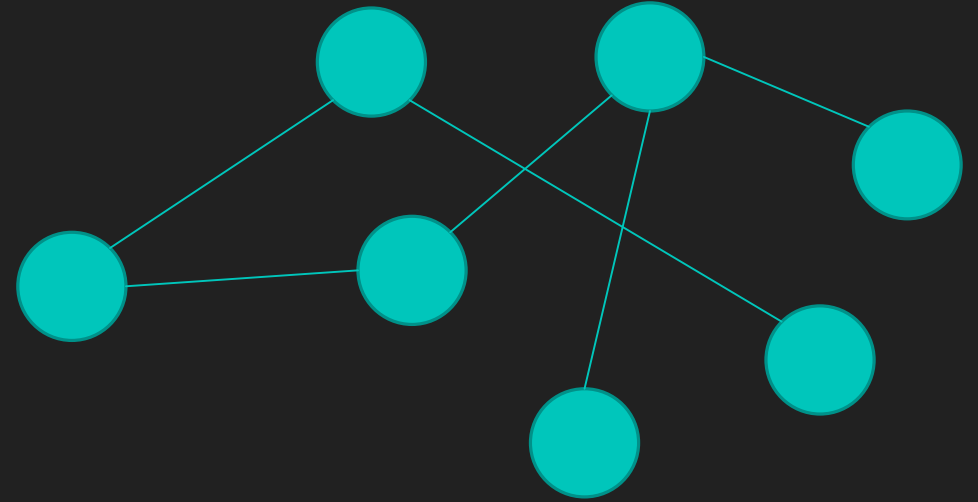
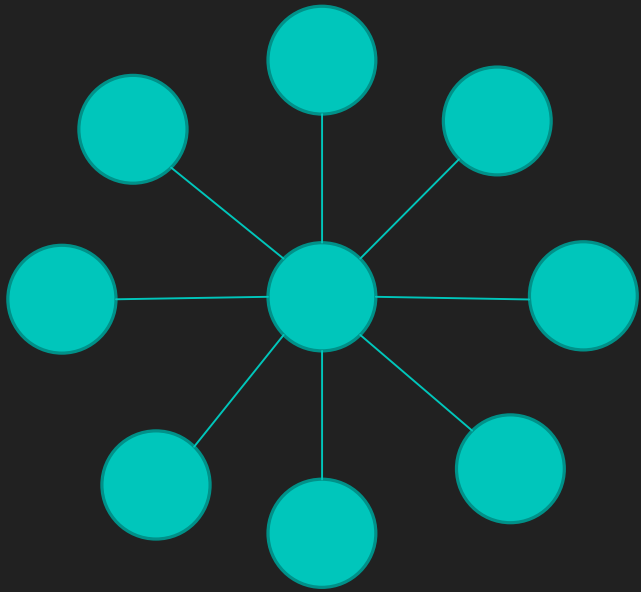
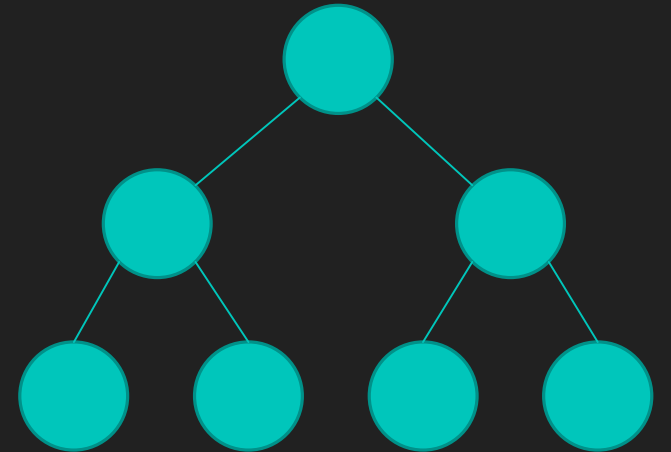
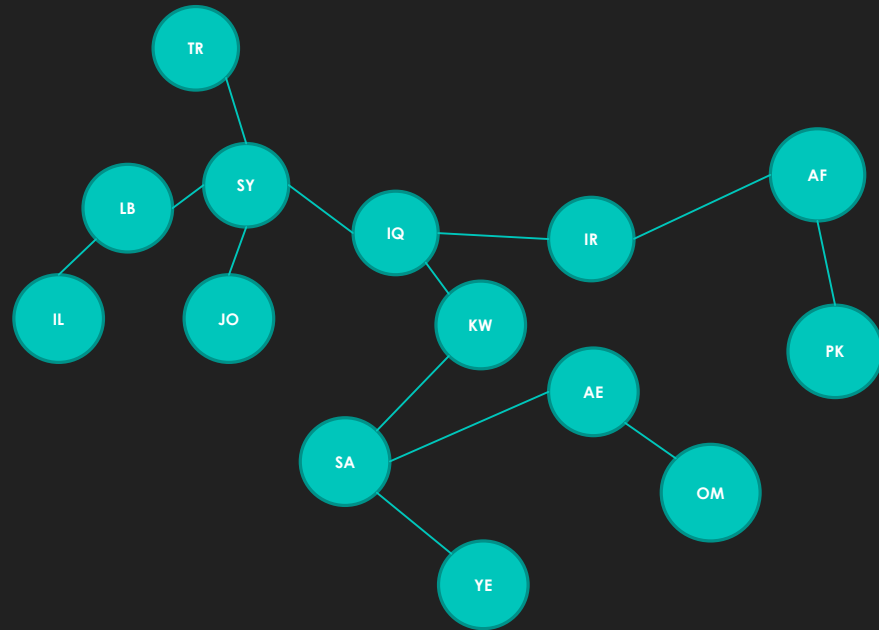


