Anamaly Detection through Probabilistic Methods: COPOD, ABOD

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January 26, 2025

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Introduction

What is anomaly detection?

Anomaly detection is the process of identifying data points, entities, or events that fall outside the normal range.

An anomaly is anything that deviates from what is standard or expected.

Types of Anamoly Detection Methods

Proximity-Based: k-Nearest Neighbors (k-NN)

Statistical: Z-Score

Density-Based: DBSCAN

Ensemble: Isolation Forest

Cluster-Based: K-Means

Probabilistic: COPOD, ABOD

What is Copula?

A copula is a mathematical function that links univariate marginal distributions to form a multivariate distribution, capturing the dependency structure between random variables independently of their marginals.

$$C_U(\mathbf{u}) = P(U_1 \leq u_1, \ldots, U_d \leq u_d)$$

Sklar's Theorem

Hence, a joint distribution can be expressed in terms of a copula and marginal distributions as:

$$F(x_1, x_2, \ldots, x_d) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_d)),$$

COPOD Methodology

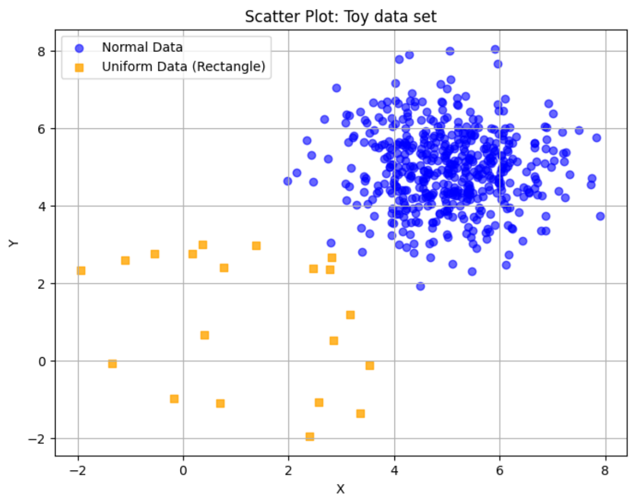
Toy Dataset

The following table shows a small example dataset used for illustration:

Index	X	Y
0	4.249385	6.316357
1	6.246140	3.395084
2	3.531856	3.284930
3	6.858784	5.087588
4	4.947678	5.555472
...		

The toy dataset is generated by taking samples from a combination of normal and uniform distributions.

Scatter Plot



COPOD Methodology

Empirical Cumulative Distribution Function

The empirical cumulative distribution function (ECDF) is a step function that estimates the cumulative distribution function (CDF) of a sample dataset. It is defined as:

$$F(x) = P((-\infty, x]) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq x),$$

- Non-parametric approach
- $U_{xi} = F(x_i)$

COPOD Methodology

Transformation of Variables into Copula Observations

Index	x	U_{xi}	y	U_{yi}
0	4.249385	0.273077	6.316357	0.921154
1	6.246140	0.909615	3.395084	0.101923
2	3.531856	0.101923	3.284930	0.086538
3	6.858784	0.957692	5.087588	0.569231
4	4.947678	0.484615	5.555472	0.726923
......				

COPOD Methodology

Estimation of Tail Probabilities

begin with

$$
\begin{aligned}
C(u_1 \dots u_d) &= \frac{1}{n}\sum_{i=1}^{n} I\left(\tilde{U}_1 \le u_1; \dots; \tilde{U}_d \le u_d\right) \\
&= \frac{1}{n}\sum_{i=1}^{n} I(\tilde{U}_1 \le u_1) \cdot \dots \cdot I(\tilde{U}_d \le u_d) \\
&\approx P(\tilde{U}_{1,i} \le u_1) \cdot \dots \cdot P(\tilde{U}_{d,i} \le u_d) \\
&= u_{1i} \cdot u_{2i} \cdot \dots \cdot u_{di}
\end{aligned}
$$

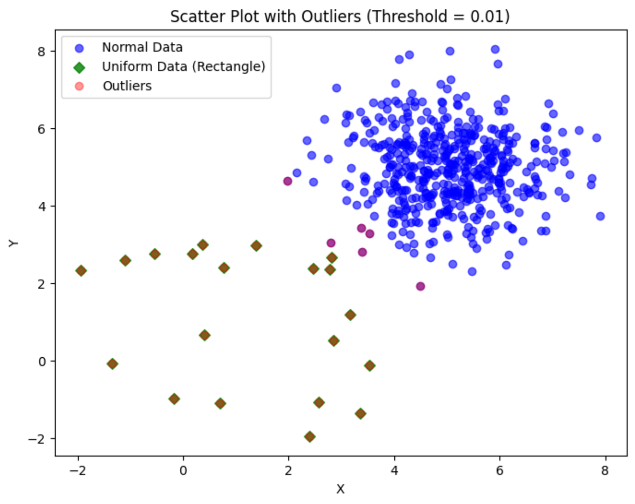
COPOD Methodology

Joint CDF :

