OUTLINE EXAMINATION MARKING SCHEME

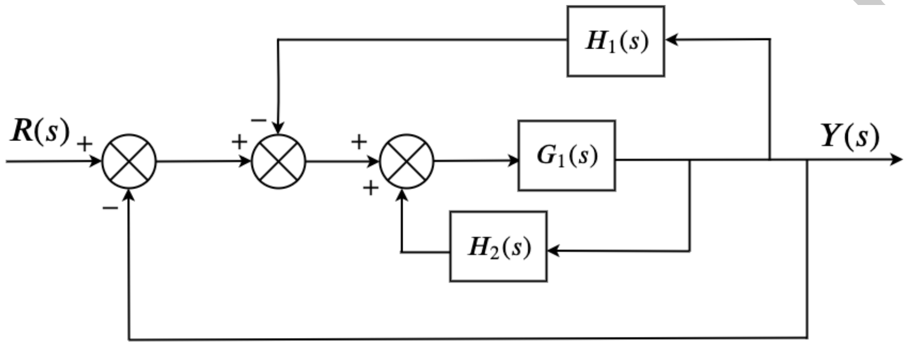
EXAM DIET: In-Class Test 2019-2020
COURSE: B.Eng. in Electronic and Computer Engineering
COURSE: B.Eng. in Mechatronic Engineering

MODULE: EE458 Control Systems Analysis

QUESTION 1

[Q1 - Read Question ≈ 5 mins]

(a) (i) One of the key findings to simplify this block diagram is to swap the second and the third summing point (looking from left to right). The modified block diagram is shown below, which is still mathematically equivalent for calculating the closed-loop transfer function $T(s)$ as in the original block diagram.



After this step, we can easily find that the structure of the new block diagram (three feedback loops) is much easier for analysing the closed-loop transfer function. The most inner loop transfer function can be obtained as:

$$T_{inner}(s) = \frac{G_1(s)}{1 - G_1(s)H_2(s)},$$

and then the middle loop transfer function is:

$$T_{middle}(s) = \frac{T_{inner}(s)}{1 + T_{inner}(s)H_1(s)},$$

and finally,

$$T(s) = \frac{Y(s)}{R(s)} = \frac{T_{middle}(s)}{1 + T_{middle}(s)}.$$

Combing all three components above, we can find that:

$$T(s) = \frac{G_1(s)}{1 - G_1(s)H_2(s) + G_1(s)H_1(s) + G_1(s)}$$

(ii) Using Matlab and the provided transfer functions for $G_1(s)$,$H_1(s)$ and $H_2(s)$ we can find that:

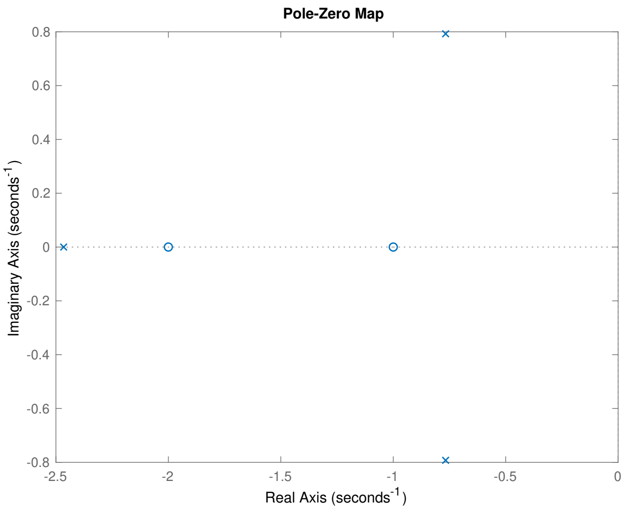
$$T(s) = \frac{s^2 + 3s + 2}{s^3 + 4s^2 + 5s + 3}.$$

Using the zpk model, we can find that:

$$T(s) = \frac{(s + 2)(s + 1)}{(s + 2.466)(s^2 + 1.534s + 1.217)}$$

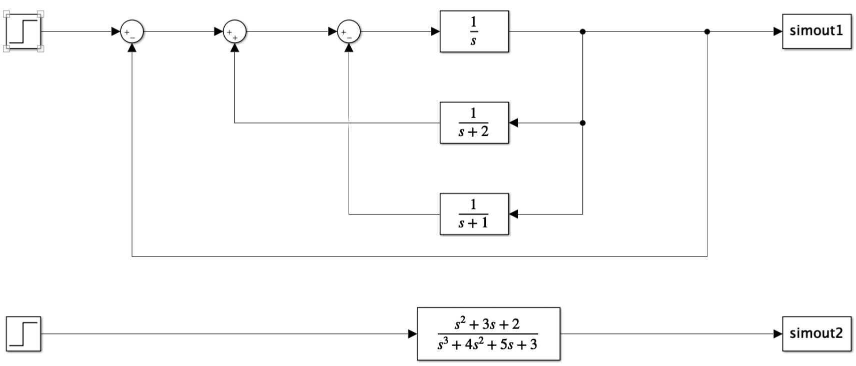
and it can easily observed that the system is 3rd order and type 0.

