Modeling 1-D FDTD Transmission Line Voltage Sources and Terminations with Parallel and Series RLC Loads

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Herein, the finite-difference time-domain (FDTD) update equations to implement voltage sources and transmission line terminations with parallel or series RLC loads (see Fig. 1) in a one-dimensional (1-D) FDTD model are presented. A 1-D FDTD model of a simple, lossless transmission line has been developed [1]. Later, it was extended to implement single circuit elements (e.g., capacitors, inductors, and resistors) [2] and parallel or series RLC loads [3] placed in parallel or series with the transmission line. In addition to the modeling of problems, the 1-D FDTD transmission line model is useful in teaching students the time-domain analysis of transmission lines. The minimal computational demands of the 1-D geometry allows for real-time displays of signal propagation [4].

Fig. 2(a)-(b) show the circuit representation and FDTD model for a voltage source with series RLC elements. From Kirchoff's Current Law (KCL) and Kirchoff's Voltage Law (KVL), we get

$$i(z,t) - i(z+\Delta z,t) - c\Delta z \frac{\partial\ v(z+\Delta z,t)}{\partial t} = 0 \text{ and}$$

$$v(z+\Delta z,t) - V_S(t) + l\Delta z \frac{\partial i(z,t)}{\partial t} + Ri(z,t) + L\frac{\partial i(z,t)}{\partial t} + V_{CS} = 0$$

where $V_{CS} = \frac{1}{C}\int i(z,t)\,dt$. Discretizing these equations yields

$$V_{CS}{}^{n} = \frac{\Delta t}{C} I^{n-0.5}(k+0.5) + V_{CS}{}^{n-1}$$

$$I^{n+0.5}(k+0.5) = B_1 I^{n-0.5}(k+0.5) - B_2\frac{1}{Z_C}\frac{v_p\Delta t}{\Delta z}\left[V^n(k+1) - V_S^n(k)\right] - B_2\frac{1}{Z_C}\frac{v_p\Delta t}{\Delta z}V_{CS}{}^{n}$$

where $B_1 = \dfrac{1+\dfrac{1}{Z_C}\dfrac{v_p\Delta t}{\Delta z}\left(\dfrac{L}{\Delta t}-\dfrac{R}{2}\right)}{1+\dfrac{1}{Z_C}\dfrac{v_p\Delta t}{\Delta z}\left(\dfrac{L}{\Delta t}+\dfrac{R}{2}\right)}$ and $B_2 = \dfrac{1}{1+\dfrac{1}{Z_C}\dfrac{v_p\Delta t}{\Delta z}\left(\dfrac{L}{\Delta t}+\dfrac{R}{2}\right)}$

$$V^{n+1}(k+1) = V^n(k+1) - Z_C\frac{v_p\Delta t}{\Delta z}\left[I^{n+0.5}(k+1.5) - I^{n+0.5}(k+0.5)\right]$$

where $v_p = 1/\sqrt{lc}$ and $Z_C = \sqrt{l/c}$.

Fig. 2(c)-(d) show the circuit representation and FDTD model for a voltage source with parallel RLC elements. By applying KCL and KVL and then discretizing, we get

$$V_{PS}{}^{n} = \left[\frac{C}{\Delta t}V_{PS}{}^{n-1} + I^{n-0.5}(k+0.5) - I_{LPS}{}^{n-1}\right] \Big/ \left(\frac{C}{\Delta t} + \frac{1}{R} + \frac{\Delta t}{L}\right)$$

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$$I^{n+0.5}(k+0.5) = I^{n-0.5}(k+0.5) - \frac{1}{Z_C}\frac{v_p \Delta t}{\Delta z}\left[V^n(k+1) - V_S^n(k)\right] - \frac{1}{Z_C}\frac{v_p \Delta t}{\Delta z}V_{PS}{}^n$$

$$I_{LPS}{}^n = \frac{\Delta t}{L}V_{PS}{}^n + I_{LPS}{}^{n-1}$$

$$V^{n+1}(k+1) = V^n(k+1) - Z_C\frac{v_p \Delta t}{\Delta z}\left[I^{n+0.5}(k+1.5) - I^{n+0.5}(k+0.5)\right]$$