Index	X	U_{X_i}	Y	U_{Y_i}	Joint CDF
0	4.249385	0.273077	6.316357	0.921154	0.251546
1	6.246140	0.909615	3.395084	0.101923	0.092711
2	3.531856	0.101923	3.284930	0.086538	0.008820
3	6.858784	0.957692	5.087588	0.569231	0.545148
4	4.947678	0.484615	5.555472	0.726923	0.352278
...					

COPOD Methodology

outlier detection



COPOD Methodology

Right tail probability

$$C(1-u) = P(U_1 \geq u_1, \ldots, U_d \geq u_d)$$

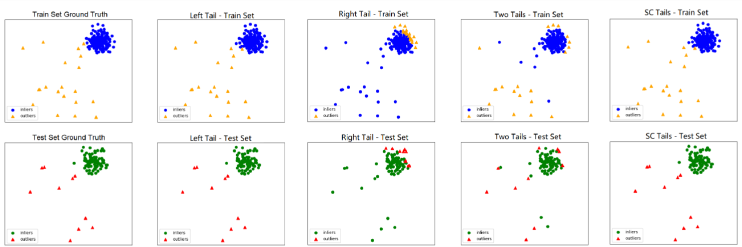
The right tail probability can be efficiently computed using:

$-\mathbf{X} = [-X_{1,i}, -X_{2,i}, \ldots, -X_{d,i}]$

$\hat{F}_d(-x) = \frac{1}{n} \sum_{i=1}^{n} I(-X_i \leq -x)$

COPOD Methodology

Scatter Plot



Source: Li, Z. et al. (n.d.). COPOD: Copula-Based Outlier Detection. Northeastern University, Carnegie Mellon University, PIK Potsdam, THD, Arima Inc., Canada.

COPOD Methodology

Skewness Correction

$$b_i = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3 \Big/ \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Definition of $\hat{W}_{d,i}$:

$$\hat{W}_{d,i} = \begin{cases} \hat{U}_{d,i}, & \text{if } b_d < 0 \\ \hat{V}_{d,i}, & \text{if } b_d > 0 \end{cases}$$

High Dimensional Case for Copula Function

Copula Function:

$$\lim_{d \to \infty} \hat{C}(u_1, u_2, \dots, u_d) = 0$$

$$-\log(\hat{C}(u)) = -\log\left(P(\hat{U}_{1,i} \leq u_1) \times \cdots \times P(\hat{U}_{d,i} \leq u_d)\right)$$

$$= -\sum_{j=1}^{d} \log(P(\hat{U}_{j,i} \leq u_j)) = -\sum_{j=1}^{d} \log(u_j)$$

