Improved 1-D FDTD Modeling of Parallel and Series RLC Loads in a Lossless Transmission Line

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This paper presents improved finite-difference time-domain (FDTD) update equations to model a parallel RLC load in series and a series RLC load in parallel with a simple, lossless transmission line. A one-dimensional (1-D) FDTD model of a simple, lossless transmission line was developed [1], and extended to model discrete circuit elements (e.g., capacitors, inductors, and resistors) [2]. Later, FDTD update equations were developed for parallel or series RLC loads placed in parallel or series with a transmission line [3], and for use with voltage sources and/or terminations [4].

Fig. 1a shows the circuit representation of an incremental section of a simple, lossless transmission line with a parallel RLC load in series. Applying Kirchoff's current law (KCL) to the top left and right nodes yields

$$i(z,t) - \frac{v_{PS}}{R} - i_{LPS} - C\frac{\partial v_{PS}}{\partial t} = 0 \qquad (1)$$

where

$$i_{LPS}(z,t) = \frac{1}{L}\int_{t_0}^{t} v_{PS}(z,t)\partial t + i_{LPS}(z,t_0) \qquad (2)$$

and

$$i(z,t) - i(z+\Delta z,t) - c\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t} = 0 \qquad (3)$$

Applying Kirchoff's voltage law (KVL) clockwise around the outside loop yields

$$v(z+\Delta z,t) - v(z,t) + v_{PS} + l\Delta z\frac{\partial i(z,t)}{\partial t} = 0 \qquad (4)$$

In these equations, c and l are the capacitance and inductance per-unit-length of the transmission line respectively.

For the FDTD method, the current and voltage are spatially discretized as shown in Fig. 2 for the incremental transmission line sections. Further, they are interleaved in time with voltage nodes placed at times $t = n\Delta t$, and current nodes placed at times $t = (n+0.5)\Delta t$. For clarity, the usual shorthand FDTD notation where $V^n(k) = v(z = k\Delta z, t = n\Delta t)$ and $I^{n+0.5}(k+0.5) = i\big(z = (k+0.5)\Delta z, t = (n+0.5)\Delta t\big)$ is used.

For the parallel RLC load in series, discretizing (1) at $t = (n-0.5)\Delta t$, using the usual second-order accurate central-difference approximation to the time derivative, yields

$$I^{n-0.5}(k+0.5) - \frac{V_{PS}^{n-0.5}(k+0.5)}{R} - I_{LPS}^{n-0.5}(k+0.5) - C\left[\frac{V_{PS}^{n}(k+0.5) - V_{PS}^{n-1}(k+0.5)}{\Delta t}\right] = 0 .$$

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Letting $V_{PS}^{n-0.5}(k+0.5) \approx \left[V_{PS}^{n}(k+0.5)+V_{PS}^{n-1}(k+0.5)\right]/2$ and solving for the voltage v_{PS} across the parallel RLC load in series, yields the update equation

$$V_{PS}^{n}(k+0.5)=\frac{\left(\frac{C}{\Delta t}-\frac{1}{2R}\right)}{\left(\frac{C}{\Delta t}+\frac{1}{2R}\right)}V_{PS}^{n-1}(k+0.5)+\frac{1}{\left(\frac{C}{\Delta t}+\frac{1}{2R}\right)}\left[I^{n-0.5}(k+0.5)-I_{LPS}^{n-0.5}(k+0.5)\right] \quad (5)$$

where the update equation for the inductor current i_{LPS}, found by discretizing (2), is

$$I_{LPS}^{n-0.5}(k+0.5)=I_{LPS}^{n-1.5}(k+0.5)+\frac{\Delta t}{L}V_{PS}^{n-1}(k+0.5) \quad (6)$$

