

Assignment 4

Part 1:

1. A) $A \Leftrightarrow B \models \neg A \vee B$

$$\begin{aligned}
 &A \Leftrightarrow B \\
 &(A \rightarrow B) \wedge (B \rightarrow A) && \text{From biconditionals 1} \\
 &(A \rightarrow B) && \text{From simplification} \\
 &\neg A \vee B && \text{From conditionals 1} \\
 &\text{Therefore } A \Leftrightarrow B \models \neg A \vee B
 \end{aligned}$$

B) $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$

$$\begin{aligned}
 &(A \wedge B) \rightarrow C \\
 &\neg(A \wedge B) \vee C && \text{From conditionals 1} \\
 &\neg A \vee \neg B \vee C && \text{From De Morgan's} \\
 &\neg A \vee \neg B \vee C \vee C && \text{From identity } C \vee C \equiv C \\
 &\neg A \vee C \vee \neg B \vee C && \text{From commutativity} \\
 &(A \Rightarrow C) \vee (B \Rightarrow C) && \text{From conditionals 1} \\
 &\text{Therefore, } (A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)
 \end{aligned}$$

2. A) $A \Leftrightarrow (B \vee E)$

$$\begin{aligned}
 &A \Leftrightarrow (B \vee E) \\
 &(A \rightarrow (B \vee E)) \wedge ((B \vee E) \rightarrow A) && \text{From biconditionals 1} \\
 &(\neg A \vee B \vee E) \wedge ((B \vee E) \rightarrow A) && \text{From conditionals 1} \\
 &(\neg A \vee B \vee E) \wedge (\neg(B \vee E) \vee A) && \text{From conditionals 1} \\
 &(\neg A \vee B \vee E) \wedge ((\neg B \wedge \neg E) \vee A) && \text{From demorgan's} \\
 &(\neg A \vee B \vee E) \wedge (A \vee \neg B) \wedge (A \vee \neg E) && \text{From Distribution.}
 \end{aligned}$$

B) $C \wedge F \Rightarrow \neg B$

$$\begin{aligned}
 &C \wedge F \Rightarrow \neg B \\
 &\neg(C \wedge F) \vee \neg B && \text{From conditionals 1} \\
 &\neg C \vee \neg F \vee \neg B && \text{From De Morgan's}
 \end{aligned}$$

C) $\neg((\neg A \Rightarrow \neg B) \wedge \neg C)$

$$\begin{aligned}
 &C \wedge F \Rightarrow \neg B \\
 &\neg(C \wedge F) \vee \neg B && \text{From conditionals 1} \\
 &\neg C \vee \neg F \vee \neg B && \text{From De Morgan's}
 \end{aligned}$$

D) $(A \Rightarrow B) \Rightarrow (\neg C \wedge B)$

$$\neg ((\neg A \Rightarrow \neg B) \wedge \neg C)$$

$$\neg ((A \vee \neg B) \wedge \neg C)$$

From conditionals 1

$$\neg (A \vee \neg B) \vee C$$

From De Morgan's

$$(\neg A \wedge B) \vee C$$

From De Morgan's

$$(\neg A \vee C) \wedge (B \vee C)$$

From Distribution

3.

A) $\exists x (S(x) \rightarrow A_1(x))$

$\hookrightarrow S(x) = x$ is a student

$A_1(x) = x$ is a1

B) $\exists x (S(x) \wedge T(x))$

$\hookrightarrow S(x) = x$ is a student

$T(x) = x$ is smart

C) $\forall x, y (B(x, y) \rightarrow S(x, y))$

$\hookrightarrow B(x, y) = x, y$ are brothers

$S(x, y) = x, y$ are siblings

D) $\forall x, y (M(x, y) \leftrightarrow P(x, y))$

$\hookrightarrow M(x, y) = x \rightarrow$ is y's mother

$P(x, y) = x \rightarrow$ is y's female parent

E) $\forall x (P(x) \rightarrow M(x))$

$\hookrightarrow P(x) = x$ is a person

$M(x) = x$ only has one mother

F) $\forall x (C(x) \leftrightarrow P(x))$

$\hookrightarrow C(x) = x$ is a cousin

$P(x) = x$ is a child of a parent's sibling

G) $\forall x (F(x) \rightarrow D(x))$

$\hookrightarrow F(x) = x$ is a farmer

$D(x) = x$ is owns a donkey

H) $\exists x (B(x) \rightarrow C(x) \wedge \neg \exists x (B(x) \rightarrow S(x)))$

