Date: Subject:

تمرین دوم سیگنال ها و سیستم ها

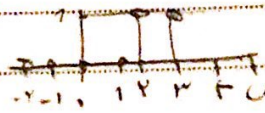
محمد چوپان ۹۸۳۱۱۲۵

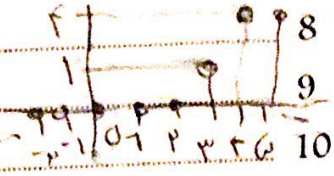
۱-

$x_1[n]$

$y_1[n]$

$x_1[n-2]$ ⇒ تغییرناپذیر ⇒ $y_1[n-2]$



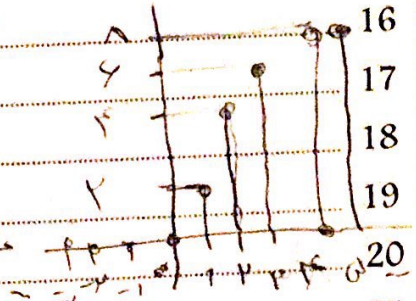


LTI

$x_2[n]$ ⇒

$x_1[n] + x_1[n-2] = x_2[n]$ ⇒ سیستم LTI پس خطی است.

$\Rightarrow y_2[n] = y_1[n] + y_1[n-2]$



۲-

$\underbrace{even(x(t))}_{K(t)} = \frac{x(t)+x(-t)}{2} = \frac{1}{2}(x(t)+x(-t))$

LTI سیستم ⇒ تغییرناپذیر ⇒ $x(t) \Rightarrow \boxed{\quad} \Rightarrow y(t) = u(t) - u(t-4)$

$x(-t) \Rightarrow \boxed{\quad} \Rightarrow y(-t) = u(-t) - u(-t-4)$

$K(t) = \frac{x(t)+x(-t)}{2}$ ⇒ خطی است ⇒ $y_K(t) = \frac{y(t)+y(-t)}{2}$

۱) از نمودار داده شده استفاده ای نکردیم

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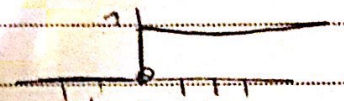
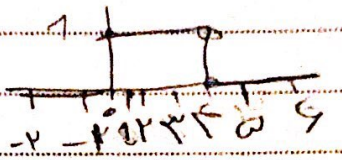
$$\frac{u(t)-u(t-4)+u(-t)+u(-t-4)}{2}$$

