

# An Abstract Framework for Deadlock Prevention in BIP\*

Paul C Attie<sup>1</sup>, Saddek Bensalem<sup>2</sup>, Marius Bozga<sup>3</sup>, Mohamad Jaber<sup>4</sup>, Joseph Sifakis<sup>5</sup>,  
and Fadi A Zaraket<sup>6</sup>

<sup>1</sup>Department of Computer Science, American University of Beirut, Beirut, Lebanon

<sup>2</sup>UJF-Grenoble 1 / CNRS VERIMAG UMR 5104, Grenoble, F-38041, France

<sup>3</sup>UJF-Grenoble 1 / CNRS VERIMAG UMR 5104, Grenoble, F-38041, France

<sup>4</sup>Department of Computer Science, American University of Beirut, Beirut, Lebanon

<sup>5</sup>Rigorous System Design Laboratory, EPFL, Lausanne, Switzerland

<sup>6</sup>Department of Electrical and Computer Engineering, American University of Beirut,  
Beirut, Lebanon

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## Abstract

We present a sound but incomplete criterion for checking deadlock freedom of finite state systems expressed in BIP: a component-based framework for the construction of complex distributed systems. Since deciding deadlock-freedom for finite-state concurrent systems is PSPACE-complete, our criterion gives up completeness in return for tractability of evaluation. Our criterion can be evaluated by model-checking subsystems of the overall large system. The size of these subsystems depends only on the local topology of direct interaction between components, and *not* on the number of components in the overall system.

We present two experiments, in which our method compares favorably with existing approaches. For example, in verifying deadlock freedom of dining philosophers, our method shows linear increase in computation time with the number of philosophers, whereas other methods (even those that use abstraction) show super-linear increase, due to state-explosion.

## 1 Introduction

Deadlock freedom is a crucial property of concurrent and distributed systems. With increasing system complexity, the challenge of assuring deadlock freedom and other correctness properties becomes even greater. In contrast to the alternatives of (1) deadlock detection and recovery, and (2) deadlock avoidance, we advocate deadlock prevention: design the system so that deadlocks do not occur.

Deciding deadlock freedom of finite-state concurrent programs is PSPACE-complete in general [15, chapter 19]. To achieve tractability, we can either make our deadlock freedom check

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incomplete (sufficient but not necessary), or we can restrict the systems that we check to special cases. We choose the first option: a system meeting our condition is free of both local and global deadlocks, while a system which fails to meet our condition may or may not be deadlock free.

We generalize previous works [2, 3, 4] by removing the requirement that interaction between processes be expressed pairwise, and also by applying to BIP [6], a framework from which efficient distributed code can be generated. In contrast, the model of concurrency in [2, 3, 4] requires shared memory read-modify-write operations with a large grain of atomicity. The full paper, including proofs for all theorems, is available on-line, as is our implementation of the method.

## 2 BIP - Behavior Interaction Priority

BIP is a component framework for constructing systems by superposing three layers of modeling: Behavior, Interaction, and Priority. A technical treatment of priority is beyond the scope of this paper. Adding priorities never introduces a deadlock, since priority enforces a choice between possible transitions from a state, and deadlock-freedom means that there is at least one transition from every (reachable) state. Hence if a BIP system without priorities is deadlock-free, then the same system with priorities added will also be deadlock-free.

**Definition 1 (Atomic Component)** *An atomic component  $B_i$  is a labeled transition system represented by a triple  $(Q_i, P_i, \rightarrow_i)$  where  $Q_i$  is a set of states,  $P_i$  is a set of communication ports, and  $\rightarrow_i \subseteq Q_i \times P_i \times Q_i$  is a set of possible transitions, each labeled by some port.*

For states  $s_i, t_i \in Q_i$  and port  $p_i \in P_i$ , write  $s_i \xrightarrow{p_i} t_i$ , iff  $(s_i, p_i, t_i) \in \rightarrow_i$ . When  $p_i$  is irrelevant, write  $s_i \rightarrow_i t_i$ . Similarly,  $s_i \xrightarrow{p_i}$  means that there exists  $t_i \in Q_i$  such that  $s_i \xrightarrow{p_i} t_i$ . In this case,  $p_i$  is *enabled* in state  $s_i$ . Ports are used for communication between different components, as discussed below.

In practice, we describe the transition system using some syntax, e.g., involving variables. We abstract away from issues of syntactic description since we are only interested in enablement of ports and actions. We assume that enablement of a port depends only on the local state of a component. In particular, it cannot depend on the state of other components. This is a restriction on BIP, and we defer to subsequent work how to lift this restriction. So, we assume the existence of a predicate  $enb_{p_i}^i$  that holds in state  $s_i$  of component  $B_i$  iff port  $p_i$  is enabled in  $s_i$ , i.e.,  $s_i(enb_{p_i}^i) = true$  iff  $s_i \xrightarrow{p_i}$ .

Figure 1(a) shows atomic components for a philosopher  $P$  and a fork  $F$  in dining philosophers. A philosopher  $P$  that is hungry (in state  $h$ ) can eat by executing *get* and moving to state  $e$  (eating). From  $e$ ,  $P$  releases its forks by executing *release* and moving back to  $h$ . Adding the thinking state does not change the deadlock behaviour of the system, since the thinking to hungry transition is internal to  $P$ , and so we omit it. A fork  $F$  is taken by either: (1) the left philosopher (transition  $get_l$ ) and so moves to state  $u_l$  (used by left philosopher), or (2) the right philosopher (transition  $get_r$ ) and so moves to state  $u_r$  (used by right philosopher). From state  $u_r$  (resp.  $u_l$ ),  $F$  is released by the right philosopher (resp. left philosopher) and so moves back to state  $f$  (free).

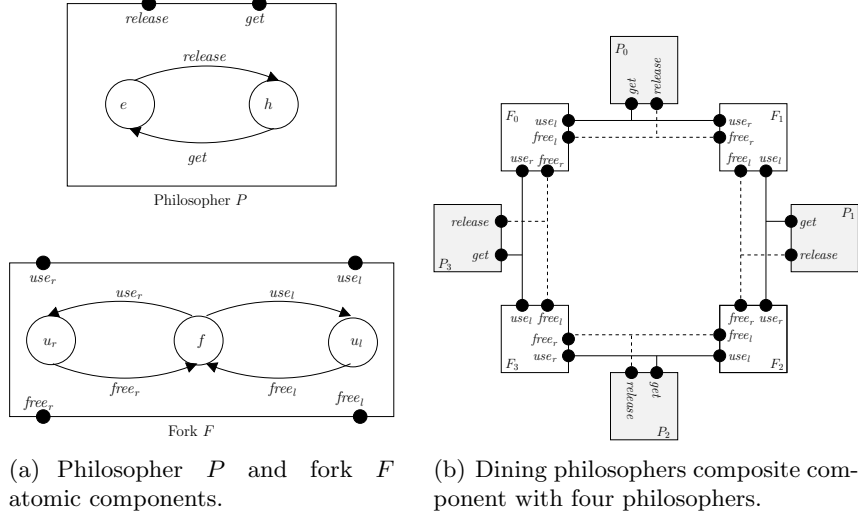


Figure 1: Dining philosophers.

**Definition 2 (Interaction)** For a given system built from a set of  $n$  atomic components  $\{B_i = (Q_i, P_i, \rightarrow_i)\}_{i=1}^n$ , we require that their respective sets of ports are pairwise disjoint, i.e., for all  $i, j$  such that  $i, j \in \{1..n\} \wedge i \neq j$ , we have  $P_i \cap P_j = \emptyset$ . An interaction is a set of ports not containing two or more ports from the same component. That is, for an interaction  $a$  we have  $a \subseteq P \wedge (\forall i \in \{1..n\} : |a \cap P_i| \leq 1)$ , where  $P = \bigcup_{i=1}^n P_i$  is the set of all ports in the system. When we write  $a = \{p_i\}_{i \in I}$ , we assume that  $p_i \in P_i$  for all  $i \in I$ , where  $I \subseteq \{1..n\}$ .

Execution of an interaction  $a$  involves all the components which have ports in  $a$ .