# Tree

Mohammad Ghoddosi



Tree

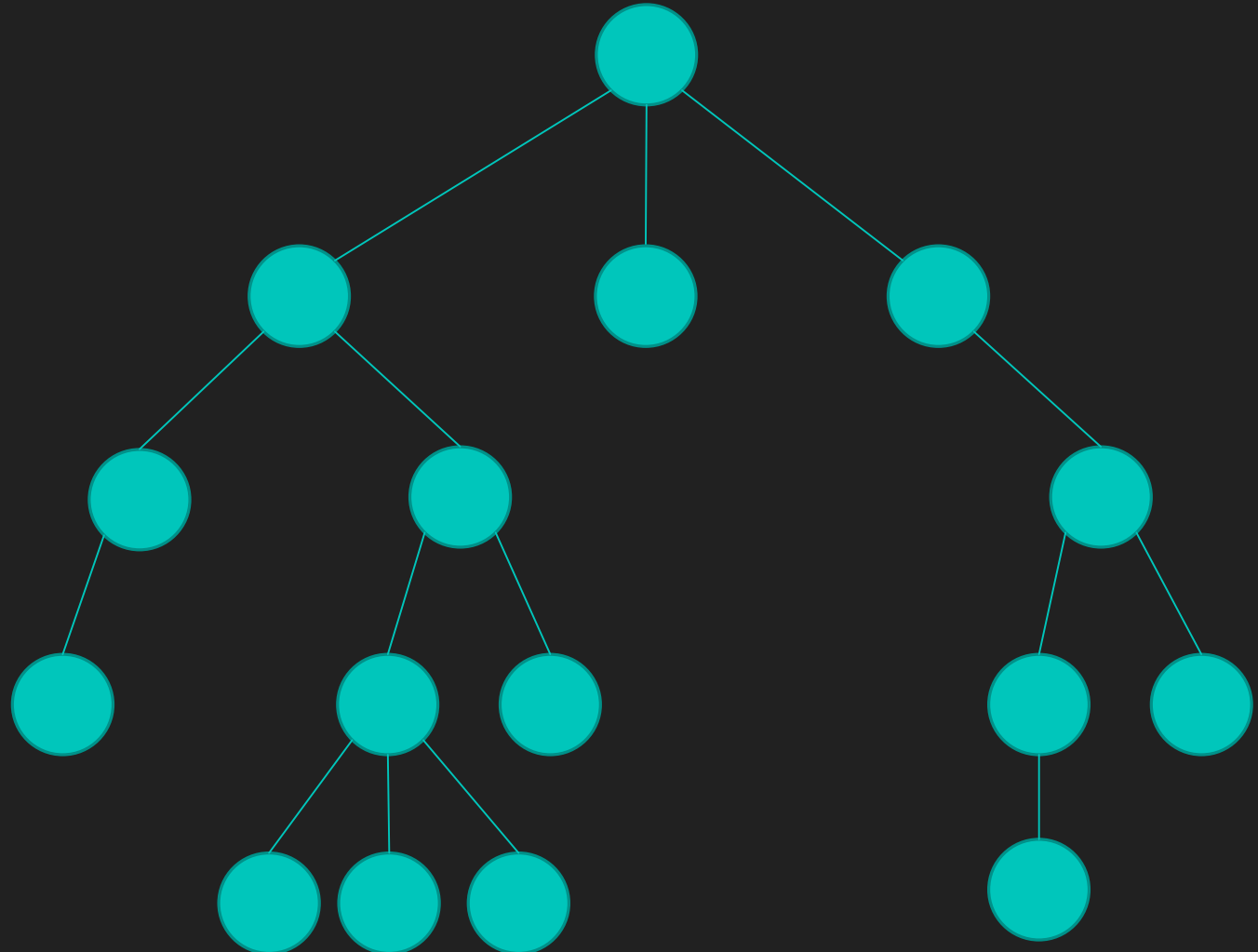


# Tree

- Undirected graph
- Any two vertices are connected with exactly one path
- No cycles
- Connected

