Assignment 2

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Problem One:

a) max(x,y)

This function returns the larger between x and y, so it is not a measure of distance.

Axiom 2: $d(x,y) = 0$ if and only if $x = y$ does hold true when $x = y = 0$, however, when $x, y > 0$, the statement is no longer true.

Example

$x = 5, y = 5 \quad \max(x,y) = 5 \neq 0$

b) diff(x,y)

This returns the absolute value between x and y, so we can say this is a distance measere.

Axiom 1: $d(x,y) \geq 0$

↳ The absolute value is returned meaning it will always be greater than or eaual to 0

Axiom 2: $d(x,y) = 0$ if and only if $x = y$

↳ This holds true because when $x = y$, there is a distance of 0

Axiom 3: $d(x,y) = d(y,x)$

↳ This holds true because the absolute value is returned by both functions

Ex. $x = 5, y = 2$

$\text{diff}(5,2) = \text{diff}(2,5)$

$|3| = |3|$

Axiom 4: $d(x,y) \leq d(x,z) + d(z,y)$

c) sum(x,y)

This function returns the sum of x and y. This is not a distance measure.

Axiom 1: $d(x,y) \geq 0$ (for + integers)

↳ This holds true

Axiom 2: $d(x,y) = 0$ if and only if $x = y$

↳ This does not hold true.

Ex. $x = 1 = y \rightarrow sum(x,y) = 2$

Therefore, this is not a distance measure.

Problem Two:

Point A → (5, 6, 7)

Point B → (8, 2, 4)

$L_1: |5-8| + |6-2| + |7-4| = 3+4+3 = 10$

$L_2: \sqrt{(5-8)^2 + (6-2)^2 + (7-4)^2} = \sqrt{9+16+9} = \sqrt{34} = 5.83095$

$L_\infty: \max(|5-8|, |6-2|, |7-4|) = \max(3,4,3) = 4$

Problem Three:

Problem Four:

a) abcdef ⟶ bdaefc

1) Delete c → abdef
2) Insert c after f → abdefc
3) Delete a → bdefc
4) Insert a after d → bdaefc

∴ Distance = 4

b) abccdabc ⟶ acbdcab

1) Delete c after b → abcdabc
2) Insert c after a → acbcdabc
3) Delete c after b → acbdabc
4) Insert c after d → acbdcabc
5) Delete c after b → acbdcab