**Definition 3 (Composite Component)** A composite component (or simply component)  $B \triangleq \gamma(B_1, \dots, B_n)$  is defined by a composition operator parameterized by a set of interactions  $\gamma \subseteq 2^P$ .  $B$  has a transition system  $(Q, \gamma, \rightarrow)$ , where  $Q = Q_1 \times \dots \times Q_n$  and  $\rightarrow \subseteq Q \times \gamma \times Q$  is the least set of transitions satisfying the rule

$$\frac{a = \{p_i\}_{i \in I} \in \gamma \quad \forall i \in I : s_i \xrightarrow{p_i} t_i \quad \forall i \notin I : s_i = t_i}{\langle s_1, \dots, s_n \rangle \xrightarrow{a} \langle t_1, \dots, t_n \rangle}$$

This inference rule says that a composite component  $B = \gamma(B_1, \dots, B_n)$  can execute an interaction  $a \in \gamma$ , iff for each port  $p_i \in a$ , the corresponding atomic component  $B_i$  can execute a transition labeled with  $p_i$ ; the states of components that do not participate in the interaction stay unchanged. Given an interaction  $a = \{p_i\}_{i \in I}$ , we denote by  $C_a$  the set of atomic components participating in  $a$ , formally:  $C_a = \{B_i \mid p_i \in a\}$ . Figure 1(b) shows a composite component consisting of four philosophers and the four forks between them. Each philosopher and its two neighboring forks share two interactions:  $Get = \{get, use_l, use_r\}$  in which the philosopher obtains the forks, and  $Rel = \{release, free_l, free_r\}$  in which the philosopher releases the forks.

**Definition 4 (Interaction enablement)** An atomic component  $B_i = (Q_i, P_i, \rightarrow_i)$  enables interaction  $a$  in state  $s_i$  iff  $s_i \xrightarrow{p_i}_i$ , where  $p_i = P_i \cap a$  is the port of  $B_i$  involved in  $a$ . Let  $B = \gamma(B_1, \dots, B_n)$  be a composite component, and let  $s = \langle s_1, \dots, s_n \rangle$  be a state of  $B$ . Then  $B$  enables  $a$  in  $s$  iff every  $B_i \in C_a$  enables  $a$  in  $s_i$ .

The definition of interaction enablement is a consequence of Definition 3. Interaction  $a$  being enabled in state  $s$  means that executing  $a$  is one of the possible transitions that can be taken from  $s$ . Let  $enb_a^i$  denote the enablement condition for interaction  $a$  in component  $B_i$ . By definition,  $enb_a^i = enb_{p_i}^i$  where  $p_i = a \cap P_i$ .

**Definition 5 (BIP System)** Let  $B = \gamma(B_1, \dots, B_n)$  be a composite component with transition system  $(Q, \gamma, \rightarrow)$ , and let  $Q_0 \subseteq Q$  be a set of initial states. Then  $(B, Q_0)$  is a BIP system.

Figure 1(b) gives a BIP-system with philosophers initially in state  $h$  (hungry) and forks initially in state  $f$  (free).

**Definition 6 (Execution)** Let  $(B, Q_0)$  be a BIP system with transition system  $(Q, \gamma, \rightarrow)$ . Let  $\rho = s_0 a_1 s_1 \dots s_{i-1} a_i s_i \dots$  be an alternating sequence of states of  $B$  and interactions of  $B$ . Then  $\rho$  is an execution of  $(B, Q_0)$  iff (1)  $s_0 \in Q_0$ , and (2)  $\forall i > 0 : s_{i-1} \xrightarrow{a_i}$   $s_i$ .

A state or transition that occurs in some execution is called *reachable*.

**Definition 7 (State Projection)** Let  $(B, Q_0)$  be a BIP system where  $B = \gamma(B_1, \dots, B_n)$  and let  $s = \langle s_1, \dots, s_n \rangle$  be a state of  $(B, Q_0)$ . Let  $\{B_{j_1}, \dots, B_{j_k}\} \subseteq \{B_1, \dots, B_n\}$ . Then  $s \upharpoonright \{B_{j_1}, \dots, B_{j_k}\} \triangleq \langle s_{j_1}, \dots, s_{j_k} \rangle$ . For a single  $B_i$ , we write  $s \upharpoonright B_i = s_i$ . We extend state projection to sets of states element-wise.

**Definition 8 (Subcomponent)** Let  $B \triangleq \gamma(B_1, \dots, B_n)$  be a composite component, and let  $\{B_{j_1}, \dots, B_{j_k}\}$  be a subset of  $\{B_1, \dots, B_n\}$ . Let  $P' = P_{j_1} \cup \dots \cup P_{j_k}$ , i.e., the union of the ports of  $\{B_{j_1}, \dots, B_{j_k}\}$ . Then the subcomponent  $B'$  of  $B$  based on  $\{B_{j_1}, \dots, B_{j_k}\}$  is as follows:

1.  $\gamma' \triangleq \{a \cap P' \mid a \in \gamma \wedge a \cap P' \neq \emptyset\}$
2.  $B' \triangleq \gamma'(B_{j_1}, \dots, B_{j_k})$

That is,  $\gamma'$  consists of those interactions in  $\gamma$  that have at least one participant in  $\{B_{j_1}, \dots, B_{j_k}\}$ , and restricted to the participants in  $\{B_{j_1}, \dots, B_{j_k}\}$ , i.e., participants not in  $\{B_{j_1}, \dots, B_{j_k}\}$  are removed.

We write  $s \upharpoonright B'$  to indicate state projection onto  $B'$ , and define  $s \upharpoonright B' \triangleq s \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$ , where  $B_{j_1}, \dots, B_{j_k}$  are the atomic components in  $B'$ .

**Definition 9 (Subsystem)** Let  $(B, Q_0)$  be a BIP system where  $B = \gamma(B_1, \dots, B_n)$ , and let  $\{B_{j_1}, \dots, B_{j_k}\}$  be a subset of  $\{B_1, \dots, B_n\}$ . Then the subsystem  $(B', Q'_0)$  of  $(B, Q_0)$  based on  $\{B_{j_1}, \dots, B_{j_k}\}$  is as follows:

1.  $B'$  is the subcomponent of  $B$  based on  $\{B_{j_1}, \dots, B_{j_k}\}$

$$2. Q'_0 = Q_0 \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$$

**Definition 10 (Execution Projection)** Let  $(B, Q_0)$  be a BIP system where  $B = \gamma(B_1, \dots, B_n)$ , and let  $(B', Q'_0)$ , with  $B' = \gamma'(B_{j_1}, \dots, B_{j_k})$  be the subsystem of  $(B, Q_0)$  based on  $\{B_{j_1}, \dots, B_{j_k}\}$ . Let  $\rho = s_0 a_1 s_1 \dots s_{i-1} a_i s_i \dots$  be an execution of  $(B, Q_0)$ . Then,  $\rho \upharpoonright (B', Q'_0)$ , the projection of  $\rho$  onto  $(B', Q'_0)$ , is the sequence resulting from:

1. replacing each  $s_i$  by  $s_i \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$ , i.e., replacing each state by its projection onto  $\{B_{j_1}, \dots, B_{j_k}\}$
2. removing all  $a_i s_i$  where  $a_i \notin \gamma'$

**Proposition 1 (Execution Projection)** Let  $(B, Q_0)$  be a BIP system where  $B = \gamma(B_1, \dots, B_n)$ , and let  $(B', Q'_0)$ , with  $B' = \gamma'(B_{j_1}, \dots, B_{j_k})$  be the subsystem of  $(B, Q_0)$  based on  $\{B_{j_1}, \dots, B_{j_k}\}$ . Let  $\rho = s_0 a_1 s_1 \dots s_{i-1} a_i s_i \dots$  be an execution of  $(B, Q_0)$ . Then,  $\rho \upharpoonright (B', Q'_0)$  is an execution of  $(B', Q'_0)$ .

*Proof.* Let  $\rho = s_0 a_1 s_1 \dots s_{i-1} a_i s_i \dots$ , be an execution of  $(B, Q_0)$ , and let  $s'_0 = s_0 \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$ .

