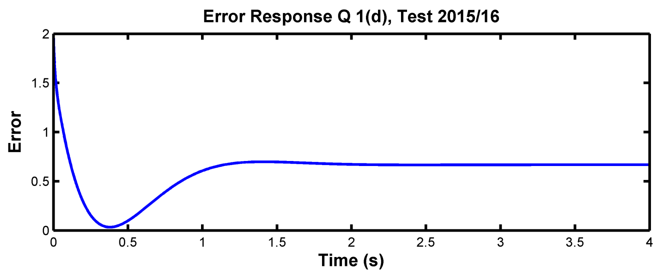
QUESTION 1 CONTINUED

(d) (i) The system was set up in SIMULINK, taking care to set the final value as 2 in the step input block for $R(s)$ and to set up an amplitude of 0.5 for $D(s)$. The simulation parameters must also be set correctly for a simulation stop time of 4 s and different blocks must be used for each transfer function.



(ii) This simulation produced the error response plot:



(iii) The steady-state error is measured in MATLAB:

```
open('s_test_q1')
Tstop = 4;Tsim = Tstop/1000;
distAmp = 0.5;
stepAmp = 2;
sim('s_test_q1')
figure(1),clf,subplot(2,1,1)
error = rin - yout;
plot(tout,error)
ess_meas = mean(error(901:end));
% ess_meas = 0.6667
```

The value is found to be $Ess_{meas} = 0.6667$. This does and should match the prescribed value in **Q 1(b)(ii)** and the predicted value from **Q 1(c)**.

[Q 1(d) 6 marks]

[Q 1(d) $\approx$ 12 mins]

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QUESTION 1 CONTINUED

(e) (i)

$$S_{KC}^{TED} = \frac{K_C}{T_{ED}} \frac{\partial T_{ED}}{\partial K_C}$$

(ii)

$$\begin{aligned}
E(s) &= \frac{-G(s)}{1 + C(s)G(s)H(s)}.D(s) \\
\Rightarrow T_{ED}(s) &= \frac{E(s)}{D(s)} = \frac{-G(s)}{1 + C(s)G(s)H(s)} \\
S_{KC}^{TED} &= \frac{K_C}{T_{ED}} \frac{\partial T_{ED}}{\partial K_C} \\
&= \frac{K_C}{\frac{-G(s)}{1+C(s)G(s)H(s)}} \cdot \left(\frac{(1 + K_C G(s)H(s)) .0 - (-G(s))G(s)H(s)}{(1 + C(s)G(s)H(s))^2} \right) \\
&= \frac{K_C}{-G(s)} \cdot \frac{G(s)G(s)H(s)}{1 + C(s)G(s)H(s)} \\
&= \frac{-K_C G(s)H(s)}{1 + C(s)G(s)H(s)} \\
&= \frac{K_C \frac{1}{(0.01s+1)} \cdot \frac{2}{(s+2)}}{1 + \frac{K_C}{(s+4)} \cdot \frac{1}{(0.01s+1)} \cdot \frac{2}{(s+2)}} \\
&= \frac{-2K_C\,(s+4)}{(s+4)\,(0.01s+1)\,.\,(s+2) + 2K_C}
\end{aligned}$$

[Q 1(e) 5 marks]
[Q 1(e) $\approx$ 10 mins]
[Total: 25 marks]

[END OF SOLUTIONS]

INDICATIVE ANS & MARKING SCHEME

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\text{Importantly:} \quad E(s) &= R(s) - Y(s) \\
&= \left(1 - \frac{C(s)G(s)}{1+C(s)G(s)H(s)}\right).R(s) - \frac{G(s)}{1+C(s)G(s)H(s)}.D(s)
\end{aligned}$$

[Q 1(a) 3 marks]

[Q 1(a) ≈ 5 mins]

(b) (i) The Final Value Theorem is:

$$E_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s.E(s)$$

(ii) The Final Value Theorem is then applied to find a value for K_C to give an overall steady-state error of 2/3.

$$\begin{aligned}
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E_{ssR} &= \lim_{s\to 0} s.\left(1 - \frac{C(s)G(s)}{1+C(s)G(s)H(s)}\right).R(s) \\
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E_{ssR} &= \frac{8}{4+K_C}
\end{aligned}$$

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E_{ssD} &= \lim_{s\to 0} s.\left(-\frac{C(s)}{1+C(s)G(s)H(s)}\right).D(s) \\
&= \lim_{s\to 0} s.0.5/s.\left(\frac{-\frac{1}{(0.01s+1)}}{1+\frac{K_C}{(s+4)}.\frac{1}{(0.01s+1)}\frac{2}{(s+2)}}\right) \\
E_{ssD} &= \frac{-2}{4+K_C}
\end{aligned}$$

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QUESTION 1 CONTINUED

$$\begin{aligned} E_{ss} &= E_{ssR} + E_{ssD} = \frac{2}{3} \\ \Rightarrow \frac{2}{3} &= \frac{8}{4+K_C} + \frac{-2}{4+K_C} \\ K_C &= 5 \end{aligned}$$

[Q 1(b) 8 marks]

[Q 1(b) $\approx$ 20 mins]

(c) The matrices for $G(s)$, $C(s)$ and $H(s)$ are entered into MATLAB and the *feedback()* function is used carefully to determine the matrices given for $E(s)$ above. The *dcgain()* function is then used with the appropriate values for the reference and disturbance inputs to determine the E_{ss}. The value for E_{ssR} is found to be 0.8889, the value for E_{ssD} is found to be -0.2222, giving a total $E_{ss} = 0.667$ which is the same as that prescribed in **Q 1(b)(ii)**.

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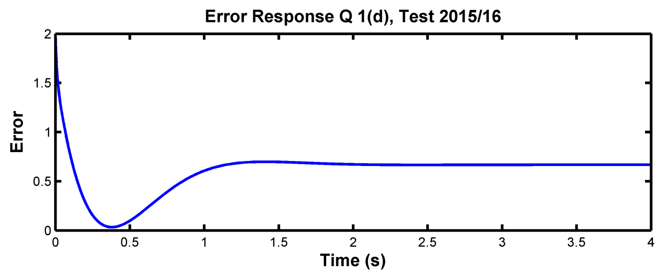

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[Q 1(d) $\approx$ 12 mins]

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S_{KC}^{TED} &= \frac{K_C}{T_{ED}} \frac{\partial T_{ED}}{\partial K_C} \\
&= \frac{K_C}{\frac{-G(s)}{1+C(s)G(s)H(s)}} . \left(\frac{(1 + K_C G(s)H(s)) .0 - (-G(s))G(s)H(s)}{(1 + C(s)G(s)H(s))^2} \right) \\
&= \frac{-K_C G(s)H(s)}{1 + C(s)G(s)H(s)} \\
&= \frac{-2K_C}{(s+4)(0.01s+1).(s+2) + 2K_C}
\end{aligned}$$

[Q 1(e) 5 marks]

[Q 1(e) $\approx$ 10 mins]

[Total: 25 marks]

[END OF SOLUTIONS]