Discretizing (3) at $t=n\Delta t$ and (4) at $t=(n+0.5)\Delta t$, using the usual second-order accurate central-difference approximation to the time derivatives, yields the update equations for the current and voltage

$$I^{n+0.5}(k+0.5)=I^{n-0.5}(k+0.5)-\frac{1}{Z_C}\frac{v_p\Delta t}{\Delta z}\left[V^{n}(k+1)-V^{n}(k)\right]-\frac{1}{Z_C}\frac{v_p\Delta t}{\Delta z}V_{PS}^{n}(k+0.5) \quad (7)$$

and

$$V^{n+1}(k+1)=V^{n}(k+1)-Z_C\frac{v_p\Delta t}{\Delta z}\left[I^{n+0.5}(k+1.5)-I^{n+0.5}(k+0.5)\right] \quad (8)$$

where $v_p=1/\sqrt{lc}$ is the phase velocity and $Z_C=\sqrt{l/c}$ is the characteristic impedance.

Fig. 1b shows the circuit representation of an incremental section of a simple, lossless transmission line with a series RLC load in parallel. Applying KVL counterclockwise around the right-hand loop yields

$$i_{SP}R+L\frac{\partial i_{SP}}{\partial t}+v_{CSP}-v(z+\Delta z,t)=0 \quad (9)$$

where

$$v_{CSP}(z+\Delta z,t)=\frac{1}{C}\int_{t_0}^{t}i_{SP}(z+\Delta z,t)\partial t+v_{CSP}(z+\Delta z,t_0) \quad (10)$$

Applying KVL clockwise around the outer loop yields

$$v(z+\Delta z,t)-v(z,t)+l\Delta z\frac{\partial i(z,t)}{\partial t}=0 \quad (11)$$

Applying KCL to the top right node yields

$$i(z,t)-i(z+\Delta z,t)-c\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}-i_{SP}=0 \quad (12)$$

Discretizing (9)-(12) yields the update equations, placed in time order,

$$V_{CSP}^{n}(k+1)=V_{CSP}^{n-1}(k+1)+\frac{\Delta t}{C}I_{SP}^{n-0.5}(k+1) \quad (13)$$

$$I_{SP}^{n+0.5}(k+1)=\frac{\left(\frac{L}{\Delta t}-\frac{R}{2}\right)}{\left(\frac{L}{\Delta t}+\frac{R}{2}\right)}I_{SP}^{n-0.5}(k+1)+\frac{1}{\left(\frac{L}{\Delta t}+\frac{R}{2}\right)}\left[V^{n}(k+1)-V_{CSP}^{n}(k+1)\right] \quad (14)$$

$$I^{n+0.5}(k+0.5)=I^{n-0.5}(k+0.5)-\frac{1}{Z_C}\frac{v_p\Delta t}{\Delta z}\left[V^{n}(k+1)-V^{n}(k)\right] \quad (15)$$

and

$$V^{n+1}(k+1) = V^{n}(k+1) - Z_C \frac{v_p \Delta t}{\Delta z}\left[I^{n+0.5}(k+1.5) - I^{n+0.5}(k+0.5) - I_{SP}^{n+0.5}(k+1) \right] \quad (16)$$

To demonstrate the validity of the improved FDTD update equations, the voltages reflected from each type of RLC load are shown in Figs. 3 and 4 along with those based on [3] and analytically derived results. For the examples, a long section of transmission line containing each type of RLC load is driven by a Gaussian voltage pulse $v_G(t) = \exp\left(-0.5\left((t-t_d)/\tau_p\right)^2\right)$ launched by a one-way voltage source injector and terminated at each end by an absorbing boundary condition [1]. For the simulations, $\Delta t/\tau_p = 0.125$ and $S = v_p \Delta t/\Delta z = 0.5$. As shown, there is excellent agreement between the analytic and improved FDTD results, an improvement in accuracy over prior FDTD updates [3], [4]. E.g., the percent error in Fig. 4 is 0.6% for the improved updates versus 2.2% using the previous updates at the first peak, and 0.25% versus 5.05% at the second.