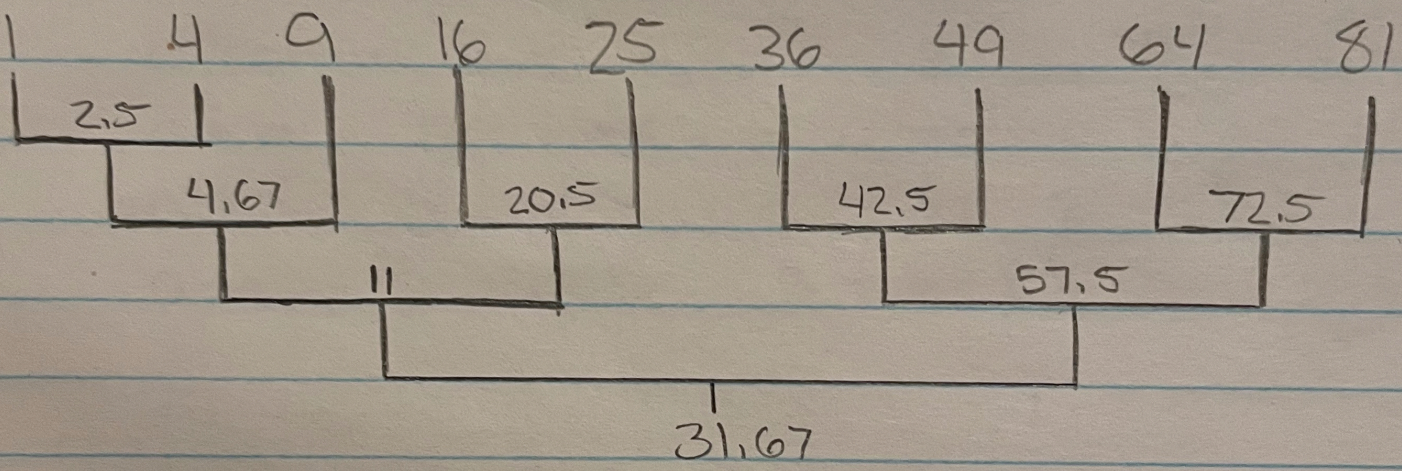
∴ Distance = 5

c) abcdef → baedfc

1) Delete a before b → bcdef
2) Insert a after b → bacdef
3) Delete e after e → bacdf
4) Insert e after a → baecdf
5) Delete c after e → baedf
6) Insert c after f → baedfc

∴ Distance = 6

Problem Five:

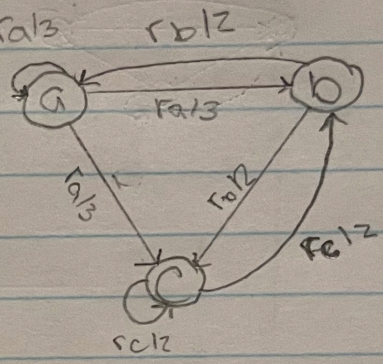


Problem Seven:

a) a connects to a, b, c → 1/3
b connects to a, c → 1/2

$a \to a = 1/3 \quad b \to a = 1/2 \quad c \to a = 0$
$a \to b = 1/3 \quad b \to b = 0 \quad c \to b = 1/2$
$a \to c = 1/3 \quad b \to c = 1/2 \quad c \to c = 1/2$

Web Graph:



$$r_1 = \begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 5/18 \\ 5/18 \\ 4/9 \end{bmatrix} = \begin{bmatrix} 2,77 \\ 2,77 \\ 2,77 \end{bmatrix}$$

$$r_2 = \begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 5/18 \\ 5/18 \\ 4/9 \end{bmatrix} = \begin{bmatrix} 25/108 \\ 34/108 \\ 49/108 \end{bmatrix} = \begin{bmatrix} 0,23 \\ 0,31 \\ 0,45 \end{bmatrix}$$

$$r_3 = \begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 25/108 \\ 34/108 \\ 49/108 \end{bmatrix} = \begin{bmatrix} 152/648 \\ 197/648 \\ 299/648 \end{bmatrix} = \begin{bmatrix} 0,23 \\ 0,30 \\ 0,46 \end{bmatrix}$$

$$r_4 = \begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 152/648 \\ 197/648 \\ 299/648 \end{bmatrix} = \begin{bmatrix} 895/3888 \\ 1201/3888 \\ 1792/3888 \end{bmatrix} = \begin{bmatrix} 0,23 \\ 0,30 \\ 0,46 \end{bmatrix} \quad \equiv$$

No change has occured between r_3 and r_4, so we may stop. The resulting page ranks are:

a = 0,23
b = 0,30
c = 0,46

} ≈ 1

b) $\beta = 0.8$

$$A = 0.8\,M + (1-\beta)\left[\tfrac{1}{N}\right]_{N\times N} = 0.8 \begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix} + 0.2 \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 4/15 & 2/5 & 0 \\ 4/15 & 0 & 2/10 \\ 4/15 & 2/5 & 2/10 \end{bmatrix} + \begin{bmatrix} 1/15 & 1/15 & 1/15 \\ 1/15 & 1/15 & 1/15 \\ 1/15 & 1/15 & 1/15 \end{bmatrix} = \begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix}$$

$$r^0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \qquad r^1 = \begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix} \cdot \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 13/45 \\ 13/45 \\ 19/45 \end{bmatrix}$$

$$r^2 = \begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix} \cdot \begin{bmatrix} 13/45 \\ 13/45 \\ 19/15 \end{bmatrix} = \begin{bmatrix} 175/675 \\ 211/675 \\ 289/675 \end{bmatrix}$$

