Additional Notes

Algorithm 6 Gradient Tree Boosting Algorithm with Exponential Loss

Initialize $f_0(x) = \arg\min_\gamma \sum_{i=1}^{N} L(y_i, \gamma)$
for $m = 1$ to M **do**
    for $i = 1$ to N **do**
        Compute the residuals $r_{im} = -\left[\frac{\partial L(y_i, f(x_i))}{\partial f(x_i)}\right]_{f=f_{m-1}}$
        For exponential loss $L(y_i, f(x_i)) = e^{-y_i f(x_i)}$, the gradient is $r_{im} = y_i e^{-y_i f_{m-1}(x_i)}$
    end for
    Fit a regression tree to the targets r_{im}, giving terminal regions R_{jm}, $j = 1, 2, \ldots, J_m$
    for $j = 1$ to J_m **do**
        Compute $\gamma_{jm} = \arg\min_\gamma \sum_{x_i \in R_{jm}} L(y_i, f_{m-1}(x_i) + \gamma)$
        For exponential loss, this simplifies to $\gamma_{jm} = \frac{1}{2} \log \frac{1-err_{jm}}{err_{jm}}$, where err_{jm} is the weighted error rate for region R_{jm}
    end for
    Update the model $f_m(x) = f_{m-1}(x) + \sum_{j=1}^{J_m} \gamma_{jm} I(x \in R_{jm})$
end for
Output $\hat{f}(x) = f_M(x)$

Algorithm 7 Random Forest with OOB Error Calculation

for $b = 1$ to B **do**
    Draw a bootstrap sample $\mathbb{Z}^*$ of size N from the training data.
    Grow a random-forest tree T_b to the bootstrapped data.
    Identify the (OOB) samples for the b-th tree.
    Predict the OOB samples using the b-th tree and save the predictions.
end for
Add the OOB predictions:
for each observation i in the dataset **do**
    Collect predictions for i from all trees where i is OOB.
    ADD these predictions to get the final OOB prediction for i (average for regression, majority vote for classification).
end for
Calculate the OOB error:
Compute the error between the OOB predictions and the actual values (MSE for regression, classification error rate for classification).
Output the ensemble of trees $\{T_b\}_1^B$ and the OOB error.