COPOD Algorithm

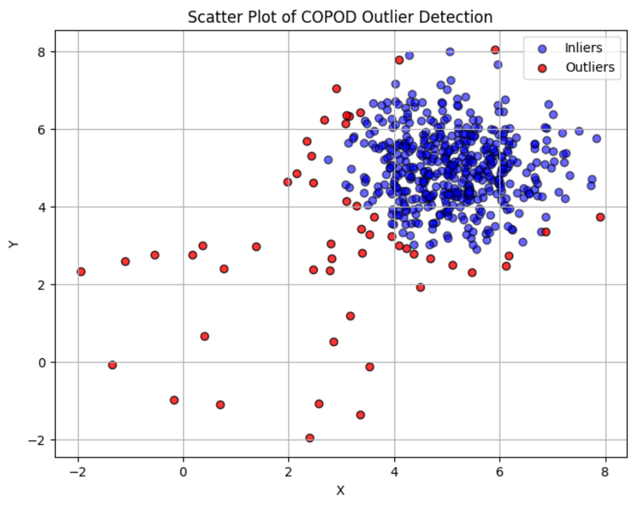
Algorithm 1 Copula Outlier Detector

Inputs: input data X
Outputs: outlier scores $O(X)$

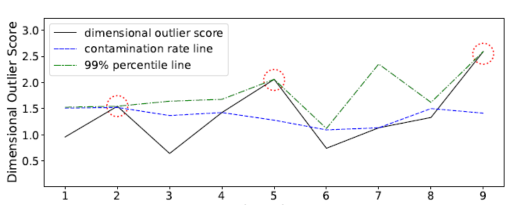
for each dimension d **do**
  Compute left tail ECDFs: $\hat{F}_d(x) = \frac{1}{n} \sum_1^n \mathbb{I}(X_i \le x)$
  Compute right tail ECDFs: $\hat{\bar{F}}_d(x) = \frac{1}{n} \sum_1^n \mathbb{I}(-X_i \le -x)$
  Compute the skewness coefficient according to Equation 11.
end for
for each i in $1, \dots n$ **do**
  Compute empirical copula observations
    $\hat{U}_{d,i} = \hat{F}_d(x_i)$,
    $\hat{V}_{d,i} = \hat{\bar{F}}_d(x_i)$
    $\hat{W}_{d,i} = \hat{U}_{d,i}$ if $b_d < 0$ otherwise $\hat{V}_{d,i}$
  Calculate tail probabilities of X_i as follows:
    $p_l = -\sum_{j=1}^{d} \log(\hat{U}_{j,i})$
    $p_r = -\sum_{j=1}^{d} \log(\hat{V}_{j,i})$
    $p_s = -\sum_{j=1}^{d} \log(\hat{W}_{j,i})$
  Outlier Score $O(x_i) = \max\{p_l,\ p_r,\ p_s\}$
end for
Return $O(X) = [O(x_1), \dots, O(x_d)]^T$

Source: Li, Z. et al. (n.d.). COPOD: Copula-Based Outlier Detection. Northeastern University, Carnegie Mellon University, PIK Potsdam, THD, Arima Inc., Canada.

Pyod implemntation



Interpretability of COPOD



Source: Li, Z. et al. (n.d.). COPOD: Copula-Based Outlier Detection. Northeastern University, Carnegie Mellon University, PIK Potsdam, THD, Arima Inc., Canada.

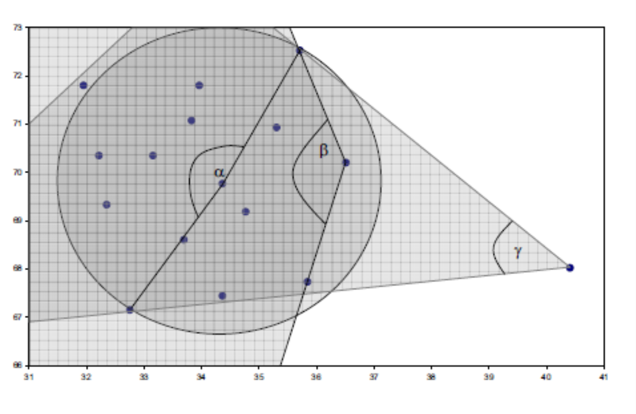
Scalability of COPOD

COPOD'S PERFORMANCE IN SECONDS UNDER DIFFERENT DIMENSION AND DATA SIZE. IT SCALES WELL REGARDING DIMENSIONS.

	d=10	d=100	d=1,000	d=10,000
n=1,000	0.0687	0.1731	1.1632	11.4605
n=10,000	0.1718	0.4686	5.2441	55.1901
n=100,000	0.6405	7.1858	70.5410	567.1058
n=1,000,000	11.4036	130.9741	694.4056	5376.5937

Source: Li, Z. et al. (n.d.). COPOD: Copula-Based Outlier Detection. Northeastern University, Carnegie Mellon University, PIK Potsdam, THD, Arima Inc., Canada.

Angle-Based Outlier Detection



Source: Kriegel, H.-P., Schubert, M., Zimek, A., et al. (2008). Angle-based outlier detection in high-dimensional data. ACM. `https://doi.org/10.1145/1401890.1401946`

Angle-Based Outlier Detection

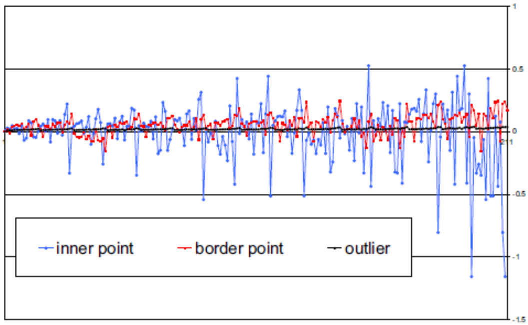


Figure 2: Spectra of angles for different types of points.