Fig. 3(a)-(b) show the circuit representation and FDTD model for a of transmission line termination with series RLC elements. By applying KCL and KVL and then discretizing, we get

$$I^{n+0.5}(k+0.5) = I^{n-0.5}(k+0.5) - \frac{1}{Z_C}\frac{v_p \Delta t}{\Delta z}\left[V^n(k+1) - V^n(k)\right]$$

$$I_{SP}{}^{n+0.5}(k+1) = \left[\frac{L}{\Delta t}I_{SP}{}^{n-0.5} - V_{CSP}{}^{n-0.5} + V^n(k+1)\right] \Big/ \left(R + \frac{L}{\Delta t} + \frac{\Delta t}{C}\right)$$

$$V^{n+1}(k+1) = V^n(k+1) + Z_C\frac{v_p \Delta t}{\Delta z}I^{n+0.5}(k+0.5) - Z_C\frac{v_p \Delta t}{\Delta z}I_{SP}{}^{n+0.5}$$

$$V_{CSP}{}^{n+0.5} = \frac{\Delta t}{C}I_{SP}{}^{n+0.5} + V_{CSP}{}^{n-0.5}$$

Fig. 3(c)-(d) show the circuit representation and FDTD model for a of transmission line termination with parallel RLC elements. By applying KCL and KVL and then discretizing, we get

$$I^{n+0.5}(k+0.5) = I^{n-0.5}(k+0.5) - \frac{1}{Z_C}\frac{v_p \Delta t}{\Delta z}\left[V^n(k+1) - V^n(k)\right]$$

$$I_{LP}{}^{n+0.5} = \frac{\Delta t}{L}V^n(k+1) + I_{LP}{}^{n-0.5}$$

$$V^{n+1}(k+1) = A_1 V^n(k+1) + A_2 Z_C\frac{v_p \Delta t}{\Delta z}I^{n+0.5}(k+0.5) - A_2 Z_C\frac{v_p \Delta t}{\Delta z}I_{LP}{}^{n+0.5} \quad \text{where}$$

$$A_1 = \frac{1 + Z_C\dfrac{v_p \Delta t}{\Delta z}\left(\dfrac{C}{\Delta t} - \dfrac{1}{2R}\right)}{1 + Z_C\dfrac{v_p \Delta t}{\Delta z}\left(\dfrac{C}{\Delta t} + \dfrac{1}{2R}\right)} \quad \text{and} \quad A_2 = \frac{1}{1 + Z_C\dfrac{v_p \Delta t}{\Delta z}\left(\dfrac{C}{\Delta t} + \dfrac{1}{2R}\right)}$$

A simple example of implementing these updates is shown in Fig. 4. It shows the source voltage and input voltage for a transmission line circuit consisting of a matched 50 Ω voltage source, 50 Ω transmission line, and 150 Ω resistive load. As expected from transmission line theory, the initial pulse launched onto the transmission line is one-half the magnitude of the source Gaussian pulse, the first reflection from the load is one-half the magnitude of the initially launched pulse (Γ_L = 0.5), and there are no further reflections (Γ_S = 0). Further, selected results will be shown to demonstrate the accuracy and validity of the update equations by comparison with analytic results.

References

[1] J.G. Maloney, K.L. Shlager, and G.S. Smith, ``A Simple FDTD Model for Transient Excitation of Antennas by Transmission Lines," *IEEE Trans. Antennas Propagation*, vol. 42, no. 2, pp. 289-292, Feb. 1994.

[2] T. P. Montoya and G. S. Smith, "Modeling Transmission Line Circuit Elements in the FDTD Method," *Microwave and Optical Technology Letters*, vol. 21, no. 7, pp.105-114, April 20, 1999.

[3] T. P. Montoya and W. R. Scott, Jr., "Modeling Parallel and Series RLC Loads in a 1-D FDTD Transmission Line," *USNC/URSI National Radio Science Meeting*, Salt Lake City, UT, p. 30, July 16-21, 2000.