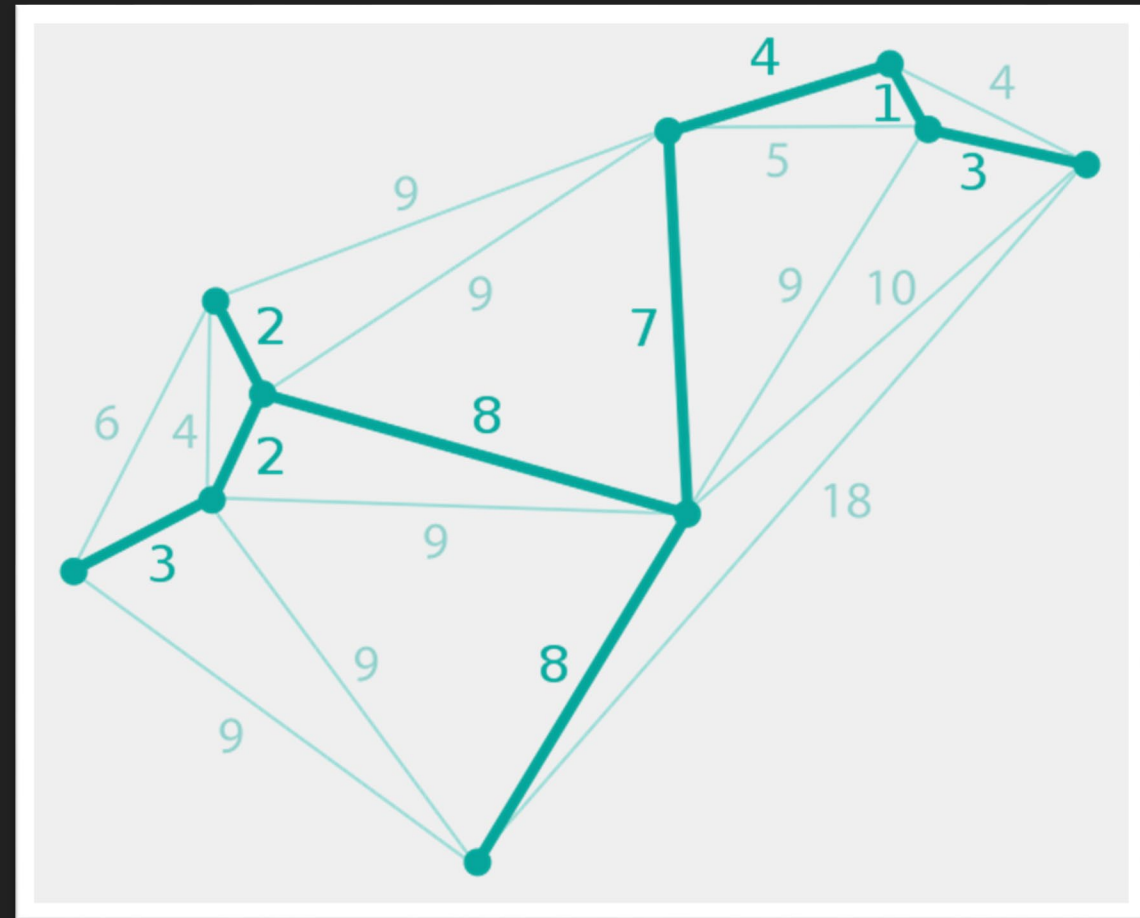
# Terminology

- Root
- Children
- Parent
- Sibling
- Uncle
- Leaf
- Height
- Level

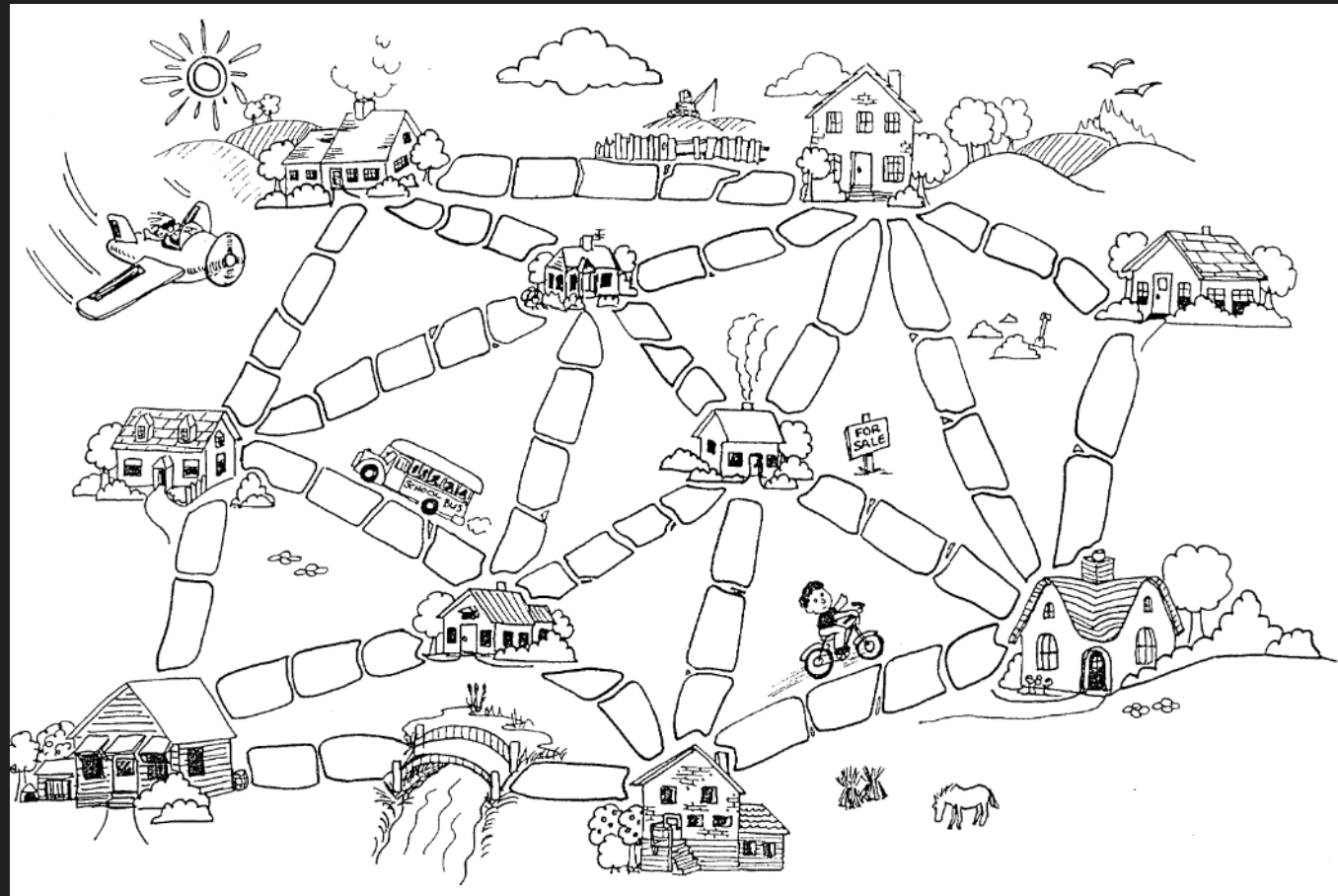


# Minimum Spanning Tree (MST)

