Stabilization of Alpha and Beta in Vanguard Index Funds

Mohagoney Moore

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1 Introduction

1.1 Total Returns, Risk-Free Returns, and Equity Premium

The total returns, risk-free returns, and equity premia are all measures of the financial return on an investment. Returns can be measured across a variety of time lengths and periods, but we will be looking at monthly returns. The total return is the rate of return of an investment or group of investments over a period of time. The monthly total return is given by

$$TR(t) = \ln\left(\frac{\text{Adjusted Close for } (t+1)}{\text{Adjusted Close for } (t)}\right),$$

where t is current month and $t+1$ is the proceeding month. Now, the monthly risk-free return is a theoretical rate of return of an investent that carries zero risk, a bond for example. The risk-free rate is given by

$$RFR(t) = \ln\left(1 + \frac{r(t-1)}{1200}\right),$$

where $r(t-1)$ is the rate of the previous month. Finally, the equity premium is the excess return returned to an investor when investing in the stock market. This is calcualted by taking the difference between the monthly total return and the risk-free return,

$$P(t) = TR(t) - RFR(t).$$

Similarly, we calculate the equity premia for our benchmark, $P_0(t)$, in the same fashion.

1.2 S&P 500 Benchmark

The Standard and Poor's 500 Index, better known as the S&P 500, is an index consisting of the 500 largest pubilcly traded companies in the United States.

Since its' inception in 1957, the S&P 500 serves as one of the leading benchmarks in the U.S. stock market. Due to the wide popularity of this large-cap index fund, other funds have mimicked its structure. We will look at a variety of Vanguard index funds and their performance against the Vanguard 500 Index Fund Investor Shares (VFINX) as our benchmark fund.

1.3 Alpha and Beta

Alpha, α, and beta, β, serve as key components in our analysis due to their effect on our equity premium. If we conduct a liner regression for our equity premium

$$P(t) = \alpha + \beta P_0(t) + e(t)$$

then our α would be our excess return and our β is the market volatility. Our index's equity premium is denoted by $P(t)$ and our benchmarked equity premium is $P_0(t)$.

Now, consider the values of α and β. An $\alpha > 1$ represents an outperformance of our benchmark, while $\alpha < 1$ is an underperformance. In the case of β, $\beta = 1$ indicates a strong correlation between the index and benchmark movement, $\beta < 1$ shows less volatility compared to the benchmark, $\beta > 1$ indicates a higher volatility to the benchmark, and $\beta \leq -1$ showing an inverse correlation to the benchmark.

2 Are Alpha and Beta Stable?

Coefficient of Variation (CV) is used to measure the spread of a dataset relative to its' mean. The CV helps many investors determine whether the excess return is worth the large variability in risk. Our CV is denoted by

$$CV = \frac{\sigma}{\mu}$$

where σ is our standard deviation and μ is our mean. A $CV \geq 1$ indicates high volatility, while a $CV < 1$ shows stability. This method is applied to α and β to measure their stability in the market over time and make decisions about which index would be considered more or less risky.

3 Data

Data was sourced from Yahoo Finance and the Federal Reserve Economic Data (FRED) websites. All data and code are stored on the Github website provided.

Yahoo Finance: https://finance.yahoo.com/
FRED: https://fred.stlouisfed.org/series/DGS3MO
GitHub: https://github.com/mohagoneym/Stabilization-of-Alpha-and-Beta-in-Vanguard-Index-Funds

4 Procedure

The following procedure was implemented after gathering our data for each Vanguard stock against the Vanguard S&P500:

1. Calculate the monthly total returns, risk-free returns, and equity premium for each index and our benchmark for October 1990 - October 2020.
2. Develop an algorithm that splits the monthly equity premia into 3 year periods and gives the linear regression summary.
3. Obtain α and β values from the summary.
4. Calculate the the coefficient of variation for the α and β values.

5 Results

Table 1: Alpha, Beta, and CV for NAESX, VBMFX, and VUSTX Indexes

	Indexes					
	NAESXvVFINX		VBMFXvVFINX		VUSTXvVFINX	
Period	α	β	α	β	α	β
1	0.0089	0.9399	0.0051	0.0837	0.0071	0.2442
2	-0.004	1.0628	-0.002	0.2886	-0.005	0.5238
3	-0.012	1.025	0.0002	0.0429	0.0003	0.0626
4	0.0056	0.7987	0.0027	-0.051	0.003	-0.186
5	0.0042	1.328	0.0024	-0.075	0.0047	-0.178
6	0.0013	1.2143	0.0001	0.0628	0.0007	0.0022
7	0.0015	1.2638	0.0072	-0.029	0.0129	-0.327
8	-0.003	1.1855	0.0015	0.0313	0.0079	-0.325
9	-0.001	0.9973	0.002	-0.048	0.0052	-0.293
10	-0.006	1.2157	0.0027	-0.001	0.0086	-0.236
CV	-13.79	0.15	1.2118	3.4327	1.1045	-3.938

From Table 1, one may notice that the coefficient of variation in α and β amongst the different indexes varies. The NAESX index shows stability in α and β due to our $CV < 1$, while our VBMFX index shows instability with $CV \geq 1$. In the case of VUSTX, we see an unstable α and a stable β. This shows stabilization variability amongst different indexes when considering the same benchmark.

6 Conclusion

To conclude, the stabilization of α and β in selected indexes when benchmarked to the Vanguard 500 Index Fund Investor Shares (VFINX) widely varies. Some indexes show strong signs of stabilization in their α and β, while some stocks will see instability in one, or both, variables. Individual indexes showing strong α and β stability over time can allow for a risk-adverse investment strategy and serve as key index choices in portfolio diversification.

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