Then, by Definition 10,  $s'_0 \in Q'_0$  and  $\rho \upharpoonright (B', Q'_0) = s'_0 b_1 s'_1 b_2 s'_2 \dots$  for some  $b_1 s'_1 b_2 s'_2 \dots$ , where  $s'_j \in Q' = Q \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$  for  $j \geq 1$ .

Consider an arbitrary step  $(s'_{j-1}, b_j, s'_j)$  of  $\rho \upharpoonright (B', Q'_0)$ . Since  $b_j s'_j$  was not removed in Clause 2 of Definition 10, we have

- (1)  $s'_j = s_k \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$  for some  $k > 0$  and such that  $a_k \in \gamma'$
- (2)  $b_j = a_k$ , and
- (3)  $s'_{j-1} = s_\ell \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$  for the smallest  $\ell$  such that  $\ell < k$  and  $\forall m : \ell + 1 \leq m < k : a_m \notin \gamma'$

From (3) and Definitions 3 and 10  $s_\ell \upharpoonright \{B_{j_1}, \dots, B_{j_k}\} = s_{k-1} \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$ . Hence  $s'_{j-1} = s_{k-1} \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$ . From  $s_{k-1} \xrightarrow{a_k} s_k$ ,  $a_k \in \gamma'$ , and Definition 3, we have  $s_{k-1} \upharpoonright \{B_{j_1}, \dots, B_{j_k}\} \xrightarrow{a_k} s_k \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$ . Hence  $s'_{j-1} \xrightarrow{b_j} s'_j$  from  $s'_{j-1} = s_{k-1} \upharpoonright \{B_{j_1}, \dots, B_{j_k}\}$  established above and (1), (2).

Since  $(s'_{j-1}, b_j, s'_j)$  was arbitrarily chosen, we conclude that every step of  $\rho \upharpoonright (B', Q'_0)$  is a step of  $(B', Q'_0)$ . Since the first state of  $\rho \upharpoonright (B', Q'_0)$  is  $s_0$ , and  $s_0 \in Q'_0$ , we have established that  $\rho \upharpoonright (B', Q'_0)$  is an execution of  $(B', Q'_0)$ .  $\square$

**Corollary 2** Let  $(B', Q'_0)$  be a subsystem of  $(B, Q_0)$ . Let  $s$  be a reachable state of  $(B, Q_0)$ . Then  $s \upharpoonright B'$  is a reachable state of  $(B', Q'_0)$ . Let  $s \xrightarrow{a} t$  be a reachable transition of  $(B, Q_0)$ , and let  $a$  be an interaction of  $(B', Q'_0)$ . Then  $s \upharpoonright B' \xrightarrow{a} t \upharpoonright B'$  is a reachable transition of  $(B', Q'_0)$ .

*Proof.* Immediate corollary of Proposition 1.  $\square$

To avoid tedious repetition, we fix, for the rest of the paper, an arbitrary BIP-system  $(B, Q_0)$ , with  $B \triangleq \gamma(B_1, \dots, B_n)$ , and transition system  $(Q, \gamma, \rightarrow)$ .

### 3 Characterizing Deadlock-freedom

**Definition 11 (Deadlock-freedom)** *A BIP-system  $(B, Q_0)$  is deadlock-free iff in every reachable state  $s$  of  $(B, Q_0)$ , some interaction  $a$  is enabled.*

We assume in the sequel that each individual component  $B_i$  is deadlock-free, when considered in isolation, with respect to the set of initial states  $Q_0 \upharpoonright B_i$ .

#### 3.1 Wait-for graphs

The wait-for-graph for a state  $s$  is a directed bipartite and-or graph which contains as nodes the atomic components  $B_1, \dots, B_n$ , and all the interactions  $\gamma$ . Edges in the wait-for-graph are from a  $B_i$  to all the interactions that  $B_i$  enables (in  $s$ ), and from an interaction  $a$  to all the components that participate in  $a$  and which do not enable it (in  $s$ ).

**Definition 12 (Wait-for-graph  $W_B(s)$ )** *Let  $B = \gamma(B_1, \dots, B_n)$  be a BIP composite component, and let  $s = \langle s_1, \dots, s_n \rangle$  be an arbitrary state of  $B$ . The wait-for-graph  $W_B(s)$  of  $s$  is a directed bipartite and-or graph, where*

1. *the nodes of  $W_B(s)$  are as follows:*
  - (a) *the and-nodes are the atomic components  $B_i$ ,  $i \in \{1..n\}$ ,*
  - (b) *the or-nodes are the interactions  $a \in \gamma$ ,*
2. *there is an edge in  $W_B(s)$  from  $B_i$  to every node  $a$  such that  $B_i \in C_a$  and  $s_i(\text{enb}_a^i) = \text{true}$ , i.e., from  $B_i$  to every interaction which  $B_i$  enables in  $s_i$ ,*
3. *there is an edge in  $W_B(s)$  from  $a$  to every  $B_i$  such that  $B_i \in C_a$  and  $s_i(\text{enb}_a^i) = \text{false}$ , i.e., from  $a$  to every component  $B_i$  which participates in  $a$  but does not enable it, in state  $s_i$ .*

A component  $B_i$  is an and-node since all of its successor actions (or-nodes) must be disabled for  $B_i$  to be incapable of executing. An interaction  $a$  is an or-node since it is disabled if any of its participant components do not enable it. An edge (path) in a wait-for-graph is called a wait-for-edge (wait-for-path). Write  $a \rightarrow B_i$  ( $B_i \rightarrow a$  respectively) for a wait-for-edge from  $a$  to  $B_i$  ( $B_i$  to  $a$  respectively). We abuse notation by writing  $e \in W_B(s)$  to indicate that  $e$  (either  $a \rightarrow B_i$  or  $B_i \rightarrow a$ ) is an edge in  $W_B(s)$ . Also  $B \rightarrow a \rightarrow B' \in W_B(s)$  for  $B \rightarrow a \in W_B(s) \wedge a \rightarrow B' \in W_B(s)$ , i.e., for a wait-for-path of length 2, and similarly for longer wait-for-paths.

Consider the dining philosophers system given in Figure 1. Figure 2(a) shows its wait-for-graph in its sole initial state. Figure 2(b) shows the wait-for-graph after execution of  $\text{get}_0$ . Edges from components to interactions are shown solid, and edges from interactions to components are shown dashed.

#### 3.2 Supercycles and deadlock-freedom

We characterize a deadlock as the existence in the wait-for-graph of a graph-theoretic construct that we call a *supercycle*:

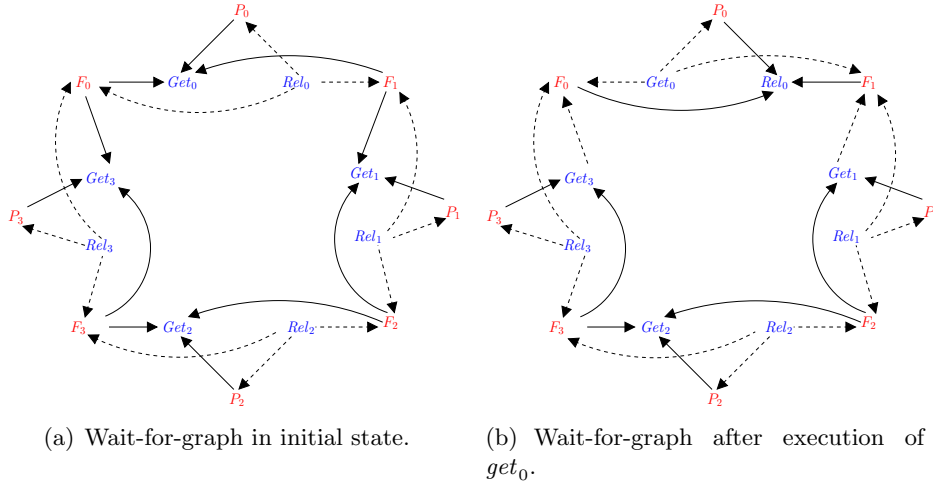


Figure 2: Example wait-for-graphs for dining philosophers system of Figure 1.