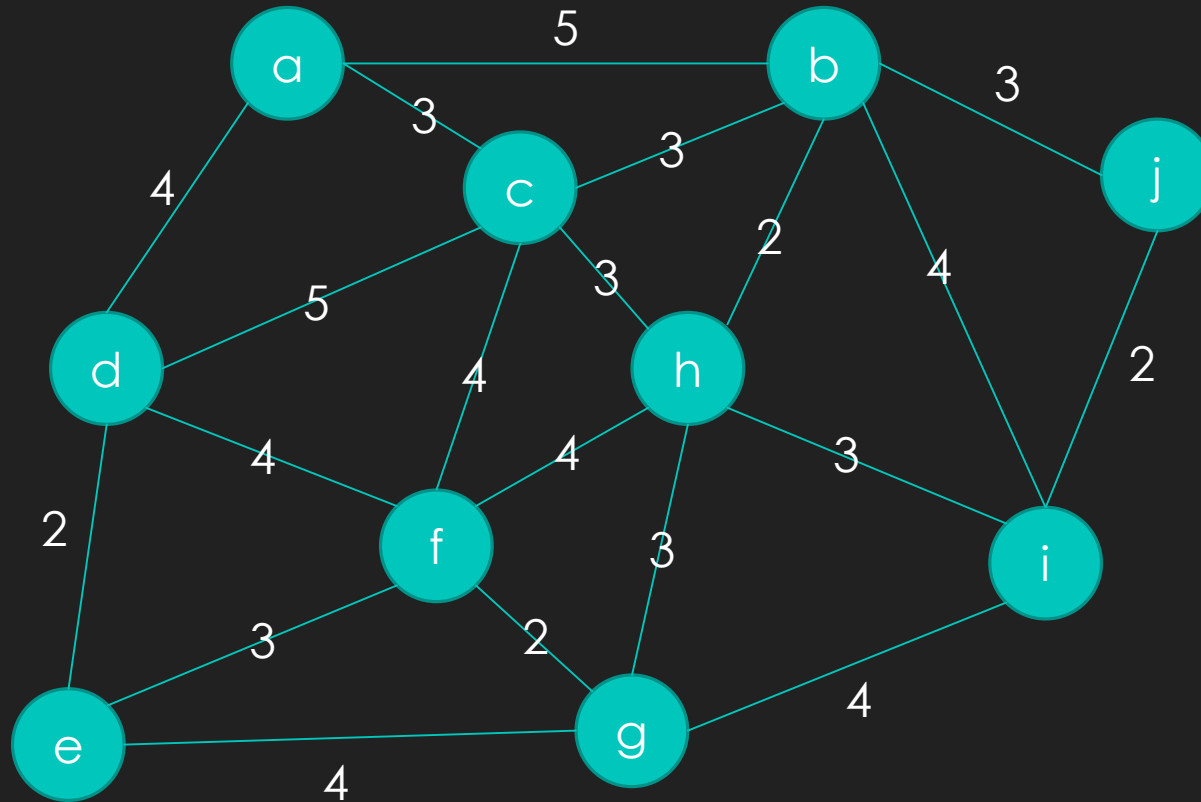
- For a weighted, connected, undirected graph
- Spanning tree
- Minimum total edge weight
- Network design
- Clustering
- Image segmentation
- Handwriting recognition
- ...