[4] W. R. Scott, Jr. et al., see link titled *Animation of Waves Traveling on Multiple Transmission Line with Reactive Loads and Networks* at http://users.ecc.gatech.edu/~wrscott/

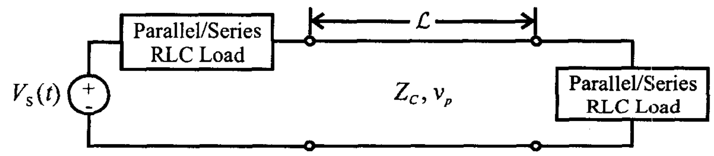

Figure 1 Simple Transmission Line Circuit

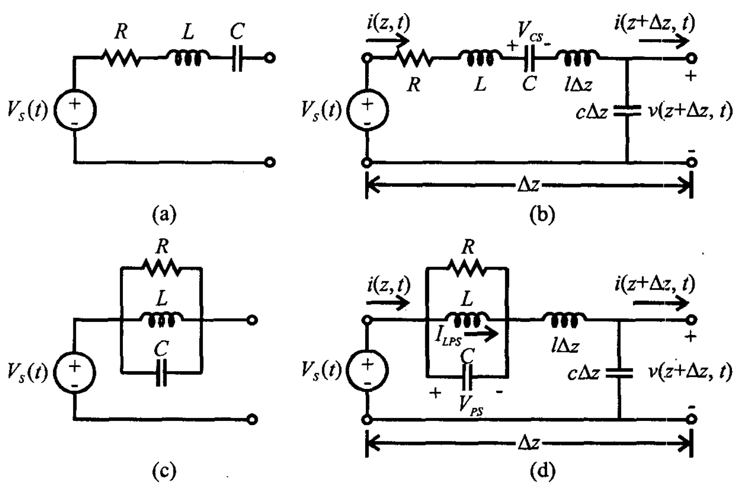

Figure 2 (a) Circuit representation and (b) FDTD model of voltage source with series RLC elements and (c) circuit representation and (d) FDTD model of voltage source with parallel RLC elements.

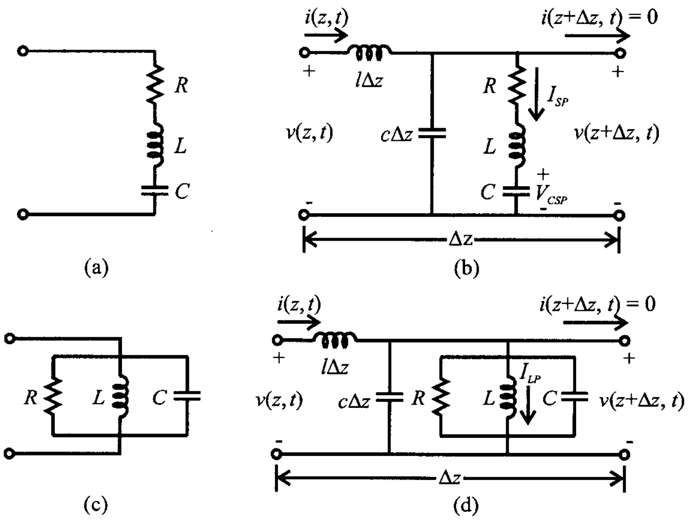

Figure 3 (a) Circuit representation and (b) FDTD model of transmission line termination with series RLC elements and (c) circuit representation and (d) FDTD model of transmission line termination with parallel RLC elements.

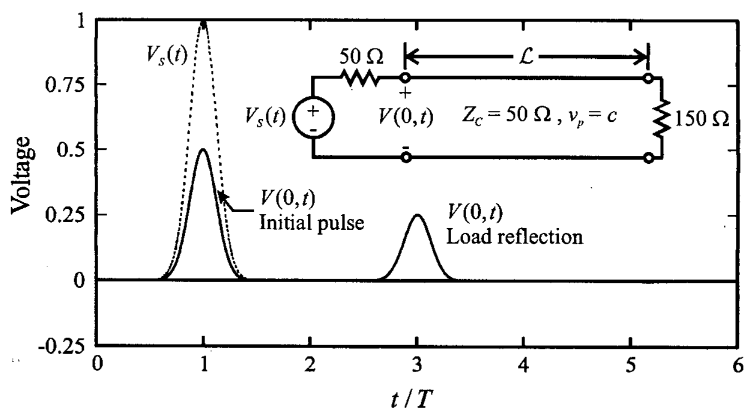

Figure 4 Source voltage, $V_S(t)$, and input voltage, $V(0, t)$, for a transmission line circuit consisting of a matched voltage source, a lossless transmission line section, and a mismatched resistive load. The time axis has been normalized by the one-way transit time of the transmission line, $T = \mathcal{L} / c$.