References

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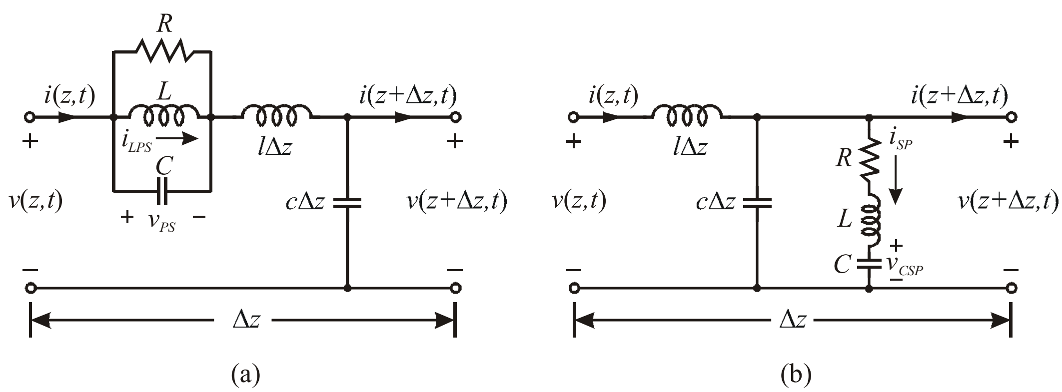

Fig. 1 Incremental sections of a simple lossless transmission line with a (a) parallel RLC load in series and (b) series RLC load in parallel.