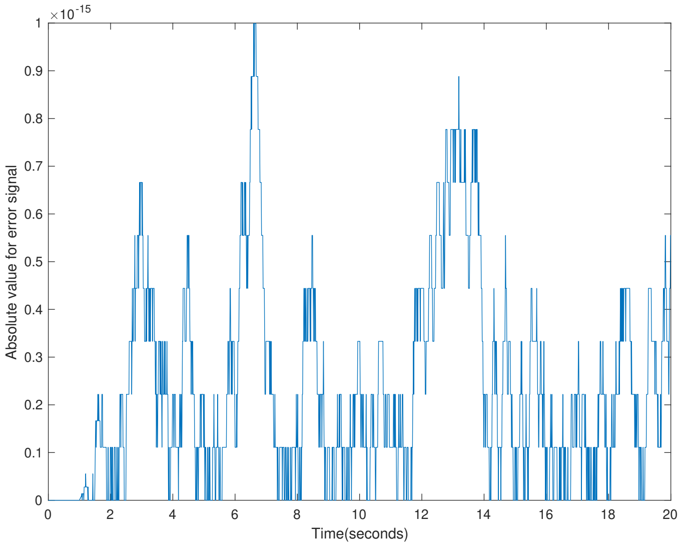
(iii) Using pzmap command, we can get the zeros and poles for the transfer function $T(s)$, and the figure is shown below.



As expected, we can find that the system has three poles and two zeros. Given these poles, we can then use the “abs” and “find” commands to find the one with the maximum magnitude. It is then found that the pole with the maximum magnitude is -2.4656.

(iv) The diagram for two simulink models is shown below.





The absolute value for the error signal between two models is shown above.

[Q 1(a) 9 marks (3 + 2 + 2 + 2)]

[Q 1(a) $\approx$ 25 mins]

(b) (i) The Matlab command to examine the stability of the system is "isstable". We can use this command to find that the closed-loop system is stable. The result is as expected since we have already found that all poles of the system are located on the left half of s-plane.

(ii) This can be done using the Final Value Theorem (FVT), i.e.

$$Y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sT(s)R(s).$$

Now, by substituting $R(s)=\frac{1}{s}$ and the expression of $T(s)$ obtained in Q1(a)(ii), we can find that:

$$Y_{ss} = \lim_{s\to 0} T(s) = \frac{2}{3}.$$

Without further complex calculation, we can present the state-steady error is

$$E_{ss} = 1 - \frac{2}{3} = \frac{1}{3}.$$

[Q 1(b) 4 marks (2 + 2)]

[Q 1(b) $\approx$ 7 mins]

(c) (i) The system is said to have zero sensitivity if changes in system parameters have no effect on the transfer function.

(ii)
- The time response can be difficult to obtain from differential equation models.
- The Lapalce transform allows for the substitution of algebraic equations for these differential equations.

(iii) According to the definition of sensitivity, we can find that:

$$S_k^T = \frac{k}{T}\frac{\delta T}{\delta k} = \frac{k}{3s+k}$$

[Q 1(c) 6 marks (2 + 2 + 2)]

[Q 1(c) $\approx$ 8 mins]

(d) Step 1: calculate $\frac{Y(s)}{R(s)}$ by setting $D_1(s)$ and $D_2(s)$ to 0. We find that:

$$\frac{Y(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H_1(s)H_2(s)}$$

Step 2: calculate $\frac{Y(s)}{D_1(s)}$ by setting $R(s)$ and $D_2(s)$ to 0. We find that:

$$\frac{Y(s)}{D_1(s)} = \frac{G_2(s)}{1 + H_1(s)H_2(s)G_1(s)G_2(s)}$$

Step 3: calculate $\frac{Y(s)}{D_2(s)}$ by setting $R(s)$ and $D_1(s)$ to 0. We find that:

$$\frac{Y(s)}{D_2(s)} = -\frac{G_1(s)G_2(s)H_1(s)}{1 + H_1(s)H_2(s)G_1(s)G_2(s)}$$

Step 4: combing all factors above to get $Y(s)$, and we get:

$$Y(s) = \frac{G_1(s)G_2(s)R(s)}{1 + G_1(s)G_2(s)H_1(s)H_2(s)} + \frac{G_2(s)D_1(s)}{1 + H_1(s)H_2(s)G_1(s)G_2(s)} - \frac{G_1(s)G_2(s)H_1(s)D_2(s)}{1 + H_1(s)H_2(s)G_1(s)G_2(s)}$$

and $E_{ss} = \lim_{s\to 0} s(R(s) - Y(s))$ can also be obtained, i.e.,

$$E_{ss} = \lim_{s\to 0}(s - \frac{G_1(s)G_2(s)s}{1 + G_1(s)G_2(s)H_1(s)H_2(s)})R(s) - \frac{G_2(s)D_1(s)s}{1 + H_1(s)H_2(s)G_1(s)G_2(s)} + \frac{G_1(s)G_2(s)H_1(s)D_2(s)s}{1 + H_1(s)H_2(s)G_1(s)G_2(s)}$$

[Q 1(d) 6 marks]

[Q 1(d) $\approx$ 15 mins]

[Total: 25 marks]

[END OF Q1 ANSWERS]