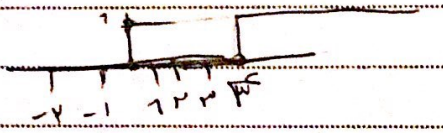
$u(t)$



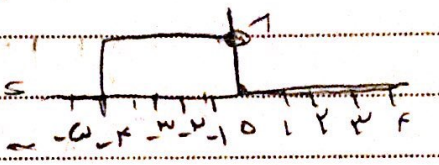
$y(t)$ =



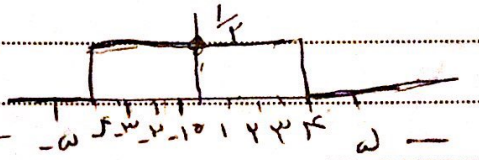
$u(t-4)$



$y(-t)$

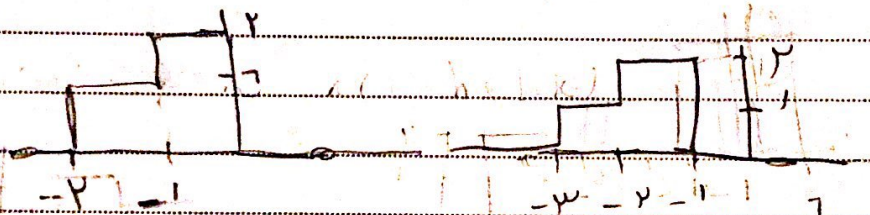


$$\frac{y(t)+y(-t)}{2} = y_e(t) \Rightarrow$$



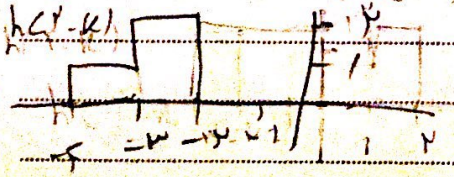
a) $y(t) = x(t) * h(t)$ (۳

$h(-k)$

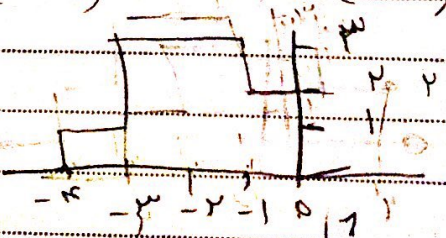


$y(t) = x(0)h(-k) + x(1)h(1-k) + x(2)h(2-k)$

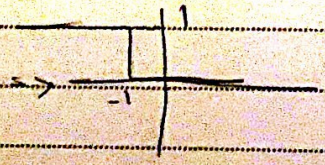
$h(2-k)$



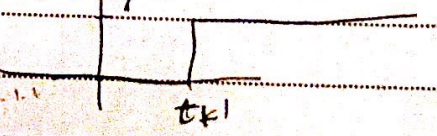
$y(t)$



b) $h(t) \Rightarrow$



$h(t-\tau) \Rightarrow$



$t+1 \geq 0 \Rightarrow y(t) = \int_{t+1}^{\infty} e^{-2\tau}\, d\tau = \frac{1}{2}e^{-2(t+1)}$

$t+1 < 0$

$$\int_{0}^{t+1} e^{-2\tau}\, d\tau = \frac{1}{2} \Rightarrow y(t)$$

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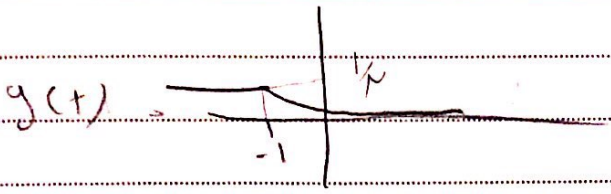
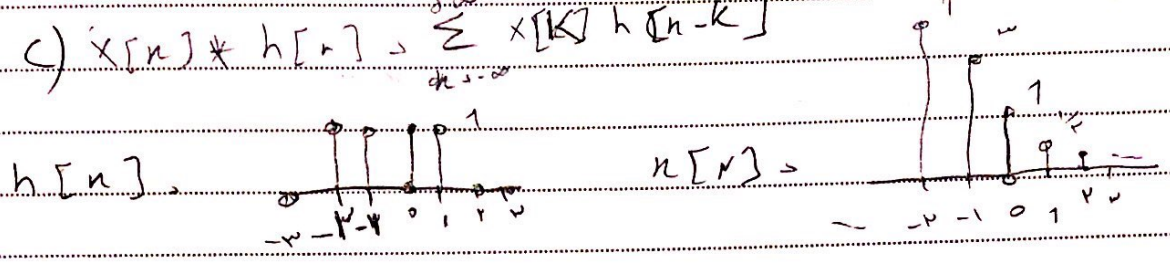


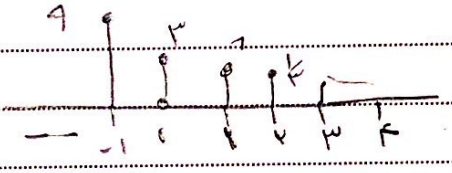
c) $x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] \Rightarrow \sum_{k=-\infty}^{+\infty} h[n-k]\, h[k]$

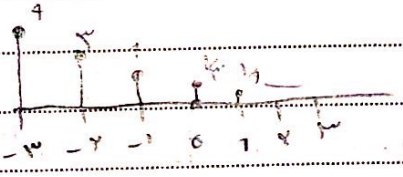


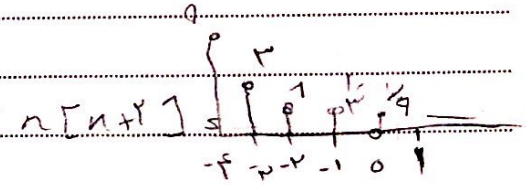
$\Rightarrow h[k] \begin{cases} 1 & -2 \le k \le 1 \\ 0 & o.w \end{cases} \Rightarrow y[n] = n[n+2] + n[n+1]$