Source: Kriegel, H.-P., Schubert, M., Zimek, A., et al. (2008). Angle-based outlier detection in high-dimensional data. ACM. `https://doi.org/10.1145/1401890.1401946`

Angle-Based Outlier Detection

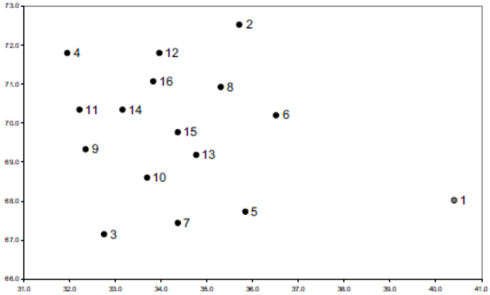
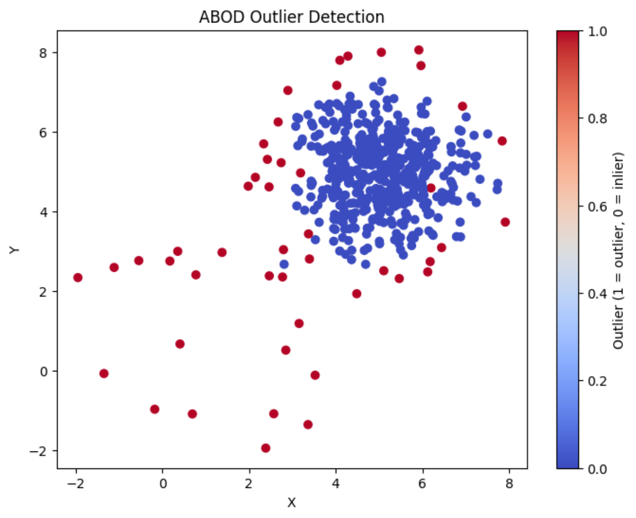


Figure 3: Ranking of points in the sample data set according to ABOF.

Source: Kriegel, H.-P., Schubert, M., Zimek, A., et al. (2008). Angle-based outlier detection in high-dimensional data. ACM. `https://doi.org/10.1145/1401890.1401946`

Pyod implementation



ABOD Approximation Complexity

ABOD: Complexity $O(n^3)$

Robust but computationally expensive for large datasets.

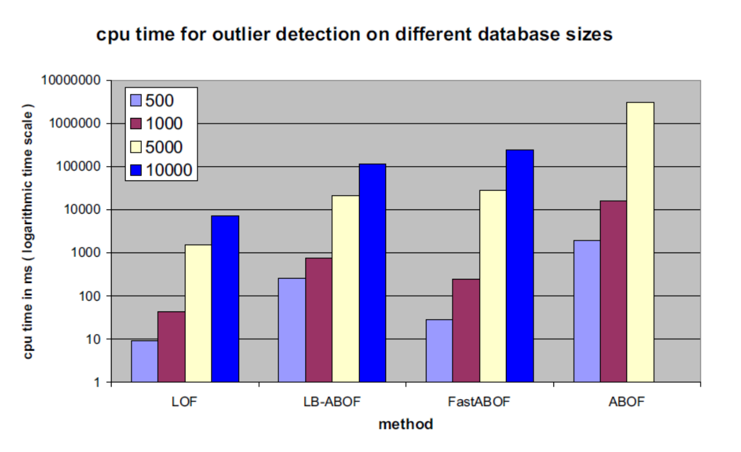
Fast ABOD: Complexity $O(n^2 + n \cdot k^2)$

Utilizes k-nearest neighbors (K-NN) to improve scalability.

LB-ABOD: Conservative Approximation with k**-NN**

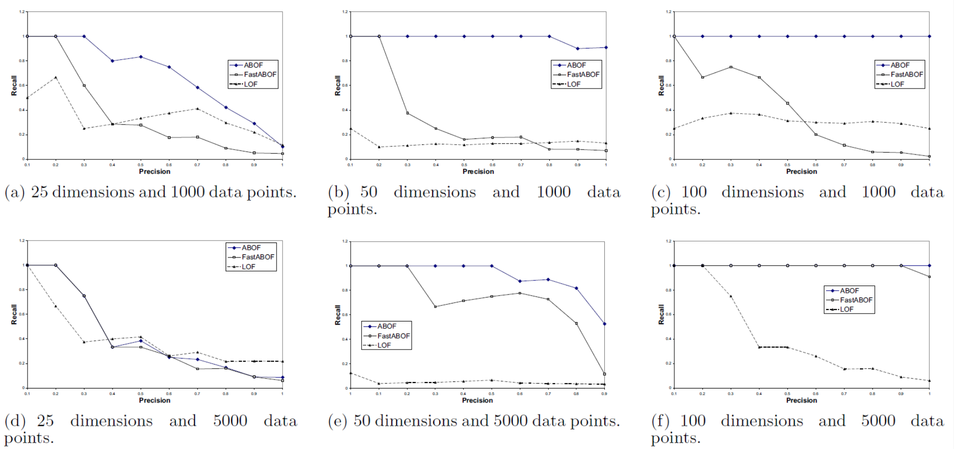
Combines the scalability of Fast ABOD with the robustness of ABOD.