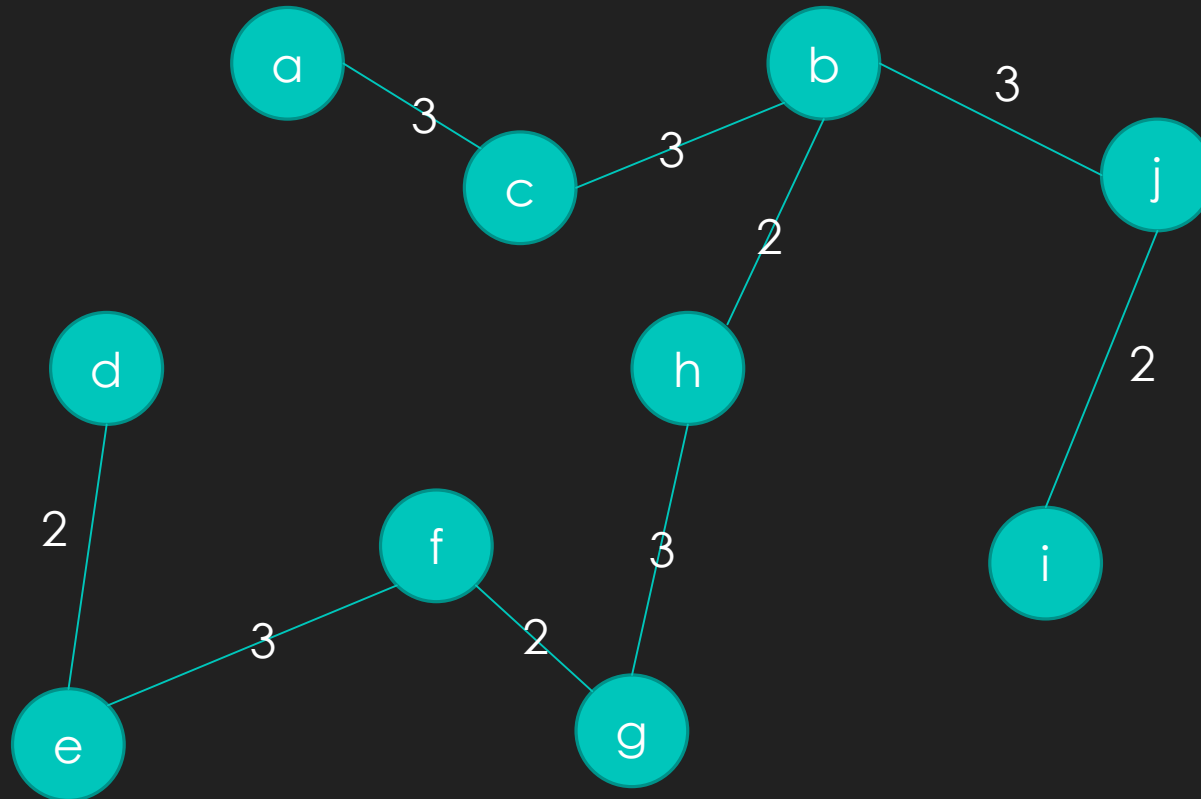
# Muddy city problem



# Muddy city problem

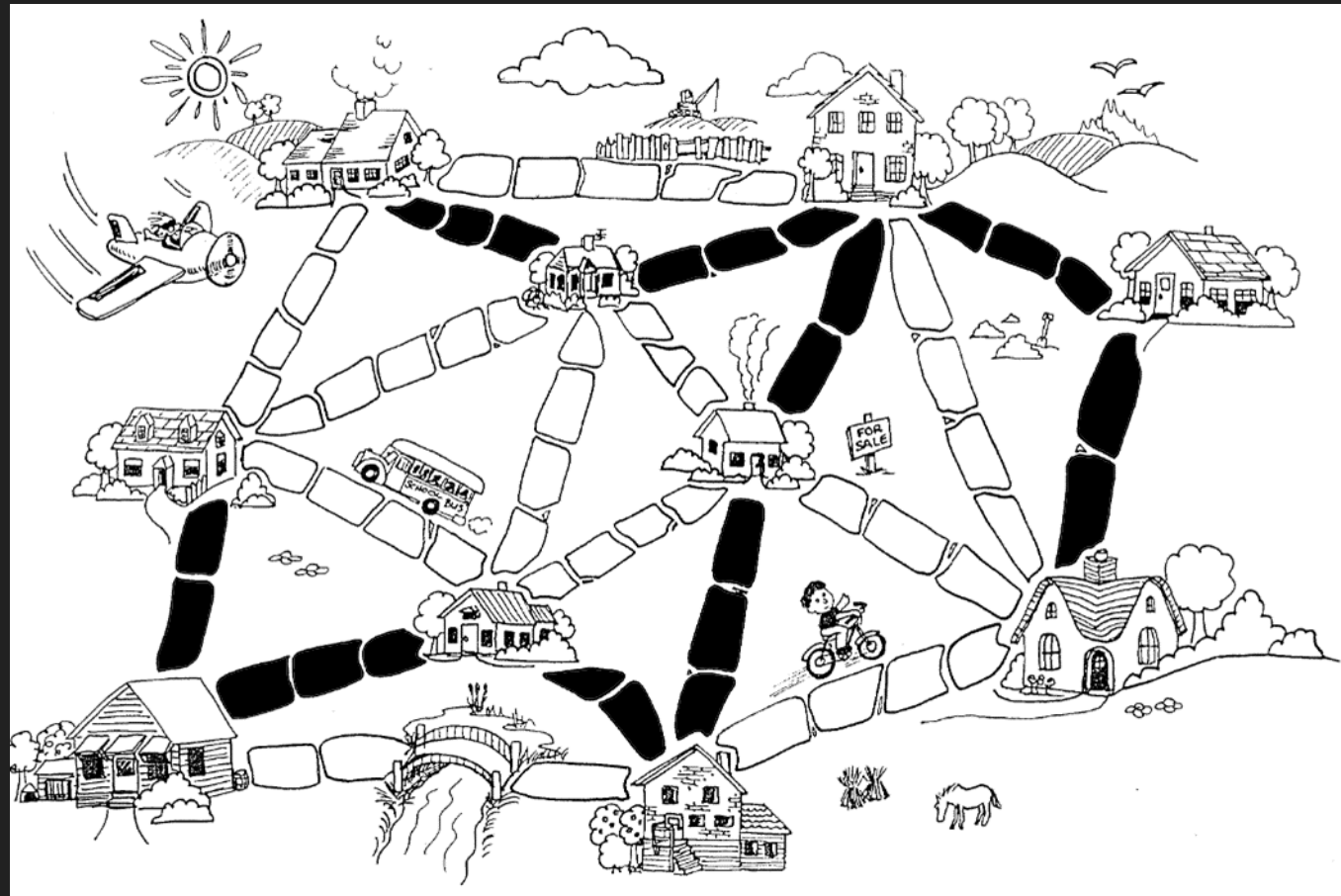


# Muddy city problem





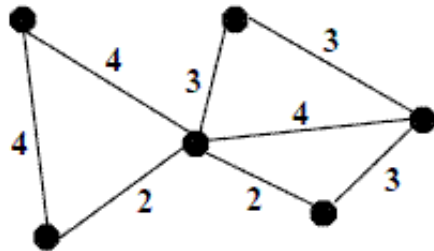
# Muddy city problem



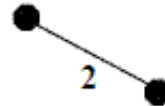
# Kruskal

## Kruskal's Algorithm

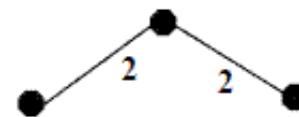
1 Given a network.....



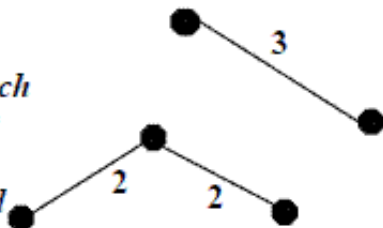
2 Choose the shortest edge (if there is more than one, choose any of the shortest).....



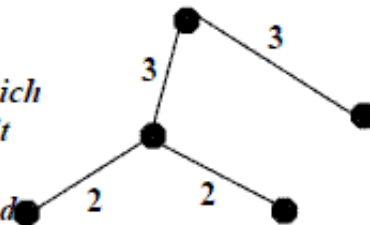
3 Choose the next shortest edge and add it.....



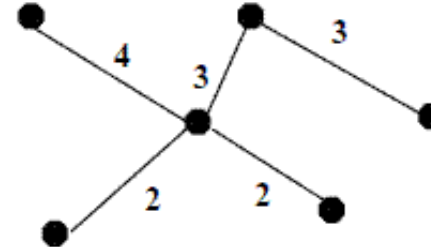