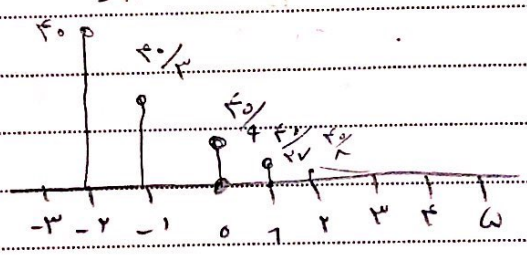
$+ n[k] + n[n-1] \Rightarrow \frac{1}{3^{n+2}} + \frac{1}{3^{n+1}} + \frac{1}{3^{n}} + \frac{1}{3^{n-1}}$

$= \frac{14/3 + 9 + 27}{3^n + 2} = \frac{40}{3^{n+2}}$









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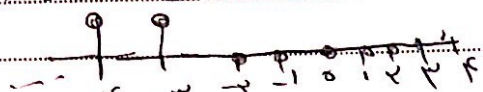
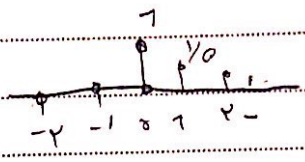
ط) $x[n]$:

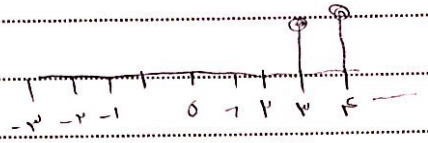


$h[n]$

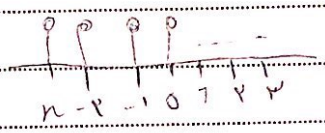


$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

$x[-k]$:

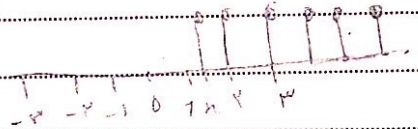


if $n \geq 3$ $x[n-K]$



$$y[N] = \sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{2}{1} = 2$$

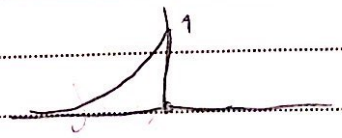
if $n < 3$ $x[n-K]$



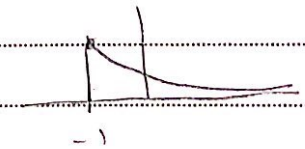
$$y[N] = \sum_{k=3-n}^{\infty} \left(\frac{1}{2}\right)^k = \frac{\left(\frac{1}{2}\right)^{3-n}}{1-\frac{1}{2}} = \frac{\left(\frac{1}{2}\right)^{3-n}}{\frac{1}{2}} = 2\left(\frac{1}{2}\right)^{3-n} = 2^{n-2}$$

ع) $y(t) = \int_{-\infty}^{+\infty} x(t-K)\, h(K)\, dK = \int_{-\infty}^{\infty} x(t)\, h(t-K)\, dK$

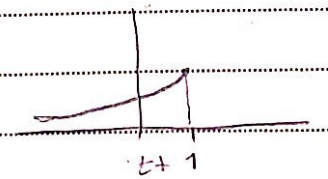
$h(t)$:



$x(t)$



$x(t-2)$



$t+1 > 0$

$$\int_{-\infty}^{0} e^{2t} \cdot e^{-t}\, dt = \int_{-\infty}^{0} e^{-t}\, dt = 1$$

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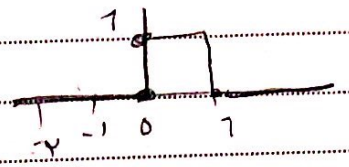
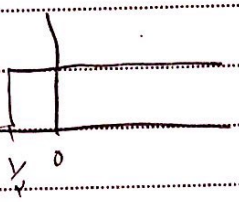
$t-1<0$ $\int_{-\infty}^{t-1} e^{t+2} e^{\tau} d\tau = e^{t-1}$