ABOD Approximation Complexity



Source: Kriegel, H.-P., Schubert, M., Zimek, A., et al. (2008). Angle-based outlier detection in high-dimensional data. ACM. https://doi.org/10.1145/1401890.1401946

ABOD Evaluation



(a) 25 dimensions and 1000 data points.

(b) 50 dimensions and 1000 data points.

(c) 100 dimensions and 1000 data points.

(d) 25 dimensions and 5000 data points.

(e) 50 dimensions and 5000 data points.

(f) 100 dimensions and 5000 data points.

Source: Kriegel, H.-P., Schubert, M., Zimek, A., et al. (2008). Angle-based outlier detection in high-dimensional data. ACM. `https://doi.org/10.1145/1401890.1401946`

Comparison of ABOD and COPOD: ROC-AUC Performance

TABLE I

ROC-AUC SCORES OF DETECTOR PERFORMANCE (AVERAGE OF 10 INDEPENDENT TRIALS, HIGHEST SCORE HIGHLIGHTED IN BOLD); RANK IS SHOWN IN PARENTHESIS (LOWER IS BETTER). COPOD OUTPERFORMS ALL BASELINES.

Data	ABOD	CBLOF	FB	HBOS	IForest	KNN	LODA	LOF	OCSVM	COPOD
Arrhythmia (mat)	0.7688 (9)	0.7838 (6)	0.7839 (5)	**0.8219 (1)**	0.802 (3)	0.7861 (4)	0.7497 (10)	0.7787 (8)	0.7812 (7)	0.8021 (2)
Breastw (mat)	0.3959 (10)	0.9651 (6)	0.396 (9)	0.9825 (4)	0.9869 (3)	0.9755 (5)	0.9873 (2)	0.47 (8)	0.9612 (7)	**0.9936 (1)**
Cardio (mat)	0.5692 (10)	0.81 (6)	0.7057 (8)	0.8351 (5)	0.9226 (2)	0.7236 (7)	0.8559 (4)	0.5736 (9)	**0.9348 (1)**	0.8974 (3)
Ionosphere (mat)	0.9248 (3)	0.8972 (4)	**0.9495 (1)**	0.5614 (10)	0.8472 (6)	0.9267 (2)	0.7975 (9)	0.8753 (5)	0.8419 (7)	0.8307 (8)
Lympho (mat)	0.911 (8)	0.9673 (7)	0.8809 (9)	**0.9957 (1)**	0.9934 (3)	0.9745 (6)	0.7303 (10)	0.9771 (4)	0.9759 (5)	0.9935 (2)
Mammography (mat)	0.5494 (10)	0.8213 (8)	**0.8775 (2)**	0.8342 (7)	0.8602 (5)	0.841 (6)	0.8809 (3)	0.7293 (9)	0.8724 (4)	**0.8942 (1)**
Optdigits (mat)	0.4667 (7)	0.7692 (3)	0.7948 (2)	**0.8732 (1)**	0.7209 (5)	0.3708 (10)	0.3982 (9)	0.45 (8)	0.4997 (6)	0.7328 (4)
Pima (mat)	0.6794 (3)	0.6578 (6)	0.3781 (10)	0.7000 (2)	0.6778 (4)	**0.7078 (1)**	0.592 (9)	0.6271 (7)	0.6215 (8)	0.6638 (5)
Satellite (mat)	0.5714 (9)	0.7494 (2)	0.6525 (7)	**0.7581 (1)**	0.7024 (3)	0.6836 (4)	0.5969 (8)	0.5573 (10)	0.6622 (5)	0.6612 (6)
Satimage-2 (mat)	0.819 (8)	**0.9992 (1)**	0.5670 (9)	0.9804 (6)	0.9947 (3)	0.9536 (7)	0.9866 (4)	0.4577 (10)	0.9978 (2)	0.9852 (5)
Shuttle (mat)	0.6591 (7)	0.6075 (8)	0.6715 (6)	0.9948 (3)	**0.9983 (1)**	0.7381 (5)	0.5892 (9)	0.5243 (10)	0.9945 (4)	0.998 (2)
Speech (mat)	**0.6267 (1)**	0.4515 (7)	0.4503 (8)	0.4538 (6)	0.447 (10)	0.4581 (4)	0.4551 (5)	0.4714 (3)	0.4472 (9)	0.4845 (2)