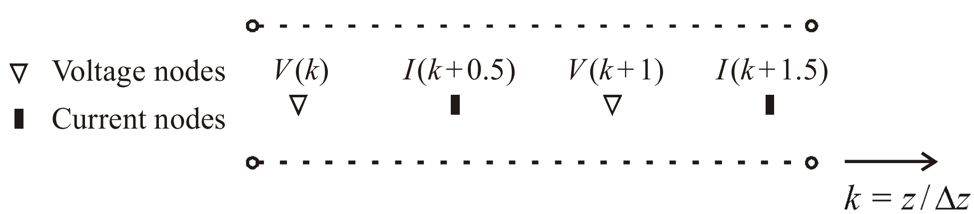

Fig. 2 FDTD model of 1-D transmission line

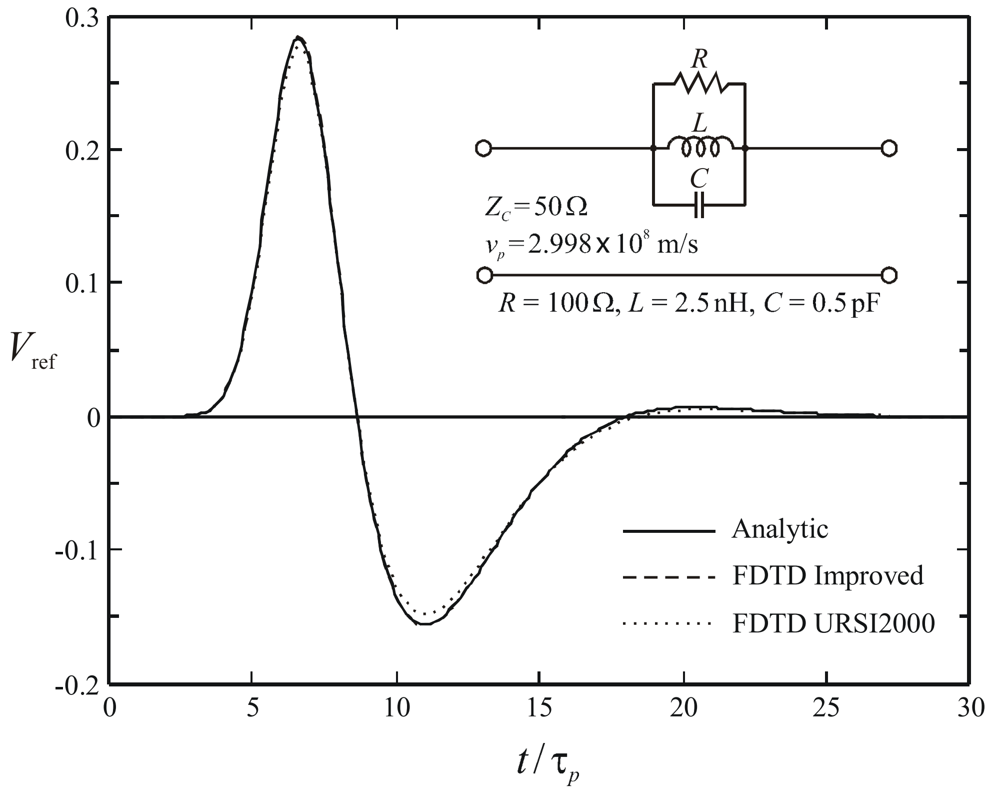

Fig. 3 Voltage reflected from parallel RLC load placed in series with a transmission line.

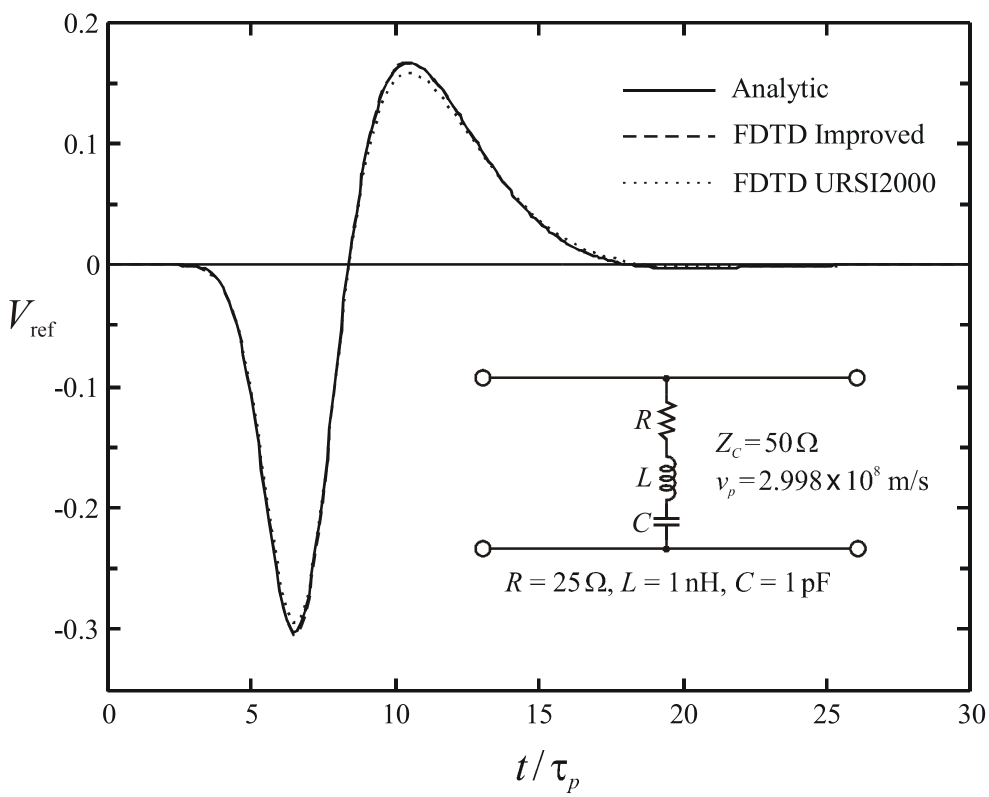

Fig. 4 Voltage reflected from series RLC load placed in parallel with a transmission line.