4 Choose the next shortest edge which wouldn't create a cycle and add it.



5 Choose the next shortest edge which wouldn't create a cycle and add it.



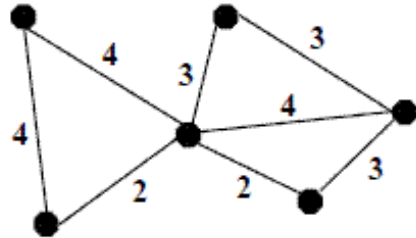
6 Repeat until you have a minimal spanning tree.



# Prime

## Prim's Algorithm

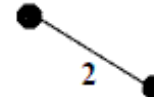
1 *Given a network.....*



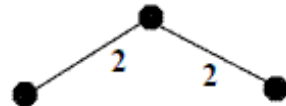
2 *Choose a vertex*



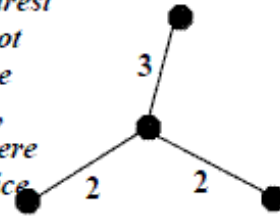
3 *Choose the shortest edge from this vertex.*



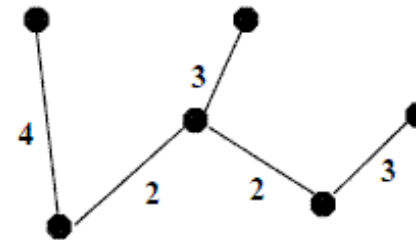
4 *Choose the nearest vertex not yet in the solution.*



5 *Choose the next nearest vertex not yet in the solution, when there is a choice choose either.*