**Definition 13 (Supercycle)** Let  $B = \gamma(B_1, \dots, B_n)$  be a composite component and  $s$  be a state of  $B$ . A subgraph  $SC$  of  $W_B(s)$  is a supercycle in  $W_B(s)$  if and only if all of the following hold:

1.  $SC$  is nonempty, i.e., contains at least one node,
2. if  $B_i$  is a node in  $SC$ , then for all interactions  $a$  such that there is an edge in  $W_B(s)$  from  $B_i$  to  $a$ :
  - (a)  $a$  is a node in  $SC$ , and
  - (b) there is an edge in  $SC$  from  $B_i$  to  $a$ ,
 that is,  $B_i \rightarrow a \in W_B(s)$  implies  $B_i \rightarrow a \in SC$ ,
3. if  $a$  is a node in  $SC$ , then there exists a  $B_j$  such that:
  - (a)  $B_j$  is a node in  $SC$ , and
  - (b) there is an edge from  $a$  to  $B_j$  in  $W_B(s)$ , and
  - (c) there is an edge from  $a$  to  $B_j$  in  $SC$ ,
 that is,  $a \in SC$  implies  $\exists B_j : a \rightarrow B_j \in W_B(s) \wedge a \rightarrow B_j \in SC$ ,

where  $a \in SC$  means that  $a$  is a node in  $SC$ , etc.  $W_B(s)$  is *supercycle-free* iff there does not exist a supercycle  $SC$  in  $W_B(s)$ . In this case, say that state  $s$  is supercycle-free.

Figure 3 shows an example supercycle (with boldened edges) for the dining philosophers system of Figure 1.  $P_0$  waits for (enables) a single interaction,  $Get_0$ .  $Get_0$  waits for (is disabled by) fork  $F_0$ , which waits for interaction  $Rel_0$ .  $Rel_0$  in turn waits for  $P_0$ . However, this supercycle occurs in a state where  $P_0$  is in  $h$  and  $F_0$  is in  $u_l$ . This state is not reachable from the initial state.

The existence of a supercycle is sufficient and necessary for the occurrence of a deadlock, and so checking for supercycles gives a sound and complete check for deadlocks. Write  $SC \subseteq W_B(s)$  when  $SC$  is a subgraph of  $W_B(s)$ . Proposition 3 states that the existence of a supercycle implies a local deadlock: all components in the supercycle are blocked forever.

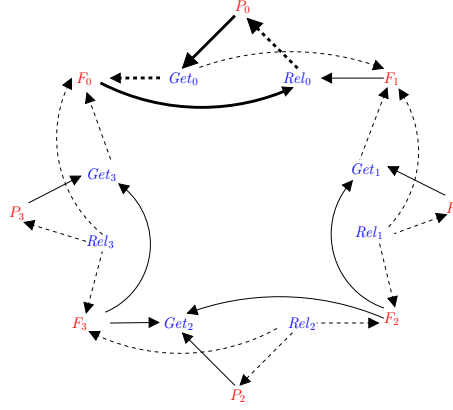


Figure 3: Example supercycle for dining philosophers system of Figure 1.

**Proposition 3** *Let  $s$  be a state of  $B$ . If  $SC \subseteq W_B(s)$  is a supercycle, then all components  $B_i$  in  $SC$  cannot execute a transition in any state reachable from  $s$ , including  $s$  itself.*

*Proof.* Let  $B_i$  be an arbitrary component in  $SC$ . By Definition 13, every interaction that  $B_i$  enables has a wait-for-edge to some other component  $B_j$  in  $SC$  and so cannot be executed in state  $s$ . Hence in any transition from  $s$  to another global state  $t$ , all of the components  $B_i$  in  $SC$  remain in the same local state. Hence  $SC \subseteq W_B(t)$ , i.e., the same supercycle  $SC$  remains in global state  $t$ . Repeating this argument from state  $t$  and onwards leads us to conclude that  $SC \subseteq W_B(u)$  for any state  $u$  reachable from  $s$ .  $\square$

Proposition 4 states that the existence of a supercycle is necessary for a local deadlock to occur: if a set of components, *considered in isolation*, are blocked, then there exists a supercycle consisting of exactly those components, together with the interactions that each component enables.

**Proposition 4** *Let  $B'$  be a subcomponent of  $B$ , and let  $s$  be an arbitrary state of  $B$  such that  $B'$ , when considered in isolation, has no enabled interaction in state  $s \upharpoonright B'$ . Then,  $W_B(s)$  contains a supercycle.*

*Proof.* Let  $B_i$  be an arbitrary atomic component in  $B'$ , and let  $a_i$  be any interaction that  $B_i$  enables. Since  $B'$  has no enabled interaction, it follows that  $a_i$  is not enabled in  $B'$ , and therefore has a wait-for-edge to some atomic component  $B_j$  in  $B'$ . Let  $SC$  be the subgraph of  $W_B(s)$  induced by:

1. the atomic components of  $B_i$  of  $B'$ ,
2. the interactions  $a$  that each atomic component  $B_i$  enables, and the edges  $B_i \rightarrow a$ , and
3. the edges  $a \rightarrow B_j$  from each interaction to some atomic component  $B_j$  in  $B'$  that does not enable  $B_j$ .

$SC$  satisfies Definition 13 and so is a supercycle.  $\square$

We consider subcomponent  $B'$  in isolation to avoid other phenomena that prevent interactions from executing, e.g., conspiracies [5]. Now the converse of Proposition 4 is that absence



of supercycles in  $W_B(s)$  means there is no locally deadlocked subsystem. Taking  $B' = B$ , this implies that  $B$  is not deadlocked, and so there is at least one interaction of  $B$  which is enabled in state  $s$ .

**Corollary 5** *If, for every reachable state  $s$  of  $(B, Q_0)$ ,  $W_B(s)$  is supercycle-free, then  $(B, Q_0)$  is deadlock-free.*

*Proof.* Take  $B'$  to be the entire system  $B$  in Proposition 4. Since  $s$  is supercycle free, it follows from the contrapositive of Proposition 4 that some interaction is enabled in  $B$ . Hence  $B$  is deadlock-free by Definition 11.  $\square$

### 3.3 Structural properties of supercycles

We present some structural properties of supercycles, which are central to our deadlock-freedom condition.

**Definition 14 (Path, path length)** *Let  $G$  be a directed graph and  $v$  a vertex in  $G$ . A path  $\pi$  in  $G$  is a finite sequence  $v_1, v_2, \dots, v_n$  such that  $(v_i, v_{i+1})$  is an edge in  $G$  for all  $i \in \{1, \dots, n-1\}$ . Write  $\text{path}_G(\pi)$  iff  $\pi$  is a path in  $G$ . Define  $\text{first}(\pi) = v_1$  and  $\text{last}(\pi) = v_n$ . Let  $|\pi|$  denote the length of  $\pi$ , which we define as follows:*

- if  $\pi$  is simple, i.e., all  $v_i$ ,  $1 \leq i \leq n$ , are distinct, then  $|\pi| = n - 1$ , i.e., the number of edges in  $\pi$
- if  $\pi$  contains a cycle, i.e., there exist  $v_i, v_j$  such that  $i \neq j$  and  $v_i = v_j$ , then  $|\pi| = \omega$  ( $\omega$  for “infinity”).

**Definition 15 (In-depth, Out-depth)** *Let  $G$  be a directed graph and  $v$  a vertex in  $G$ . Define the in-depth of  $v$  in  $G$ , notated as  $\text{in\_depth}_G(v)$ , as follows:*

- if there exists a path  $\pi$  in  $G$  that contains a cycle and ends in  $v$ , i.e.,  $|\pi| = \omega \wedge \text{last}(\pi) = v$ , then  $\text{in\_depth}_G(v) = \omega$ ,
- otherwise, let  $\pi$  be a longest path ending in  $v$ . Then  $\text{in\_depth}_G(v) = |\pi|$ .

Formally,  $\text{in\_depth}_G(v) = (\text{MAX } \pi : \text{path}_G(\pi) \wedge \text{last}(\pi) = v : |\pi|)$ .

Likewise define  $\text{out\_depth}_G(v) = (\text{MAX } \pi : \text{path}_G(\pi) \wedge \text{first}(\pi) = v : |\pi|)$ , the out-depth of  $v$  in  $G$ , i.e., we consider paths starting (rather than ending) in  $v$ .