WBC (mat)	0.9047 (9)	0.9201 (7)	0.4615 (10)	0.9516 (2)	0.9314 (6)	0.9366 (3)	0.9126 (8)	0.9349 (4)	0.9319 (5)	**0.9747 (1)**
Wine (mat)	0.4305 (9)	0.2835 (10)	0.9346 (2)	0.9001 (4)	0.8163 (5)	0.5177 (8)	0.7873 (6)	0.9045 (3)	0.6363 (7)	**0.949 (1)**
Arrhythmia (arff)	0.7396 (9)	0.7514 (5)	0.7614 (2)	0.7493 (7)	0.7567 (3)	0.7508 (6)	0.7028 (10)	0.749 (8)	0.7557 (4)	**0.7618 (1)**
Cardiotocography (arff)	0.4593 (10)	0.5673 (6)	0.5184 (7)	0.5936 (5)	0.6828 (2)	0.4904 (9)	0.6738 (3)	0.5045 (8)	**0.6976 (1)**	0.637 (4)
HeartDisease (arff)	0.5969 (6)	0.5926 (7)	0.6286 (3)	**0.6893 (1)**	0.602 (5)	0.6136 (4)	0.4821 (10)	0.5626 (8)	0.5548 (9)	0.6728 (2)
Hepatitis (arff)	0.7155 (9)	0.7585 (8)	0.8420 (2)	0.7961 (5)	0.7935 (6)	0.8111 (4)	0.6075 (10)	0.8305 (3)	0.7644 (7)	**0.8432 (1)**
InternetAds (arff)	0.6427 (4)	0.6166 (7)	0.6132 (8)	**0.6994 (1)**	0.6851 (2)	0.6216 (6)	0.5276 (10)	0.6057 (9)	0.6233 (5)	0.6763 (3)
Ionosphere (arff)	**0.9250 (1)**	0.8859 (3)	0.8800 (4)	0.552 (10)	0.836 (7)	0.9187 (2)	0.8222 (8)	0.8604 (5)	0.8381 (6)	0.8219 (9)
KDDCup99 (arff)	0.6925 (8)	0.9970 (2)	0.391 (10)	0.995 (4)	0.9968 (3)	0.7611 (6)	0.7277 (7)	0.5774 (9)	0.9948 (5)	**0.9972 (1)**
Lymphography (arff)	0.9864 (9)	0.9956 (6)	0.9961 (5)	0.9984 (3)	0.9994 (1)	0.9965 (4)	0.8385 (10)	0.995 (7)	0.9915 (8)	0.9985 (2)
Pima (arff)	0.6666 (3)	0.6658 (4)	0.6544 (7)	0.6591 (5)	0.6756 (2)	**0.7063 (1)**	0.5974 (10)	0.6582 (6)	0.6403 (9)	0.6466 (8)
Shuttle (arff)	0.8338 (7)	0.9802 (2)	0.3671 (10)	0.7924 (8)	0.8661 (6)	0.9607 (4)	0.7066 (9)	**0.9857 (1)**	0.9658 (3)	0.8795 (5)
SpamBase (arff)	0.4349 (9)	0.5476 (5)	0.6795 (2)	0.6698 (3)	0.6328 (4)	0.5399 (6)	0.4449 (8)	0.4255 (10)	0.5386 (7)	**0.7007 (1)**
Stamps (arff)	0.7617 (7)	0.7299 (10)	0.7419 (8)	0.9182 (2)	0.9132 (3)	0.8738 (6)	0.9014 (4)	0.739 (9)	0.88 (5)	**0.9388 (1)**
Waveform (arff)	0.6522 (9)	0.7143 (4)	0.6966 (5)	0.683 (6)	0.6813 (7)	0.7403 (2)	0.622 (10)	0.7354 (3)	0.6649 (8)	**0.7865 (1)**
WBC (arff)	0.9559 (8)	0.9658 (7)	0.9432 (9)	0.9819 (3)	**0.9906 (1)**	0.9715 (6)	0.9811 (4)	0.9315 (10)	0.9751 (5)	0.9904 (2)
WDBC (arff)	0.9032 (9)	0.9088 (8)	0.5388 (10)	0.9674 (2)	0.9462 (3)	0.9272 (6)	0.9455 (4)	0.9204 (7)	0.938 (5)	**0.9826 (1)**
WPBC (arff)	0.457 (10)	0.4918 (9)	0.5196 (4)	0.5357 (2)	0.5164 (5)	0.526 (3)	0.5029 (7)	0.5108 (6)	0.4979 (8)	**0.5465 (1)**
AVG	0.6900 (9)	0.7617 (5)	0.6792 (10)	0.7974 (3)	0.8092 (2)	0.7601 (6)	0.7135 (7)	0.6974 (8)	0.7826 (4)	**0.8247 (1)**

Source: Li, Z. et al. (n.d.). *COPOD: Copula-Based Outlier Detection*. Northeastern University, Carnegie Mellon University, PIK Potsdam, THD, Arima Inc., Canada.

