Include frequency in AWE

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Firstly, let’s move AWE to a separate file as a function as follows:

function [h\_impulse, y\_step, t] = AWE(A,B,C,D,w,time)

t = linspace(0,time,250);

q = length(B);

num\_moments = 2 \* q;

% s=1i\*w;

moments = zeros(1, num\_moments);

for i = 1:num\_moments

moments(i) = -C' \* (A)^(-i) \* B;

end

moments(1)=moments(1)+D;

approx\_order = length(B);

% Construct the moment matrix

moment\_matrix = zeros(approx\_order);

Vector\_c = -moments(approx\_order+1:2\*approx\_order)';

for i = 1:approx\_order

moment\_matrix(i, :) = moments(i:i+approx\_order-1);

end

% Solve for denominator coefficients

b\_matrix = inv(moment\_matrix) \* Vector\_c;

% Compute poles

poles = roots([b\_matrix', 1]);

% Compute residues

V = zeros(approx\_order);

for i = 1:approx\_order

for j = 1:approx\_order

V(i, j) = 1 / poles(j)^(i-1);

end

end

A\_diag = diag(1 ./ poles);

r\_moments = moments(1:approx\_order);

residues = -1 \* (A\_diag \ (V \ r\_moments'));

% Impulse response

h\_impulse = zeros(size(t));

for i = 1:approx\_order

h\_impulse = h\_impulse + residues(i) \* exp(poles(i) \* t);

end

% Step response using recursive convolution

y\_step = zeros(size(t));

y = zeros(length(poles), 1);

for n = 2:length(t)

dt = t(n) - t(n-1);

exp\_term = exp(poles \* dt);

for i = 1:length(poles)

y(i) = residues(i) \* (1 - exp\_term(i))/(-poles(i)) \* 1 + exp\_term(i) \* y(i);

end

y\_step(n) = sum(y);

end

end

This function will take A,B,C and D matrices as the input and the end point in time, and w( frequency) and will return the impulse and unit responses with respect to t.

* Adjusting AWE to include frequency:

Looking at the general form of Y(s):

For impulse input, U(s) = 1.

Expand this about

So,

So, adjusting the code as follows.

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moments = zeros(1, num\_moments);

for k = 1:num\_moments

moments(k) = (-1)^(k-1) \* C' \* (s0 \* eye(size(A)) - A)^-(k) \* B;

end

moments(1)=moments(1)+D;

 …..

Which can be tested as follows for the impulse response:

clear

clc

% test AWE

A = [-2, 1, 0, 0; 1, -2, 1, 0; 0, 1, -2, 1;0, 0, 1, -1];

B = [1; 0; 0; 0];

C = [1; 0; 0; 0];

D = 0;

t\_end = 2;

wo = 10\*pi;

[h,y,t]=AWE(A,B,C,D,wo,t\_end);

plot(t,h)

xlabel('time s')

ylabel('H(t)')

title(['Impulse response at w= ', num2str(wo)]);

grid on