We use  $\text{in\_depth}_B(v, s)$  for  $\text{in\_depth}_{W_B(s)}(v)$ , and also  $\text{out\_depth}_B(v, s)$  for  $\text{out\_depth}_{W_B(s)}(v)$ .

**Proposition 6** *A supercycle  $SC$  contains no nodes with finite out-depth.*

*Proof.* By contradiction. Let  $v$  be a node in  $SC$  with finite out-depth. Hence by Definition 15 all outgoing paths from  $v$  are simple (and finite), and end in a sink node  $w$ , so  $w$  has no outgoing wait-for-edges. By assumption, all atomic components are individually deadlock-free, i.e., they

always enable at least one interaction. So if  $w$  is an atomic component  $B_i$ , we have a wait-for-edge  $B_i \rightarrow a$  for some interaction  $a$ , contradicting the fact that  $w$  is a sink node. Hence  $w$  is some interaction  $a$ . Since  $a$  has no outgoing edges, it violates clause 3 in Definition 13, contradicting the assumption that  $SC$  is a supercycle.  $\square$

**Proposition 7** *Every supercycle  $SC$  contains at least one cycle.*

*Proof.* By contradiction. Suppose that  $SC$  is a supercycle and is also acyclic. Then every path in  $SC$  is simple, and therefore finite. Hence every node in  $SC$  has finite out-depth. By Proposition 6,  $SC$  cannot be a supercycle.  $\square$

**Proposition 8** *Let  $B = \gamma(B_1, \dots, B_n)$  be a composite component and  $s$  a state of  $B$ . Let  $SC$  be a supercycle in  $W_B(s)$ , and let  $SC'$  be the graph obtained from  $SC$  by removing all vertices of finite in-depth and their incident edges. Then  $SC'$  is also a supercycle in  $W_B(s)$ .*

*Proof.* A vertex with finite in-depth cannot lie on a cycle in  $SC$ . Hence by Proposition 7,  $SC' \neq \emptyset$ . Thus  $SC'$  satisfies clause (1) of the supercycle definition (13). Let  $v$  be an arbitrary vertex of  $SC'$ . Thus  $v \in SC$  and  $in\_depth_{SC}(v) = \omega$  by definition of  $SC'$ . Let  $w$  be an arbitrary successor of  $v$  in  $SC$ .  $in\_depth_{SC}(w) = \omega$  by Definition 15. Hence  $w \in SC'$ . Furthermore,  $w$  is a successor of  $v$  in  $SC'$ , by definition of  $SC'$ . Thus every vertex  $v$  of  $SC'$  is also a vertex of  $SC$ , and the successors of  $v$  in  $SC'$  are the same as the successors of  $v$  in  $SC$ . Now since  $SC$  is a supercycle, every vertex  $v$  in  $SC$  has enough successors in  $SC$  to satisfy clauses (2) and (3) of the supercycle definition (13). It follows that every vertex  $v$  in  $SC'$  has enough successors in  $SC'$  to satisfy clauses (2) and (3) of the supercycle definition (13).  $\square$

## 4 A Global Condition for Deadlock Freedom

Consider a reachable transition  $s \xrightarrow{a} t$  of  $(B, Q_0)$ . Suppose that the execution of this transition creates a supercycle  $SC$ , i.e.,  $SC \not\subseteq W_B(s) \wedge SC \subseteq W_B(t)$ . The only components that can change state along this transition are the participants of  $a$ , i.e., the  $B_i \in C_a$ , and so they are the only components that can cause a supercycle to be created in going from  $s$  to  $t$ . There are three relevant possibilities for each  $B_i \in C_a$ :

1.  $B_i$  has finite in-depth in  $W_B(t)$ : then, if  $B_i \in SC$ , it can be removed and still leave a supercycle  $SC'$ , by Proposition 8. Hence  $SC'$  exists in  $W_B(s)$ , and so  $B_i$  is not essential to the creation of a supercycle.
2.  $B_i$  has finite out-depth in  $W_B(t)$ : by Proposition 6,  $B_i$  cannot be part of a supercycle, and so  $SC \subseteq W_B(s)$ .
3.  $B_i$  has infinite in-depth and infinite out-depth in  $W_B(t)$ : in this case,  $B_i$  is possibly an essential part of  $SC$ , i.e.,  $SC$  was created in going from  $s$  to  $t$ .

We thus impose a condition which guarantees that only case 1 or case 2 occur.

**Definition 16** ( $DFC(a)$ ) *Let  $s \xrightarrow{a} t$  be a reachable transition of BIP-system  $(B, Q_0)$ . Then, in  $t$ , the following holds. For every component  $B_i$  of  $C_a$ : either  $B_i$  has finite in-depth, or finite*

out-depth, in  $W_B(t)$ . Formally,

$$\forall B_i \in C_a : in\_depth_B(B_i, t) < \omega \vee out\_depth_B(B_i, t) < \omega.$$

To proceed, we show that wait-for-edges not involving some interaction  $a$  and its participants  $B_i \in C_a$  are unaffected by the execution of  $a$ . Say that edge  $e$  in a wait-for-graph is  $B_i$ -incident iff  $B_i$  is one of the endpoints of  $e$ .

**Proposition 9 (Wait-for-edge preservation)** *Let  $s \xrightarrow{a} t$  be a transition of composite component  $B = \gamma(B_1, \dots, B_n)$ , and let  $e$  be a wait-for edge that is not  $B_i$ -incident, for every  $B_i \in C_a$ . Then  $e \in W_B(s)$  iff  $e \in W_B(t)$ .*

*Proof.* Fix  $e$  to be an arbitrary wait-for-edge that is not  $B_i$ -incident.  $e$  is either  $B_j \rightarrow b$  or  $b \rightarrow B_j$ , for some component  $B_j$  of  $B$  that is not in  $C_a$ , and an interaction  $b$  (different from  $a$ ) that  $B_j$  participates in. Now  $s|B_j = t|B_j$ , since  $s \xrightarrow{a} t$  and  $B_j \notin C_a$ . Hence  $s(enb_b^j) = t(enb_b^j)$ . It follows from Definition 12 that  $e \in W_B(s)$  iff  $e \in W_B(t)$ .  $\square$

We show, by induction on the length of finite exeuctions, that every reachable state is supercycle-free. Assume that every initial state is supercycle-free, for the base case. Assuming  $\mathcal{DFC}(a)$  for all  $a \in \gamma$  provides, by the above discussion, the induction step.

**Theorem 10 (Deadlock-freedom)** *If (1) for all  $s_0 \in Q_0$ ,  $W_B(s_0)$  is supercycle-free, and (2) for all interactions  $a$  of  $B$  (i.e.,  $a \in \gamma$ ),  $\mathcal{DFC}(a)$  holds, then for every reachable state  $u$  of  $(B, Q_0)$ :  $W_B(u)$  is supercycle-free.*

*Proof.* We only need show the induction step: for every reachable transition  $s \xrightarrow{a} t$ ,  $W_B(s)$  is supercycle-free implies that  $W_B(t)$  is supercycle-free. We establish the contrapositive: if  $W_B(t)$  contains a supercycle, then so does  $W_B(s)$ .

Let  $SC$  be a supercycle in  $W_B(t)$ , and let  $SC'$  be  $SC$  with all nodes of finite in-depth removed.  $SC'$  is a supercycle in  $W_B(t)$  by Proposition 8. Let  $e$  be an arbitrary edge in  $SC'$ . Hence  $e \in W_B(t)$ . Also, both nodes of  $e$  have infinite in-depth (by construction of  $SC'$ ) and infinite out-depth (by Proposition 6) in  $W_B(t)$ . Let  $B_i$  be an arbitrary component in  $C_a$ . By  $\mathcal{DFC}(a)$ ,  $B_i$  has finite in-depth or finite out-depth in  $W_B(t)$ :  $in\_depth_B(B_i, t) < \omega \vee out\_depth_B(B_i, t) < \omega$ . Hence  $e$  is not  $B_i$ -incident. So,  $e \in W_B(s)$ , by Proposition 9. Hence  $SC' \subseteq W_B(s)$ , and so  $W_B(s)$  contains a supercycle.  $\square$

## 5 A Local Condition for Deadlock Freedom

Evaluating  $\mathcal{DFC}(a)$  requires checking all reachable transitions of  $(B, Q_0)$ , which is subject to state-explosion. We need a condition which implies  $\mathcal{DFC}(a)$  and can be checked efficiently. Observe that if  $in\_depth_B(B_i, t) < \omega \vee out\_depth_B(B_i, t) < \omega$ , then there is some finite  $\ell$  such that  $in\_depth_B(B_i, t) = \ell \vee out\_depth_B(B_i, t) = \ell$ . This can be verified in a subsystem whose size depends on  $\ell$ , as follows.

