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Stewart platform

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Industrial plant

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Introduction:

The purpose of this report is to analyze and simulate the effects of applying a force at the center of a Stewart Platform. The Stewart Platform is a parallel manipulator consisting of six linear actuators connected to a movable platform. Understanding the platform's response to external forces is crucial for design optimization, stability assessment, and safety considerations.

Simulation Setup:

To investigate the impact of applying a force at the center of the platform, a detailed simulation model was developed using appropriate software such as Simscape Multibody. The model incorporated the geometric properties, inertial characteristics, and mechanical constraints of the Stewart Platform. The force was applied as a known magnitude and direction at the center of the platform.

Kinematic Analysis:

The simulation allowed for the analysis of the platform's kinematic response to the applied force. By observing the changes in position, orientation, and velocities of the platform and its links, the impact on the overall system kinematics was assessed. The simulation provided insights into how the platform translated and rotated in response to the applied force, thereby enabling the examination of displacement and angular changes.

Dynamic Analysis:

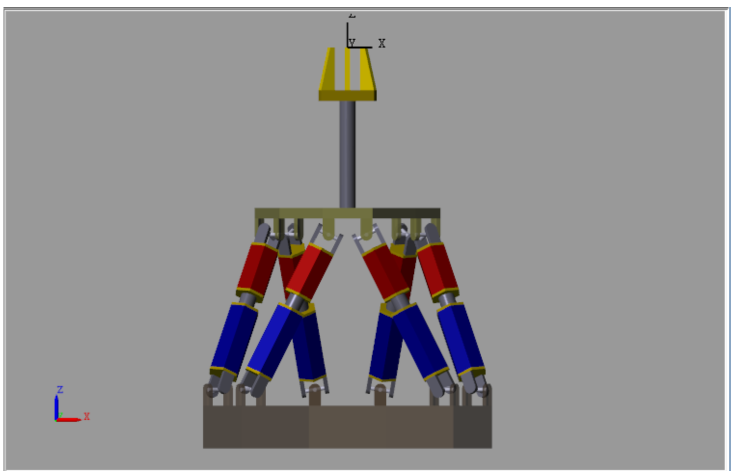
The dynamic behavior of the Stewart Platform under the influence of the applied force was thoroughly investigated through simulation. By considering the interaction between the force, the platform's links, and the actuators, the resulting forces and torques in the system were determined. The simulation provided valuable information regarding the internal forces and stress distribution within the platform structure, aiding in the evaluation of structural integrity and potential failure modes.

Force Transmission Analysis:

The simulation also facilitated the analysis of force transmission within the Stewart Platform. By examining the forces transmitted through the links, joints, and actuators, the load distribution and potential points of stress concentration were identified. This analysis helped determine the critical components that experience higher loads due to the applied force and guided the design of robust and reliable structures.

Control System Response:

The response of the control system to the applied force was assessed through simulation. By considering the control algorithms and feedback mechanisms implemented in the platform's control system, the simulation allowed for the evaluation of the platform's ability to maintain stability and desired performance under the influence of the force. The control system response analysis provided insights into the need for adjustments or enhancements to ensure safe and stable operation.

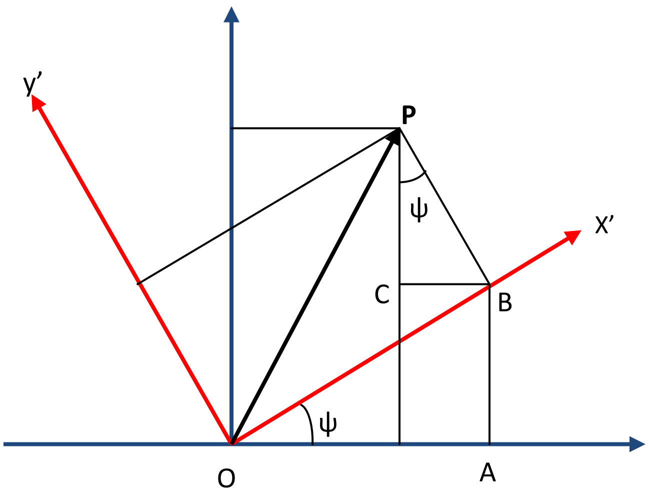


Calculations:

Three angular displacements then define the orientation of the platform with respect to the Base. A set of Euler angles are used in the following sequence:

1. Rotate an angle ψ (yaw) around the z-axis
2. Rotate an angle θ (pitch) around the y-axis
3. Rotate an angle φ (roll) around the x-axis