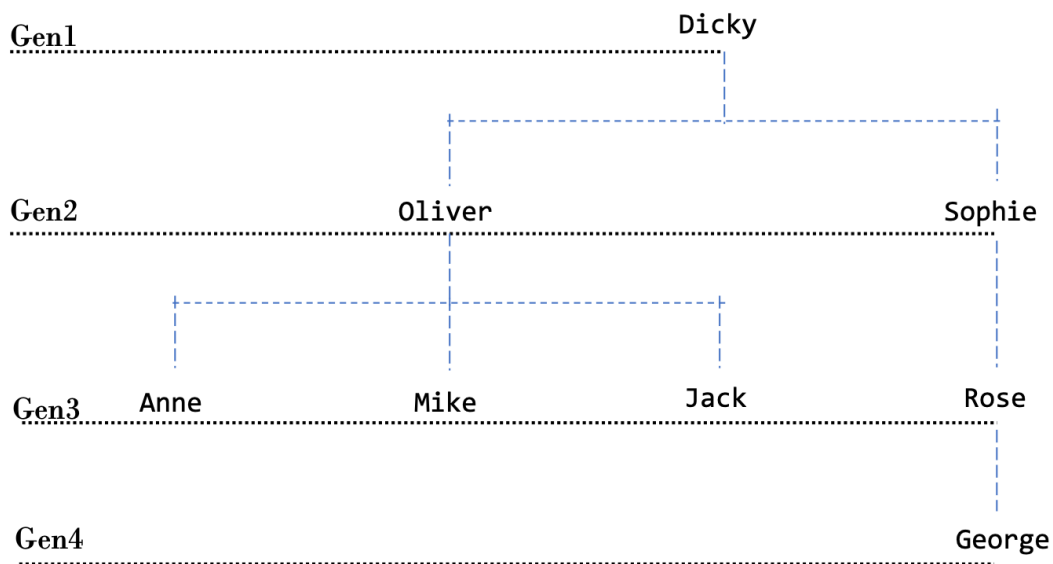
$\hookrightarrow B(x) = x$ is a bird

$C(x) = x$ is a crow

$S(x) = x$ is a squirrel

4. Natural English Translation
 - a. Everyone is male or female
 - b. All people who speak the same language understand each other
 - c. Joan is a mother of one child
 - d. Joan and Kevin are parents of the same child
 - e. There exists a planet in which all humans live on
 - f. All students take a course
 - g. No two individuals do not have the same SSN
 - h. All people's SSN's contain 9 digits
5. General Unifier
 - a. $\{x/A, y/B, z/B\}$
 - b. Cannot unify A with B
 - c. $\{y/\text{John}, x/\text{John}\}$
 - d. Cannot unify y with father()

Part 2:



- 1) Was George the parent of Oliver?

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[?- parent(X,george).
X = rose.
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- a)
2) Who was Oliver's parent?

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[?- parent(X,oliver).
X = dicky.
```

- a)
3) Who were the children of Oliver?

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[?- parent(oliver,X).  
X = anne ;  
X = mike ;  
X = jack.
```

- a)
4) Who were the brothers of Anne?

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[?- brother(X,anne).  
X = mike ;  
X = jack ;
```

- a)
5) Who were the cousins of Rose?

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[?- cousin(X,rose).  
X = anne ;  
X = mike ;  
X = jack ;
```

a)

Custom Queries:

- 1) Who is Rose's uncle?

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[?- uncle(X,rose).  
X = oliver ;
```

a)

- 2) Who is Oliver's daughter(s)?

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[?- daughter(X,oliver).  
X = anne ;
```

a)

- 3) Who is Rose's son(s)?

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[?- son(X,rose).  
X = george.
```

a)

- 4) Who is George's Grandmother?

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[?- grandmother(X,george).  
X = sophie.
```

a)

- 5) Who are Rose's ancestors?

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[?- ancestor(X,rose).  
X = sophie ;  
X = dicky ;
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a)