Comparison of ABOD and COPOD: Average Precision

TABLE II

AVERAGE PRECISION OF DETECTOR PERFORMANCE (AVERAGE OF 10 INDEPENDENT TRIALS, HIGHEST SCORE HIGHLIGHTED IN BOLD); RANK IS SHOWN IN PARENTHESIS (LOWER IS BETTER). COPOD OUTPERFORMS ALL BASELINES.

Data	ABOD	CBLOF	FB	HBOS	IForest	KNN	LODA	LOF	OCSVM	COPOD
Arrhythmia (mat)	0.3585 (10)	0.399 (6)	0.3872 (8)	0.4928 (2)	**0.5062 (1)**	0.3968 (7)	0.4359 (4)	0.3742 (9)	0.4045 (5)	0.4727 (3)
Breastw (mat)	0.2945 (9)	0.9141 (7)	0.2927 (10)	0.953 (4)	0.9721 (3)	0.9269 (6)	0.9777 (2)	0.3215 (8)	0.9343 (5)	**0.9877 (1)**
Cardio (mat)	0.1936 (9)	0.4136 (6)	0.2091 (8)	0.4602 (4)	0.5763 (2)	0.3451 (7)	0.4237 (5)	0.1626 (10)	0.5324 (3)	**0.5793 (1)**
Ionosphere (mat)	0.9141 (2)	0.8712 (3)	0.1198 (10)	0.3664 (9)	0.7889 (6)	**0.9238 (1)**	0.7156 (8)	0.8211 (5)	0.8229 (4)	0.7194 (7)
Lympho (mat)	0.5168 (9)	0.8078 (8)	0.8266 (4)	0.9252 (2)	**0.9516 (1)**	0.82 (6)	0.4165 (10)	0.8252 (5)	0.8144 (7)	0.8936 (3)
Mammography (mat)	0.0232 (10)	0.1369 (7)	0.4258 (2)	0.1222 (8)	0.2329 (4)	0.1695 (6)	0.2808 (3)	0.1179 (9)	0.188 (5)	**0.429 (1)**
Optdigits (mat)	0.0279 (7)	0.0624 (3)	0.1092 (2)	**0.1957 (1)**	0.0553 (4)	0.0223 (10)	0.0244 (9)	0.0288 (6)	0.0275 (8)	0.0532 (5)
Pima (mat)	0.5107 (4)	0.4771 (6)	0.0234 (10)	**0.5693 (1)**	0.5031 (5)	**0.5145 (3)**	0.4081 (9)	0.4299 (8)	0.4608 (7)	0.5407 (2)
Satellite (mat)	0.3973 (9)	**0.6897 (1)**	0.4518 (8)	0.6866 (2)	0.6539 (3)	0.5431 (7)	0.5805 (6)	0.3898 (10)	0.6529 (4)	0.5854 (5)
Satimage-2 (mat)	0.1874 (9)	**0.9777 (1)**	0.3984 (8)	0.758 (6)	0.9285 (3)	0.4186 (7)	0.8695 (4)	0.0267 (10)	0.9746 (2)	0.8599 (5)
Shuttle (mat)	0.1705 (8)	0.1945 (7)	0.0353 (10)	0.9796 (3)	**0.9857 (1)**	0.2036 (6)	0.3786 (5)	0.1421 (9)	0.9015 (4)	0.9807 (2)
Speech (mat)	**0.0395 (1)**	0.0216 (5)	0.0223 (4)	0.0272 (2)	0.0177 (9)	0.0216 (5)	0.0165 (10)	0.0243 (3)	0.0214 (7)	0.0195 (8)
WBC (mat)	0.3545 (10)	0.4995 (9)	0.562 (5)	0.6627 (2)	0.5898 (3)	0.5294 (7)	0.5641 (4)	0.558 (6)	0.5137 (8)	**0.7827 (1)**
Wine (mat)	0.0838 (9)	0.0598 (10)	0.3563 (4)	0.405 (2)	0.2794 (5)	0.0946 (8)	0.2783 (6)	0.3611 (3)	0.1414 (7)	**0.6082 (1)**
Arrhythmia (arff)	0.6987 (9)	0.7118 (6)	0.7314 (4)	0.7504 (2)	0.7463 (3)	0.7118 (6)	0.697 (10)	0.704 (8)	0.7164 (5)	**0.7519 (1)**