**Definition 17 (Structure Graph  $G_B, G_i^\ell, G_a^\ell$ )** *The structure graph  $G_B$  of composite component  $B = \gamma(B_1, \dots, B_n)$  is a bipartite graph whose nodes are the  $B_1, \dots, B_n$  and all the  $a \in \gamma$ .*

There is an edge between  $B_i$  and interaction  $a$  iff  $B_i$  participates in  $a$ , i.e.,  $B_i \in C_a$ . Define the distance between two nodes to be the number of edges in a shortest path between them. Let  $G_i^\ell$  ( $G_a^\ell$  respectively) be the subgraph of  $G_B$  that contains  $B_i$  ( $a$  respectively) and all nodes of  $G_B$  that have a distance to  $B_i$  ( $a$  respectively) less than or equal to  $\ell$ .

Then  $\text{in\_depth}_B(B_i, t) = \ell \vee \text{out\_depth}_B(B_i, t) = \ell$  can be verified in the wait-for-graph of  $G_i^{\ell+1}$ , since we verify either that all wait-for-paths ending in  $B_i$  have length  $\leq \ell$ , or that all wait-for-paths starting in  $B_i$  have length  $\leq \ell$ . These conditions can be checked in  $G_i^{\ell+1}$ , since  $G_i^{\ell+1}$  contains every node in a wait-for-path of length  $\ell+1$  or less and which starts or ends in  $B_i$ . Since  $G_i^{\ell+1} \subseteq G_a^{\ell+2}$  for  $B_i \in C_a$ , we use  $G_a^{\ell+2}$  instead of the set of subsystems  $\{G_i^{\ell+1} : B_i \in C_a\}$ . We leave analysis of the tradeoff between using one larger system ( $G_a^{\ell+2}$ ) versus several smaller ones ( $G_i^{\ell+1}$ ) to another paper. Define  $D_a^\ell$ , the *deadlock-checking subsystem for interaction  $a$  and depth  $\ell$* , to be the subsystem of  $(B, Q_0)$  based on  $G_a^{\ell+2}$ .

**Definition 18** ( $\mathcal{LDFC}(a, \ell)$ ) Let  $s_a \xrightarrow{a} t_a$  be a reachable transition of  $D_a^\ell$ . Then, in  $t_a$ , the following holds. For every component  $B_i$  of  $C_a$ : either  $B_i$  has in-depth at most  $\ell$ , or out-depth at most  $\ell$ , in  $W_{D_a^\ell}(t_a)$ . Formally,

$$\forall B_i \in C_a : \text{in\_depth}_{D_a^\ell}(B_i, t_a) \leq \ell \vee \text{out\_depth}_{D_a^\ell}(B_i, t_a) \leq \ell.$$

To infer deadlock-freedom in  $(B, Q_0)$  by checking  $\mathcal{LDFC}(a, \ell)$ , we show that wait-for behavior in  $B$  “projects down” to any subcomponent  $B'$ , and that wait-for behavior in  $B'$  “projects up” to  $B$ .

**Proposition 11 (Wait-for-edge projection)** Let  $(B', Q'_0)$  be a subsystem of  $(B, Q_0)$ . Let  $s$  be a state of  $(B, Q_0)$ , and  $s' = s \upharpoonright B'$ . Let  $a$  be an interaction of  $(B', Q'_0)$ , and  $B_i \in C_a$  an atomic component of  $B'$ . Then (1)  $a \rightarrow B_i \in W_B(s)$  iff  $a \rightarrow B_i \in W_{B'}(s')$ , and (2)  $B_i \rightarrow a \in W_B(s)$  iff  $B_i \rightarrow a \in W_{B'}(s')$ .

*Proof.* By Definition 12,  $a \rightarrow B_i \in W_B(s)$  iff  $s \upharpoonright i(\text{enb}_a^{B_i}) = \text{false}$ . Since  $s' = s \upharpoonright B'$ , we have  $s' \upharpoonright i = s \upharpoonright i$ . Hence  $s \upharpoonright i(\text{enb}_a^{B_i}) = s' \upharpoonright i(\text{enb}_a^{B_i})$ . By Definition 12,  $a \rightarrow B_i \in W_{B'}(s')$  iff  $s' \upharpoonright i(\text{enb}_a^{B_i}) = \text{false}$ . Putting together these three equalities gives us clause (1).

By Definition 12,  $B_i \rightarrow a \in W_B(s)$  iff  $s \upharpoonright i(\text{enb}_a^{B_i}) = \text{true}$ . Since  $s' = s \upharpoonright B'$ , we have  $s' \upharpoonright i = s \upharpoonright i$ . Hence  $s \upharpoonright i(\text{enb}_a^{B_i}) = s' \upharpoonright i(\text{enb}_a^{B_i})$ . By Definition 12,  $B_i \rightarrow a \in W_{B'}(s')$  iff  $s' \upharpoonright i(\text{enb}_a^{B_i}) = \text{true}$ . Putting the above three equalities together gives us clause (2).  $\square$

Since wait-for-edges project up and down, it follows that wait-for-paths project up and down, provided that the subsystem contains the entire wait-for-path.

**Proposition 12 (In-projection, Out-projection)** Let  $\ell \geq 0$ , let  $B_i$  be an atomic component of  $B$ , and let  $(B', Q'_0)$  be a subsystem of  $(B, Q_0)$  which is based on a superset of  $G_i^{\ell+1}$ . Let  $s$  be a state of  $(B, Q_0)$ , and  $s' = s \upharpoonright B'$ . Then (1)  $\text{in\_depth}_B(B_i, s) \leq \ell$  iff  $\text{in\_depth}_{B'}(B_i, s') \leq \ell$ , and (2)  $\text{out\_depth}_B(B_i, s) \leq \ell$  iff  $\text{out\_depth}_{B'}(B_i, s') \leq \ell$ .

*Proof.* We establish clause (1). The proof of clause (2) is analogous, except we replace paths ending in  $B_i$  by paths starting from  $B_i$ . The proof of clause (1) is by double implication.

$\text{in\_depth}_B(B_i, s) \leq \ell$  implies  $\text{in\_depth}_{B'}(B_i, s') \leq \ell$ : Let  $\pi$  be an arbitrary wait-for-path in  $W_{B'}(s')$  ending in  $B_i$ . Since  $(B', Q'_0)$  is a subsystem of  $(B, Q_0)$ , by Definition 12 and  $s' = s \upharpoonright B'$ ,

$W_{B'}(s')$  is a subgraph of  $W_B(s)$ . Hence  $\pi$  is a wait-for-path in  $W_B(s)$ . By  $\text{in\_depth}_B(B_i, s) \leq \ell$ , we have  $|\pi| \leq \ell$ . Hence  $\text{in\_depth}_{B'}(B_i, s') \leq \ell$  since  $\pi$  was arbitrarily chosen.

$\text{in\_depth}_B(B_i, s) \leq \ell$  follows from  $\text{in\_depth}_{B'}(B_i, s') \leq \ell$ : Let  $\pi$  be an arbitrary wait-for-path ending in  $B_i$ . Suppose that  $|\pi| > \ell$ . Since  $(B', Q'_0)$  is a subsystem of  $(B, Q_0)$ , by Definition 12 and  $s' = s|B'$ ,  $W_{B'}(s')$  is a subgraph of  $W_B(s)$ . Hence every edge  $e$  in  $\pi$  that is within distance  $\ell + 1$  from  $B_i$  is also an edge in  $W_{B'}(s')$ , since  $(B', Q'_0)$  is based on a superset of  $G_i^{\ell+1}$ . Hence  $\text{in\_depth}_B(B_i, s) > \ell$  implies  $\text{in\_depth}_{B'}(B_i, s') > \ell$ . The contrapositive is our desired result.  $\square$

We now show that  $\mathcal{LDFC}(a, \ell)$  implies  $\mathcal{DFC}(a)$ , which in turn implies deadlock-freedom.