If we consider the first rotation ψ (yaw) around the z-axis:



y

$P = i'x' + j'y' +$

$k'z' = ix + jy + kz$

$x = OA - BC$

$= x'\cos\psi - y'\sin\psi$

$y = AB + PC$

$= x'\sin\psi + y'\cos\psi$

x $z = z'$

We define the rotation matrix $\mathbf{R}_z(\psi)$ where

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R}_z(\psi) \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad \text{and} \quad \mathbf{R}_z(\psi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Similarly If we consider the second rotation θ (pitch) around the y-axis we can show

$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix},$$

And for the third rotation φ (roll) around the x-axis:

$$\mathbf{R}_x(\varphi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

The full rotation matrix of the Platform relative to the Base is then given by:

$${}^{P}\mathbf{R}_{B} = \mathbf{R}_z(\psi).\mathbf{R}_y(\theta).\mathbf{R}_x(\varphi)$$

$$= \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\psi\cos\theta & -\sin\psi & \cos\psi\sin\theta \\ \sin\psi\cos\theta & \cos\psi & \sin\psi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\psi\cos\theta & -\sin\psi\cos\varphi + \cos\psi\sin\theta\sin\varphi & \sin\psi\sin\varphi + \cos\psi\sin\theta\cos\varphi \\ \sin\psi\cos\theta & \cos\psi\cos\varphi + \sin\psi\sin\theta\sin\varphi & -\cos\psi\sin\varphi + \sin\psi\sin\theta\cos\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{pmatrix}$$

Now consider a Stewart Platform.

For the i^{th} leg:

The coordinates $\boldsymbol{q}_i$ of the anchor point P_i with respect to the Base reference framework are given by the equation

$$\boldsymbol{q_i} = T + {}^{P}\mathbf{R}_{\boldsymbol{B}} \cdot \boldsymbol{p_i}$$

Where T is the translation vector, giving the positional linear displacement of the origin of the Platform frame with respect to the Base reference framework, and $\boldsymbol{p}_i$ is the vector defining the coordinates of the anchor point P_i with respect to the Platform framework.

Similarly the length of the i^{th} leg is given by

$$\boldsymbol{l}_i = T + {}^{P}\mathbf{R}_{\boldsymbol{B}} \cdot \boldsymbol{p_i} - \boldsymbol{b_i}$$

where $\boldsymbol{b_i}$ is the vector defining the coordinates of the lower anchor point B_i. These 6 equations give the lengths of the 6 legs to achieve the desired position and attitude of the platform.

When considering the Forward Kinematics, this expression represents 18 simultaneous nonlinear equations in the 6 unknowns representing the position and attitude of the platform. Much work has been done on finding the solutions to these equations; in the general case there are 40 possible solutions, although in practice many of these solutions would not be practical.

If the leg lengths are achieved via rotational servos, rather than linear servos, a further calculation is required to determine the angle of rotation of the servo. Each servo / leg combination can be represented as follows:

Where: a = length of the servo operating arm $A_{\boldsymbol{i}}$ are the points of the arm/leg joint on the i^{th} servo with coordinates

$\boldsymbol{a} = [x_a \quad y_a \quad z_a]^T$ in the base framework.

$B_{\boldsymbol{i}}$ are the points of rotation of the servo arms with the coordinates

$\boldsymbol{b} = [x_b \quad y_b \quad z_b]^T$ in the base framework.

P_i are the points the joints between the operating rods and the platform, with coordinates $\boldsymbol{p} = [x_p \quad y_p \quad z_p]^T$ in the platform framework

S = length of operating leg

$\boldsymbol{l_i}$ = length of the i^{th} leg as calculated from $\boldsymbol{l_i}$ = T + ${}^{P}\mathbf{R}_{\boldsymbol{B}}$. $\boldsymbol{p_i}$ - $\boldsymbol{b_i}$ α = angle of servo

operating arm from horizontal

β = angle of servo arm plane relative to the x-axis. Note that the shaft axis lies in the x-y plane where z = 0

Point A is constrained to be on the servo arm, but the arrangement of the servos means that the odd and even arms are a reflection of each other. So for the even legs we have:

x_a= a $\cos\alpha\ \cos\beta + x_b$, and

y_a= a $\cos\alpha\ \sin\beta + y_b$, and

z_a= a $\sin\alpha + z_b$

And for the odd legs we have:

x_a= a $\cos(\pi-\alpha)\ \cos(\pi+\beta) + x_b$, and

y_a= a $\cos(\pi-\alpha)\ \sin(\pi+\beta) + y_b$, and

z_a= a $\sin(\pi-\alpha) + z_b$

But $\sin(\pi-\alpha) = \sin\propto$, and $\cos(\pi-\alpha) = -\cos\propto$

And $\sin(\pi+\beta) = -\sin\beta$, and $\cos(\pi+\beta) = -\cos\beta$

Substituting these values into the equations for the odd legs, we get the same equations as (4), (5), and (6) for the even legs.

