General instructions: *Put your name on* ***every page*** *of the exam. Write as neatly as possible – we must be able to read your responses. Complete all problems to the best of your ability. Be sure to clearly explain your answers, especially in your proofs. Keep your eyes on* ***your own*** *paper. When you are done, raise your hand and wait for your exam and crib sheet to be collected before leaving the room.*

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1. Prove the following statements via induction:

(a) $\forall n \in \mathbb{N}$, $9^n + 3$ is divisible by 12.

(b) $\forall n \in \mathbb{N}$ where $n \geq 8, n$ can be written as $3a + 5b$, where $a, b \in \mathbb{N}_0$. (That is, all integers greater than or equal to 8 can be written as a sum of 3s and 5s.)

2. Use the following predicates to translate the English sentences below into predicate logic. Assume the universe of discourse is "people who work at RPI." *(Some of the sentences might be false; that's fine, since we're just doing translation here.)*

E(x) = x owns an electric car.

C(x) = x works in the computer science department.

P(x) = x is a professor.

B(x) = x rides a bicycle to work.

S(x,y) = x works in the same building as y.

(a) Everyone who owns an electric car is a professor.

(b) There is a professor who is not in computer science who rides a bicycle to work.

(c) No people who work in the same building as Dan ride bicycles to work.

(d) Everyone in the computer science department works in the same building.

3. Now do the reverse – translate the logic sentences into "plain" English, using the same predicates. Avoid phrases like "it is not the case that" – write what someone would actually <u>say</u> if talking about this in conversation.

(a) $\forall x \left(P(x) \wedge E(x)\right) \Rightarrow \neg B(x)$

(b) $\forall x \left[P(x) \Rightarrow \exists y \left(S(x,y) \wedge E(y)\right)\right]$

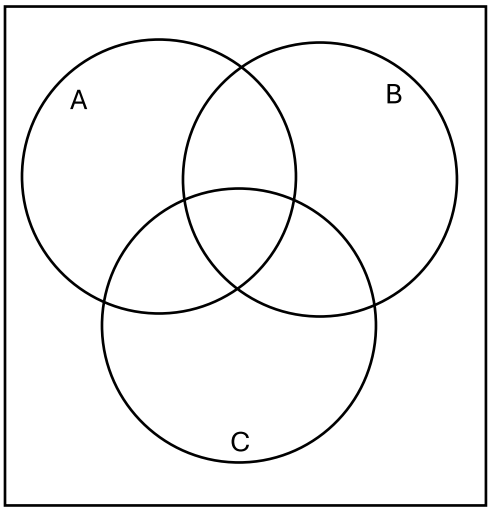
4. Give truth tables for these propositional logic expressions. Use the correct order of operations. (Remember, $\oplus$ means exclusive-OR.) You may add auxiliary columns if you wish. (Indeed, doing so is recommended.)

(a) $\neg p \vee q \wedge r$

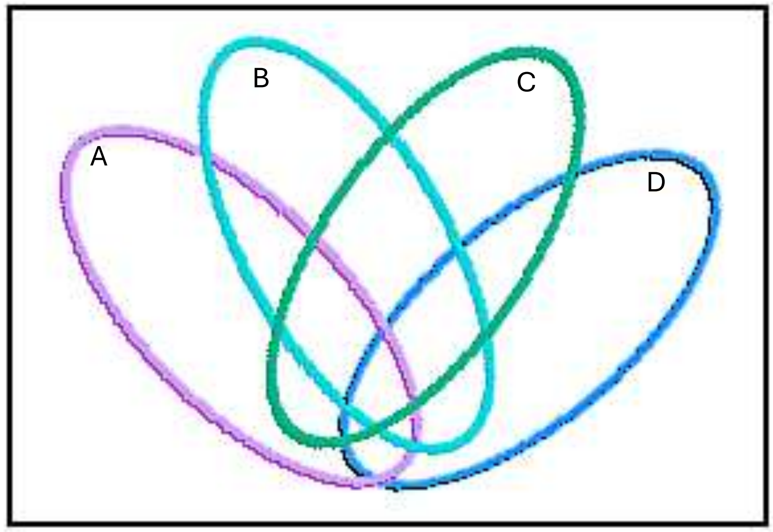
(b) $(p \oplus q) \vee (\neg q \Rightarrow r)$

5. On the Venn diagrams below, shade in the areas corresponding to the following sets.

(a) $(\bar{A} \cap \bar{C}) \cup (B \cap C)$



(b) $(A \cap \bar{B} \cap C) \cup (B \cap \bar{C} \cap D) \cup (C \cap \bar{D} \cap \bar{A})$



6. Give a recursive definition for the set of all binary strings containing *at least* two more 1s than 0s.

7. Prove the following statements using any desired method.

(a) If x and y are positive integers such that $xy < 10000$, then either $x < 100$ or $y < 100$.

(b) Every odd positive integer is the difference of two perfect squares.

(Advance note: all numbers in this problem are completely made up – this is a puzzle/thought exercise, not a realistic scenario.)

8. You have 10 drones, each of which can store 100 watt-hours (Wh) of energy. Traveling one mile costs a drone 1Wh of energy. You have a small package (e.g. a letter) that can be carried on a single drone. You can also, at any time, land the drones to freely transfer energy between drones. (Assume this costs no energy.)

If there is no requirement to fly the drones back to base (i.e. someone will come by to pick them up later), what is the farthest distance away that you can deliver the package? Explain how you came to your answer and show your work.