**Lemma 13** *Let  $a$  be an interaction of  $B$ , i.e.,  $a \in \gamma$ . If  $\mathcal{LDFC}(a, \ell)$  holds for some finite  $\ell \geq 0$ , then  $\mathcal{DFC}(a)$  holds.*

*Proof.* Let  $s \xrightarrow{a} t$  be a reachable transition of  $(B, Q_0)$  and let  $B_i \in C_a$ ,  $s_a = s|D_a^\ell$ ,  $t_a = t|D_a^\ell$ . Then  $s_a \xrightarrow{a} t_a$  is a reachable transition of  $D_a^\ell$  by Corollary 2. By  $\mathcal{LDFC}(a, \ell)$ ,  $\text{in\_depth}_{D_a^\ell}(B_i, t_a) \leq \ell \vee \text{out\_depth}_{D_a^\ell}(B_i, t_a) \leq \ell$ . Hence by Proposition 12,  $\text{in\_depth}_B(B_i, t) \leq \ell \vee \text{out\_depth}_B(B_i, t) \leq \ell$ . So  $\text{in\_depth}_B(B_i, t) < \omega \vee \text{out\_depth}_B(B_i, t) < \omega$ . Hence  $\mathcal{DFC}(a)$ .  $\square$

**Theorem 14 (Deadlock-freedom)** *If (1) for all  $s_0 \in Q_0$ ,  $W_B(s_0)$  is supercycle-free, and (2) for all interactions  $a$  of  $B$  ( $a \in \gamma$ ),  $\mathcal{LDFC}(a, \ell)$  holds for some  $\ell \geq 0$ , then for every reachable state  $u$  of  $(B, Q_0)$ :  $W_B(u)$  is supercycle-free.*

*Proof.* Let  $a$  be an arbitrary interaction in  $\gamma$ . By  $(\exists \ell : \mathcal{LDFC}(a, \ell))$  and Lemma 13, we have  $\mathcal{DFC}(a)$ . By Theorem 10, for every reachable state  $u$  of  $(B, Q_0)$ :  $W_B(u)$  is supercycle-free.  $\square$

## 6 Implementation and Experimentation

LDFC-BIP, ( $\sim 1500$  LOC Java) implements our method for finite-state BIP-systems. Pseudocode for LDFC-BIP is shown in Figure 4.  $\text{checkDF}(B, Q_0)$  iterates over each interaction  $a$  of  $(B, Q_0)$ , and checks  $(\exists \ell \geq 0 : \mathcal{LDFC}(a, \ell))$  by starting with  $\ell = 0$  and incrementing  $\ell$  until either  $\mathcal{LDFC}(a, \ell)$  is found to hold, or  $D_a^\ell$  has become the entire system and  $\mathcal{LDFC}(a, \ell)$  does not hold. In the latter case,  $\mathcal{LDFC}(a, \ell)$  does not hold for any finite  $\ell$ , and, in practice, computation would halt before  $D_a^\ell$  had become the entire system, due to exhaustion of resources.

$\text{locLDFC}(a, \ell)$  checks  $\mathcal{LDFC}(a, \ell)$  by examining every reachable transition that executes  $a$ , and checking that the final state satisfies Definition 18.

The running time of our implementation is  $O(\sum_{a \in \gamma} |D_a^{\ell_a}|)$ , where  $\ell_a$  is the smallest value of  $\ell$  for which  $\mathcal{LDFC}(a, \ell)$  holds, and where  $|D_a^{\ell_a}|$  denotes the size of the transition system of  $D_a^{\ell_a}$ .

### 6.1 Experiment: Dining Philosophers

We consider  $n$  philosophers in a cycle, based on the components of Figure 1. Figure 6(a) provides experimental results. The  $x$  axis gives the number  $n$  of philosophers (and also the number of forks), and the  $y$  axis gives the verification time (in milliseconds). We verified that  $\mathcal{LDFC}(a, \ell)$  holds for  $\ell = 1$  and all interactions  $a$ . Hence dining philosophers is deadlock-free. We increase  $n$  and plot the verification time for both LDFC-BIP and D-Finder 2 [8]. D-Finder 2 implements

```

checkDF( $B, Q_0$ ), where  $B \triangleq \gamma(B_1, \dots, B_n)$ 
1. forall interactions  $a \in \gamma$ 
2.   //check ( $\exists \ell \geq 0 : \mathcal{LDFC}(a, \ell)$ )
3.    $\ell \leftarrow 0$ ;                                     //start with  $\ell = 0$ 
4.   while (true)
5.     if ( $\text{locLDFC}(a, \ell) = \text{true}$ ) break endif;           //success, so go on to next  $a$ 
6.     if ( $D_a^\ell = \gamma(B_1, \dots, B_n)$ ) return(false) endif;
7.      $\ell \leftarrow \ell + 1$                                //increment  $\ell$  until success or intractable or failure
8.   endwhile
9. endfor;
10. return(true)                                     //return true if check succeeds for all  $a \in \gamma$ 

locLDFC( $a, \ell$ )
1. forall reachable transitions  $s_a \xrightarrow{a} t_a$  of  $D_a^\ell$ 
2.   if ( $\neg(\forall B_i \in C_a : \text{in\_depth}_{D_a^\ell}(B_i, t_a) = \ell \vee \text{out\_depth}_{D_a^\ell}(B_i, t_a) = \ell)$ )
3.     return(false)                                     //check Definition 18
4. endfor;
5. return(true)                                     //return true if check succeeds for all transitions

```

Figure 4: Pseudocode for the implementation of our method.

a compositional and incremental method for the verification of BIP-systems. D-Finder (the precursor of D-Finder 2) has been compared favorably with NuSmv and SPIN, outperforming both NuSmv and SPIN on dining philosophers, and outperforming NuSmv on the gas station example [7], treated next. Our results show that LDFC-BIP has a linear increase of computation time with the system size ( $n$ ), and so outperforms D-Finder 2.

## 6.2 Experiment: Gas Station

A gas station [13] consists of an operator, a set of pumps, and a set of customers. Before using a pump, a customer has to prepay. Then the customer uses the pump, collects his change and starts a new transaction. Before being used by a customer, a pump has to be activated by the operator. When a pump is shut off, it can be re-activated for the next operation.

Figure 5 gives the model for a gas station system for one pump and two customers. The operator has two control locations and three ports. The transition labeled with *prepay* accepts a customer's prepay and activates the pump for the customer. When a customer is served, the transition labeled with *finish* synchronizes the pump and the customer. A pump has three control locations and three ports. Besides the synchronization between the operator and customer through activate and finish ports, a pump and a customer are synchronized through *start* ports.

We verified  $\mathcal{LDFC}(a, \ell)$  for  $\ell = 2$  and all interactions  $a$ . Hence gas station is deadlock-free. Figures 6(b), 6(c), and 6(d) present the verification times using LDFC-BIP and D-Finder 2. We consider a system with 3 pumps and variable number of customers. In these figures, the  $x$  axis gives the number  $n$  of customers, and the  $y$  axis gives the verification time (in seconds).