6 *Repeat until you have a minimal spanning tree.*

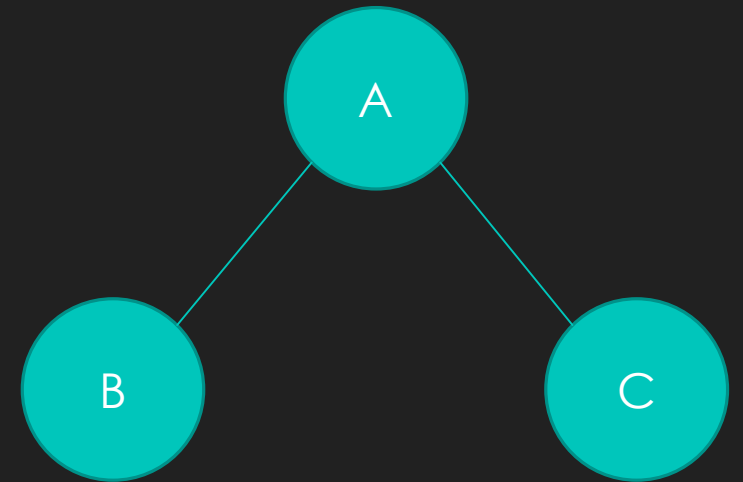


# Binary tree

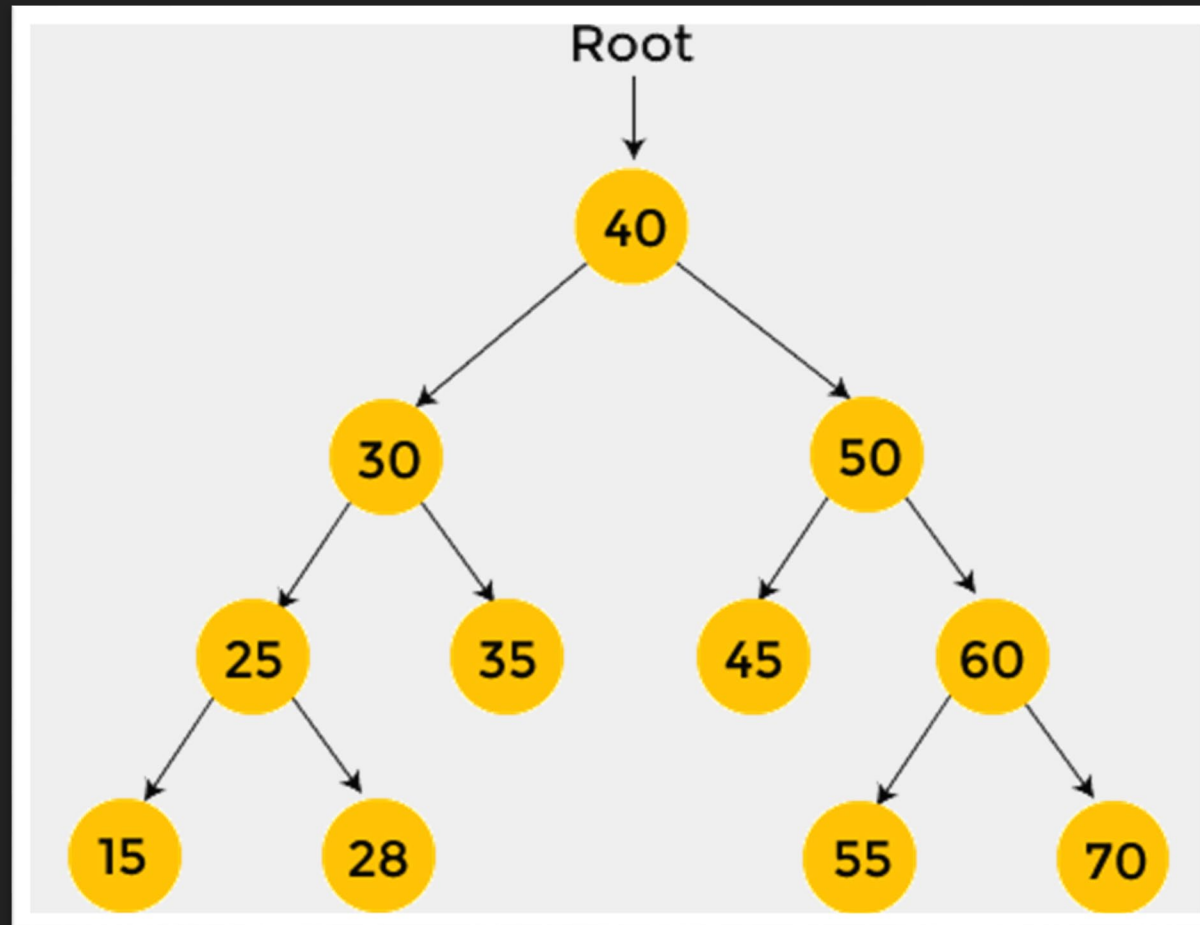
- Each nodes has max of 2 children
- Full binary tree
  - Every node has 0 or 2 children
- Used for:
  - Searching
  - Sorting
  - Compression algorithm
  - Decision tree
- Array implementation