By Pythagoras we also have:

$$a^2 = (x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2$$

$$= (x_a^2 + y_a^2 + z_a^2) + (x_b^2 + y_b^2 + z_b^2) - 2(x_a x_b + y_a y_b + z_a z_b)$$

$$l^2 = (x_p - x_b)^2 + (y_p - y_b)^2 + (z_p - z_b)^2$$

$$= (x_p^2 + y_p^2 + z_p^2) + (x_b^2 + y_b^2 + z_b^2) - 2(x_p x_b + y_p y_b + z_p z_b)$$

$$s^2 = (x_p - x_a)^2 + (y_p - y_a)^2 + (z_p - z_a)^2$$

$$= (x_p^2 + y_p^2 + z_p^2) + (x_a^2 + y_a^2 + z_a^2) - 2(x_p x_a + y_p y_a + z_p z_a)$$

And substituting from equations (7) & (8)

$s^2 = l^2 -\ \ x_b^2 + y_b^2 + z_b^2) \quad (x_p x \quad y_p y_b + z_p z_b) \quad ^2 \quad a \ (\quad +2 \quad b+ \quad + \quad -(x$

$(x_a x_b + y_a y_b + \quad {}_a z_b) \quad (x_p x_a + y_p y_a + \quad {}_p z_a z \quad {}_b^2 + y_b^2 + z_b^2) +$

$2 \quad z \quad -2 \quad)$

re-arranging gives

$l^2 - (s^2 - a^2)$ = 2 $+ y_b^2 + z_b^2)$ $x_a(x_p$ $(x_b^2$ + 2 $- x_b)$ + 2
$y_a(y_p - y_b)$ + 2 $(x_p x_b + y_p y_b$ $z_p z_b)$ $z_a(z_p - z_b)$ –
2 $+$

And substituting for x_a, y_a, z_a from equations (4), (5) & (6) then gives

$l^2 - (s^2 - a^2)$ = 2 $(x_b^2 + y_b^2 + z_b^2)$ + 2$(a \cos\alpha \cos\beta + x_b)\,(x_p - x_b)$+
$(y(a \cos\alpha \sin\beta + y_b)$ 2
$_p - y_b)$ + 2$(a \sin\alpha + z_b)(z_p - z_b)$ - $)\,(x_p x_b + y_p y_b + z_p z_b$
2

= 2 $(x_b^2 + y_b^2 + z_b^2)$ $a \cos\alpha \cos$ $(x_p$ $x_b)$ $x_p x_b$ x_b^2 + 2 β
$-$ $\cos\alpha \sin\beta\,(y_p - y_b)$ $y_p y_b$ y_b^2 $\sin$ $z_p -$ $)$ + 2 - 2
+ ^2_b $(x_p x_b + y_p y$ z_p $)$
2a + 2 - 2 + 2a $\alpha($
z_b + 2$z_p z_b$ -
2z - $_b + z_b$

Which reduces to

$l^2 - (s^2 - a^2)$ = 2a$\sin\alpha\,(z_p - z_b)$ +2$a \cos\alpha \cos\beta\,(x_p - x_b)$ + 2a$\cos\alpha \sin\beta\,(y_p - y_b)$

= 2a$\sin\alpha\,(z_p - z_b)$ +2a $\cos\alpha\,[\cos\beta\,(x_p - x_b) + \sin\beta(y_p - y_b)]$

Which is an equation of the form:-

L = $\mathrm{M} \sin\alpha + \mathrm{N} \cos\alpha$

Using the Trig identity for the sum of sine waves

$$a \sin x + b \cos x = c \sin(x + v)$$

where $c = \sqrt{a^2 + b^2}$ and $\tan v = \frac{b}{a}$

We therefore have another sine function of α with a phase shift δ

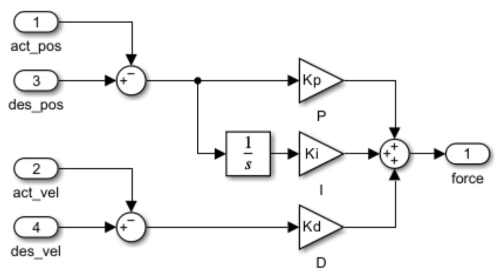
L $= \sqrt{\mathrm{M}^2 + \mathrm{N}^2}\,\sin(\alpha + \delta)$ where δ = $\tan^{-1}\frac{\mathrm{N}}{\mathrm{M}}$