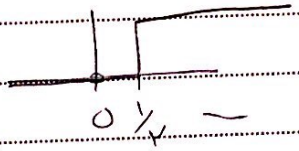
F) $h(t) = u(t) - u(t-1)$ $h(t)$:



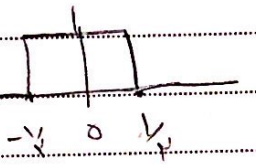
$u(t+\frac{1}{2})$



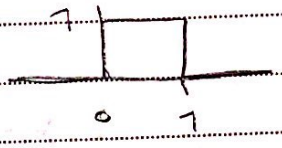
$u(t-\frac{1}{2})$



$\Pi(t)$

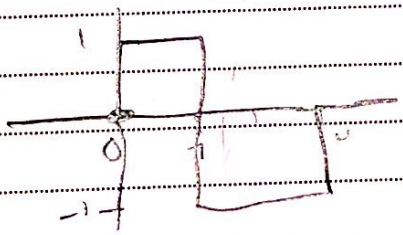


$\Pi(t-\frac{1}{2})$

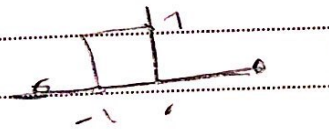


$\Pi(t-\frac{3}{2})$ =

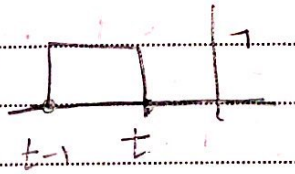
$x(t)$ =>



$h(-k)$ =



$h(t-k)$



$t<0$ صفر $\Leftrightarrow 0<t<1$ $\Rightarrow \int_{0}^{t} 2\,d\tau \le t$

$1<t<2$ $\int_{1}^{t} -1\,d\tau + \int_{t-1}^{1} 1\,d\tau = -2t+3$

$2<t<3$ $\int_{t-1}^{2} -1\,d\tau = t-3$

$t>3$ صفر $\Leftrightarrow 0$

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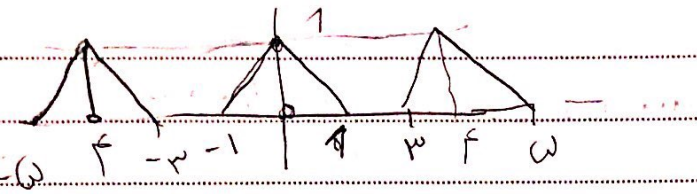
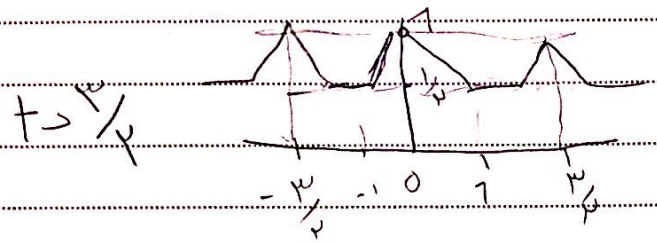
۲) برای $T=۴$

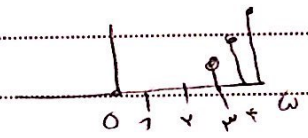




(۵) $h[n] = ۲^{-n} u[۳-n]$ $h[-۳] = ۲^{۳} = ۸$

$h[n]$

$h(-k)$



$h(n-k)$

برای $n<۵$, k ای

$h(n-k) \neq ۰$

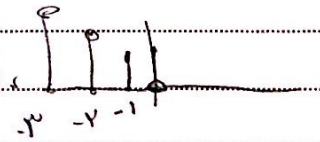
پس علی نیست.

$$\sum_{n=-\infty}^{+\infty} |h[n]| = ۸ < \infty$$

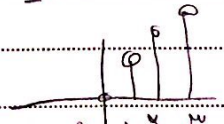
پس پایدار است.