# Traversal

- Pre-order
  - ABC
- Post-order
  - BCA
- In-order
  - BAC



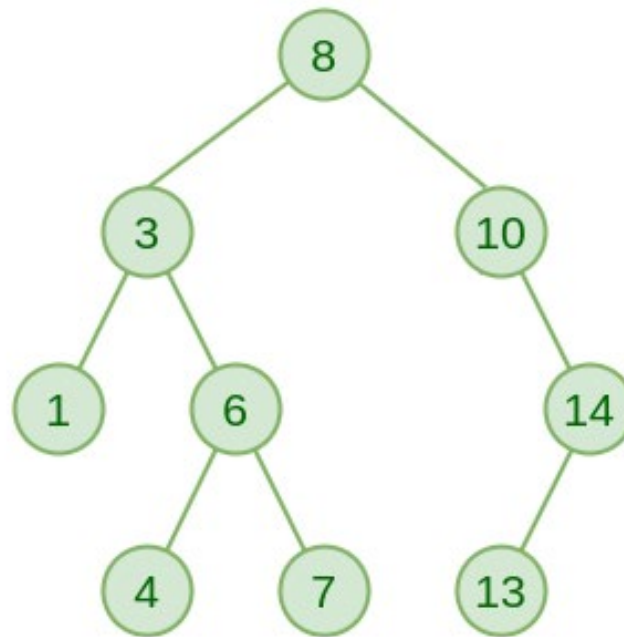
# Traversal



# Binary search tree

- Fast insertion and removal of elements
- Fast search
- Binary search
- Binary tree
  - Each node has 2 children
- Left subtree: only values less than the node
- Right subtree: only values greater than the node

# Binary search tree



Binary Search Tree



# Binary search tree

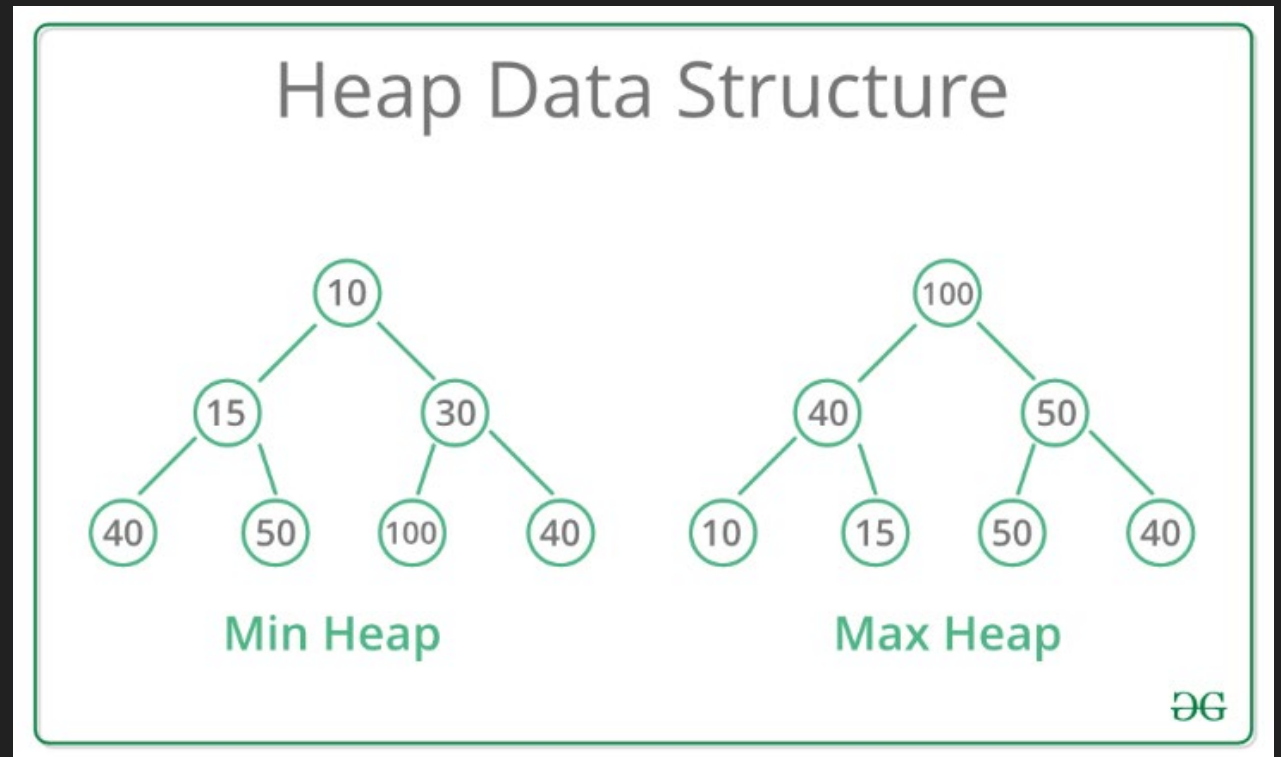
- Insertion
- Search
  - Binary search
- Deletion
  - In-order successor

# Self balancing Binary Search Tree

- Search, Insert, Delete, ... is  $O(h)$
- Sort in  $O(n)$  (without insertion cost)
- Height problem in BST
- Self balancing BSTs
  - AVL
  - Red and black trees
  - $H = O(\log n)$
- Simple to find all numbers greater than ... or smaller than ...

# Heap

- Complete binary tree
- Max heap
  - Root is always greater than its children
- Min heap
  - Root is always smaller than its children
- Operations:
  - Heapify
  - Insertion
  - Deletion
  - peak



# Heap

- Fast access to maximum/minimum in  $O(1)$
- Efficient insertion and deletion  $O(\log n)$
- Efficient implementation with arrays
- Priority queue
- Not good at searching for a value  $O(n)$