Cardiotocography (arff)	0.247 (10)	0.3625 (6)	0.2702 (8)	0.3664 (5)	**0.4342 (1)**	0.3108 (7)	0.4315 (2)	0.2582 (9)	0.4185 (3)	0.3778 (4)
HeartDisease (arff)	0.5341 (5)	0.521 (7)	0.5506 (3)	0.6247 (2)	0.5337 (6)	0.5384 (4)	0.445 (10)	0.4779 (9)	0.5128 (8)	**0.64 (1)**
Hepatitis (arff)	0.3304 (10)	0.3739 (9)	**0.5660 (2)**	0.4732 (5)	0.4424 (6)	0.4752 (4)	0.3957 (8)	0.4986 (3)	0.4274 (7)	**0.5849 (1)**
InternetAds (arff)	0.276 (7)	0.3153 (5)	0.2737 (8)	**0.5347 (1)**	0.4895 (3)	0.2805 (6)	0.2417 (10)	0.2622 (9)	0.3157 (4)	0.5102 (2)
Ionosphere (arff)	**0.5055 (6)**	0.4839 (7)	0.5286 (2)	0.5281 (3)	0.5146 (5)	0.525 (4)	0.4199 (10)	0.4584 (9)	0.4779 (8)	**0.5302 (1)**
KDDCup99 (arff)	0.3798 (7)	0.4143 (5)	0.3018 (10)	0.5249 (2)	0.4923 (3)	0.4219 (4)	0.3752 (8)	0.3489 (9)	0.4115 (6)	**0.5667 (1)**
Lymphography (arff)	0.2465 (8)	0.2409 (9)	0.4352 (2)	0.3983 (4)	0.3936 (5)	0.3412 (7)	0.4038 (3)	0.2291 (10)	0.3464 (6)	**0.4637 (1)**
Pima (arff)	**0.9153 (1)**	0.8619 (3)	0.4545 (9)	0.3637 (10)	0.7767 (6)	**0.9145 (2)**	0.7631 (7)	0.811 (5)	0.8215 (4)	0.7031 (8)
Shuttle (arff)	0.0182 (10)	0.1977 (5)	**0.3171 (1)**	0.2781 (2)	0.2733 (3)	0.0457 (8)	0.1352 (6)	0.028 (9)	0.1249 (7)	0.2389 (4)
SpamBase (arff)	0.8013 (8)	0.925 (6)	0.4128 (10)	0.9783 (3)	**0.9917 (1)**	0.9417 (4)	0.4914 (9)	0.9254 (5)	0.8367 (7)	0.9817 (2)
Stamps (arff)	0.2626 (5)	0.3583 (3)	0.1195 (8)	0.0804 (10)	0.1297 (7)	0.3597 (2)	0.1316 (6)	**0.3945 (1)**	0.3042 (4)	0.0854 (9)
Waveform (arff)	0.0656 (7)	0.1293 (3)	**0.1925 (1)**	0.0547 (9)	0.0559 (8)	0.1343 (2)	0.0521 (10)	0.108 (4)	0.0702 (6)	0.0785 (5)
WBC (arff)	0.5422 (9)	0.5561 (8)	0.7534 (3)	0.6935 (6)	**0.863 (1)**	0.5806 (7)	0.7242 (4)	0.3303 (10)	0.7018 (5)	0.8381 (2)
WDBC (arff)	0.43 (9)	0.6669 (6)	0.2973 (10)	0.7952 (2)	0.7178 (4)	0.6535 (7)	0.7935 (3)	0.7038 (5)	0.6123 (8)	**0.8407 (1)**
WPBC (arff)	0.2095 (10)	0.2244 (7)	**0.2412 (2)**	0.2303 (5)	0.2313 (4)	0.2328 (3)	0.2231 (8)	0.2248 (6)	0.2226 (9)	**0.2438 (1)**
AVG	0.3512 (10)	0.4623 (5)	0.3715 (9)	0.5093 (3)	0.5376 (2)	0.4466 (6)	0.4365 (7)	0.3782 (8)	0.4904 (4)	**0.5649 (1)**

Source: Li, Z. et al. (n.d.). *COPOD: Copula-Based Outlier Detection*. Northeastern University, Carnegie Mellon University, PIK Potsdam, THD, Arima Inc., Canada.

Questions?

Thank you for your attention!