ب) $h[n] = ۲^{n} u[-n]$

$h[n]$



$h(n-k)$



$h[n-k] \neq ۰ \quad k>n$ پس علی نیست

$h[n] \neq ۰ \quad n<۰$

$$\sum_{n=-\infty}^{+\infty} |h[n]| = \infty$$ ناپایدار است

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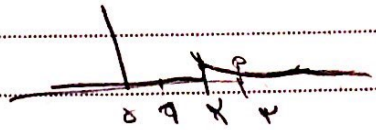
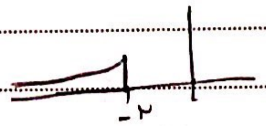
ج) $h(t) = e^{-2t} u(t-2)$

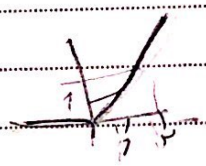
$h(t)$ 

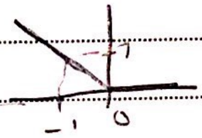
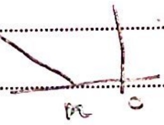
$h(-k)$  $h(n-k) = 0 \quad k > n$

$h(n) = 0 \quad n < 0$ علّی است.

$\sum_{-\infty}^{\infty} |h(k)| < \infty$ پس پایدار است

د) $y(t) = \int_{-\infty}^{\infty} (t-\tau)\, u(t-\tau)\, x(\tau)\, d\tau$

$\Rightarrow h(t) = t\, u(t) \Rightarrow r(t)$ $h(t)$ 

$h(-k)$  $h(n-k)$ 

$h(n-k) = 0 \quad k > n$ پس علّی است

$h(n) = 0 \quad n < 0$

$\sum_{-\infty}^{\infty} |h(k)| = \infty$ پس پایدار نیست

ه) $x[n] + z[n-1] \times \frac{1}{2} = z[n]$ (الف)

$\Rightarrow z[n] - z[n-1] \times \frac{1}{2} = x[n]$

ب) $Z(z)\left[1 - \frac{1}{2} z^{-1}\right] = X(z) \Rightarrow H(z) = \frac{1}{1 - \frac{1}{2} z^{-1}}$

ج) $y(z) = z[n-1] + \frac{1}{2} z[n] \Rightarrow Y(z) = z^{-1} + \frac{1}{2}$

$H(z) = (z^{-1} + \frac{1}{2})(1 - \frac{1}{2} z^{-1})$

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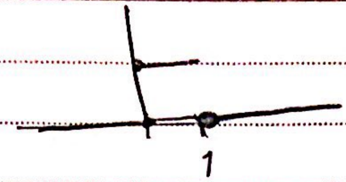
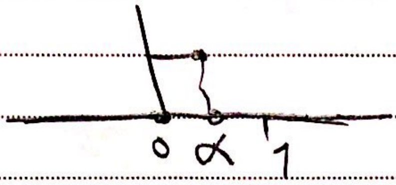
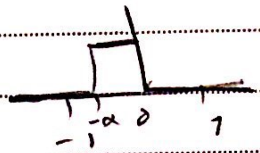
$x(t) =$



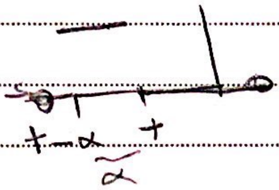
$h(t)$



$h(-t)$



$h(t-\tau)$



$t<0 \quad y(t)=0$, $0<t-\alpha<1$, $t>1$: $\int_{t-\alpha}^{1} d\tau = 1-t+\alpha$

$0<t-\alpha<1$, $t<1$: $\int_{t-\alpha}^{t} d\tau = \alpha$

$t-\alpha>1$, $t>1$: 0 , $t-\alpha<0$, $0<t<1$: $\int_{0}^{t} d\tau = t$

ب)

$\frac{d(y(t))}{dt}$: $t<0$: 0

$t-\alpha<0$, $0<t<1$: 1

$0<t-\alpha<1$, $t<1$: 0 (I)

$0<t-\alpha<1$, $t>1$: -1 ۴ نقطه دارد.

$t-\alpha>1$: 0

اگر $\alpha=1$ باشد.

بازه (I) را نداریم پس ۲ دامنه هست و ۳ بازه داریم

و ۲ نقطه ناپیوستگی داریم