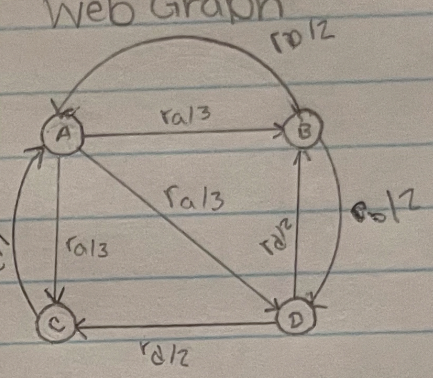
$$r^3 = \begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix} \cdot \begin{bmatrix} 175/675 \\ 211/675 \\ 289/675 \end{bmatrix} = \begin{bmatrix} 2641/10125 \\ 3109/10125 \\ 4375/10125 \end{bmatrix}$$

$$r^4 = \begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix} \cdot \begin{bmatrix} 2641/10125 \\ 3109/10125 \\ 4375/10125 \end{bmatrix} = \begin{bmatrix} 39343/151875 \\ 46939/151875 \\ 65593/151875 \end{bmatrix} = \begin{matrix} 0.26 \\ 0.31 \\ 0.43 \end{matrix} \Big\} \approx 1$$

Problem Eight:

a) $A = 0.8M + 0.2[1/N]_{N \times N}$

Web Graph



$$= 0.8\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} + 0.2\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2/5 & 4/15 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2/5 & 4/15 & 0 \\ 7/15 & 1/5 & 1/5 & 3/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix}$$

$$r^0 = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} \quad r^1 = \begin{bmatrix} 0 & 2/5 & 4/15 & 0 \\ 7/15 & 1/5 & 1/5 & 3/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 18/60 \\ 22/60 \\ 10/60 \\ 10/60 \end{bmatrix}$$

$$r^2 = \begin{bmatrix} 0 & 2/5 & 4/15 & 0 \\ 7/15 & 1/5 & 1/5 & 3/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 18/60 \\ 22/60 \\ 10/60 \\ 10/60 \end{bmatrix} = \begin{bmatrix} 42/150 \\ 52/150 \\ 34/150 \\ 34/150 \end{bmatrix}$$

$$r^3 = \begin{bmatrix} 0 & 2/5 & 4/15 & 0 \\ 7/15 & 1/5 & 1/5 & 3/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 42/150 \\ 52/150 \\ 22/150 \\ 34/150 \end{bmatrix} = \begin{bmatrix} 96/375 \\ 137/375 \\ 67/375 \\ 80/375 \end{bmatrix}$$

$$r^4 = \begin{bmatrix} 0 & 2/5 & 4/15 & 0 \\ 7/15 & 1/5 & 1/5 & 3/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 96/375 \\ 137/375 \\ 67/375 \\ 80/375 \end{bmatrix} = \begin{bmatrix} 542/1875 \\ 668/1875 \\ 268/1875 \\ 402/1875 \end{bmatrix} = \begin{bmatrix} r\,A \\ r\,B \\ r\,C \\ r\,D \end{bmatrix}$$

Problem Ten:

1 100110100 110110 1011011 011 011 0 0 1
8 4 2 2 1 1

2 10011010011011010110110110110 11 00 1
2 1

3 100110100110110101101101, 1011001

1 seems to be the only one that work because of the constraint "buckets are sorted by size" on slide 16 of lecture 10.

Problem Eleven:

initial state

... 101, 1011001, 0 11101, 1001, 0 11 0 1
8 4 4 2 2 1

add 1 - No change

... 101, 1011001, 0 11101, 1001, 0 11 0 1 1
8 4 4 2 2 1 1

add 1 - merge 4 groups, merge 2 groups, merge 1 group

... 101, 1011001011101, 1001011, 0 11 1
8 8 4 2 1

add last 1, no change

... 101, 1011001011101, 1001011 0 11 1 1
8 8 4 2 1 1