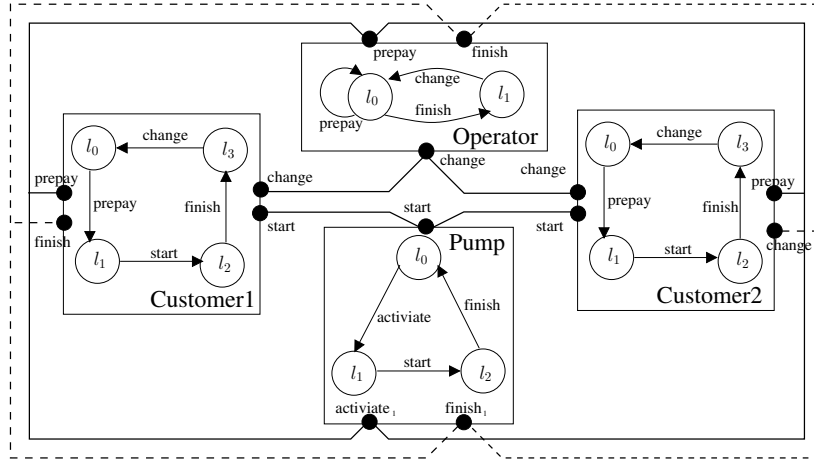


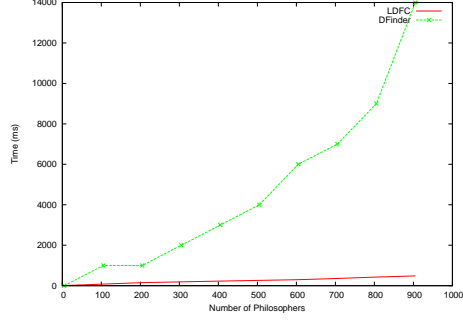
Figure 5: Sketch of Gas Station

D-Finder 2 suffers state-explosion at  $n = 1800$ , because we consider only three pumps, and so the incremental method used by D-Finder 2 deteriorates. LDFC-BIP outperforms D-Finder 2 as the number of customers increases.

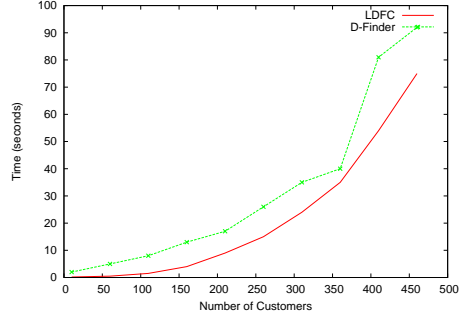
## 7 Discussion, Related Work, and Further Work

**Related work.** The notions of wait-for-graph and supercycle [3, 4] were initially defined for a shared memory program  $P = P_1 \parallel \dots \parallel P_K$  in *pairwise normal form*: a binary symmetric relation  $I$  specifies the directly interacting pairs (“neighbors”)  $\{P_i, P_j\}$ . If  $P_i$  has neighbors  $P_j$  and  $P_k$ , then the code in  $P_i$  that interacts with  $P_j$  is expressed separately from the code in  $P_i$  that interacts with  $P_k$ . These synchronization codes are executed synchronously and atomically, so the grain of atomicity is proportional to the degree of  $I$ . Attie and Chockler [3] give two polynomial time methods for deadlock freedom. The first checks subsystems consisting of three processes. The second computes the wait-for-graphs of all pair subsystems  $P_i \parallel P_j$ , and takes their union, for all pairs and all reachable states of each pair. The first method considers only wait-for-paths of length  $\leq 2$ . The second method is prone to false negatives, because wait-for edges generated by different states are all merged together, which can result in spurious supercycles.

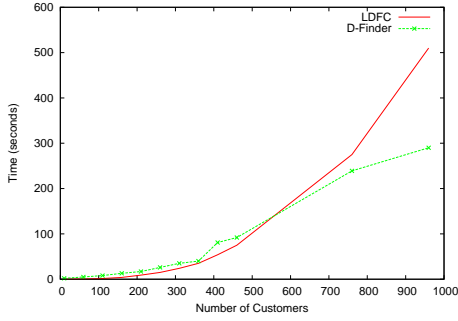
Gössler and Sifakis [12] use a BIP-like formalism, Interaction Models. They present a criterion for global deadlock freedom, based on an and-or graph with components and constraints as the two sets of nodes. A constraint gives the condition under which a component is blocked. Edges are labeled with conjuncts of the constraints. Deadlock freedom is checked by traversing every cycle, taking the conjunction of all the conditions labeling its edges, and verifying that this conjunction is always false, i.e., verifying the absence of cyclical blocking. No complexity bounds are given. Martens and Majster-Cederbaum [14] present a polynomial time checkable deadlock freedom condition based on structural restrictions: “the communication structure between the components is given by a tree.” This restriction allows them to analyze only pair systems. Brookes and Roscoe [11] provide criteria for deadlock freedom of CSP programs based



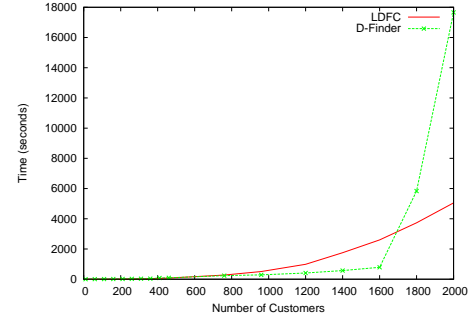
(a) Dining philosophers benchmark.



(b) Gas station benchmark 1.



(c) Gas station benchmark 2.



(d) Gas station benchmark 3.

Figure 6: Benchmarks generated by our experiments.

on structural and behavioral restrictions combined with analysis of pair systems. No implementation, or complexity bounds, are given. Aldini and Bernardo [1] use a formalism based on process algebra. They check deadlock by analysing cycles in the connections between software components, and claim scalability, but no complexity bounds are given.

We compared our implementation LDFC-BIP to D-Finder 2 [8]. D-Finder 2 computes a finite-state abstraction for each component, which it uses to compute a global invariant  $I$ . It then checks if  $I$  implies deadlock freedom. Unlike LDFC-BIP, D-Finder 2 handles infinite state systems. However, LDFC-BIP had superior running time for dining philosophers and gas station (both finite-state).

All the above methods verify global (and not local) deadlock-freedom. Our method verifies both. Also, our approach makes no structural restriction at all on the system being checked for deadlock.

**Discussion.** Our approach has the following advantages:

**Local and global deadlock** Our method shows that no subset of processes can be deadlocked, i.e., absence of both local and global deadlock.



**Check works for realistic formalism** By applying the approach to BIP, we provide an efficient deadlock-freedom check within a formalism from which efficient distributed implementations can be generated [9].

**Locality** If a component  $B_i$  is modified, or is added to an existing system, then  $\mathcal{LDFC}(a, \ell)$  only has to be re-checked for  $B_i$  and components within distance  $\ell$  of  $B_i$ . A condition whose evaluation considers the entire system at once, e.g., [1, 8, 12] would have to be re-checked for the entire system.

**Easily parallelizable** Since the checking of each subsystem  $D_a^\ell$  is independent of the others, the checks can be carried out in parallel. Hence our method can be easily parallelized and distributed, for speedup, if needed. Alternatively, performing the checks sequentially minimizes the amount of memory needed.

**Framework aspect** Supercycles and in/out-depth provide a *framework* for deadlock-freedom. Conditions more general and/or discriminating than the one presented here should be devisable in this framework. This is a topic for future work.

**Further work.** Our implementation uses explicit state enumeration. Using BDD's may improve the running time when  $\mathcal{LDFC}(a, \ell)$  holds only for large  $\ell$ . An enabled port  $p$  enables all interactions containing  $p$ . Deadlock-freedom conditions based on ports could exploit this interdependence among interaction enablement. Our implementation should produce *counterexamples* when a system fails to satisfy  $\mathcal{LDFC}(a, \ell)$ . *Design rules* for ensuring  $\mathcal{LDFC}(a, \ell)$  will help users to produce deadlock-free systems, and also to interpret counterexamples. A *fault* may create a deadlock, i.e., a supercycle, by creating wait-for-edges that would not normally arise. Tolerating a fault that creates up to  $f$  such spurious wait-for-edges requires that there do not arise during normal (fault-free) operation subgraphs of  $W_B(s)$  that can be made into a supercycle by adding  $f$  edges. We will investigate criteria for preventing formation of such subgraphs. Methods for evaluating  $\mathcal{LDFC}(a, \ell)$  on *infinite state* systems will be devised, e.g., by extracting proof obligations and verifying using SMT solvers. We will extend our method to *Dynamic BIP*, [10], where participants can add and remove interactions at run time.

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