Therefore $\sin(\alpha + \delta)$ = $\frac{\mathrm{L}}{\sqrt{\mathrm{M}^2 + \mathrm{N}^2}}$

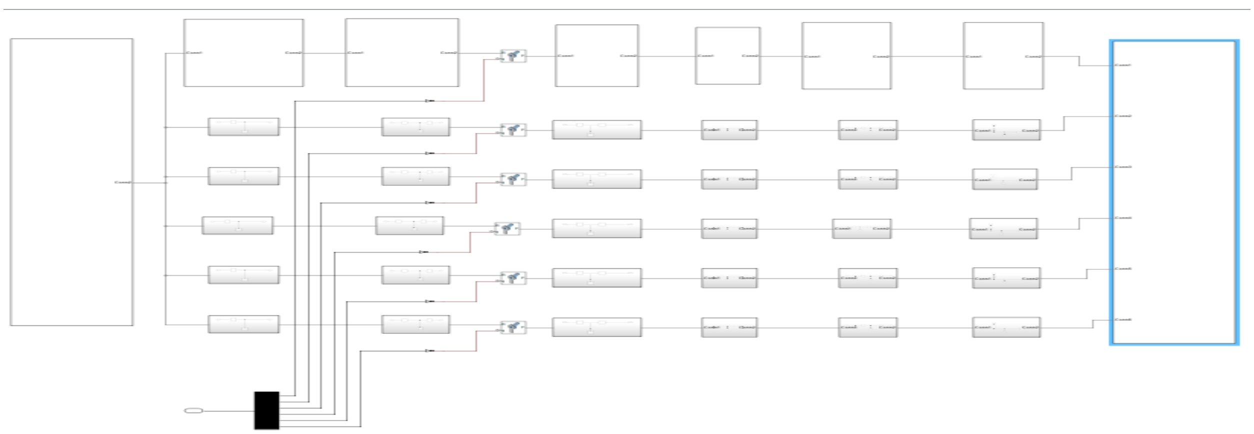
Validation and Experimental Corroboration:

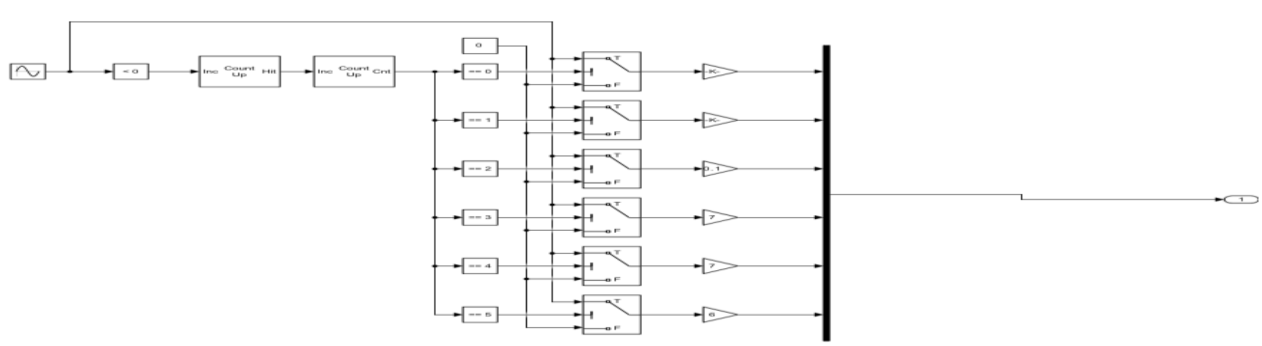
To validate the simulation results, experimental data were collected from a physical prototype of the Stewart Platform subjected to the applied force. By comparing the simulation outputs with the experimental measurements, the accuracy and reliability of the simulation were assessed. Any discrepancies were analyzed to identify potential sources of error and refine the simulation model for improved accuracy.

Simple PID Low-Level Controller



Modeling the Physical Components with Simscape





Conclusion:

The simulation and analysis of applying a force at the center of a Stewart Platform revealed valuable insights into the platform's kinematic and dynamic response. The simulation enabled the evaluation of displacements, rotations, internal forces, stress distribution, and control system behavior. The results obtained from the simulation were validated against experimental data, affirming the reliability of the simulation model. This analysis aids in optimizing the design, assessing structural integrity, and ensuring safe operation of the Stewart Platform in the presence of external forces.

References

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